

More about Superparticle Sum Rules in Grand Unified Theories

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Abstract

Sum rules among superparticle masses are derived under the assumption that models beyond the MSSM are four-dimensional supersymmetric grand unified theories or five-dimensional supersymmetric orbifold grand unified theories. Sfermion sum rules are classified into four types and those sum rules can be useful probes of the MSSM and beyond.

1 Introduction

The supersymmetric (SUSY) extension of the standard model has been attractive as physics beyond the weak scale.[1, 2] The naturalness problem can be partially solved by the introduction of supersymmetry. The gauge coupling unification can be realized within the framework of the minimal supersymmetric standard model (MSSM), under the assumption of ‘desert’ between the TeV scale and the unification scale.[3, 4, 5, 6] If the magnitude of soft SUSY breaking parameters is the weak or TeV scale, superpartners will be discovered at the Large Hadron Collider (LHC) in the near future. Then a next task is to understand the structure of the MSSM (or an extension of

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the MSSM, e.g., the NMSSM) and disclose physics beyond the MSSM. The MSSM contains many parameters, whose magnitudes can be determined by precision measurements. Those experimental data will give a hint to explore the MSSM and beyond, including a mechanism of SUSY breaking and gauge symmetry breaking. Hence it is important to investigate the particle spectrum and interactions beyond the standard model (SM). Much works using mass relations among scalar particles have been carried out based on such a motivation.[7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17]

If the grand unification is indeed the case, then the theory can be described as a supersymmetric grand unified theory (SUSY GUT).[18, 19] The theory is fascinating, but, in general, it suffers from problems related to Higgs multiplets, e.g., triplet-doublet splitting problem and the problem of proton stability[20, 21] in the minimal SUSY $SU(5)$ GUT. Many interesting unified models have been constructed by an extension of gauge group and/or Higgs sector. There is a possibility to solve the problem by the introduction of extra dimension. The triplet-doublet splitting of Higgs multiplets, in fact, is elegantly realized based on SUSY $SU(5)$ GUT in a five dimension.[22, 23]¹ A variety of higher-dimensional SUSY GUTs on an orbifold have been proposed towards a construction of realistic model. Recently, the possibility of complete family unification has been studied in five-dimensional SUSY $SU(N)$ orbifold GUTs.[25] It is important to make distinctive predictions in order to specify models by using experimental data.

In this paper, we study the superparticle (sparticle) spectrum in the MSSM and derive sum rules among sparticle masses under the assumption that models beyond the MSSM are four-dimensional SUSY GUTs or five-dimensional SUSY orbifold GUTs. We classify sfermion sum rules and show that those sum rules can be useful probes of the MSSM and beyond.

The contents of this paper are as follows. In section 2, general arguments regarding sparticle spectrum and sum rules are given based on the MSSM. Sparticle sum rules are also examined under the assumption that scalar particles take a common soft SUSY breaking mass at some high energy scale as an example. In section 3, specific sum rules among sparticle masses are derived from various SUSY GUTs. Section 4 is devoted to conclusions and discussions.

¹In four-dimensional heterotic string models, it has been pointed out that extra color triplets are projected out by the Wilson line mechanism.[24]

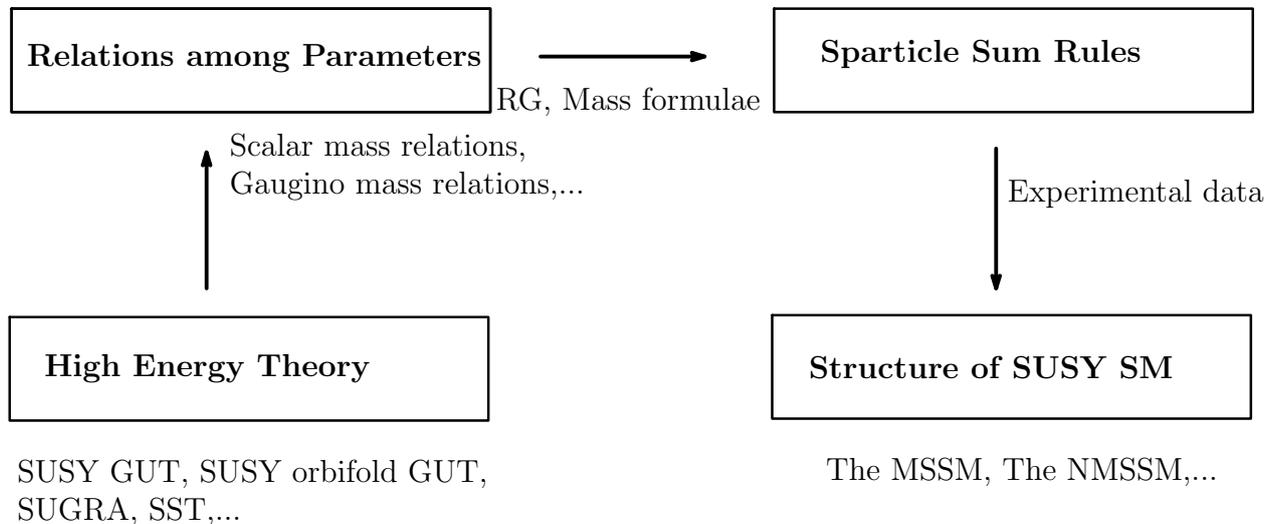
2 MSSM sparticle spectrum and sum rules

2.1 Basic assumptions and strategy

First we list assumptions adopted in our analysis.

1. The theory beyond the SM is the MSSM. Here the MSSM means the SUSY extension of the SM with the minimal particle contents, without specifying the structure of soft SUSY breaking terms. The superpartners and Higgs bosons have a mass whose magnitude is, at most, of order TeV scale. We neglect the threshold correction at the TeV scale due to the mass difference among the MSSM particles. Further the TeV scale is often identified with the weak scale (M_{EW}) for simplicity.
2. The MSSM holds from TeV scale to a high energy scale (M_U). Above M_U , there is a new physics. Possible candidates are SUSY GUTs and/or SUSY orbifold GUTs. There is a big desert between M_{EW} and M_U . In a simple scenario, the grand unification occurs at $M_U = 2.1 \times 10^{16} \text{GeV}$. We also consider a partial unification or the case that the grand unified gauge group is broken down to the SM one (G_{SM}) in several steps, e.g., $G_U \rightarrow \dots \rightarrow G_n \rightarrow G_{SM}$. In such a case, the M_U is the partial unification scale or the scale of gauge symmetry breaking at the final stage $G_n \rightarrow G_{SM}$.
3. The SUSY is broken in a SUSY breaking sector and the effect is mediated to our visible sector as the appearance of soft SUSY breaking terms. The pattern of soft SUSY breaking parameters reflects on the mechanism of SUSY breaking and/or the way of its mediation. We do not specify the mechanism of SUSY breaking. In most cases, the gravity mediation is assumed. We also assume that soft SUSY breaking terms respect the gauge invariance. After the breakdown of gauge symmetry, there appear extra contributions to soft SUSY breaking parameters, which do not respect the gauge symmetry any more. For example, there are renormalization group (RG) effects, threshold corrections at the breaking scale, contributions from non-renormalizable interaction terms, F -term contributions (contributions from interactions in superpotential) and D -term contributions. We assume that 1-loop RG effects and D -term contributions to soft SUSY breaking scalar masses dominate for the mass splitting in our analysis. The D -term contributions, in general, originates from D -terms related to broken gauge symmetries when soft SUSY breaking parameters possess a non-universal structure and the rank of gauge group lowers after the breakdown of gauge symmetry.[26, 27, 9, 10] In most cases, the magnitude of D -term condensation is, at most, of order

Figure 1: Outline of strategy



TeV scale squared and hence D -term contributions can induce sizable effects on sfermion spectrum.

4. The pattern of Yukawa couplings reflects flavor structure in unified theory. We assume that a suitable pattern of Yukawa couplings is obtained in the low-energy effective theory. We neglect effects of Yukawa couplings concerning to the first two generations and those of the off-diagonal ones because they are small compared with the third generation ones.

5. The sufficient suppression of flavor-changing neutral currents (FCNC) processes requires the mass degeneracy for each squark and slepton species in the first two generations unless those masses are rather heavy or fermion and its superpartner mass matrices are aligned. We assume that the generation-changing entries in the sfermion mass matrices are sufficiently small in the basis where fermion mass matrices are diagonal. At first, we derive sum rules without the requirement of mass degeneracy and after that we give a brief comment on the case with the degenerate masses.

6. After some parameters are made real by the rephasing of fields, CP violation occurs if the rest are complex. We assume that Yukawa couplings are dominant as a source of CP violation and other parameters are real.

Next we explain the outline of our strategy according to Figure 1. Let us construct a high energy theory with particular particle contents and (unified gauge) symmetries. The theory, in general, contains free parameters including unknown quantities related to symmetry breakings, e.g., D -term contributions. We can derive specific relations among soft SUSY breaking parameters at M_U by eliminating free parameters. Soft SUSY breaking parameters receive RG effects, and those values at M_{EW} can be calculated by using RG equations. After the breakdown of electroweak symmetry, mass formulae of the physical masses are written, in terms of parameters in the SUSY SM. Peculiar sum rules among sparticle masses at M_{EW} are obtained by rewriting specific relations at M_U in terms of physical masses and parameters. In the near future, if superpartners and Higgs bosons were discovered and those masses and interactions were precisely measured, the presumed high energy theory can be tested by checking whether peculiar sum rules hold or not.

2.2 MSSM sparticle spectrum

We review the particle spectrum in the MSSM.[28, 29, 30, 31] The Higgs sector consists of two Higgs doublets $H_1(= H_d)$ and $H_2(= H_u)$. The stationary conditions of Higgs potential are given by

$$M_Z^2 = -\frac{m_1^2 - m_2^2}{\cos 2\beta} - (m_1^2 + m_2^2), \quad m_1^2 + m_2^2 = -\frac{2\mu B}{\sin 2\beta}, \quad (1)$$

where m_1 and m_2 are defined by $m_1^2 \equiv \mu^2 + m_{H_1}^2$ and $m_2 \equiv \mu^2 + m_{H_2}^2$, and B is the Higgs mass mixing parameter. Here m_{H_1} and m_{H_2} are soft SUSY breaking masses of Higgs doublets. The angle β is defined by the ratio of the vacuum expectation values (VEVs) of neutral components of Higgs bosons such that $\tan \beta \equiv v_2/v_1$. After the breakdown of electroweak symmetry, one physical charged Higgs boson (H^\pm) and three kinds of neutral Higgs bosons (H^0, h^0, A^0) appear. The mass-squared of pseudoscalar Higgs boson (A^0) is given by

$$M_A^2 = m_1^2 + m_2^2 = 2\mu^2 + m_{H_1}^2 + m_{H_2}^2. \quad (2)$$

Using Eqs.(1) and (2), we derive relations²

$$B = -\frac{\sin 2\beta}{2\mu}M_A^2, \quad (3)$$

$$m_1^2 = \frac{1}{2} \left(M_A^2 - (M_A^2 + M_Z^2) \cos 2\beta \right), \quad (4)$$

$$m_2^2 = \frac{1}{2} \left(M_A^2 + (M_A^2 + M_Z^2) \cos 2\beta \right). \quad (5)$$

Using the above relations (3)-(5), values of B , m_1 and m_2 can be fixed. There exist sum rules among Higgs masses such that

$$M_{H^\pm}^2 = M_A^2 + M_W^2, \quad (6)$$

$$M_h^2 + M_H^2 = M_A^2 + M_Z^2 + \delta_h, \quad (7)$$

$$M_h^2 M_H^2 = M_A^2 M_Z^2 \cos^2 2\beta + \frac{\delta_h}{2} \left(M_A^2 + M_Z^2 + (M_A^2 - M_Z^2) \cos 2\beta \right), \quad (8)$$

where M_{H^\pm} , M_H and M_h are masses of H^\pm , H^0 and h^0 , and 1-loop corrections to the scalar potential are considered. The δ_h represents the radiative corrections,

$$\delta_h = \frac{3g^2 m_t^4}{16\pi^2 M_W^2 \sin^2 \beta} \ln \left(\left(1 + \frac{M_{\tilde{t}_L}^2}{m_t^2} \right) \left(1 + \frac{M_{\tilde{t}_R}^2}{m_t^2} \right) \right). \quad (9)$$

Here m_t and $M_{\tilde{t}_{L(R)}}$ stand for top quark mass and scalar top quark masses, respectively. For simplicity, we ignore the intra-generational mixing such as \tilde{t}_L - \tilde{t}_R mixing and contributions from bottom Yukawa coupling. Hence the Higgs sector in the MSSM can be tested by the above mass formulae and sum rules.

Next we consider superpartners of gauge and Higgs bosons. The gaugino masses relating to the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group are denoted M_i ($i = 3, 2, 1$) with a GUT normalization for weak hypercharge and the SUSY Higgs mass (or higgsino mass) parameter called ‘ μ parameter’ is denoted μ . The mass eigenstates in gaugino and higgsino sector consist of gluino (\tilde{g})

²In the case that $\tan \beta$ is large, i.e., $\cos 2\beta = \frac{1-\tan^2 \beta}{1+\tan^2 \beta} \approx -1$ and $\sin 2\beta = \frac{2 \tan \beta}{1+\tan^2 \beta} \approx \frac{2}{\tan \beta}$, we have the approximation formulae $B \approx -\frac{M_A^2}{\mu \tan \beta}$, $m_1^2 \approx M_A^2 + \frac{1}{2}M_Z^2$ and $m_2^2 \approx -\frac{1}{2}M_Z^2$.

whose mass is given by $M_{\tilde{g}} = M_3$ at tree level,³ and two kinds of mixture of the gauginos and higgsinos, i.e., charginos ($\tilde{\chi}_i^\pm; i = 1, 2$) and neutralinos ($\tilde{\chi}_i^0; i = 1, 2, 3, 4$). The mass-squareds of charginos are given by

$$M_{\tilde{\chi}_1^\pm}^2 = \frac{1}{2} \left(M_2^2 + \mu^2 + 2M_W^2 - \Delta_C \right), \quad (10)$$

$$M_{\tilde{\chi}_2^\pm}^2 = \frac{1}{2} \left(M_2^2 + \mu^2 + 2M_W^2 + \Delta_C \right), \quad (11)$$

where Δ_C is defined as

$$\Delta_C \equiv \sqrt{(M_2^2 + \mu^2 + 2M_W^2)^2 - 4(M_2\mu - M_W^2 \sin 2\beta)^2}. \quad (12)$$

From Eqs.(10) and (11), the following relations are derived as

$$M_{\tilde{\chi}_1^\pm}^2 + M_{\tilde{\chi}_2^\pm}^2 = M_2^2 + \mu^2 + 2M_W^2, \quad (13)$$

$$M_{\tilde{\chi}_1^\pm}^2 M_{\tilde{\chi}_2^\pm}^2 = \left(M_2\mu - M_W^2 \sin 2\beta \right)^2. \quad (14)$$

In the same way, relations among neutralino masses are given by

$$M_{\tilde{\chi}_1^0}^2 + M_{\tilde{\chi}_2^0}^2 + M_{\tilde{\chi}_3^0}^2 + M_{\tilde{\chi}_4^0}^2 = M_1^2 + M_2^2 + 2\mu^2 + 2M_Z^2, \quad (15)$$

$$\begin{aligned} & M_{\tilde{\chi}_1^0}^2 M_{\tilde{\chi}_2^0}^2 + M_{\tilde{\chi}_1^0}^2 M_{\tilde{\chi}_3^0}^2 + M_{\tilde{\chi}_1^0}^2 M_{\tilde{\chi}_4^0}^2 + M_{\tilde{\chi}_2^0}^2 M_{\tilde{\chi}_3^0}^2 + M_{\tilde{\chi}_2^0}^2 M_{\tilde{\chi}_4^0}^2 + M_{\tilde{\chi}_3^0}^2 M_{\tilde{\chi}_4^0}^2 \\ &= \left(M_1 M_2 - \mu^2 - M_Z^2 \right)^2 + 2(M_1 + M_2) \left(\mu^2 (M_1 + M_2) \right. \\ &\quad \left. + M_W^2 (M_1 + M_2 \tan^2 \theta_W) - \mu M_Z^2 \sin 2\beta \right) \\ &\quad - 2\mu^2 M_1 M_2 + 2\mu M_W^2 (M_1 + M_2 \tan^2 \theta_W) \sin 2\beta, \end{aligned} \quad (16)$$

$$\begin{aligned} & M_{\tilde{\chi}_1^0}^2 M_{\tilde{\chi}_2^0}^2 M_{\tilde{\chi}_3^0}^2 + M_{\tilde{\chi}_1^0}^2 M_{\tilde{\chi}_2^0}^2 M_{\tilde{\chi}_4^0}^2 + M_{\tilde{\chi}_1^0}^2 M_{\tilde{\chi}_3^0}^2 M_{\tilde{\chi}_4^0}^2 + M_{\tilde{\chi}_2^0}^2 M_{\tilde{\chi}_3^0}^2 M_{\tilde{\chi}_4^0}^2 \\ &= \left(\mu^2 (M_1 + M_2) + M_W^2 (M_1 + M_2 \tan^2 \theta_W) - \mu M_Z^2 \sin 2\beta \right)^2 \\ &+ 2 \left(\mu^2 M_1 M_2 - \mu M_W^2 (M_1 + M_2 \tan^2 \theta_W) \sin 2\beta \right) \left(M_1 M_2 - \mu^2 - M_Z^2 \right), \end{aligned} \quad (17)$$

$$M_{\tilde{\chi}_1^0}^2 M_{\tilde{\chi}_2^0}^2 M_{\tilde{\chi}_3^0}^2 M_{\tilde{\chi}_4^0}^2 = \left(-\mu^2 M_1 M_2 + \mu M_W^2 (M_1 + M_2 \tan^2 \theta_W) \sin 2\beta \right)^2. \quad (18)$$

Using the above relations, values of M_2^2 , M_1^2 , μ^2 and $\tan \beta$ can be determined. Hence the gaugino and higgsino sector in the MSSM can be tested by those sum rules.

