On Low-Energy Theory from General Supergravity

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Starting from non-minimal supergravity theory with unified gauge symmetry, we obtain the low-energy effective theory by taking the flat limit and integrating out the superheavy fields in a model-independent manner. The scalar potential has extra non-universal contributions to soft supersymmetry breaking terms which can give an impact on phenomenological study.

§1. Introduction

The standard model (SM) has been established as an effective theory below the weak scale, although at present there are some measurements inconsistent with the SM predictions. The search for the theory beyond SM is one of the most important subjects in elementary particle physics. SM has a problem what is called ‘naturalness problem’. This problem essentially means that there is no natural mechanism to keep the value of Higgs field’s mass the weak scale one against radiative corrections, and it can be a key to explore new physics. In fact, ‘naturalness problem’ is elegantly solved by the introduction of ‘supersymmetry’ (SUSY).

The minimal SUSY extension of SM (MSSM) is regarded as a candidate of realistic theory beyond SM. The Lagrangian density of MSSM consists of two parts,

\[
\mathcal{L}_{\text{MSSM}} = \mathcal{L}_{\text{SUSY, MSSM}} + \mathcal{L}_{\text{Soft, MSSM}},
\]

\[
\mathcal{L}_{\text{SUSY, MSSM}} = -\frac{1}{2} \sum \lambda_a^a \lambda^a - \text{H.c.} - \sum (m^2)_{k,l} z^k z^l
\]

\[
- \sum A_{kml} z^k z^l z^m - \sum B_{kl} z^k z^l - \text{H.c.},
\]

where \( \mathcal{L}_{\text{SUSY, MSSM}} \) is the SUSY part and \( \mathcal{L}_{\text{Soft, MSSM}} \) is the soft SUSY breaking part. Here \( \lambda^a \) (\( a = 1, 2, 3 \)) are gauginos (bino, wino, gluino) and \( z^k \)’s are scalars (squarks, sleptons and Higgs doublets). The parameters \( (M_a, (m^2)_k, A_{kml}, B_{kl}) \) are called ‘soft SUSY breaking parameters’ and they are arbitrary and the origin is unknown in the MSSM.

It is expected that these parameters originate in more fundamental theories. We have quite an interesting scenario for the origin of soft SUSY breaking terms based on supergravity (SUGRA). The SUSY is spontaneously or dynamically broken in the so-called hidden sector and the effect is transported to our observable sector by the gravitational interaction. As a result, soft SUSY breaking terms appear in our sector. In this scenario, the pattern of soft SUSY breaking terms is determined by the structure

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**) In this paper, we do not assume the universality on the soft SUSY breaking parameters from the beginning when we use the terminology ‘MSSM’.
of SUGRA. For example, it is well-known that the minimal SUGRA leads to a universal type of soft SUSY breaking parameters. The scalar potential $V$ is given as follows,

$$ V = V_{\text{SUSY}} + V_{\text{soft}}, $$

(1.3)

$$ V_{\text{SUSY}} = \left| \frac{\partial \tilde{W}}{\partial z^i} \right|^2 + \frac{1}{2} g_a^2 (z^i_k (T^a)_l z^j)^2, $$

(1.4)

$$ V_{\text{soft}} = A \tilde{W} + B z^k \frac{\partial \tilde{W}}{\partial z^k} + \text{H.c.} + |B|^2 z^*_k z^k, $$

(1.5)

where $\tilde{W}$ is a superpotential, $g_a$'s are gauge coupling constants and $T^a$'s are gauge generators. $V_{\text{SUSY}}$ stands for the SUSY part, while $V_{\text{soft}}$ contains the soft SUSY breaking terms. The parameters $A$ and $B$ are written as

$$ A = \frac{\langle F^i \rangle \langle \tilde{z}_i^* \rangle}{M^2} - 3m_{3/2}^*, $$

(1.6)

$$ B = m_{3/2}^*, $$

(1.7)

where $F_i$'s and $\tilde{z}_i$'s are $F$-components and scalar components of chiral supermultiplets in the hidden sector, respectively. The bracket $\langle \cdots \rangle$ denotes the vacuum expectation value (VEV) of the quantity, $M$ is a gravitational scale and $m_{3/2}$ is a gravitino mass.

