An Explicit Formula of the Solution of Constant Coefficients Partial Differential Equation with the Meromorphic Cauchy Data

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The analytic Cauchy problem with the meromorphic data has been studied by Hamade ([2] cf. [3]). In this note we give an explicit formula of the solution of constant coefficients equation with the meromorphic data by means of the extended Borel transformation ([1]).

In [1], we give an explicit formula of the solution of constant coefficients partial differential equation $P \left( \frac{\partial}{\partial \zeta} \right) u = 0$ with the Cauchy data

$$\frac{\partial^k u}{\partial \zeta^k}(0, \zeta_2, \ldots, \zeta_n) = g_{k+1}(\zeta_2, \ldots, \zeta_n), \quad 0 \leq k \leq m - 1,$$

in the form

$$u(z) = \mathcal{B} \left[ \langle 1 - z_1 \sigma(z_2^{-1}, \ldots, z_n^{-1}) \rangle^r, \theta_{\sigma(z_1^{-1}, \ldots, z_n^{-1})}^{-1}, \mathcal{B}^{-1}[g] \rangle \right](z),$$

where $P(z) = \prod_{i=1}^s (z_1 - \sigma_i(z_2, \ldots, z_n)^r)$ and $(1 - z_1 \sigma(z_2^{-1}, \ldots, z_n^{-1}))^r$ and $\mathcal{B}^{-1}[g]$ are vectors such that

$$\langle 1 - z_1 \sigma(z_2^{-1}, \ldots, z_n^{-1}) \rangle^r = \langle (1 - z_1 \sigma(z_2^{-1}, \ldots, z_n^{-1}))^{-1}, (1 - z_1 \sigma(z_2^{-1}, \ldots, z_n^{-1}))^{-2}, \ldots, (1 - z_1 \sigma(z_2^{-1}, \ldots, z_n^{-1}))^{-s}, (1 - z_1 \sigma(z_2^{-1}, \ldots, z_n^{-1}))^{-s+1}, \ldots, (1 - z_1 \sigma(z_2^{-1}, \ldots, z_n^{-1}))^{-s+r} \rangle,$$

$$\mathcal{B}^{-1}[g] = \langle \mathcal{B}^{-1}[g_1], \ldots, \mathcal{B}^{-1}[g_m] \rangle,$$

and $\langle F, G \rangle = \sum F_i G_i$, $F = (F_1, \ldots, F_m)$, $G = (G_1, \ldots, G_m)$.

On the other hand, in [1], we also show that to define
\[ B \log z \] is well defined and most of the properties of Borel transformation is preserved. Especially, by (2), we get

\[ B \left[ \log z \right] (\zeta) = \log \zeta + \gamma, \quad \gamma \text{ is Euler's constant}, \]

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By (4), we get

\[ B \left[ z^{-m-1} \log z \right] (\zeta) = (-1)^{n-1}(n-1)! \zeta^{-n}, \quad n \geq 1. \]

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Hence, since any element of \( \hat{\mathcal{N}} \) can be expressed as a Puiseaux series [cf. [1]], 

\( B^{-1} \) is defined on \( \hat{\mathcal{N}} \). Therefore, (1) also expresses the solution of \( P \frac{\partial}{\partial z} u = 0 \) with the meromorphic Cauchy data.

Similarly, we obtain the solution of \( P \frac{\partial}{\partial z} u = f \) for \( f \in \hat{\mathcal{N}} \). For example, for \( f = 1 / \zeta_1 \cdots \zeta_n \), we get

\[ u(\zeta) = \left[ \zeta_1^{m_1} \cdots \zeta_n^{m_n} \right] \frac{1}{P(z_1^{-1}, \ldots, z_n^{-1})} \frac{\log z_1 \cdots \log z_n}{z_1 \cdots z_n} (\zeta). \]

(6)\[ u(\zeta) = \left[ \zeta_1^{m_1} \cdots \zeta_n^{m_n} \right] \frac{1}{P(z_1^{-1}, \ldots, z_n^{-1})} \frac{\log z_1 \cdots \log z_n}{z_1 \cdots z_n} (\zeta). \]

References