³It is known that the gluino mass receives relatively large radiative corrections such that $M_{\tilde{g}} = M_3(Q) \left(1 + \frac{\alpha_s}{4\pi} (15 + 6 \ln(Q/M_3) + \sum A_q) \right)$ where Q is the renormalization scale and $\sum A_q$ is a sum of contributions from quark and squark loops.[32]

Finally we consider the matter sector. After the breakdown of electroweak symmetry, two kinds of contributions are added to sfermion masses, i.e., fermion masses (m_f) and the D -term contribution ($D_W(\tilde{f})$) relating to the generator of the broken symmetry ($SU(2)_L \times U(1)_Y / U(1)_{EM}$). The diagonal elements ($M_{\tilde{f}}^2$) of sfermion mass-squared matrices are written as

$$M_{\tilde{f}}^2 = m_{\tilde{F}}^2 + m_f^2 + D_W(\tilde{f}), \quad (19)$$

where f labels fermion species such that

$$f = \begin{cases} u_L, & d_L, & u_R, & d_R, & \nu_{eL}, & e_L, & e_R, \\ c_L, & s_L, & c_R, & s_R, & \nu_{\mu L}, & \mu_L, & \mu_R, \\ t_L, & b_L, & t_R, & b_R, & \nu_{\tau L}, & \tau_L, & \tau_R, \end{cases} \quad (20)$$

and \tilde{f} means the scalar partner of f . The $m_{\tilde{F}}^2$ is the soft SUSY breaking mass of scalar particle \tilde{F} . The \tilde{F} represents a multiplet of G_{SM} , which contains the scalar partner of f , and are given by,

$$\tilde{F} = \begin{cases} \tilde{q}_1, & \tilde{u}_R^*, & \tilde{d}_R^*, & \tilde{l}_1, & \tilde{e}_R^*, \\ \tilde{q}_2, & \tilde{c}_R^*, & \tilde{s}_R^*, & \tilde{l}_2, & \tilde{\mu}_R^*, \\ \tilde{q}_3, & \tilde{t}_R^*, & \tilde{b}_R^*, & \tilde{l}_3, & \tilde{\tau}_R^*, \end{cases} \quad (21)$$

where \tilde{q}_1 means the first generation scalar quark (squark) doublet, \tilde{u}_R^* up squark singlet, \tilde{d}_R^* down squark singlet, \tilde{l}_1 the first generation scalar lepton (slepton) doublet, \tilde{e}_R^* selectron singlet and so on. The astrisk means its complex conjugate. The $D_W(\tilde{f})$ are given by

$$\begin{aligned} D_W(\tilde{f}) &= \left(T_L^3(\tilde{f}) - Q(\tilde{f}) \sin^2 \theta_W \right) M_Z^2 \cos 2\beta \\ &= \left((T_L^3 - Q(\tilde{f})) M_Z^2 + Q(\tilde{f}) M_W^2 \right) \cos 2\beta \quad (f = u_L, \dots, \tau_L), \end{aligned} \quad (22)$$

$$\begin{aligned} D_W(\tilde{f}) &= Q(\tilde{f}) \sin^2 \theta_W M_Z^2 \cos 2\beta \\ &= Q(\tilde{f}) \left(M_Z^2 - M_W^2 \right) \cos 2\beta \quad (f = u_R, \dots, \tau_R). \end{aligned} \quad (23)$$

The off-diagonal elements of sfermion mass-squared matrices are proportional to the corresponding fermion mass. For the first two generations, the diagonal ones $M_{\tilde{f}}^2$ are regarded as ‘physical masses’ which are eigenvalues of mass-squared matrices because the off-diagonal ones are negligibly small. Using

the mass formula (19), values of m_F^2 can be determined for the first two generations. For the third generation, mass-squared matrices are given by

$$\begin{pmatrix} m_{\tilde{t}_L}^2 + m_t^2 + D_W(\tilde{t}_L) & -m_t(A_t + \mu \cot \beta) \\ -m_t(A_t + \mu \cot \beta) & m_{\tilde{t}_R}^2 + m_t^2 + D_W(\tilde{t}_R) \end{pmatrix} \quad (\text{for top squarks}), \quad (24)$$

$$\begin{pmatrix} m_{\tilde{b}_L}^2 + m_b^2 + D_W(\tilde{b}_L) & -m_b(A_b + \mu \tan \beta) \\ -m_b(A_b + \mu \tan \beta) & m_{\tilde{b}_R}^2 + m_b^2 + D_W(\tilde{b}_R) \end{pmatrix} \quad (\text{for bottom squarks}), \quad (25)$$

$$\begin{pmatrix} m_{\tilde{\tau}_L}^2 + m_\tau^2 + D_W(\tilde{\tau}_L) & -m_\tau(A_\tau + \mu \tan \beta) \\ -m_\tau(A_\tau + \mu \tan \beta) & m_{\tilde{\tau}_R}^2 + m_\tau^2 + D_W(\tilde{\tau}_R) \end{pmatrix} \quad (\text{for tau sleptons}), \quad (26)$$

where A_t , A_b and A_τ are trilinear coupling parameters called ‘ A parameter’. By diagonalized the above mass-squared matrices, we obtain mass eigenstates whose masses are physical ones, $(M_{\tilde{t}_1}, M_{\tilde{t}_2})$ for top squarks, $(M_{\tilde{b}_1}, M_{\tilde{b}_2})$ for bottom squarks and $(M_{\tilde{\tau}_1}, M_{\tilde{\tau}_2})$ for tau sleptons. By using the feature of trace, we have the relations,

$$\begin{aligned} M_{\tilde{t}_1}^2 + M_{\tilde{t}_2}^2 &= M_{\tilde{t}_L}^2 + M_{\tilde{t}_R}^2, & M_{\tilde{b}_1}^2 + M_{\tilde{b}_2}^2 &= M_{\tilde{b}_L}^2 + M_{\tilde{b}_R}^2, \\ M_{\tilde{\tau}_1}^2 + M_{\tilde{\tau}_2}^2 &= M_{\tilde{\tau}_L}^2 + M_{\tilde{\tau}_R}^2. \end{aligned} \quad (27)$$

By diagonalizing the mass squared matrices, we have the relations,

$$(M_{\tilde{t}_1}^2 - M_{\tilde{t}_2}^2)^2 = (M_{\tilde{t}_L}^2 - M_{\tilde{t}_R}^2)^2 + 4m_t^2 (A_t + \mu \cot \beta)^2, \quad (28)$$

$$(M_{\tilde{b}_1}^2 - M_{\tilde{b}_2}^2)^2 = (M_{\tilde{b}_L}^2 - M_{\tilde{b}_R}^2)^2 + 4m_b^2 (A_b + \mu \tan \beta)^2, \quad (29)$$

$$(M_{\tilde{\tau}_1}^2 - M_{\tilde{\tau}_2}^2)^2 = (M_{\tilde{\tau}_L}^2 - M_{\tilde{\tau}_R}^2)^2 + 4m_\tau^2 (A_\tau + \mu \tan \beta)^2, \quad (30)$$

If A parameters were measured precisely, m_F^2 s (and M_f^2 s) in the third generation can be fixed by using the mass-squared matrices (24) - (26). From the fact that left-handed fermions (and its superpartners) form $SU(2)_L$ doublets, e.g., $q_1 = (u_L, d_L)$ (and $\tilde{q}_1 = (\tilde{u}_L, \tilde{d}_L)$), we obtain following sum rules among $SU(2)_L$ doublet sfermions:

$$M_{\tilde{u}_L}^2 - M_{\tilde{d}_L}^2 = m_u^2 - m_d^2 + M_W^2 \cos 2\beta \simeq M_W^2 \cos 2\beta, \quad (31)$$

$$M_{\tilde{\nu}_{eL}}^2 - M_{\tilde{e}_L}^2 = m_{\nu_{eL}}^2 - m_e^2 + M_W^2 \cos 2\beta \simeq M_W^2 \cos 2\beta, \quad (32)$$

$$M_{\tilde{c}_L}^2 - M_{\tilde{s}_L}^2 = m_c^2 - m_s^2 + M_W^2 \cos 2\beta \simeq M_W^2 \cos 2\beta, \quad (33)$$

$$M_{\tilde{\nu}_{\mu L}}^2 - M_{\tilde{\mu}_L}^2 = m_{\nu_{\mu L}}^2 - m_\mu^2 + M_W^2 \cos 2\beta \simeq M_W^2 \cos 2\beta, \quad (34)$$

$$M_{\tilde{t}_L}^2 - M_{\tilde{b}_L}^2 = m_t^2 - m_b^2 + M_W^2 \cos 2\beta \simeq m_t^2 + M_W^2 \cos 2\beta, \quad (35)$$

$$M_{\tilde{\nu}_{\tau L}}^2 - M_{\tilde{\tau}_L}^2 = m_{\nu_{\tau L}}^2 - m_\tau^2 + M_W^2 \cos 2\beta \simeq M_W^2 \cos 2\beta, \quad (36)$$

where we neglect fermion masses except for the top quark mass in the final expressions. The above sum rules (31) - (36) are irrelevant to the structure of models beyond the MSSM, and hence the sfermion sector (and the breakdown of electroweak symmetry) in the MSSM can be tested by using them. We refer these sum rules (31) - (36) as the type I sfermion sum rules.

2.3 Renormalization group evolution of parameters

We review RG evolution of coupling constants and mass parameters at 1-loop level.[28, 29, 30, 31] We use conventional RG equations (RGEs) of soft SUSY breaking parameters in the case that the dynamics in the hidden sector do not give sizable effects.⁴

RGEs regarding gauge couplings g_i are given by

$$\frac{d}{dt}\alpha_i \equiv -2\pi Q \frac{\partial}{\partial Q}\alpha_i = -b_i\alpha_i^2, \quad (37)$$

where $\alpha_i \equiv g_i^2/(4\pi)$, Q is the renormalization scale, b_i s are coefficients of beta functions at 1-loop level, i.e., $b_3 = -3, b_2 = 1, b_1 = 33/5$ in the MSSM. By solving Eq.(37), we have evolution of gauge couplings such that

$$\alpha_i(M_U) = \frac{\alpha_i(Q)}{1 - \frac{b_i}{2\pi}\alpha_i(Q)\ln(M_U/Q)}. \quad (38)$$

The RGEs of gaugino masses are given by

$$\frac{d}{dt}\left(\frac{M_i}{\alpha_i}\right) = 0 \quad (39)$$

and Eq.(39) is easily solved as

$$M_i(Q) = \frac{\alpha_i(Q)}{\alpha_i(M_U)}M_i(M_U) = \left(1 - \frac{b_i}{2\pi}\alpha_i(Q)\ln\frac{M_U}{Q}\right)M_i(M_U). \quad (40)$$

⁴In Ref. [33], Dine et al. have pointed out that hidden sector interactions can give rise to sizable effects on RG evolutions of soft SUSY breaking parameters if hidden sector fields are treated as dynamical. In Ref. [34], Cohen et al. have derived mass relations among scalar fields by using RGEs modified by the hidden dynamics from the GUT scale to an intermediate scale where auxiliary fields in the hidden sector freeze into their VEV.

In the first two generations, RGEs concerning soft SUSY breaking sfermion masses are given by

$$\frac{d}{dt}m_{\tilde{F}}^2 = 4 \sum_{i=1}^3 C_2^{(i)}(\tilde{F})\alpha_i M_i^2 - \frac{3}{5}Y(\tilde{F})\alpha_1 S, \quad (41)$$

$$\frac{d}{dt}S = -b_1\alpha_1 S, \quad S \equiv \sum_{\tilde{F}} Y(\tilde{F})n_{\tilde{F}}m_{\tilde{F}}^2, \quad (42)$$

where $C_2^{(i)}(\tilde{F})$ and $Y(\tilde{F})$ represent the eigenvalues of second Casimir operator (e.g., $C_2^{(1)}(\tilde{q}_1) = 4/3$, $C_2^{(2)}(\tilde{q}_1) = 3/4$ and $C_2^{(1)}(\tilde{q}_1) = 1/60$) and hypercharge for \tilde{F} , respectively. In Eq.(42), $n_{\tilde{F}}$ represents degrees of freedom for \tilde{F} . The solutions of (41) are written by

$$\begin{aligned} m_{\tilde{F}}^2(Q) &= m_{\tilde{F}}^2(M_U) + \sum_{i=1}^3 \frac{2C_2^{(i)}(\tilde{F})}{b_i} (M_i^2(M_U) - M_i^2(Q)) \\ &\quad + \frac{3}{5b_1}Y(\tilde{F}) (S(Q) - S(M_U)) \\ &= m_{\tilde{F}}^2(M_U) + \sum_i \xi_i(\tilde{F})M_i^2(Q) + \xi_S(\tilde{F})S(Q), \end{aligned} \quad (43)$$

where the coefficients $\xi_i(k)$ and $\xi_S(k)$ are defined by

$$\xi_i(\tilde{F}) \equiv \frac{2C_2^{(i)}(\tilde{F})}{b_i} \left(\left(\frac{\alpha_i(M_U)}{\alpha_i(Q)} \right)^2 - 1 \right), \quad \xi_S(\tilde{F}) \equiv \frac{3Y(\tilde{F})}{5b_1} \left(1 - \frac{\alpha_1(M_U)}{\alpha_1(Q)} \right). \quad (44)$$

Here we use the solution of $S(Q)$ given by

$$S(Q) = \frac{\alpha_1(Q)}{\alpha_1(M_U)} S(M_U). \quad (45)$$

Values of $m_{\tilde{F}}^2(M_U)$ can be determined by using the solutions (43) and the value $\alpha_i(M_U)$. If we do not know the value of M_U , we can use one of solutions to fix it.