On the other hand, SUSY-Grand Unified Theory (SUSY-GUT) has been hopeful as a realistic theory. In fact, the precision measurements at LEP have shown that the gauge coupling constants $g_3$, $g_2$ and $g_1$ of 'SM gauge group' $G_{\text{SM}} = SU(3)_C \times SU(2)_L \times U(1)_Y$ meet at about $10^{16}$ GeV within the framework of MSSM. SUSY SU(5) GUT is the simplest unification scenario and predicts the long lifetime of nucleon consistent with the present data. However various unification scenarios consistent with the LEP data have been known within SUSY-GUTs. For example, the direct breaking of the larger group down to $G_{\text{SM}}$ and the models with extra heavy generations. Non-trivial examples are the models of SUSY SO(10) GUT with chain breaking. So it is important to specify the realistic SUSY-GUT model by using some observables in addition to gauge couplings.

Here let us emphasize that the soft SUSY breaking parameters can be powerful probes for physics beyond the MSSM such as SUSY-GUTs, SUGRAs, and superstring theories (SSTs). The reason is as follows. The SUSY spectrum at the weak scale, which is expected to be measured in the near future, is translated into the soft SUSY breaking parameters. And the values of these parameters at higher energy scales are obtained by using the renormalization group equations (RGEs). In many cases, there exist, at some energy scale, some relations among these parameters. They reflect the structure of high-energy physics. Hence we can specify the high-energy physics by checking these relations.

We give some examples.*

1. We can know whether the 'SM gauge group' is grand-unified or not by checking the 'GUT relation' among gaugino masses $M_a (a = 1, 2, 3)$

$$ \frac{M_1}{5/3g_Y^2} = \frac{M_2}{g_2^2} = \frac{M_3}{g_3^2}. $$

(1.8)

* We neglect the threshold corrections, the effect of higher dimensional operators, the mass mixing effect and so on.
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It is shown that the gaugino mass spectrum satisfies the ‘GUT relation’ at any energy scale between the unification scale and the weak scale as far as the ‘SM model gauge group’ is embedded into a simple group, irrespective of the symmetry breaking pattern.12)

2. The pattern of gauge symmetry breakdown can be specified by checking certain sum rules among scalar masses. For example, the scalar masses satisfy the following mass relations for the breaking $SU(5) \rightarrow G_{SM}$

$$m_{ar{q}}^2 = m_{ar{u}}^2 = m_{ar{e}}^2 \equiv m_2^2,$$

$$m_{ar{l}}^2 = m_{ar{d}}^2 \equiv m_3^2,$$

at the breaking scale. Here $m_{ar{q}}, m_{ar{u}}, \ldots$ are soft SUSY breaking scalar masses of squark doublet $\bar{q}$, up-type singlet squark $\bar{u}$ and so on. Scalar mass relations are derived for $SO(10)$ breakings12) and for $E_6$ breakings.15)

3. We can know the structure of SUGRA and SST by checking some specific relations among soft SUSY breaking parameters. For example, the SST with the SUSY breaking due to dilaton $F$-term leads to the highly restricted pattern such as16)

$$|A| = |M_{1/2}| = \sqrt{3}|m_{3/2}|,$$

where gauginos and scalars get masses with common values $M_{1/2}$ and $m_{3/2}$, respectively.

In this way, the soft SUSY breaking parameters can play important roles to probe new physics, but here we should note that the features of these parameters have not been completely investigated based on SUGRA with a general structure yet.

We have two important consequences so far.

(1) The precision measurements of the SUSY spectrum are very important. We hope that projects using next-generation colliders are developed and advanced quickly.

(2) But first it is important to place the low-energy theory within a more general framework as it relates to SUGRA. This is the motivation of our work.17)

The content of this paper is as follows. In §2, we briefly show the procedure of the derivation and the result of our low-energy theory. We give a conclusion in §3.