For the third generation sfermions and Higgs doublets,⁵ the relevant RGEs of their mass are given by

$$\frac{d}{dt}m_{\tilde{F}}^2 = 4 \sum_{i=1}^3 C_2^{(i)}(\tilde{F})\alpha_i M_i^2 - \frac{3}{5}Y(\tilde{F})\alpha_1 S + \mathcal{Y}_{\tilde{F}}, \quad (46)$$

⁵Hereafter we add Higgs bosons (H_1, H_2) to \tilde{F} .

where the $\mathcal{Y}_{\tilde{F}}$ stands for contributions from Yukawa interactions such that

$$\mathcal{Y}_{\tilde{q}_3} = -\frac{f_t^2}{4\pi} (m_{\tilde{q}_3}^2 + m_{\tilde{t}_R^*}^2 + m_{H_2}^2 + A_t^2) - \frac{f_b^2}{4\pi} (m_{\tilde{q}_3}^2 + m_{\tilde{b}_R^*}^2 + m_{H_1}^2 + A_b^2), \quad (47)$$

$$\mathcal{Y}_{\tilde{t}_R^*} = -\frac{f_t^2}{2\pi} (m_{\tilde{q}_3}^2 + m_{\tilde{t}_R^*}^2 + m_{H_2}^2 + A_t^2), \quad (48)$$

$$\mathcal{Y}_{\tilde{b}_R^*} = -\frac{f_b^2}{2\pi} (m_{\tilde{q}_3}^2 + m_{\tilde{b}_R^*}^2 + m_{H_1}^2 + A_b^2), \quad (49)$$

$$\mathcal{Y}_{\tilde{l}_3} = -\frac{f_\tau^2}{4\pi} (m_{\tilde{l}_3}^2 + m_{\tilde{\tau}_R^*}^2 + m_{H_1}^2 + A_\tau^2), \quad (50)$$

$$\mathcal{Y}_{\tilde{\tau}_R^*} = -\frac{f_\tau^2}{2\pi} (m_{\tilde{l}_3}^2 + m_{\tilde{\tau}_R^*}^2 + m_{H_1}^2 + A_\tau^2), \quad (51)$$

$$\mathcal{Y}_{H_1} = -\frac{f_\tau^2}{4\pi} (m_{\tilde{l}_3}^2 + m_{\tilde{\tau}_R^*}^2 + m_{H_1}^2 + A_\tau^2) - \frac{3f_b^2}{4\pi} (m_{\tilde{q}_3}^2 + m_{\tilde{b}_R^*}^2 + m_{H_1}^2 + A_b^2), \quad (52)$$

$$\mathcal{Y}_{H_2} = -\frac{3f_t^2}{4\pi} (m_{\tilde{q}_3}^2 + m_{\tilde{t}_R^*}^2 + m_{H_2}^2 + A_t^2), \quad (53)$$

where f_t , f_b and f_τ are Yukawa couplings. The solutions of (46) are formally written by⁶

$$m_{\tilde{F}}^2(Q) = m_{\tilde{F}}^2(M_U) + \sum_i \xi_i(\tilde{F}) M_i^2(Q) + \xi_S(\tilde{F}) S(Q) + \mathcal{F}_{\tilde{F}}, \quad (54)$$

where $d\mathcal{F}_{\tilde{F}}/dt = \mathcal{Y}_{\tilde{F}}$. By using RG Eq. (46) and RG Eqs. of Yukawa couplings and A parameters, values of $\mathcal{F}_{\tilde{F}}$ and $m_{\tilde{F}}^2(M_U)$ can be determined numerically.

2.4 Sparticle sum rules – general arguments

In the previous subsection, we encounter several sum rules among sparticles and Higgs bosons, which are irrelevant to the structure of models beyond the MSSM, and can be the touchstone of MSSM. In this subsection, we give general arguments on sum rules among sparticles which are relevant to physics beyond the MSSM.

First we consider gaugino masses. We assume that gaugino masses take the following values at some high energy scale M_U :

$$M_i(M_U) = l_i M_{1/2}, \quad (55)$$

⁶The exact analytical expressions for $\mathcal{F}_{\tilde{F}}$ have not been known in case with whole third generation Yukawa couplings.

where l_i s are some constant numbers and $M_{1/2}$ is a mass parameter. Then the values of M_i at M_{EW} are given by

$$M_i(M_{EW}) = l_i \frac{\alpha_i(M_{EW})}{\alpha_i(M_U)} M_{1/2} = l_i \left(1 - \frac{b_i}{2\pi} \alpha_i \ln \frac{M_U}{M_{EW}} \right) M_{1/2}. \quad (56)$$

Hereafter we abbreviate $M_i(M_{EW})$ and $\alpha_i(M_{EW})$ as M_i and α_i , respectively. Same applies to $M_{\tilde{f}}^2$ and $m_{\tilde{f}}^2$. Using Eq.(56), we obtain the sum rule,

$$\frac{\alpha_i(M_U)}{l_i \alpha_i} M_i = \frac{\alpha_j(M_U)}{l_j \alpha_j} M_j. \quad (57)$$

As a simple but interesting case, we consider the case with a universal gaugino mass, i.e., $M_i(M_U) = M_{1/2}$, and the gauge coupling unification, i.e., $\alpha_i(M_U) = \alpha_U$. In this case, we obtain so-called GUT-relation,

$$\frac{M_1}{\alpha_1} = \frac{M_2}{\alpha_2} = \frac{M_3}{\alpha_3} = \frac{M_{1/2}}{\alpha_U}. \quad (58)$$

This relation holds on at the 1-loop level even when the gauge symmetry breaks to the SM one in several steps, if the gauge group is unified into a simple group at some high energy scale.[9, 10]

Next we consider scalar masses. We assume that there exist the following n kinds of relations among scalar mass parameters at M_U :

$$\sum_{\tilde{F}} a_{\tilde{F}}^{(k)} m_{\tilde{F}}^2(M_U) = 0, \quad (59)$$

where $a_{\tilde{F}}^{(k)}$ ($k = 1, \dots, n$) are some constants. Then the following specific sum rules are derived:

$$\begin{aligned} & a_{H_1}^{(k)} m_{H_1}^2 + a_{H_2}^{(k)} m_{H_2}^2 + \sum_{\tilde{f}} a_{\tilde{f}}^{(k)} M_{\tilde{f}}^2 - \sum_f a_{\tilde{f}}^{(k)} m_f^2 - \sum_{\tilde{f}} a_{\tilde{f}}^{(k)} D_W(\tilde{f}) \\ & - \sum_{\tilde{F}} a_{\tilde{F}}^{(k)} \sum_i \xi_i(\tilde{F}) M_i^2 - \sum_{\tilde{F}} a_{\tilde{F}}^{(k)} \xi_S(\tilde{F}) S - \sum_{\tilde{F}} a_{\tilde{F}}^{(k)} \mathcal{F}_{\tilde{F}} = 0 \end{aligned} \quad (60)$$

by using Eqs.(19), (43) and (54). If the GUT-relation (58) holds on, the above sum rules are rewritten by

$$\begin{aligned} & a_{H_1}^{(k)} m_{H_1}^2 + a_{H_2}^{(k)} m_{H_2}^2 + \sum_{\tilde{f}} a_{\tilde{f}}^{(k)} M_{\tilde{f}}^2 - \sum_f a_{\tilde{f}}^{(k)} m_f^2 - \sum_{\tilde{f}} a_{\tilde{f}}^{(k)} D_W(\tilde{f}) \\ & - \sum_{\tilde{F}} a_{\tilde{F}}^{(k)} \sum_i \left(\frac{\alpha_i}{\alpha_3} \right)^2 \xi_i(\tilde{F}) M_g^2 - \sum_{\tilde{F}} a_{\tilde{F}}^{(k)} \xi_S(\tilde{F}) S - \sum_{\tilde{F}} a_{\tilde{F}}^{(k)} \mathcal{F}_{\tilde{F}} = 0. \end{aligned} \quad (61)$$

The (60) or (61) is the master formula of our analysis.

We discuss the case with anomaly mediation[35, 36], in which the hypothesis of desert violates on the face of it. When we take effects of anomaly mediation in SUGRA into account, gaugino masses are given by

$$M_i(Q) = \left(1 - \frac{b_i}{2\pi}\alpha_i \ln \frac{M_U}{Q}\right) M_{1/2} + \frac{b_i}{4\pi}\alpha_i m_{3/2}, \quad (62)$$

where the second term represents the contribution from anomaly mediation. The first one is an ordinary contribution and here we take a common gaugino mass $M_{1/2}$. The above formula is rewritten as

$$M_i = \left(1 - \frac{b_i}{2\pi}\alpha_i \ln \frac{Q_*}{M_{EW}}\right) M_{1/2} = \frac{\alpha_i}{\alpha_i(Q_*)} M_{1/2}, \quad (63)$$

where Q_* is a new scale defined by

$$Q_* \equiv \frac{M_U}{\left(M_U/m_{3/2}\right)^x}, \quad x \equiv \frac{m_{3/2}}{2M_{1/2} \ln(M_U/m_{3/2})}. \quad (64)$$

At the scale Q_* , gaugino masses unify, but it does not necessarily mean the existence of new physics there. Hence the unification is regarded as a mirage and this type of SUSY breaking mediation is called ‘mirage mediation’.[37, 38, 39, 40] If some conditions for parameters fulfills, this kind of unification occurs for scalar masses. It is possible to select models beyond the MSSM by checking sum rules such as (58) and (61). Using one of sum rules, the scale M_U is determined and other sum rules are tested. If the value of M_U is not an expected one from a theoretical prediction, there is a possibility that mirage mediation occurs.

2.5 Sparticle sum rules – examples

We derive specific sum rules among scalar particles taking two simple examples such as a universal type of soft SUSY breaking scalar masses at M_U and a generation dependent type there. Former one has been intensively studied in Ref.[7, 8]. Here we study more generic situation with the case that $\tan\beta$ is large, i.e., the bottom and tau Yukawa couplings are not negligible.

2.5.1 Universal type

Let us discuss the case with a universal type of soft SUSY breaking scalar masses at M_U , i.e., $m_F^2(M_U) = m_0^2$.⁷ By using the mass formula (19), the solutions (43) and (54), the scalar masses at M_{EW} are given by

$$M_{\tilde{u}_L}^2 = M_{\tilde{c}_L}^2 = m_0^2 + \zeta_3 M_3^2 + \zeta_2 M_2^2 + \zeta_1 M_1^2 + \left(\frac{2}{3} M_W^2 - \frac{1}{6} M_Z^2 \right) \cos 2\beta, \quad (65)$$

$$M_{\tilde{d}_L}^2 = M_{\tilde{s}_L}^2 = m_0^2 + \zeta_3 M_3^2 + \zeta_2 M_2^2 + \zeta_1 M_1^2 + \left(-\frac{1}{3} M_W^2 - \frac{1}{6} M_Z^2 \right) \cos 2\beta, \quad (66)$$

$$M_{\tilde{u}_R}^2 = M_{\tilde{c}_R}^2 = m_0^2 + \zeta_3 M_3^2 + 16\zeta_1 M_1^2 + \left(-\frac{2}{3} M_W^2 + \frac{2}{3} M_Z^2 \right) \cos 2\beta, \quad (67)$$

$$M_{\tilde{d}_R}^2 = M_{\tilde{s}_R}^2 = m_0^2 + \zeta_3 M_3^2 + 4\zeta_1 M_1^2 + \left(\frac{1}{3} M_W^2 - \frac{1}{3} M_Z^2 \right) \cos 2\beta, \quad (68)$$

$$M_{\tilde{\nu}_{eL}}^2 = M_{\tilde{\nu}_{\mu L}}^2 = m_0^2 + \zeta_2 M_2^2 + 9\zeta_1 M_1^2 + \frac{1}{2} M_Z^2 \cos 2\beta, \quad (69)$$

$$M_{\tilde{e}_L}^2 = M_{\tilde{\mu}_L}^2 = m_0^2 + \zeta_2 M_2^2 + 9\zeta_1 M_1^2 + \left(-M_W^2 + \frac{1}{2} M_Z^2 \right) \cos 2\beta, \quad (70)$$

$$M_{\tilde{e}_R}^2 = M_{\tilde{\mu}_R}^2 = m_0^2 + 36\zeta_1 M_1^2 + \left(M_W^2 - M_Z^2 \right) \cos 2\beta, \quad (71)$$

$$M_{\tilde{t}_L}^2 = m_0^2 + \zeta_3 M_3^2 + \zeta_2 M_2^2 + \zeta_3 M_3^2 + \left(\frac{2}{3} M_W^2 - \frac{1}{6} M_Z^2 \right) \cos 2\beta - F_t - F_b + m_t^2, \quad (72)$$

$$M_{\tilde{b}_L}^2 = m_0^2 + \zeta_3 M_3^2 + \zeta_2 M_2^2 + \zeta_1 M_1^2 + \left(-\frac{1}{3} M_W^2 - \frac{1}{6} M_Z^2 \right) \cos 2\beta - F_t - F_b + m_b^2, \quad (73)$$

$$M_{\tilde{t}_R}^2 = m_0^2 + \zeta_3 M_3^2 + 16\zeta_1 M_1^2 + \left(-\frac{2}{3} M_W^2 + \frac{2}{3} M_Z^2 \right) \cos 2\beta - 2F_t + m_t^2, \quad (74)$$

$$M_{\tilde{b}_R}^2 = m_0^2 + \zeta_3 M_3^2 + 4\zeta_1 M_1^2 + \left(\frac{1}{3} M_W^2 - \frac{1}{3} M_Z^2 \right) \cos 2\beta - 2F_b + m_b^2, \quad (75)$$

⁷In the SUGRA, M_U is regarded as the gravitational scale $M \equiv M_{pl}/\sqrt{8\pi}$. Such a mass degeneracy originates from the minimal SUGRA[41, 42] or, in general, the SUGRA which possesses $SU(n)$ symmetry in the Kähler potential concerning n kinds of matter multiplets.[43]

$$M_{\tilde{\nu}_{\tau L}}^2 = m_0^2 + \zeta_2 M_2^2 + 9\zeta_1 M_1^2 + \frac{1}{2} M_Z^2 \cos 2\beta - F_\tau, \quad (76)$$

$$M_{\tilde{\tau}_L}^2 = m_0^2 + \zeta_2 M_2^2 + 9\zeta_1 M_1^2 + \left(-M_W^2 + \frac{1}{2} M_Z^2\right) \cos 2\beta - F_\tau + m_\tau^2, \quad (77)$$

$$M_{\tilde{\tau}_R}^2 = m_0^2 + 36\zeta_1 M_1^2 + \left(M_W^2 - M_Z^2\right) \cos 2\beta - 2F_\tau + m_\tau^2, \quad (78)$$

$$m_{H_1}^2 = m_0^2 + \zeta_2 M_2^2 + 9\zeta_1 M_1^2 - F_\tau - 3F_b, \quad (79)$$

$$m_{H_2}^2 = m_0^2 + \zeta_2 M_2^2 + 9\zeta_1 M_1^2 - 3F_t, \quad (80)$$

where we neglect effects of Yukawa couplings in the first two generations and we use $S = 0$. Hereafter we neglect m_b and m_τ for simplicity. The ζ_3 , ζ_2 and ζ_1 are defined by

$$\zeta_3 = -\frac{8}{9} \left(\left(\frac{\alpha_3(M_U)}{\alpha_3} \right)^2 - 1 \right), \quad \zeta_2 = \frac{3}{2} \left(\left(\frac{\alpha_2(M_U)}{\alpha_2} \right)^2 - 1 \right),$$

$$\zeta_1 = \frac{1}{198} \left(\left(\frac{\alpha_1(M_U)}{\alpha_1} \right)^2 - 1 \right). \quad (81)$$

The F_t , F_b and F_τ stands for effects of Yukawa interactions in the third generation and they satisfy the following equations,

$$\frac{dF_t}{dt} = \frac{f_t^2}{4\pi} \left(m_{\tilde{q}_3}^2 + m_{\tilde{t}_R}^2 + m_{H_2}^2 + A_t^2 \right), \quad (82)$$

$$\frac{dF_b}{dt} = \frac{f_b^2}{4\pi} \left(m_{\tilde{q}_3}^2 + m_{\tilde{b}_R}^2 + m_{H_1}^2 + A_b^2 \right), \quad (83)$$

$$\frac{dF_\tau}{dt} = \frac{f_\tau^2}{4\pi} \left(m_{\tilde{l}_3}^2 + m_{\tilde{\tau}_R}^2 + m_{H_1}^2 + A_\tau^2 \right). \quad (84)$$

Values of F_t , F_b and F_τ are determined numerically by solving RG Eqs. of sparticle masses and coupling constants.

Twenty-two kinds of sum rules must exist because there are twenty-three kinds of scalar mass parameters ($M_{\tilde{u}_L}^2$, $M_{\tilde{d}_L}^2$, \dots , $M_{\tilde{\tau}_R}^2$, $m_{H_1}^2$, $m_{H_2}^2$) and one unknown parameter m_0 . By eliminating m_0 , we, in fact, obtain the following twenty-two sum rules,

$$M_{\tilde{u}_L}^2 - M_{\tilde{d}_L}^2 = M_{\tilde{c}_L}^2 - M_{\tilde{s}_L}^2 = M_{\tilde{t}_L}^2 - M_{\tilde{b}_L}^2 - m_t^2 = M_W^2 \cos 2\beta, \quad (85)$$

$$M_{\tilde{\nu}_{eL}}^2 - M_{\tilde{e}_L}^2 = M_{\tilde{\nu}_{\mu L}}^2 - M_{\tilde{\mu}_L}^2 = M_{\tilde{\nu}_{\tau L}}^2 - M_{\tilde{\tau}_L}^2 = M_W^2 \cos 2\beta, \quad (86)$$

$$\begin{aligned}
M_{\tilde{u}_L}^2 - M_{\tilde{u}_R}^2 &= M_{\tilde{c}_L}^2 - M_{\tilde{c}_R}^2 = M_{\tilde{t}_L}^2 - M_{\tilde{t}_R}^2 - F_t + F_b \\
&= \zeta_2 M_2^2 - 15\zeta_1 M_1^2 + \left(\frac{4}{3}M_W^2 - \frac{5}{6}M_Z^2\right) \cos 2\beta, \tag{87}
\end{aligned}$$

$$\begin{aligned}
M_{\tilde{d}_L}^2 - M_{\tilde{e}_R}^2 &= M_{\tilde{s}_L}^2 - M_{\tilde{\mu}_R}^2 = M_{\tilde{b}_L}^2 - M_{\tilde{\tau}_R}^2 + F_t + F_b - 2F_\tau \\
&= \zeta_3 M_3^2 + \zeta_2 M_2^2 - 35\zeta_1 M_1^2 + \left(-\frac{4}{3}M_W^2 + \frac{5}{6}M_Z^2\right) \cos 2\beta, \tag{88}
\end{aligned}$$

$$\begin{aligned}
M_{\tilde{e}_L}^2 - M_{\tilde{d}_R}^2 &= M_{\tilde{\mu}_L}^2 - M_{\tilde{s}_R}^2 = M_{\tilde{\tau}_L}^2 - M_{\tilde{b}_R}^2 + F_\tau - 2F_b \\
&= -\zeta_3 M_3^2 + \zeta_2 M_2^2 + 5\zeta_1 M_1^2 + \left(-\frac{4}{3}M_W^2 + \frac{5}{6}M_Z^2\right) \cos 2\beta, \tag{89}
\end{aligned}$$

$$\begin{aligned}
M_{\tilde{u}_R}^2 - M_{\tilde{d}_R}^2 &= M_{\tilde{c}_R}^2 - M_{\tilde{s}_R}^2 = M_{\tilde{t}_R}^2 - M_{\tilde{b}_R}^2 - m_t^2 + 2F_t - 2F_b \\
&= 12\zeta_1 M_1^2 + \left(-M_W^2 + M_Z^2\right) \cos 2\beta, \tag{90}
\end{aligned}$$

$$M_{\tilde{u}_L}^2 = M_{\tilde{c}_L}^2 = M_{\tilde{t}_L}^2 - m_t^2 + F_t + F_b, \tag{91}$$

$$m_{H_1}^2 - m_{H_2}^2 = -F_\tau - 3F_b + 3F_t, \tag{92}$$

$$m_{H_2}^2 = M_{e_L}^2 + \left(M_W^2 - \frac{1}{2}M_Z^2\right) \cos 2\beta - 3F_t. \tag{93}$$

They are also expressed as

$$M_{\tilde{u}_L}^2 - M_{\tilde{d}_L}^2 = M_W^2 \cos 2\beta, \tag{94}$$

$$M_{\tilde{\nu}_{eL}}^2 - M_{\tilde{e}_L}^2 = M_W^2 \cos 2\beta, \tag{95}$$

$$M_{\tilde{u}_L}^2 - M_{\tilde{u}_R}^2 = \zeta_2 M_2^2 - 15\zeta_1 M_1^2 + \left(\frac{4}{3}M_W^2 - \frac{5}{6}M_Z^2\right) \cos 2\beta, \tag{96}$$

$$\begin{aligned}
M_{\tilde{d}_L}^2 - M_{\tilde{e}_R}^2 &= \zeta_3 M_3^2 + \zeta_2 M_2^2 - 35\zeta_1 M_1^2 \\
&\quad + \left(-\frac{4}{3}M_W^2 + \frac{5}{6}M_Z^2\right) \cos 2\beta, \tag{97}
\end{aligned}$$

$$\begin{aligned}
M_{\tilde{e}_L}^2 - M_{\tilde{d}_R}^2 &= -\zeta_3 M_3^2 + \zeta_2 M_2^2 + 5\zeta_1 M_1^2 \\
&\quad + \left(-\frac{4}{3}M_W^2 + \frac{5}{6}M_Z^2\right) \cos 2\beta, \tag{98}
\end{aligned}$$

$$M_{\tilde{u}_R}^2 - M_{\tilde{d}_R}^2 = 12\zeta_1 M_1^2 + \left(-M_W^2 + M_Z^2\right) \cos 2\beta, \tag{99}$$

$$M_{\tilde{u}_L}^2 = M_{\tilde{c}_L}^2 = M_{\tilde{t}_L}^2 - m_t^2 + F_t + F_b, \tag{100}$$

$$M_{\tilde{d}_L}^2 = M_{\tilde{s}_L}^2 = M_{\tilde{b}_L}^2 + F_t + F_b, \tag{101}$$

$$M_{\tilde{u}_R}^2 = M_{\tilde{c}_R}^2 = M_{\tilde{t}_R}^2 - m_t^2 + 2F_t, \tag{102}$$

$$M_{\tilde{d}_R}^2 = M_{\tilde{s}_R}^2 = M_{\tilde{b}_L}^2 + 2F_b, \tag{103}$$

$$M_{\tilde{\nu}_{eL}}^2 = M_{\tilde{\nu}_{\mu L}}^2 = M_{\tilde{\nu}_{\tau L}}^2 + F_\tau, \tag{104}$$

$$M_{\tilde{e}_L}^2 = M_{\tilde{\mu}_L}^2 = M_{\tilde{\tau}_L}^2 + F_\tau, \quad (105)$$

$$M_{\tilde{e}_R}^2 = M_{\tilde{\mu}_R}^2 = M_{\tilde{\tau}_R}^2 + 2F_\tau, \quad (106)$$

$$m_{H_1}^2 - m_{H_2}^2 = -F_\tau - 3F_b + 3F_t, \quad (107)$$

$$m_{H_2}^2 = M_{e_L}^2 + \left(M_W^2 - \frac{1}{2} M_Z^2 \right) \cos 2\beta - 3F_t. \quad (108)$$

If all parameters were measured precisely enough, these sum rules can be powerful tools to test the universality of scalar mass parameters at M_U . If any parameters have ambiguities, we should use sum rules in which such parameters are absent. Here we discuss three typical cases that parameters are eliminated.

(i) Elimination of $\cos 2\beta$

In this case, there exist twenty-one sum rules, i.e., fifteen relations (100) - (107) and six relations such that

$$M_{\tilde{u}_L}^2 - M_{\tilde{d}_L}^2 = M_{\tilde{\nu}_{eL}}^2 - M_{\tilde{e}_L}^2, \quad (109)$$

$$M_{\tilde{u}_L}^2 - M_{\tilde{u}_R}^2 + M_{\tilde{d}_L}^2 - M_{\tilde{e}_R}^2 = \zeta_3 M_3^2 + 2\zeta_2 M_2^2 - 50\zeta_1 M_1^2, \quad (110)$$

$$M_{\tilde{d}_L}^2 - M_{\tilde{e}_R}^2 - M_{\tilde{e}_L}^2 + M_{\tilde{d}_R}^2 = 2\zeta_3 M_3^2 - 40\zeta_1 M_1^2, \quad (111)$$

$$3 \left(M_{\tilde{\nu}_e}^2 + M_{\tilde{e}_L}^2 \right) - M_{\tilde{d}_R}^2 - 5M_{\tilde{u}_R}^2 = -6\zeta_3 M_3^2 + 6\zeta_2 M_2^2 - 30\zeta_1 M_1^2, \quad (112)$$

$$M_{\tilde{u}_R}^2 - M_{\tilde{d}_R}^2 = 12\zeta_1 M_1^2 - \left(1 - \frac{M_Z^2}{M_W^2} \right) \left(M_{\tilde{u}_L}^2 - M_{\tilde{d}_L}^2 \right), \quad (113)$$

$$m_{H_2}^2 = M_{e_L}^2 + \left(1 - \frac{M_Z^2}{2M_W^2} \right) \left(M_{\tilde{u}_L}^2 - M_{\tilde{d}_L}^2 \right) - 3F_t. \quad (114)$$

(ii) Elimination of ζ_i s

In this case, there exist nineteen sum rules, i.e., eighteen relations (94) , (95), (100) - (108) and one relation such that

$$2M_{\tilde{u}_R}^2 - M_{\tilde{d}_R}^2 - M_{\tilde{d}_L}^2 + M_{\tilde{e}_L}^2 - M_{\tilde{e}_R}^2 = \frac{10}{3} \left(M_Z^2 - M_W^2 \right) \cos 2\beta. \quad (115)$$

(iii) Elimination of $F_t, F_b, F_\tau, A_t, A_b$ and A_τ

In this case, there exist sixteen sum rules, i.e., thirteen relations such as the first and second generation part of (85) - (91), and three sum rules such

that

$$M_{\tilde{\tau}_1}^2 + M_{\tilde{\tau}_2}^2 - 3M_{\tilde{\nu}_\tau}^2 = M_{\tilde{e}_L}^2 + M_{\tilde{e}_R}^2 - 3M_{\tilde{\nu}_{eL}}^2, \quad (116)$$

$$\begin{aligned} & 3 \left(M_{\tilde{t}_1}^2 + M_{\tilde{t}_2}^2 - M_{\tilde{b}_1}^2 - M_{\tilde{b}_2}^2 \right) + 2 \left(m_{H_1}^2 - m_{H_2}^2 - M_{\tilde{\nu}_{\tau L}}^2 \right) \\ &= 3 \left(M_{\tilde{u}_L}^2 + M_{\tilde{u}_R}^2 + 2m_t^2 - M_{\tilde{d}_L}^2 - M_{\tilde{d}_R}^2 \right) - 2M_{\tilde{\nu}_{eL}}^2, \end{aligned} \quad (117)$$

$$\begin{aligned} & 3 \left(M_{\tilde{t}_1}^2 + M_{\tilde{t}_2}^2 + M_{\tilde{b}_1}^2 + M_{\tilde{b}_2}^2 \right) - 4 \left(m_{H_1}^2 + m_{H_2}^2 - M_{\tilde{\nu}_{\tau L}}^2 \right) \\ &= 3 \left(M_{\tilde{u}_L}^2 + M_{\tilde{u}_R}^2 + 2m_t^2 + M_{\tilde{d}_L}^2 + M_{\tilde{d}_R}^2 \right) \\ &\quad - 8M_{\tilde{e}_L}^2 + 4M_{\tilde{\nu}_{eL}}^2 - \left(8M_W^2 - 4M_Z^2 \right) \cos 2\beta. \end{aligned} \quad (118)$$

2.5.2 Generation dependent type

We study the case that soft SUSY breaking scalar masses are generation dependent ones at M_U , i.e., there are five independent mass parameters such that

$$m_{\tilde{q}_1}^2(M_U) = m_{\tilde{u}_R^*}^2(M_U) = m_{\tilde{d}_R^*}^2(M_U) = m_{\tilde{l}_1}^2(M_U) = m_{\tilde{e}_R^*}^2(M_U) \equiv m_0^{(1)2}, \quad (119)$$

$$m_{\tilde{q}_2}^2(M_U) = m_{\tilde{c}_R^*}^2(M_U) = m_{\tilde{s}_R^*}^2(M_U) = m_{\tilde{l}_2}^2(M_U) = m_{\tilde{\mu}_R^*}^2(M_U) \equiv m_0^{(2)2}, \quad (120)$$

$$m_{\tilde{q}_3}^2(M_U) = m_{\tilde{t}_R^*}^2(M_U) = m_{\tilde{b}_R^*}^2(M_U) = m_{\tilde{l}_3}^2(M_U) = m_{\tilde{\tau}_R^*}^2(M_U) \equiv m_0^{(3)2}, \quad (121)$$

$$m_{H_1}^2(M_U) \equiv m_0^{(H_1)2}, \quad m_{H_2}^2(M_U) \equiv m_0^{(H_2)2}. \quad (122)$$

Note that $S(M_U) = m_{H_2}^2(M_U) - m_{H_1}^2(M_U)$ and hence S is non-vanishing if $m_{H_1}^2(M_U) \neq m_{H_2}^2(M_U)$. In this case, we have eighteen sum rules, i.e., six relations (85) and (86), and twelve ones

$$\begin{aligned} M_{\tilde{u}_L}^2 - M_{\tilde{u}_R}^2 &= M_{\tilde{c}_L}^2 - M_{\tilde{c}_R}^2 = M_{\tilde{t}_L}^2 - M_{\tilde{t}_R}^2 - F_t + F_b \\ &= \zeta_2 M_2^2 - 15\zeta_1 M_1^2 + \left(\frac{4}{3} M_W^2 - \frac{5}{6} M_Z^2 \right) \cos 2\beta + 5\mathcal{S}, \end{aligned} \quad (123)$$

$$\begin{aligned} M_{\tilde{d}_L}^2 - M_{\tilde{e}_R}^2 &= M_{\tilde{s}_L}^2 - M_{\tilde{\mu}_R}^2 = M_{\tilde{b}_L}^2 - M_{\tilde{\tau}_R}^2 + F_t + F_b - 2F_\tau \\ &= \zeta_3 M_3^2 + \zeta_2 M_2^2 - 35\zeta_1 M_1^2 + \left(-\frac{4}{3} M_W^2 + \frac{5}{6} M_Z^2 \right) \cos 2\beta - 5\mathcal{S}, \end{aligned} \quad (124)$$

$$\begin{aligned} M_{\tilde{e}_L}^2 - M_{\tilde{d}_R}^2 &= M_{\tilde{\mu}_L}^2 - M_{\tilde{s}_R}^2 = M_{\tilde{\tau}_L}^2 - M_{\tilde{b}_R}^2 + F_\tau - 2F_b \\ &= -\zeta_3 M_3^2 + \zeta_2 M_2^2 + 5\zeta_1 M_1^2 + \left(-\frac{4}{3} M_W^2 + \frac{5}{6} M_Z^2 \right) \cos 2\beta - 5\mathcal{S}, \end{aligned} \quad (125)$$

$$\begin{aligned} M_{\tilde{u}_R}^2 - M_{\tilde{d}_R}^2 &= M_{\tilde{c}_R}^2 - M_{\tilde{s}_R}^2 = M_{\tilde{t}_R}^2 - M_{\tilde{b}_R}^2 - m_t^2 + 2F_t - 2F_b \\ &= 12\zeta_1 M_1^2 + \left(-M_W^2 + M_Z^2 \right) \cos 2\beta - 6\mathcal{S}, \end{aligned} \quad (126)$$

where \mathcal{S} is defined by

$$\mathcal{S} \equiv \frac{1}{6} \frac{3}{5b_1} \left(1 - \frac{\alpha_1(M_U)}{\alpha_1} \right) S = \frac{1}{10b_1} \left(1 - \frac{\alpha_1(M_U)}{\alpha_1} \right) (m_{H_2}^2 - m_{H_1}^2). \quad (127)$$

In place of (123) or (124), we can use the following one,

$$\begin{aligned} M_{\tilde{u}_R}^2 - M_{\tilde{e}_R}^2 &= M_{\tilde{c}_R}^2 - M_{\tilde{\mu}_R}^2 \\ &= M_{\tilde{t}_R}^2 - M_{\tilde{\tau}_R}^2 - m_t^2 + 2F_t - 2F_\tau \\ &= \zeta_3 M_3^2 - 20\zeta_1 M_1^2 + \left(-\frac{5}{3} M_W^2 + \frac{5}{3} M_Z^2 \right) \cos 2\beta - 10\mathcal{S}. \end{aligned} \quad (128)$$

By eliminating \mathcal{S} , we have following relations,

$$M_{\tilde{u}_L}^2 - M_{\tilde{u}_R}^2 + M_{\tilde{d}_L}^2 - M_{\tilde{e}_R}^2 = \zeta_3 M_3^2 + 2\zeta_2 M_2^2 - 50\zeta_1 M_1^2, \quad (129)$$

$$M_{\tilde{u}_L}^2 - M_{\tilde{u}_R}^2 + M_{\tilde{e}_L}^2 - M_{\tilde{d}_R}^2 = -\zeta_3 M_3^2 + 2\zeta_2 M_2^2 - 10\zeta_1 M_1^2, \quad (130)$$

$$6 \left(M_{\tilde{u}_L}^2 - M_{\tilde{u}_R}^2 \right) + 5 \left(M_{\tilde{u}_R}^2 - M_{\tilde{d}_R}^2 \right) = 6\zeta_2 M_2^2 - 30\zeta_1 M_1^2 + 3M_W^2 \cos 2\beta. \quad (131)$$

If we require the mass degeneracy between the first two generations, i.e., $m_0^{(1)2} = m_0^{(2)2}$, we obtain extra sum rules $M_{\tilde{u}_L}^2 = M_{\tilde{c}_L}^2$, $M_{\tilde{d}_L}^2 = M_{\tilde{s}_L}^2$ and so on, and the sufficient suppression of FCNC processes is realized.