§2. The derivation and the result

Various types of low-energy theories have been derived based on the hidden sector SUSY breaking scenario in a model-dependent or model-independent way.6), 19) - 22) The difference among their structures arises from what type of SUGRA has been taken as a starting point. Four types of SUGRAs occur to us, that is, the minimal one, the minimal one with GUT, non-minimal one and non-minimal one with GUT. The first three cases have been energetically investigated.6), 19), 20) The study of the last case has also been started in a model-independent manner.22)

Let us explain the work of Ref. 22) briefly. The starting theory is a SUSY-GUT with non-universal soft SUSY breaking terms, which is derived from non-minimal SUGRA with a hidden ansatz by taking the flat limit first. Here the hidden ansatz means that the superpotential is separate from hidden sector to the observable one such as $W_{SG} = W(z) + \tilde{W}(\tilde{z})$. It is shown that there exist extra non-universal contributions
to soft SUSY breaking terms and some phenomenological implications are discussed. The results are written down in terms of SUSY-GUT, so it might be relatively easy to compare the values of measurements with the parameters in SUSY-GUT in the future. But we could have wished to know the information on the structure of SUGRA directly. Hence we would like to carry out the following subjects (1) to take a more general SUGRA, e.g., to take off the hidden ansatz (2) to write down the low-energy theory in terms of SUGRA in order to connect the experiments with SUGRA directly.

Our setting is SUGRA with non-minimal Kähler potential and a certain unified gauge symmetry. And our goal is to obtain its low-energy theory by taking the flat limit and integrating out heavy fields in a model-independent way.

First we give some basic assumptions.

1. The SUSY is spontaneously broken by the $F$-term condensation in the hidden sector. The Planck scale physics plays an essential role in the SUSY breaking. The hidden fields $\tilde{z}^i$ are gauge singlets and they have the VEVs of $O(M)$. The magnitude of $W_{SG}$ and $\tilde{F}^i$ is $O(m_{3/2}M^2)$ and $O(m_{3/2}M)$, respectively. We identify the gravitino mass with the weak scale.*

2. The unified gauge symmetry is broken down at the unification scale $M_U$ independent of the SUSY breaking. Some observable scalar fields have the VEVs of $O(M_U)$.

3. All fields are classified into two categories by using the values of those masses. One is a set of heavy fields with mass of $O(M_U)$. The other is a set of light fields with mass of $O(m_{3/2})$. There are no light singlet observable fields which induce a large tadpole contribution to Higgs masses by coupling to Higgs doublets renormalizably in superpotential.

Next we explain the procedure to obtain the low-energy theory.

1. We calculate the VEVs of derivatives and write down the scalar potential by using the fluctuations $\Delta z$.

2. When there exists a mass mixing, we need to diagonalize the scalar mass matrix to identify the heavy fields and the light ones correctly.

3. Then we solve the stationary conditions of the potential for the heavy fields while keeping the light fields arbitrary and integrate out the heavy fields by inserting the solutions into the scalar potential.

On the derivation of the scalar potential, we come across a problem related to the stability of the weak scale. The problem is as follows. Some light fields, which contain weak Higgs doublets, classified by using SUSY fermionic masses generally would get intermediate masses at tree level after the SUSY is broken down. We explain it by taking SUGRA without a unified symmetry as an example. When the hidden ansatz is taken off, the following extra terms should be added,

$$\frac{\partial \tilde{W}^*}{\partial \tilde{z}^i} \langle (K^{-1})_j^i \frac{\partial \tilde{W}}{\partial \tilde{z}^j} \rangle + \Delta C(z, z^*) + \langle \tilde{F}^i \frac{\partial \tilde{W}}{\partial \tilde{z}^i} \rangle + \text{H.c.,}$$

where $\Delta C(z, z^*)$ is a bilinear polynomial of $z$ and $z^*$. The magnitude of the third term