3 Scalar sum rules in grand unification

We derive peculiar sum rules among scalar masses in cases with several kinds of non-universal soft SUSY breaking scalar mass parameters at M_U based on SUSY GUTs in four and five dimensions.

3.1 Grand unification in four dimension

Scalar mass relations have been derived for various symmetry breaking patterns based on $SO(10)$ SUSY GUTs [9, 10] and E_6 SUSY GUTs [13]. Sfermion sum rules at the TeV scale have been derived based on $SO(10)$ SUSY GUTs [14, 16, 17]. We re-examine scalar sum rules at the TeV scale for various kinds of symmetry breaking patterns and particle assignments by assuming that 1-loop RG effects and D -term contributions are dominant as a source to violate the scalar mass degeneracy.

(1) $SU(5) \rightarrow G_{SM}$

From $SU(5)$ gauge symmetry, there exist following relations at the grand unification scale $M_U (= 2.1 \times 10^{16} \text{GeV})$,

$$m_{\tilde{q}_1}^2(M_U) = m_{\tilde{u}_R^*}^2(M_U) = m_{\tilde{e}_R^*}^2(M_U) \equiv m_{10}^{(1)2}, \quad (132)$$

$$m_{\tilde{l}_1}^2(M_U) = m_{\tilde{d}_R^*}^2(M_U) \equiv m_{\bar{5}}^{(1)2}, \quad (133)$$

$$m_{\tilde{q}_2}^2(M_U) = m_{\tilde{c}_R^*}^2(M_U) = m_{\tilde{\mu}_R^*}^2(M_U) \equiv m_{10}^{(2)2}, \quad (134)$$

$$m_{\tilde{l}_2}^2(M_U) = m_{\tilde{s}_R^*}^2(M_U) \equiv m_{\bar{5}}^{(2)2}, \quad (135)$$

$$m_{\tilde{q}_3}^2(M_U) = m_{\tilde{t}_R^*}^2(M_U) = m_{\tilde{\tau}_R^*}^2(M_U) \equiv m_{10}^{(3)2}, \quad (136)$$

$$m_{\tilde{l}_3}^2(M_U) = m_{\tilde{b}_R^*}^2(M_U) \equiv m_0^{(3)2}, \quad (137)$$

$$m_{H_1}^2(M_U) \equiv m_{5H}^2, \quad m_{H_2}^2(M_U) \equiv m_{5H}^2, \quad (138)$$

where $m_{10}^{(1)}$, $m_{\bar{5}}^{(1)}$, $m_{10}^{(2)}$, $m_{\bar{5}}^{(2)}$, $m_{10}^{(3)}$, $m_{\bar{5}}^{(3)}$, m_{5H} and m_{5H} are soft SUSY breaking scalar mass parameters. Here we use the fact that \tilde{q}_1 , \tilde{u}_R^* and \tilde{e}_R^* belong to $\mathbf{10}$, and \tilde{l}_1 and \tilde{d}_R^* belong to $\bar{\mathbf{5}}$ and the same assignment holds on for second and third generations. By eliminating those eight mass parameters, we obtain fifteen sum rules, the type I sfermion sum rules ((85) and (86)) and three kinds of sum rules ((123), (128) and (125) with a common gaugino mass at M_U) such that,

$$\begin{aligned} M_{\tilde{u}_L}^2 - M_{\tilde{u}_R}^2 &= M_{\tilde{c}_L}^2 - M_{\tilde{c}_R}^2 = M_{\tilde{t}_L}^2 - M_{\tilde{t}_R}^2 - F_t + F_b \\ &= \left(\zeta_2 \left(\frac{\alpha_2}{\alpha_3} \right)^2 - 15\zeta_1 \left(\frac{\alpha_1}{\alpha_3} \right)^2 \right) M_{\tilde{g}}^2 \\ &\quad + \left(\frac{4}{3}M_W^2 - \frac{5}{6}M_Z^2 \right) \cos 2\beta + 5\mathcal{S}, \end{aligned} \quad (139)$$

$$\begin{aligned} M_{\tilde{u}_R}^2 - M_{\tilde{e}_R}^2 &= M_{\tilde{c}_R}^2 - M_{\tilde{\mu}_R}^2 \\ &= M_{\tilde{t}_R}^2 - M_{\tilde{\tau}_R}^2 - m_t^2 + 2F_t - 2F_\tau \\ &= \left(\zeta_3 - 20\zeta_1 \left(\frac{\alpha_1}{\alpha_3} \right)^2 \right) M_{\tilde{g}}^2 + \left(-\frac{5}{3}M_W^2 + \frac{5}{3}M_Z^2 \right) \cos 2\beta - 10\mathcal{S}. \end{aligned} \quad (140)$$

$$\begin{aligned} M_{\tilde{e}_L}^2 - M_{\tilde{d}_R}^2 &= M_{\tilde{\mu}_L}^2 - M_{\tilde{s}_R}^2 = M_{\tilde{\tau}_L}^2 - M_{\tilde{b}_R}^2 + F_\tau - 2F_b \\ &= \left(-\zeta_3 + \zeta_2 \left(\frac{\alpha_2}{\alpha_3} \right)^2 + 5\zeta_1 \left(\frac{\alpha_1}{\alpha_3} \right)^2 \right) M_{\tilde{g}}^2 \end{aligned}$$

$$+ \left(-\frac{4}{3}M_W^2 + \frac{5}{6}M_Z^2 \right) \cos 2\beta - 5\mathcal{S}. \quad (141)$$

The above sum rules (139) - (141) holds on in the case with the direct breakdown of a grand unified symmetry to the SM one at M_U , and hence the sfermion sector (and the grand unification) in SUSY GUT can be tested by using them. We refer these sum rules (139) - (141) as the type IIA sfermion sum rules.

(2) $SO(10) \rightarrow G_{SM}$

After the breakdown of $SO(10)$ gauge symmetry to the SM one, there exist following relations at the grand unification scale M_U ,

$$m_{\tilde{q}_1}^2(M_U) = m_{\tilde{u}_R^*}^2(M_U) = m_{\tilde{e}_R^*}^2(M_U) = m_{16}^{(1)2} - D, \quad (142)$$

$$m_{\tilde{l}_1}^2(M_U) = m_{\tilde{d}_R^*}^2(M_U) = m_{16}^{(1)2} + 3D, \quad (143)$$

$$m_{\tilde{q}_2}^2(M_U) = m_{\tilde{c}_R^*}^2(M_U) = m_{\tilde{\mu}_R^*}^2(M_U) = m_{16}^{(2)2} - D, \quad (144)$$

$$m_{\tilde{l}_2}^2(M_U) = m_{\tilde{s}_R^*}^2(M_U) = m_{16}^{(2)2} + 3D, \quad (145)$$

$$m_{\tilde{q}_3}^2(M_U) = m_{\tilde{t}_R^*}^2(M_U) = m_{\tilde{\tau}_R^*}^2(M_U) = m_{16}^{(3)2} - D, \quad (146)$$

$$m_{\tilde{l}_3}^2(M_U) = m_{\tilde{b}_R^*}^2(M_U) = m_{16}^{(3)2} + 3D, \quad (147)$$

$$m_{H_1}^2(M_U) = m_{10}^2 - 2D, \quad m_{H_2}^2(M_U) = m_{10}^2 + 2D, \quad (148)$$

where $m_{16}^{(1)}$, $m_{16}^{(2)}$, $m_{16}^{(3)}$ and m_{10} are soft SUSY breaking scalar mass parameters and D is a parameter which represents D -term condensation related to $SO(10)/SU(5)$ generator. The coefficients of D represent the values of broken $U(1)$ charge. Here we assume that the particle assignment is the ordinary one where particles in each generation belong to **16** and two Higgs doublets belong to **10**. By eliminating those five unknown parameters, we obtain eighteen sum rules, i.e., fifteen ones (85), (86), (139), (140) and (141), which are same as those derived from $SU(5)$ breaking, and the following three sum rules,

$$\begin{aligned} m_{H_1}^2 - m_{H_2}^2 - M_{\tilde{u}_R}^2 + M_{\tilde{d}_R}^2 \\ = -12\zeta_1 M_1^2 + (M_W^2 - M_Z^2) \cos 2\beta - F_\tau - 3F_b + 3F_t, \end{aligned} \quad (149)$$

$$M_{\tilde{u}_R}^2 - M_{\tilde{d}_R}^2 = M_{\tilde{c}_R}^2 - M_{\tilde{s}_R}^2 = M_{\tilde{t}_R}^2 - M_{\tilde{b}_R}^2 - m_t^2 + 2F_t - 2F_b. \quad (150)$$

In Ref.[44], scalar sum rules are examined in the case that some low-energy particles appear as a linear combination of several distinct fields. We

refer ‘particle twisting’ as such a linear combination of some particles.[45, 46, 47, 48, 49, 50] In the case with particle twisting concerning $(\tilde{l}_1, \tilde{d}_R^*)$, $(\tilde{l}_2, \tilde{s}_R^*)$ and $(\tilde{l}_3, \tilde{b}_R^*)$, i.e., each one constitutes as a combination of particles in **16** and those in **10**, the sum rules (149) and (150) do not hold on.

(3) $SU(5) \times U(1)_F \rightarrow G_{SM}$

The particle assignment is different from the ordinary $SU(5)$ GUT, and the model is called ‘flipped $SU(5)$ GUT’. In this model, \tilde{q}_1 , \tilde{d}_R^* and $\tilde{\nu}_R^*$ belong to **10**, \tilde{l}_1 and \tilde{u}_R^* belong to $\bar{\mathbf{5}}$ and \tilde{e}_R^* belongs to **1** in $SU(5)$. After the breakdown of $SU(5) \times U(1)_F$ gauge symmetry to the SM one, there exist following relations among the first generation sfermions at the breaking scale M_U ,

$$m_{\tilde{q}_1}^2(M_U) = m_{10}^{(1)2} - (4g_5^2 + g_F^2)D_F, \quad (151)$$

$$m_{\tilde{d}_R^*}^2(M_U) = m_{10}^{(1)2} + (16g_5^2 - g_F^2)D_F, \quad (152)$$

$$m_{\tilde{u}_R^*}^2(M_U) = m_{\bar{5}}^{(1)2} + (-8g_5^2 + 3g_F^2)D_F, \quad (153)$$

$$m_{\tilde{l}_1}^2(M_U) = m_{\bar{5}}^{(1)2} + (12g_5^2 + 3g_F^2)D_F, \quad (154)$$

$$m_{\tilde{e}_R^*}^2(M_U) = m_1^{(1)2} - 5g_F^2 D_F, \quad (155)$$

where $m_{10}^{(1)}$, $m_{\bar{5}}^{(1)}$ and $m_1^{(1)}$ are soft SUSY breaking scalar mass parameters and D_F is a parameter which represents the D -term condensation related to $SU(5) \times U(1)_F/G_{SM}$ generator. The g_5 and g_F are gauge coupling constants of $SU(5)$ and $U(1)_F$, respectively. The same type of relations hold on for second and third generations and Higgs bosons. In this case, there are twelve arbitrary parameters, i.e., eleven soft SUSY breaking scalar mass parameters and one parameter from the D -term contribution. By eliminating these parameters, we obtain eleven sum rules, i.e., the type I sfermion sum rules and five ones such that

$$\begin{aligned} & M_{\tilde{u}_L}^2 + M_{\tilde{d}_L}^2 - 2M_{\tilde{u}_R}^2 - 2M_{\tilde{d}_R}^2 + M_{\tilde{\nu}_{eL}}^2 + M_{\tilde{e}_L}^2 \\ &= M_{\tilde{c}_L}^2 + M_{\tilde{s}_L}^2 - 2M_{\tilde{c}_R}^2 - 2M_{\tilde{s}_R}^2 + M_{\tilde{\nu}_{\mu L}}^2 + M_{\tilde{\mu}_L}^2 \\ &= M_{\tilde{t}_L}^2 + M_{\tilde{b}_L}^2 - 2M_{\tilde{t}_R}^2 - 2M_{\tilde{b}_R}^2 + M_{\tilde{\nu}_{\tau L}}^2 + M_{\tilde{\tau}_L}^2 + m_t^2 \\ &\quad - 2F_t - 2F_b + 2F_\tau \\ &= -2 \left(\zeta_3 - 2\zeta_2 \left(\frac{\alpha_2}{\alpha_3} \right)^2 \right) M_g^2 - 20\zeta_1 M_1^2, \end{aligned} \quad (156)$$

$$M_{\tilde{d}_L}^2 - M_{\tilde{d}_R}^2 = M_{\tilde{s}_L}^2 - M_{\tilde{s}_R}^2 = M_{\tilde{b}_L}^2 - M_{\tilde{b}_R}^2 + F_t - F_b, \quad (157)$$

where we use the relation $M_{\tilde{g}} = \alpha_3 M_2 / \alpha_2$ due to the partial unification of gaugino masses, i.e., $M_3(M_U) = M_2(M_U)$. Note that $M_1(M_U)$ does not necessarily equal to $M_3(M_U)$.

(4) $SU(4) \times SU(2)_L \times SU(2)_R \rightarrow G_{SM}$

In this case, there are eight arbitrary parameters, i.e., seven soft SUSY breaking scalar mass parameters and one parameter from a D -term contribution. By eliminating them, we obtain fifteen sum rules, i.e., the type I sfermion sum rules and nine ones such that

$$\begin{aligned}
& M_{\tilde{u}_L}^2 + M_{\tilde{d}_L}^2 - M_{\tilde{\nu}_{eL}}^2 - M_{\tilde{e}_L}^2 - 2M_{\tilde{e}_R}^2 + 2M_{\tilde{d}_R}^2 \\
&= M_{\tilde{c}_L}^2 + M_{\tilde{s}_L}^2 - M_{\tilde{\nu}_{\mu L}}^2 - M_{\tilde{\mu}_L}^2 - 2M_{\tilde{\mu}_R}^2 + 2M_{\tilde{s}_R}^2 \\
&= M_{\tilde{t}_L}^2 + M_{\tilde{b}_L}^2 - M_{\tilde{\nu}_{\tau L}}^2 - M_{\tilde{\tau}_L}^2 - 2M_{\tilde{\tau}_R}^2 + 2M_{\tilde{b}_R}^2 - m_t^2 \\
&\quad + 2F_t + 6F_b - 6F_\tau \\
&= 2\zeta_3 M_3^2 - 40\zeta_1 M_1^2, \tag{158}
\end{aligned}$$

$$\begin{aligned}
& g_R^2 \left(M_{\tilde{u}_L}^2 + M_{\tilde{d}_L}^2 - M_{\tilde{\nu}_{eL}}^2 - M_{\tilde{e}_L}^2 \right) - 2g_4^2 \left(M_{\tilde{u}_R}^2 - M_{\tilde{d}_R}^2 \right) \\
&= g_R^2 \left(M_{\tilde{c}_L}^2 + M_{\tilde{s}_L}^2 - M_{\tilde{\nu}_{\mu L}}^2 - M_{\tilde{\mu}_L}^2 \right) - 2g_4^2 \left(M_{\tilde{c}_R}^2 - M_{\tilde{s}_R}^2 \right) \\
&= g_R^2 \left(M_{\tilde{t}_L}^2 + M_{\tilde{b}_L}^2 - M_{\tilde{\nu}_{\tau L}}^2 - M_{\tilde{\tau}_L}^2 \right) - 2g_4^2 \left(M_{\tilde{t}_R}^2 - M_{\tilde{b}_R}^2 \right) \\
&\quad - g_R^2 \left(m_t^2 - 2F_t - 2F_b + 2F_\tau \right) + 2g_4^2 \left(m_t^2 - 2F_t + 2F_b \right) \\
&= 2g_R^2 \zeta_3 M_3^2 - \left(16g_R^2 + 24g_4^2 \right) \zeta_1 M_1^2 \\
&\quad + \left(\frac{4}{3}g_R^2 - 2g_4^2 \right) \left(M_W^2 - M_Z^2 \right) \cos 2\beta + \left(8g_R^2 + 12g_4^2 \right) \mathcal{S}, \tag{159}
\end{aligned}$$

$$\begin{aligned}
& m_{H_1}^2 - m_{H_2}^2 - M_{\tilde{u}_R}^2 - M_{\tilde{d}_R}^2 = m_{H_1}^2 - m_{H_2}^2 - M_{\tilde{c}_R}^2 - M_{\tilde{s}_R}^2 \\
&= m_{H_1}^2 - m_{H_2}^2 - M_{\tilde{t}_R}^2 - M_{\tilde{b}_R}^2 + m_t^2 - 2F_t - 2F_b \\
&= -12\zeta_1 M_1^2 + \left(M_W^2 - M_Z^2 \right) \cos 2\beta + 3F_t - 3F_b - F_\tau, \tag{160}
\end{aligned}$$

where we assume that H_1 and H_2 belong to $(\mathbf{1}, \mathbf{2}, \mathbf{2})$ of $SU(4) \times SU(2)_L \times SU(2)_R$.

(5) $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \rightarrow G_{SM}$

In this case, there are fourteen arbitrary parameters, i.e., thirteen soft SUSY breaking scalar mass parameters and one parameter from a D -term contribution. By eliminating these parameters, we obtain nine sum rules (85), (86) and (160).

(6) $E_6 \rightarrow G_{SM}$

In the case without particle twisting, there are two kinds of particle assignment in $SO(10)$ subgroup. One is that \tilde{l}_1 and \tilde{d}_R^* belong to $\mathbf{16}$ and H_1 belongs to $\mathbf{10}$. The other is that \tilde{l}_1 and \tilde{d}_R^* belong to $\mathbf{10}$ and H_1 belongs to $\mathbf{16}$. For both ones, we can obtain the same eighteen sum rules as those with $SO(10) \rightarrow G_{SM}$.

In the case with particle twisting concerning $(\tilde{l}_1, \tilde{d}_R^*)$, $(\tilde{l}_2, \tilde{s}_R^*)$ and $(\tilde{l}_3, \tilde{b}_R^*)$, sum rules are reduced to fifteen ones, the type I ((85) and (86)) and the type IIA ((139), (140) and (141)). As pointed out in [44], the twisting angles are determined from sparticle spectrum, and hence the flavor structure of GUT can be probed.

(7) $SU(6) \times SU(2)_R \rightarrow G_{SM}$

For sfermions in the first generation, \tilde{q}_1 and \tilde{l}_1 belong to $(\mathbf{15}, \mathbf{1})$ and \tilde{u}_R^* , \tilde{d}_R^* and \tilde{e}_R^* belong to $(\mathbf{6}, \mathbf{2})$ of $SU(6) \times SU(2)_R$. The same assignment holds on for other generations. In this case, we can obtain the same sum rules as those with $SU(4) \times SU(2)_L \times SU(2)_R \rightarrow G_{SM}$ by replacing g_4 by the gauge coupling of $SU(6)$.

(8) $SU(6) \times SU(2)_L \rightarrow G_{SM}$

For sfermions in the first generation, \tilde{q}_1 and \tilde{l}_1 belong to $(\mathbf{6}, \mathbf{2})$ and \tilde{u}_R^* , \tilde{d}_R^* and \tilde{e}_R^* belong to $(\mathbf{15}, \mathbf{1})$ of $SU(6) \times SU(2)_L$. The same assignment holds on for other generations. In this case, there are eight arbitrary parameters, i.e., six soft SUSY breaking scalar mass parameters and two parameters from D -term contributions. By eliminating these parameters, we obtain fifteen sum rules, i.e., twelve relations (85), (86), (158), and (160), and three ones such that

$$\begin{aligned} M_{\tilde{u}_R}^2 - M_{\tilde{e}_R}^2 &= M_{\tilde{c}_R}^2 - M_{\tilde{\mu}_R}^2 \\ &= M_{\tilde{t}_R}^2 - M_{\tilde{\tau}_R}^2 - m_t^2 + 2F_t - 2F_\tau \\ &= \zeta_3 M_3^2 - 20\zeta_1 M_1^2 + \left(-\frac{5}{3}M_W^2 + \frac{5}{3}M_Z^2\right) \cos 2\beta - 10S. \end{aligned} \quad (161)$$

(9) $SU(3)_c \times SU(3)_L \times SU(3)_R \rightarrow G_{SM}$

For sfermions in the first generation, \tilde{q}_1 belongs to $(\mathbf{3}, \mathbf{3}, \mathbf{1})$, \tilde{u}_R^* and \tilde{d}_R^* belong to $(\mathbf{3}, \mathbf{1}, \mathbf{3})$ and \tilde{l}_L and \tilde{e}_R^* belong to $(\mathbf{1}, \mathbf{3}, \mathbf{3})$ of $SU(3)_c \times SU(3)_L \times SU(3)_R$.

The same assignment holds on for other generations. In this case, there are twelve arbitrary parameters, i.e., ten soft SUSY breaking scalar mass parameters and two parameters from D -term contributions. By eliminating these parameters, we obtain eleven sum rules, i.e., nine relations (85), (86) and (160), and two ones such that

$$M_{\tilde{e}_L}^2 - M_{\tilde{e}_R}^2 = M_{\tilde{\mu}_L}^2 - M_{\tilde{\mu}_R}^2 = M_{\tilde{\tau}_L}^2 - M_{\tilde{\tau}_R}^2 - F_\tau. \quad (162)$$

3.2 Orbifold grand unification in five dimension

Higher-dimensional SUSY GUTs on an orbifold have attractive features as a realistic model beyond the MSSM. The triplet-doublet splitting of Higgs multiplets is elegantly realized in SUSY $SU(5)$ GUT in a five dimension.[22][23] Four-dimensional chiral fermions are generated through the dimensional reduction. Those phenomena originates from the fact that a part of zero modes are projected out by orbifolding, i.e., by non-trivial boundary conditions (BCs) concerning extra dimensions on bulk fields. Therefore we expect that some specific sum rules obtained in the previous section can be modified by splitting a bulk multiplet on the orbifold breaking. Chiral anomalies may arise at the boundaries with the advent of chiral fermions. Those anomalies must be cancelled in the four-dimensional effective theory by the contribution of brane chiral fermions and/or counter terms such as the Chern-Simons term.[51, 52, 53]

Recently, the possibility of complete family unification has been studied in SUSY $SU(N)$ GUTs defined on a five-dimensional space-time $M^4 \times (S^1/Z_2)$. [25] Here M^4 is the four-dimensional Minkowski space-time and S^1/Z_2 is the one-dimensional orbifold. Interesting models have been found, in which bulk fields from a single hypermultiplet and a few brane fields compose three families.

In this section, we derive peculiar sum rules among sfermion masses in two kinds of models where a partial family unification is realized, after studying $SU(5)$ SUSY orbifold GUT as a warm-up.⁸ We assume that extra gauge symmetries are broken at the same time as the orbifold breaking, and we do not specify the origin of soft SUSY breaking terms though some SUSY breaking mechanisms are known on the orbifold.⁹ We also assume that the

⁸Sfermion masses have been studied from the viewpoint of flavor symmetry and its violation in $SU(5)$ SUSY orbifold GUT.[54]

⁹The typical one is Scherk-Schwarz mechanism, in which SUSY is broken by the dif-

four-dimensional effective theory is anomaly free due to the presence of appropriate brane fields and/or the Chern-Simons term.

(a) $SU(5) \rightarrow G_{SM}$

The orbifold breaking $SU(5) \rightarrow G_{SM}$ is realized by the Z_2 parity assignment,

$$P_0 = \text{diag}(+1, +1, +1, +1, +1) , \quad (163)$$

$$P_1 = \text{diag}(+1, +1, +1, -1, -1) . \quad (164)$$

After the breakdown of $SU(5)$, $\bar{\mathbf{5}}$ and $\mathbf{10}$ are decomposed into a sum of multiplets regarding to the maximal subgroup $SU(3)_C \times SU(2)_L \times U(1)_Y$,

$$\bar{\mathbf{5}} = (\bar{\mathbf{3}}, \mathbf{1}, 1/3) + (\mathbf{1}, \mathbf{2}, -1/2), \quad (165)$$

$$\mathbf{10} = (\mathbf{3}, \mathbf{2}, 1/6) + (\bar{\mathbf{3}}, \mathbf{1}, -2/3) + (\mathbf{1}, \mathbf{1}, 1), \quad (166)$$

respectively. The Z_2 parity ($\mathcal{P}_0, \mathcal{P}_1$) of multiplets is listed in Table 1. In the first column, the subscript L (R) represents the left-handedness (right-handedness) for Weyl fermions. In the second and third column, $(\eta_{\bar{\mathbf{5}}}, \eta_{\mathbf{10}})$ and $(\eta'_{\bar{\mathbf{5}}}, \eta'_{\mathbf{10}})$ are intrinsic Z_2 parities which take a value $+1$ or -1 .

Whatever intrinsic Z_2 parity for $\bar{\mathbf{5}}$ we choose, either zero mode of $(\bar{\mathbf{3}}, \mathbf{1}, 1/3)$ or $(\mathbf{1}, \mathbf{2}, -1/2)$ is projected out and hence we, in general, have no scalar mass relation from a bulk field whose representation is $\bar{\mathbf{5}}$. For a bulk fields with $\mathbf{10}$, $(\bar{\mathbf{3}}, \mathbf{1}, -2/3)_L$ and $(\mathbf{1}, \mathbf{1}, 1)_L$ have zero mode when we take $\eta_{\mathbf{10}} = 1$ and $\eta'_{\mathbf{10}} = 1$. In this case, there exists the following relation at a breaking scale M_U ,

$$m_{\tilde{u}_R^*}^2(M_U) = m_{\tilde{e}_R^*}^2(M_U), \quad (167)$$

where zero mode of superpartners for $(\bar{\mathbf{3}}, \mathbf{1}, -2/3)_L$ and $(\mathbf{1}, \mathbf{1}, 1)_L$ are identified with \tilde{u}_R^* and \tilde{e}_R^* , respectively. The relation (167) leads to the sum rule (a piece of type IIA sfermion sum rules),

$$M_{\tilde{u}_R}^2 - M_{\tilde{e}_R}^2 = \left(\zeta_3 - 20\zeta_1 \left(\frac{\alpha_1}{\alpha_3} \right)^2 \right) M_g^2 - \frac{5}{3} (M_W^2 - M_Z^2) \cos 2\beta - 10\mathcal{S}, \quad (168)$$

ference of BCs between bosons and fermions.[55, 56] This mechanism on S^1/Z_2 leads to a restricted type of soft SUSY breaking parameters such as $M_i = \beta/R$ for bulk gauginos and $m_{\tilde{F}}^2 = (\beta/R)^2$ for bulk scalar particles where β is a real parameter and R is a radius of S^1 .

Table 1: Z_2 parity ($\mathcal{P}_0, \mathcal{P}_1$) for Weyl fermions from $\bar{\mathbf{5}}$ and $\mathbf{10}$

<i>Representation</i>	\mathcal{P}_0	\mathcal{P}_1
$(\mathbf{3}, \mathbf{1}, 1/3)_L$	$\eta_{\bar{\mathbf{5}}}$	$\eta'_{\bar{\mathbf{5}}}$
$(\bar{\mathbf{3}}, \mathbf{1}, 1/3)_R$	$-\eta_{\bar{\mathbf{5}}}$	$-\eta'_{\bar{\mathbf{5}}}$
$(\mathbf{1}, \mathbf{2}, -1/2)_L$	$\eta_{\bar{\mathbf{5}}}$	$-\eta'_{\bar{\mathbf{5}}}$
$(\mathbf{1}, \mathbf{2}, -1/2)_R$	$-\eta_{\bar{\mathbf{5}}}$	$\eta'_{\bar{\mathbf{5}}}$
$(\mathbf{3}, \mathbf{2}, 1/6)_L$	$\eta_{\mathbf{10}}$	$-\eta'_{\mathbf{10}}$
$(\bar{\mathbf{3}}, \mathbf{2}, 1/6)_R$	$-\eta_{\mathbf{10}}$	$\eta'_{\mathbf{10}}$
$(\bar{\mathbf{3}}, \mathbf{1}, -2/3)_L$	$\eta_{\mathbf{10}}$	$\eta'_{\mathbf{10}}$
$(\mathbf{3}, \mathbf{1}, -2/3)_R$	$-\eta_{\mathbf{10}}$	$-\eta'_{\mathbf{10}}$
$(\mathbf{1}, \mathbf{1}, 1)_L$	$\eta_{\mathbf{10}}$	$\eta'_{\mathbf{10}}$
$(\mathbf{1}, \mathbf{1}, 1)_R$	$-\eta_{\mathbf{10}}$	$-\eta'_{\mathbf{10}}$

where we use the GUT relation $M_3(M_U) = M_2(M_U)$. This type of sum rule generally holds on orbifold breaking of $SU(N)$ gauge symmetry for bulk fields with an antisymmetric representation if the bulk field contains $\mathbf{10}_L$ or $\bar{\mathbf{10}}_R$ under the subgroup $SU(5)$, and those $SU(2)_L$ singlets have even Z_2 parities. We refer a piece of type IIA such as (168) as the type IIB sfermion sum rules.