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* This assumption may be a little too strong since we only need to require that the soft SUSY breaking masses are of order of weak scale. In fact, there is quite an interesting scenario that the gravitino mass is decoupled to the soft parameters and the magnitude of the SUSY breaking is determined by the gaugino masses.
and its hermitian conjugate can be of order $m_{3/2}^3 M$ if the Yukawa couplings between the hidden sector fields and the observable sector light fields are of order unity, and so a large mixing mass of Higgs doublets can be introduced. In the presence of such a large $B$-parameter, the electro-weak symmetry breaking does not work at the weak scale. Hence we require that such dangerous terms are suppressed as

$$\langle \hat{F}^i \rangle \frac{\partial \hat{W}}{\partial \hat{z}^i} = O(m_{3/2}^4),$$  \hspace{1cm} (2.2)$$

by some mechanism. This requirement gives a constraint on the total Kähler potential. Of course, models with the hidden ansatz fulfill this requirement trivially. In the same way, we must impose some conditions to keep the gauge hierarchy in the case of SUGRA with unified gauge symmetry.\textsuperscript{23}, \textsuperscript{17}

Our SUGRA consists of the Kähler potential $K$, the superpotential $W_{SG}$ and the gauge kinetic function $f_{\alpha\beta}$, which are written down in terms of the variations $\Delta \hat{z}^I$ of mass eigenstates as follows,

\begin{align*}
K &= \langle \hat{K} \rangle + \langle \hat{K}_I \rangle \Delta \hat{z}^I + \frac{1}{2} \langle \hat{K}_{ij} \rangle \Delta \hat{z}^i \Delta \hat{z}^j + \cdots, \\ W_{SG} &= \langle \hat{W} \rangle + \langle \hat{W}_I \rangle \Delta \hat{z}^I + \frac{1}{2} \langle \hat{W}_{ij} \rangle \Delta \hat{z}^i \Delta \hat{z}^j \\
&\quad + \frac{1}{3!} \langle \hat{W}_{ijk} \rangle \Delta \hat{z}^i \Delta \hat{z}^j \Delta \hat{z}^k + \cdots
\end{align*}

(2.3)

and

$$f_{\alpha\beta} = f_{\alpha\beta}(\Delta \hat{z}),$$

(2.5)

where $\hat{I} = (I, \bar{I})$ and the ellipses represent higher order terms. Here we shall explain our notations for the field’s indices. The index $I, J, \ldots$ run all scalar species. In them, $i, j, \ldots$ and $\kappa, \lambda, \ldots$ run the hidden fields and the observable ones, respectively. Furthermore, in the observable sector fields, $k, l, \ldots, K, L, \ldots$ and $A, B, \ldots$ run the light non-singlet fields, the heavy complex ones and the heavy real ones related to the broken generators, respectively.

Under the above-mentioned assumptions and requirements, we can obtain the scalar potential $V_{\text{eff}}$ by the straightforward calculation. The result can be compactly expressed if we define the effective superpotential $\hat{W}_{\text{eff}}$ as

$$\hat{W}_{\text{eff}}(z) = \frac{1}{2!} \hat{\mu}_{kl} \delta \hat{z}^k \delta \hat{z}^l + \frac{1}{3!} \hat{h}_{klm} \delta \hat{z}^k \delta \hat{z}^l \delta \hat{z}^m,$$

(2.6)

where

$$\hat{\mu}_{kl} \equiv E^{1/2} \left( \langle \hat{W}_{kl} \rangle + \frac{\langle \hat{W} \rangle}{M^2} \langle \hat{K}_{kl} \rangle - \langle \hat{K}_{kl} \rangle \langle \hat{K}^{-1} \rangle \delta \hat{G}_j \right) + (m''_{3/2})_{kl},$$

(2.7)

$$\hat{h}_{klm} \equiv E^{1/2} \langle \hat{W}_{klm} \rangle.$$          \hspace{1cm} (2.8)

Here $E = \langle \exp(K/M^2) \rangle$, $\langle \hat{K}^{-1} \rangle$ is the inverse matrix of $\langle \hat{K}_{ij} \rangle$ and $\delta \hat{G}_j = \langle \hat{W}_j \rangle + \langle \hat{W} \rangle \langle \hat{K}_j \rangle / M^2$. Then we can write down the scalar potential $V_{\text{eff}}$ as

$$V_{\text{eff}} = V_{\text{SUSY}} + V_{\text{Soft}},$$

(2.9)