(b) $SU(8) \rightarrow G_{SM} \times SU(3) \times U(1)'$

We study sfermion mass relations stemmed from [8, 4] of $SU(8)$ after the orbifold breaking $SU(8) \rightarrow G_{SM} \times SU(3) \times U(1)'$, which is realized by the Z_2 parity assignment,

$$P_0 = \text{diag}(+1, +1, +1, +1, +1, -1, -1, -1) , \quad (169)$$

$$P_1 = \text{diag}(+1, +1, +1, -1, -1, +1, +1, +1) . \quad (170)$$

After the breakdown of $SU(8)$, the fourth antisymmetric representation [8, 4] with ${}_8C_4$ components are decomposed into a sum of multiplets regarding to the subgroup $SU(3)_C \times SU(2)_L \times SU(3)$,

$$[8, 4] = \sum_{l_1=0}^4 \sum_{l_2=0}^{4-l_1} ({}_3C_{l_1}, {}_2C_{l_2}, {}_3C_{4-l_1-l_2}) , \quad (171)$$

where l_1 and l_2 are intergers. The Z_2 parity of $({}_3C_{l_1}, {}_2C_{l_2}, {}_3C_{4-l_1-l_2})$ is given

Table 2: Z_2 parity ($\mathcal{P}_0, \mathcal{P}_1$) and the $U(1)'$ charge for Weyl fermions from [8, 4]

<i>Representation</i>	\mathcal{P}_0	\mathcal{P}_1	$U(1)'$
$({}_3C_1, {}_2C_0, {}_3C_3)_L$	$-\eta_{[8,4]}$	$\eta'_{[8,4]}$	-12
$({}_3C_1, {}_2C_0, {}_3C_3)_R$	$\eta_{[8,4]}$	$-\eta'_{[8,4]}$	-12
$({}_3C_0, {}_2C_1, {}_3C_3)_L$	$-\eta_{[8,4]}$	$-\eta'_{[8,4]}$	-12
$({}_3C_0, {}_2C_1, {}_3C_3)_R$	$\eta_{[8,4]}$	$\eta'_{[8,4]}$	-12
$({}_3C_2, {}_2C_0, {}_3C_2)_L$	$\eta_{[8,4]}$	$\eta'_{[8,4]}$	-4
$({}_3C_2, {}_2C_0, {}_3C_2)_R$	$-\eta_{[8,4]}$	$-\eta'_{[8,4]}$	-4
$({}_3C_0, {}_2C_2, {}_3C_2)_L$	$\eta_{[8,4]}$	$\eta'_{[8,4]}$	-4
$({}_3C_0, {}_2C_2, {}_3C_2)_R$	$-\eta_{[8,4]}$	$-\eta'_{[8,4]}$	-4
$({}_3C_1, {}_2C_1, {}_3C_2)_L$	$\eta_{[8,4]}$	$-\eta'_{[8,4]}$	-4
$({}_3C_1, {}_2C_1, {}_3C_2)_R$	$-\eta_{[8,4]}$	$\eta'_{[8,4]}$	-4
$({}_3C_3, {}_2C_0, {}_3C_1)_L$	$-\eta_{[8,4]}$	$\eta'_{[8,4]}$	4
$({}_3C_3, {}_2C_0, {}_3C_1)_R$	$\eta_{[8,4]}$	$-\eta'_{[8,4]}$	4
$({}_3C_1, {}_2C_2, {}_3C_1)_L$	$-\eta_{[8,4]}$	$\eta'_{[8,4]}$	4
$({}_3C_1, {}_2C_2, {}_3C_1)_R$	$\eta_{[8,4]}$	$-\eta'_{[8,4]}$	4
$({}_3C_2, {}_2C_1, {}_3C_1)_L$	$-\eta_{[8,4]}$	$-\eta'_{[8,4]}$	4
$({}_3C_2, {}_2C_1, {}_3C_1)_R$	$\eta_{[8,4]}$	$\eta'_{[8,4]}$	4
$({}_3C_3, {}_2C_1, {}_3C_0)_L$	$\eta_{[8,4]}$	$-\eta'_{[8,4]}$	12
$({}_3C_3, {}_2C_1, {}_3C_0)_R$	$-\eta_{[8,4]}$	$\eta'_{[8,4]}$	12
$({}_3C_2, {}_2C_2, {}_3C_0)_L$	$\eta_{[8,4]}$	$\eta'_{[8,4]}$	12
$({}_3C_2, {}_2C_2, {}_3C_0)_R$	$-\eta_{[8,4]}$	$-\eta'_{[8,4]}$	12

by

$$\mathcal{P}_0 = (-1)^{l_1+l_2}\eta_{[8,4]}, \quad \mathcal{P}_1 = (-1)^{l_2}\eta'_{[8,4]}, \quad (172)$$

where $\eta_{[8,4]}$ and $\eta'_{[8,4]}$ are intrinsic Z_2 parities which take a value +1 or -1. We assume that the Z_2 parity (172) is assigned for the left-handed Weyl fermions. The corresponding right-handed ones have opposite Z_2 parities. In Table 2, the Z_2 parity ($\mathcal{P}_0, \mathcal{P}_1$) and the $U(1)'$ charge are listed for representations of Weyl fermions. In the first column, the subscript L (R) represents the left-handedness (right-handedness) for Weyl fermions.

Let us take $\eta_{[8,4]} = 1$ and $\eta'_{[8,4]} = 1$. In this case, particles with even Z_2

Table 3: Sfermions with even Z_2 parity from [8, 4]

<i>Representation</i>	<i>Species</i>	$U(1)'$
$(\bar{\mathbf{3}}, \mathbf{1}, \mathbf{1})$	\tilde{d}_R^* (or $\tilde{s}_R^*, \tilde{b}_R^*$)	12
$(\mathbf{3}, \mathbf{2}, \bar{\mathbf{3}})$	$\tilde{q}_1, \tilde{q}_2, \tilde{q}_3$	-4
$(\mathbf{1}, \mathbf{1}, \bar{\mathbf{3}})$	$\tilde{e}_R^*, \tilde{\mu}_R^*, \tilde{\tau}_R^*$	-4
$(\bar{\mathbf{3}}, \mathbf{1}, \bar{\mathbf{3}})$	$\tilde{u}_R^*, \tilde{c}_R^*, \tilde{t}_R^*$	-4
$(\mathbf{1}, \mathbf{2}, \mathbf{1})$	\tilde{l}_1 (or \tilde{l}_2, \tilde{l}_3)	12

parities are given in Table 3. Each particle possesses a zero mode whose scalar component is identified with one of the MSSM particles in four dimension.

After the breakdown of $SU(3) \times U(1)'$ gauge symmetry, there exist the following relations at a breaking scale M_U ,

$$m_{\tilde{d}_R^*}^2(M_U) = m_{[8,4]}^2 + 12D', \quad (173)$$

$$m_{\tilde{q}_1}^2(M_U) = m_{[8,4]}^2 - D_1 - D_2 - 4D', \quad (174)$$

$$m_{\tilde{q}_2}^2(M_U) = m_{[8,4]}^2 + D_1 - D_2 - 4D', \quad (175)$$

$$m_{\tilde{q}_3}^2(M_U) = m_{[8,4]}^2 + 2D_2 - 4D', \quad (176)$$

$$m_{\tilde{u}_R^*}^2(M_U) = m_{\tilde{e}_R^*}^2(M_U) = m_{[8,4]}^2 - D_1 - D_2 - 4D', \quad (177)$$

$$m_{\tilde{c}_R^*}^2(M_U) = m_{\tilde{\mu}_R^*}^2(M_U) = m_{[8,4]}^2 + D_1 - D_2 - 4D', \quad (178)$$

$$m_{\tilde{t}_R^*}^2(M_U) = m_{\tilde{\tau}_R^*}^2(M_U) = m_{[8,4]}^2 + 2D_2 - 4D', \quad (179)$$

$$m_{\tilde{l}_1}^2(M_U) = m_{[8,4]}^2 + 12D', \quad (180)$$

where $m_{[8,4]}$ is a soft SUSY breaking scalar mass parameter, D_1 and D_2 are parameters which represent D -term condensations related to $SU(3)$ generator and D' stands for the D -term contribution of $U(1)'$. By eliminating those four unknown parameters, we obtain the following relations at M_U ,

$$m_{\tilde{u}_R^*}^2(M_U) = m_{\tilde{e}_R^*}^2(M_U) = m_{\tilde{q}_1}^2(M_U), \quad (181)$$

$$m_{\tilde{c}_R^*}^2(M_U) = m_{\tilde{\mu}_R^*}^2(M_U) = m_{\tilde{q}_2}^2(M_U), \quad (182)$$

$$m_{\tilde{t}_R^*}^2(M_U) = m_{\tilde{\tau}_R^*}^2(M_U) = m_{\tilde{q}_3}^2(M_U), \quad (183)$$

$$m_{\tilde{d}_R^*}^2(M_U) = m_{\tilde{l}_1}^2(M_U). \quad (184)$$

We need two right-handed down-type quark chiral supermultiplets and two $SU(2)$ lepton doublet chiral supermultiplets in order to obtain three generations. We assume that they are brane fields. When our four-dimensional world is a boundary on the point $y = 0$, each chiral multiplet must form a multiplet of $SU(5)$. Hence we have complete $SU(5)$ type of sum rules in the case that three generations come from $[8, 4]$ and two brane fields with $\bar{5}$. This result occurs from the fact that $[8, 4]$ is a real representation.

(c) $SU(9) \rightarrow G_{SM} \times SU(3) \times U(1) \times U(1)'$

We study sfermion mass relations stemmed from $[9, 6]$ of $SU(9)$ after the orbifold breaking $SU(9) \rightarrow G_{SM} \times SU(3) \times U(1) \times U(1)'$, which is realized by the Z_2 parity assignment,

$$P_0 = \text{diag}(+1, +1, +1, +1, +1, -1, -1, -1, -1), \quad (185)$$

$$P_1 = \text{diag}(+1, +1, +1, -1, -1, +1, +1, +1, -1). \quad (186)$$

After the breakdown of $SU(9)$, the sixth antisymmetric representation $[9, 6]$ with ${}_9C_6$ components are decomposed into a sum of multiplets regarding to the subgroup $SU(3)_C \times SU(2)_L \times SU(3) \times U(1)$,

$$[9, 6] = \sum_{l_1=0}^6 \sum_{l_2=0}^{6-l_1} \sum_{l_3=0}^{6-l_1-l_2} ({}_3C_{l_1}, {}_2C_{l_2}, {}_3C_{l_3}, {}_1C_{6-l_1-l_2-l_3}), \quad (187)$$

where l_1 and l_2 are intergers. The Z_2 parity of $({}_3C_{l_1}, {}_2C_{l_2}, {}_3C_{l_3}, {}_1C_{6-l_1-l_2-l_3})$ is given by

$$\mathcal{P}_0 = (-1)^{l_1+l_2} \eta_{[9,6]}, \quad \mathcal{P}_1 = (-1)^{l_1+l_3} \eta'_{[9,6]}, \quad (188)$$

where $\eta_{[9,6]}$ and $\eta'_{[9,6]}$ are intrinsic Z_2 parities which takes a value $+1$ or -1 . We assume that the Z_2 parity (188) is assigned for the left-handed Weyl fermions. The corresponding right-handed ones have opposite Z_2 parities. In Table 4, the Z_2 parity ($\mathcal{P}_0, \mathcal{P}_1$) and the $U(1)'$ charge are listed for left-handed Weyl fermions. Each right-handed Weyl fermion has the same $U(1)'$ charge but opposite parity of corresponding left-handed one.

Let us take $\eta_{[9,6]} = 1$ and $\eta'_{[9,6]} = 1$. In this case, sfermions with even Z_2 parities are listed in Table 5.

After the breakdown of $SU(3) \times U(1) \times U(1)'$ gauge symmetry, there exist the following relations at a breaking scale M_U ,

$$m_{\tilde{t}_1}^2(M_U) = m_{[9,6]}^2 + D_1 + D_2 - 2D + 6D', \quad (189)$$

Table 4: Z_2 parity ($\mathcal{P}_0, \mathcal{P}_1$) and the $U(1)'$ charge for Weyl fermions from [9, 6]

<i>Representation</i>	\mathcal{P}_0	\mathcal{P}_1	$U(1)$	$U(1)'$
$({}^3C_3, 2C_2, 3C_1, 1C_0)_L$	$-\eta_{[9,6]}$	$\eta'_{[9,6]}$	1	15
$({}^3C_3, 2C_2, 3C_1, 1C_0)_R$	$\eta_{[9,6]}$	$-\eta'_{[9,6]}$	1	15
$({}^3C_3, 2C_2, 3C_0, 1C_1)_L$	$-\eta_{[9,6]}$	$-\eta'_{[9,6]}$	-3	15
$({}^3C_3, 2C_2, 3C_0, 1C_1)_R$	$\eta_{[9,6]}$	$\eta'_{[9,6]}$	-3	15
$({}^3C_3, 2C_1, 3C_2, 1C_0)_L$	$\eta_{[9,6]}$	$-\eta'_{[9,6]}$	-2	6
$({}^3C_3, 2C_1, 3C_2, 1C_0)_R$	$-\eta_{[9,6]}$	$\eta'_{[9,6]}$	-2	6
$({}^3C_3, 2C_1, 3C_1, 1C_1)_L$	$\eta_{[9,6]}$	$\eta'_{[9,6]}$	2	6
$({}^3C_3, 2C_1, 3C_1, 1C_1)_R$	$-\eta_{[9,6]}$	$-\eta'_{[9,6]}$	2	6
$({}^3C_2, 2C_2, 3C_2, 1C_0)_L$	$\eta_{[9,6]}$	$\eta'_{[9,6]}$	-2	6
$({}^3C_2, 2C_2, 3C_2, 1C_0)_R$	$-\eta_{[9,6]}$	$-\eta'_{[9,6]}$	-2	6
$({}^3C_2, 2C_2, 3C_1, 1C_1)_L$	$\eta_{[9,6]}$	$-\eta'_{[9,6]}$	2	6
$({}^3C_2, 2C_2, 3C_1, 1C_1)_R$	$-\eta_{[9,6]}$	$\eta'_{[9,6]}$	2	6
$({}^3C_3, 2C_0, 3C_3, 1C_0)_L$	$-\eta_{[9,6]}$	$\eta'_{[9,6]}$	3	-3
$({}^3C_3, 2C_0, 3C_3, 1C_0)_R$	$\eta_{[9,6]}$	$-\eta'_{[9,6]}$	3	-3
$({}^3C_3, 2C_0, 3C_2, 1C_1)_L$	$-\eta_{[9,6]}$	$-\eta'_{[9,6]}$	-1	-3
$({}^3C_3, 2C_0, 3C_2, 1C_1)_R$	$\eta_{[9,6]}$	$\eta'_{[9,6]}$	-1	-3
$({}^3C_1, 2C_2, 3C_3, 1C_0)_L$	$-\eta_{[9,6]}$	$\eta'_{[9,6]}$	3	-3
$({}^3C_1, 2C_2, 3C_3, 1C_0)_R$	$\eta_{[9,6]}$	$-\eta'_{[9,6]}$	3	-3
$({}^3C_1, 2C_2, 3C_2, 1C_1)_L$	$-\eta_{[9,6]}$	$-\eta'_{[9,6]}$	-1	-3
$({}^3C_1, 2C_2, 3C_2, 1C_1)_R$	$\eta_{[9,6]}$	$\eta'_{[9,6]}$	-1	-3
$({}^3C_2, 2C_1, 3C_3, 1C_0)_L$	$-\eta_{[9,6]}$	$-\eta'_{[9,6]}$	3	-3
$({}^3C_2, 2C_1, 3C_3, 1C_0)_R$	$\eta_{[9,6]}$	$\eta'_{[9,6]}$	3	-3
$({}^3C_2, 2C_1, 3C_2, 1C_1)_L$	$-\eta_{[9,6]}$	$\eta'_{[9,6]}$	-1	-3
$({}^3C_2, 2C_1, 3C_2, 1C_1)_R$	$\eta_{[9,6]}$	$-\eta'_{[9,6]}$	-1	-3
$({}^3C_2, 2C_0, 3C_3, 1C_1)_L$	$\eta_{[9,6]}$	$-\eta'_{[9,6]}$	0	-12
$({}^3C_2, 2C_0, 3C_3, 1C_1)_R$	$-\eta_{[9,6]}$	$\eta'_{[9,6]}$	0	-12
$({}^3C_0, 2C_2, 3C_3, 1C_1)_L$	$\eta_{[9,6]}$	$-\eta'_{[9,6]}$	0	-12
$({}^3C_0, 2C_2, 3C_3, 1C_1)_R$	$-\eta_{[9,6]}$	$\eta'_{[9,6]}$	0	-12
$({}^3C_1, 2C_1, 3C_3, 1C_1)_L$	$\eta_{[9,6]}$	$\eta'_{[9,6]}$	0	-12
$({}^3C_1, 2C_1, 3C_3, 1C_1)_R$	$-\eta_{[9,6]}$	$-\eta'_{[9,6]}$	0	-12

Table 5: Sfermions with even Z_2 parity from [9, 6]

<i>Representation</i>	<i>Species</i>	$U(1)$	$U(1)'$
$(\mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{1})$	$\tilde{l}_1, \tilde{l}_2, \tilde{l}_3$	-2	6
$(\bar{\mathbf{3}}, \mathbf{1}, \bar{\mathbf{3}}, \mathbf{1})$	$\tilde{d}_R^*, \tilde{s}_R^*, \tilde{b}_R^*$	2	6
$(\mathbf{1}, \mathbf{1}, \mathbf{3}, \mathbf{1})$	$\tilde{e}_R^*, \tilde{\mu}_R^*, \tilde{\tau}_R^*$	1	3
$(\bar{\mathbf{3}}, \mathbf{1}, \mathbf{3}, \mathbf{1})$	$\tilde{u}_R^*, \tilde{c}_R^*, \tilde{t}_R^*$	1	3
$(\mathbf{3}, \mathbf{2}, \mathbf{1}, \mathbf{1})$	\tilde{q}_1 (or \tilde{q}_2, \tilde{q}_3)	-3	3
$(\mathbf{3}, \mathbf{2}, \mathbf{1}, \mathbf{1})$	\tilde{q}_2 (or \tilde{q}_1, \tilde{q}_3)	0	-12

$$m_{\tilde{l}_2}^2(M_U) = m_{[9,6]}^2 - D_1 + D_2 - 2D + 6D', \quad (190)$$

$$m_{\tilde{l}_3}^2(M_U) = m_{[9,6]}^2 - 2D_2 - 2D + 6D', \quad (191)$$

$$m_{\tilde{d}_R^*}^2(M_U) = m_{[9,6]}^2 - D_1 - D_2 + 2D + 6D', \quad (192)$$

$$m_{\tilde{s}_R^*}^2(M_U) = m_{[9,6]}^2 + D_1 - D_2 + 2D + 6D', \quad (193)$$

$$m_{\tilde{b}_R^*}^2(M_U) = m_{[9,6]}^2 + 2D_2 + 2D + 6D', \quad (194)$$

$$m_{\tilde{e}_R^*}^2(M_U) = m_{\tilde{u}_R^*}^2(M_U) = m_{[9,6]}^2 + D_1 + D_2 + D + 3D', \quad (195)$$

$$m_{\tilde{\mu}_R^*}^2(M_U) = m_{\tilde{c}_R^*}^2(M_U) = m_{[9,6]}^2 - D_1 + D_2 + D + 3D', \quad (196)$$

$$m_{\tilde{\tau}_R^*}^2(M_U) = m_{\tilde{t}_R^*}^2(M_U) = m_{[9,6]}^2 - 2D_2 + D + 3D', \quad (197)$$

$$m_{\tilde{q}_1}^2(M_U) = m_{[9,6]}^2 - 3D + 3D', \quad (198)$$

$$m_{\tilde{q}_2}^2(M_U) = m_{[9,6]}^2 - 12D', \quad (199)$$

where $m_{[9,6]}$ is a soft SUSY breaking scalar mass parameter, D_1 , D_2 and D are parameters which represent D -term condensations related to $SU(3) \times U(1)$ generator and D' stands for the D -term contribution of $U(1)'$. By eliminating those five unknown parameters, we obtain the following relations at M_U ,

$$m_{\tilde{u}_R^*}^2(M_U) = m_{\tilde{e}_R^*}^2(M_U), \quad m_{\tilde{c}_R^*}^2(M_U) = m_{\tilde{\mu}_R^*}^2(M_U),$$

$$m_{\tilde{t}_R^*}^2(M_U) = m_{\tilde{\tau}_R^*}^2(M_U), \quad (200)$$

$$m_{\tilde{d}_R^*}^2(M_U) + m_{\tilde{l}_1}^2(M_U) = m_{\tilde{s}_R^*}^2(M_U) + m_{\tilde{l}_2}^2(M_U)$$

$$= m_{\tilde{b}_R^*}^2(M_U) + m_{\tilde{l}_3}^2(M_U), \quad (201)$$

$$m_{\tilde{d}_R^*}^2(M_U) + m_{\tilde{u}_R^*}^2(M_U) = m_{\tilde{s}_R^*}^2(M_U) + m_{\tilde{c}_R^*}^2(M_U)$$

$$= m_{b_R^*}^2(M_U) + m_{t_R^*}^2(M_U), \quad (202)$$

$$\begin{aligned} & \sum_{i=1}^3 m_{\tilde{l}_i}^2(M_U) + m_{\tilde{u}_R^*}^2(M_U) + m_{\tilde{c}_R^*}^2(M_U) + m_{\tilde{t}_R^*}^2(M_U) \\ &= 3m_{\tilde{q}_1}^2(M_U) + m_{\tilde{d}_R^*}^2(M_U) + m_{\tilde{s}_R^*}^2(M_U) + m_{\tilde{b}_R^*}^2(M_U), \end{aligned} \quad (203)$$

$$\begin{aligned} & \sum_{i=1}^3 m_{\tilde{l}_i}^2(M_U) + 4 \left(m_{\tilde{d}_R^*}^2(M_U) + m_{\tilde{s}_R^*}^2(M_U) + m_{\tilde{b}_R^*}^2(M_U) \right) \\ &+ 3m_{\tilde{q}_2}^2(M_U) = 6 \left(m_{\tilde{u}_R^*}^2(M_U) + m_{\tilde{c}_R^*}^2(M_U) + m_{\tilde{t}_R^*}^2(M_U) \right). \end{aligned} \quad (204)$$

The relation (200) leads to the type IIB sfermion sum rules,

$$\begin{aligned} M_{\tilde{u}_R}^2 - M_{\tilde{e}_R}^2 &= M_{\tilde{c}_R}^2 - M_{\tilde{\mu}_R}^2 \\ &= M_{\tilde{t}_R}^2 - M_{\tilde{\tau}_R}^2 - m_t^2 + 2F_t - 2F_\tau \\ &= \left(\zeta_3 - 20\zeta_1 \left(\frac{\alpha_1}{\alpha_3} \right) \right) M_{\tilde{g}}^2 + \left(-\frac{5}{3}M_W^2 + \frac{5}{3}M_Z^2 \right) \cos 2\beta - 10\mathcal{S}. \end{aligned} \quad (205)$$

The other relations lead to following six sum rules,

$$M_{\tilde{d}_R}^2 + M_{\tilde{e}_L}^2 = M_{\tilde{s}_R}^2 + M_{\tilde{\mu}_L}^2 = M_{\tilde{b}_R}^2 + M_{\tilde{\tau}_L}^2 + 2F_b + F_\tau, \quad (206)$$

$$M_{\tilde{d}_R}^2 + M_{\tilde{u}_R}^2 = M_{\tilde{s}_R}^2 + M_{\tilde{c}_R}^2 = M_{\tilde{b}_R}^2 + M_{\tilde{t}_R}^2 - m_t^2 + 2F_t + 2F_b, \quad (207)$$

$$\begin{aligned} & M_{\tilde{e}_L}^2 + M_{\tilde{\mu}_L}^2 + M_{\tilde{\tau}_L}^2 + M_{\tilde{u}_R}^2 + M_{\tilde{c}_R}^2 + M_{\tilde{t}_R}^2 \\ &= 3M_{\tilde{u}_L}^2 + M_{\tilde{d}_R}^2 + M_{\tilde{s}_R}^2 + M_{\tilde{b}_R}^2 - 3 \left(\zeta_3 - 20\zeta_1 \left(\frac{\alpha_1}{\alpha_3} \right) \right) M_{\tilde{g}}^2 \\ &\quad - \left(8M_W^2 - 5M_Z^2 \right) \cos 2\beta + m_t^2 - 2F_t + 2F_b - F_\tau, \end{aligned} \quad (208)$$

$$\begin{aligned} & M_{\tilde{e}_L}^2 + M_{\tilde{\mu}_L}^2 + M_{\tilde{\tau}_L}^2 + 4 \left(M_{\tilde{d}_R}^2 + M_{\tilde{s}_R}^2 + M_{\tilde{b}_R}^2 \right) + 3M_{\tilde{c}_L}^2 \\ &= 6 \left(M_{\tilde{u}_R}^2 + M_{\tilde{c}_R}^2 + M_{\tilde{t}_R}^2 \right) - 3 \left(\zeta_3 - 2\zeta_2 \left(\frac{\alpha_2}{\alpha_3} \right)^2 + 70\zeta_1 \left(\frac{\alpha_1}{\alpha_3} \right)^2 \right) M_{\tilde{g}}^2 \\ &\quad - 9 \left(M_W^2 - M_Z^2 \right) \cos 2\beta - 6m_t^2 + 12F_t - 8F_b - F_\tau. \end{aligned} \quad (209)$$

The brane fields live on the orbifold fix points and respect with the gauge symmetry on the brane. Therefore a part of type IIA sum rules hold on for brane fields if our world is a boundary with $y = 0$.

4 Conclusions

We have studied the sparticle spectrum in the MSSM and derived sum rules among sparticle masses under the assumption that models beyond the MSSM are four-dimensional SUSY GUTs or five-dimensional SUSY orbifold GUTs. We have obtained various kinds of scalar sum rules and we can classify them into following four types.

(Type I) Sfermion sum rules inherent the MSSM:

$$\begin{aligned} M_{\tilde{u}_L}^2 - M_{\tilde{d}_L}^2 &= M_{\tilde{\nu}_{eL}}^2 - M_{\tilde{e}_L}^2 = M_{\tilde{c}_L}^2 - M_{\tilde{s}_L}^2 = M_{\tilde{\nu}_{\mu L}}^2 - M_{\tilde{\mu}_L}^2 \\ &= M_{\tilde{t}_L}^2 - M_{\tilde{b}_L}^2 - m_t^2 = M_{\tilde{\nu}_{\tau L}}^2 - M_{\tilde{\tau}_L}^2 = M_W^2 \cos 2\beta. \end{aligned} \quad (210)$$

These sum rules are derived from the fact that left-handed fermions (and its superpartner) form $SU(2)_L$ doublets, and they are irrelevant to the structure of models beyond the MSSM. The sfermion sector (and the breakdown of electroweak symmetry) in the MSSM can be tested by using them.

(Type IIA) Sfermion sum rules from four-dimensional grand unification:

$$\begin{aligned} M_{\tilde{u}_L}^2 - M_{\tilde{u}_R}^2 &= M_{\tilde{c}_L}^2 - M_{\tilde{c}_R}^2 = M_{\tilde{t}_L}^2 - M_{\tilde{t}_R}^2 - F_t + F_b \\ &= \left(\zeta_2 \left(\frac{\alpha_2}{\alpha_3} \right)^2 - 15\zeta_1 \left(\frac{\alpha_1}{\alpha_3} \right)^2 \right) M_{\tilde{g}}^2 \\ &\quad + \left(\frac{4}{3}M_W^2 - \frac{5}{6}M_Z^2 \right) \cos 2\beta + 5\mathcal{S}, \end{aligned} \quad (211)$$

$$\begin{aligned} M_{\tilde{u}_R}^2 - M_{\tilde{e}_R}^2 &= M_{\tilde{c}_R}^2 - M_{\tilde{\mu}_R}^2 = M_{\tilde{t}_R}^2 - M_{\tilde{\tau}_R}^2 - m_t^2 + 2F_t - 2F_\tau \\ &= \left(\zeta_3 - 20\zeta_1 \left(\frac{\alpha_1}{\alpha_3} \right)^2 \right) M_{\tilde{g}}^2 \\ &\quad + \left(-\frac{5}{3}M_W^2 + \frac{5}{3}M_Z^2 \right) \cos 2\beta - 10\mathcal{S}, \end{aligned} \quad (212)$$

$$\begin{aligned} M_{\tilde{e}_L}^2 - M_{\tilde{d}_R}^2 &= M_{\tilde{\mu}_L}^2 - M_{\tilde{s}_R}^2 = M_{\tilde{\tau}_L}^2 - M_{\tilde{b}_R}^2 + F_\tau - 2F_b \\ &= \left(-\zeta_3 + \zeta_2 \left(\frac{\alpha_2}{\alpha_3} \right)^2 + 5\zeta_1 \left(\frac{\alpha_1}{\alpha_3} \right)^2 \right) M_{\tilde{g}}^2 \\ &\quad + \left(-\frac{4}{3}M_W^2 + \frac{5}{6}M_Z^2 \right) \cos 2\beta - 5\mathcal{S}. \end{aligned} \quad (213)$$

These sum rules are derived in the case with the direct breakdown of a grand

unified symmetry to the SM one, and hence the sfermion sector (and the grand unification) in SUSY GUT can be tested by using them.

(Type IIB) Sfermion sum rules from five-dimensional orbifold grand unification:

$$M_{\tilde{u}_R}^2 - M_{\tilde{e}_R}^2 = \left(\zeta_3 - 20\zeta_1 \left(\frac{\alpha_1}{\alpha_3} \right)^2 \right) M_g^2 - \frac{5}{3} (M_W^2 - M_Z^2) \cos 2\beta - 10\mathcal{S}. \quad (214)$$

This sum rule is a piece of type IIA ones and it is derived on the orbifold breaking of $SU(N)$ gauge symmetry for bulk fields with an antisymmetric representation if the bulk field contains $\mathbf{10}_L$ or $\overline{\mathbf{10}}_R$ under the subgroup $SU(5)$, and $SU(2)_L$ singlets have even Z_2 parities. The orbifold breaking (of $SU(N)$ gauge symmetry) can be tested by using it.

(Type III) Sfermion sum rules specific to each model:

There are sfermion sum rules specific to physics beyond the MSSM such as the chain breaking of gauge symmetry or representations of gauge group in SUSY orbifold GUT. We refer them as type III sfermion sum rules.

(Type IV) Sfermion sum rules favored by phenomenology:

There are sfermion sum rules favored from the viewpoint of particle phenomenology. For example, if sum rules such as $M_{\tilde{u}_L}^2 = M_{\tilde{c}_L}^2$, $M_{\tilde{d}_L}^2 = M_{\tilde{s}_L}^2$ and so on hold on, the sufficient suppression of FCNC processes can be realized. We refer them as type IV sfermion sum rules.

Four-dimensional superstring models, in general, possess extra gauge symmetries including several $U(1)$ s and sfermions could have a different quantum numbers. Therefore sum rules different from those in our analysis could be derived. In fact, some exotic scalar mass relations have been obtained.[57, 58] Superstring theory can be tested by using them. In any case, sparticle sum rules can play as useful probes of the MSSM and beyond.

Some assumptions in our analysis might be too strong to describe a high-energy physics correctly. The resultant sum rules can be modified by relaxing some assumptions. For example, if extra gauge symmetries are broken at different scales from the grand unification scale or the orbifold breaking scale, or if extra particles exist in the intermediate scale, soft SUSY breaking parameters receive extra renormalization effects and then our analysis should be modified. In the case that effects such as threshold corrections, higher loop

RG evolution, F -term contributions and/or higher dimensional operators are sizable, we should consider them.

In our analysis, we have assumed the gravity-mediated SUSY breaking in the case that the dynamics in the hidden sector do not give sizable effects. It is also important to study sum rules specific to the way of mediation and the hidden dynamics. Sum rules have been examined in the gauge-mediated SUSY breaking models[59, 60], the anomaly-mediated SUSY breaking models[61] and models with strong dynamics in the hidden sector.[34]

In most cases, we have not required the mass degeneracy for each squark and slepton species in the first two generations and then dangerous FCNC processes can be induced unless those masses are rather heavy or fermion and its superpartner mass matrices are aligned. In orbifold family unification models, a non-abelian subgroup such as $SU(3)$ of $SU(N)$ plays the role of family symmetry and its D -term contributions spoil the mass degeneracy. Conversely, the requirement of degenerate masses would give a constraint on the D -term condensations and lead to extra sfermion sum rules. It is important to study phenomenological aspects of orbifold GUTs and construct a realistic model which explains family structure. It is interesting to derive sfermion sum rules specific to each model and classify them. These will be presented in a separate publication.

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