\textsuperscript{*} Here we omitted the terms irrelevant to the gauge non-singlet fields $\delta \hat{z}^k$ and the terms whose magnitudes are less than $O(m_{3/2}^3)$.\textsuperscript{17}
\[ V_{\text{SUSY}} = \left| \frac{\partial \hat{W}_{\text{eff}}}{\partial \hat{z}^k} \right|^2 + \frac{1}{2} g_\alpha^2 (\hat{z}^k (T^a)_{kl} \hat{z}^l)^2, \]  

\[ V_{\text{Soft}} = A \hat{W}_{\text{eff}} + B^k (\hat{z}) \frac{\partial \hat{W}_{\text{eff}}}{\partial \hat{z}^k} + \text{H.c.} + B^k (\hat{z}) \text{eff} B_k (\hat{z}) \text{eff} + C (\hat{z}) \text{eff} + \Delta V, \]

where \( \Delta V \) is a sum of contributions from a unified symmetry breaking and a mass mixing. The parameters \( A, B^k (\hat{z}) \text{eff} \) and \( C (\hat{z}) \text{eff} \) are given as

\[ A = m_{3/2}^* - 3 m_{3/2}, \]

\[ B^k (\hat{z}) \text{eff} = (m_{3/2}^* + m_{3/2}' + m_{3/2}''_{kl}) \delta^{kk} \delta^l \]

and

\[ C (\hat{z}) \text{eff} = E \delta \hat{G}_i ((\hat{K}^{-1})^{ij}) \left( \frac{1}{3!} \langle \hat{W}_{ji, kl} \rangle \delta \hat{z}^l \delta \hat{z}^i \delta \hat{z}^j + \frac{\langle \hat{W} \rangle}{M^2} \delta \hat{z}^2 \delta \hat{K}_j \right) + \text{H.c.} \]

\[ + E \left( \delta \hat{G}_i \delta^2 \left( \hat{K}^{-1} \right)^{ij} \delta \hat{G}_j + \frac{\langle V \rangle}{M^2} \delta \hat{z}^2 \hat{K}_j \right) \]

\[ - (m_{3/2})_{i} \delta^{kl} (m_{3/2})_{kl} \delta \hat{z}^k \delta \hat{z}^l - (m_{3/2})_{kl} \delta^{kl} (m_{3/2})_{i} \delta \hat{z}^k \delta \hat{z}^l \]

\[ - (m_{3/2})_{i} \delta^{kl} (m_{3/2})_{k} \delta \hat{z}^k \delta \hat{z}^l \text{ + H.c.} \}

\[ + A \left[ E^{1/2} \left( \frac{\langle \hat{W} \rangle}{M^2} \langle \hat{K}_{kl} \rangle - \langle \hat{K}_{kl} \rangle \langle \hat{K}^{-1} \rangle^{ij} \delta \hat{G}_j \right) \right. \]

\[ + (m_{3/2})_{kl} \right] \delta \hat{z}^k \delta \hat{z}^l, \]

where

\[ (m_{3/2})_{kl} = E^{1/2} \langle \hat{W} \rangle \delta_{kl}, \]

\[ m_{3/2}^* = E^{1/2} \langle \hat{K}_{kl} \rangle \langle \hat{K}^{-1} \rangle^{ij} \delta \hat{G}_j, \]

\[ (m_{3/2}'_{kl}) = - E^{1/2} (\hat{K}_{kl}) \langle \hat{K}^{-1} \rangle^{ij} \delta \hat{G}_j, \]

\[ (m_{3/2}'_{kl}) = - E^{1/2} \langle \hat{K}_{kl} \rangle \langle \hat{K}^{-1} \rangle^{ij} \delta \hat{G}_j, \]

\[ \text{where } \hat{G}_\lambda = \hat{W}_\lambda + \hat{K}_\lambda \hat{W} / M^2 + (\hat{K})_{\lambda\rho}(\hat{K}^{-1})^{\rho i} \hat{G}_j. \text{ And the quantities with a prime such as } \delta \hat{z}^j \text{ mean that the terms proportional to } \delta \hat{z}^j \text{ are omitted.} \]

There exist extra chirality-conserving scalar mass terms in \( \Delta \hat{V} \). The formula of the scalar masses is given as

\[ (m^2)_{kl} = (m_0^2)_{kl} + (\text{Ref}_A F_{AB}^1) \langle \hat{D}^A \rangle (T^B)_{kl} + (F-\text{term contributions}), \]

\[ \langle \hat{D}^A \rangle = 2 (M^2_{AB})^{AB} E \delta \hat{G}_\lambda \delta \hat{G}_\lambda \{ G_{\mu\lambda}^\lambda (\hat{z} T^B)_{\mu} + \tilde{G}_{\mu\lambda} (T^B)_{\mu} \}, \]

where \((m_0^2)_{kl}\)'s are present before the heavy sector is integrated out and so they respect the original unified gauge symmetry. And \((M^2_{AB})^{AB}\)'s are heavy gauge boson masses.
and $G = K + M^2 \ln(|W_{SG}|^2/M^6)$. The most important one comes from the $D$-term condensation of the heavy gauge sector. It is the second term in Eq. (2·19) and referred to as the $D$-term contribution.\(^{1}\) The sizable $D$-term contribution can appear at $M$ when the Kähler potential has a non-minimal structure and the rank of gauge group is reduced by the symmetry breaking. We can see that the $D$-term condensations $\langle D^A \rangle$ vanish up to $O(m^4_{3/2}/M^6)$ at $M$.\(^{2}\) The $D$-term contribution is proportional to the charge of the broken $U(1)$ factor and gives mass splittings within the same multiplet in the full theory. So its existence will give an impact on the phenomenological study on the scalar masses.\(^{12),22),26}\)

The scalar potential obtained should be regarded as the effective theory renormalized at the scale $M_U$. This potential serves a matching condition when we solve one-loop renormalization group equations above and below the scale $M_X$. The potential is written down in terms of SUGRA, so it will be useful to disclose the structure of SUGRA from the measurement of SUSY spectrum.

We should consider the renormalization effects for the soft SUSY breaking parameters and diagonalize the scalar mass matrix $\langle V_{kk} \rangle$ to derive the weak scale SUSY spectrum.

§3. Conclusion

We have derived the low-energy effective theory starting from non-minimal SUGRA with unified gauge symmetry under some physical assumptions and requirements in a model-independent manner. The result is summarized in Eqs.(2·6)-(2·20). We state chief results in correspondence with the assumptions.

The starting SUGRA consists of a non-minimal Kähler potential and a superpotential without the hidden ansatz based on the hidden sector SUSY breaking scenario. The non-minimality leads to non-universal soft SUSY breaking terms as pointed out in Ref. 20. The dangerous $B$ term, which destabilizes the weak scale, can exist if any conditions are not imposed on Yukawa couplings in $W_{SG}$.

The SUGRA has a unified gauge symmetry which is broken down at a scale $M_U$. Some scalar fields get the VEVs of $O(M_U)$. There exist heavy fields with the masses of $O(M_U)$ in addition to light fields with the masses of $O(m_{3/2})$. In such a situation, there appear extra non-universal contributions to the soft SUSY breaking terms reflected to the combination of the non-minimality of Kähler potential and the breakdown of extra gauge symmetry. The most important one comes from the $D$-term condensations of the heavy gauge sector. This contribution is proportional to the charge of broken diagonal generators, so we can know the large gauge symmetry by the precision measurement of scalar masses. There can exist many dangerous terms which threaten to the gauge hierarchy, so we required that such terms are suppressed.

It is expected that low-energy theories are checked by the precision measurements

\(^{1}\) Historically, it was demonstrated that the $D$-term contribution occurs when the gauge symmetry is broken at an intermediate scale due to the non-universal soft scalar masses in Ref. 24) and its existence in a more general situation was suggested in Ref. 25).

\(^{2}\) The $D$-term contribution can be sizable at $M_U$ by radiative corrections even when it vanish at $M$.\(^{25}\)
of the SUSY spectrum at the weak scale. From an optimistic point of view, if the SUSY is realized in nature, low-energy theories can be a touchstone in elementary particle physics at the beginning of the next century.

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