

家蠶繭形の數學的表示と營繭中に於ける蠶の運動(I)

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Nobufusa YANAGISAWA: On the mathematical expression
of the forms of the cocoons and the motion of the silk-worms
during the cocooning.

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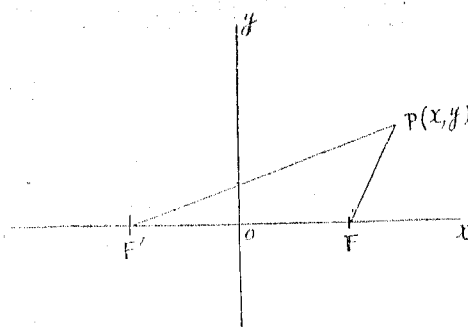
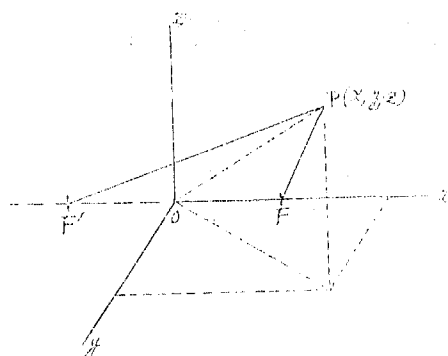
緒 言

繭形に就きては、池田榮太郎、服部達吉、中村弘、横山忠雄、樺井博、山口定次郎諸氏の多數研究がある。筆者は繭形が全く自然的曲線の美しさを持ち、圓、橢圓、依諾形等も蠶體の略略同形(目測)なるこより推して、數學的諸ファクターも亦略々同類のものでなくてはならぬし、又その形が長い年月の環境例へば、風、氣候、害蟲に對する抵抗、防禦とか、蛹自體の重みに對する力學的安定等何等かの物理學的條件を具へた形であらねばならぬと考へ、繭形のカーブトレーシングを試み、本文に掲ぐる Oval of CASSINI を得た。繭形上、又繭絲上等より、今後力學的研究を續行せんとするものであるが、茲には營繭作業が如何に數學的であるかを掲げ、合せて營繭中の蠶體運動の一考察を記さんとする。

尙本研究に當り本校校長井上博士初め、製絲科、養蠶科、物理學教室の各先生方、南佐久農蠶學校等より格別な便宜さ、貴重なる實驗試料を與へられた事を厚く感謝するに共に、御懇篤なる指導と鞭撻を與へられた東京帝國大學理學部長寺澤寛一教授、同工學部山本武藏教授同理學部西川正治教授、同理學部平田森三助教授の諸先生に對し深く感謝の意を表する。

1. 二定點よりの距離の積が一定なる點の軌跡に就きて

二定點を F, F' とし、その中點を O, FF' を x 軸とする。 $FP \times F'P$ が一定なる點 P の描く空間曲面は、 $\{(x+a)^2 + y^2 + z^2\} \{(x-a)^2 + y^2 + z^2\} = m^4$ にて表はされる。爰に a は OF 及び OF' の長さの絶對値、 m は $FP \times F'P$ なる面積の平方根である。煩雜を避け P 點の軌跡を平面上に限定すれば、 $\{(x+a)^2 + y^2\} \{(x-a)^2 + y^2\} = m^4$ 或は $(x^2 + y^2 + a^2)^2 - 4a^2 x^2 = m^4 \dots\dots(1)$ 之は CASSINI 卵形線 (Oval of CASSINI) を表す。



2. Oval of CASSINI の極大、極小

$$(x^2 + y^2 + a^2)^2 - 4a^2 x^2 = m^4$$

$$\therefore y = \{(4a^2 x^2 + m^4)^{\frac{1}{2}} - x^2 - a^2\}^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2} \{(4a^2 x^2 + m^4)^{\frac{1}{2}} - x^2 - a^2\}^{-\frac{1}{2}} \left\{ \frac{1}{2} (4a^2 x^2 + m^4)^{-\frac{1}{2}} (8a^2 x) - 2x \right\}$$

$$= \frac{4a^2 x - 2x \sqrt{4a^2 x^2 + m^4}}{2 \sqrt{4a^2 x^2 + m^4} - x^2 - a^2 \sqrt{4a^2 x^2 + m^4}}$$

$$\text{今 } \frac{dy}{dx} = 0 \text{ と置く } x \{4a^2 - 2\sqrt{4a^2 x^2 + m^4}\} = 0$$

$$\therefore x = 0 \quad \text{或は} \quad x = \pm \frac{\sqrt{4a^4 - m^4}}{2a} \quad \left(\text{但し } \sqrt{2a} \geq m \right)$$

$$\begin{aligned} \frac{d^2 y}{dx^2} &= -\frac{1}{4} \{(4a^2 x^2 + m^4)^{\frac{1}{2}} - x^2 - a^2\}^{-\frac{3}{2}} \left\{ \frac{1}{2} (4a^2 x^2 + m^4)^{-\frac{1}{2}} (8a^2 x) - 2x \right\} \{(4a^2 x^2 + m^4)^{-\frac{1}{2}} \\ &\quad (4a^2 x) - 2x\} + \frac{1}{2} \{(4a^2 x^2 + m^4)^{\frac{1}{2}} - x^2 - a^2\}^{-\frac{1}{2}} \left\{ -\frac{1}{4} (4a^2 x^2 + m^4)^{-\frac{3}{2}} (8a^2 x)^2 + 4a^2 (4a^2 x^2 + m^4)^{-\frac{1}{2}} - 2 \right\} \end{aligned}$$

$$= -\frac{1}{4} \frac{(4a^2 x - 2x \sqrt{4a^2 x^2 + m^4})^2}{\{(4a^2 x^2 + m^4)^{\frac{1}{2}} - x^2 - a^2\} \sqrt{4a^2 x^2 + m^4} - x^2 - a^2 (4a^2 x^2 + m^4)}$$

$$+ \frac{1}{2} \frac{-16a^4 x^2 + 4a^2 (4a^2 x^2 + m^4) - 2(4a^2 x^2 + m^4) \sqrt{4a^2 x^2 + m^4}}{\sqrt{4a^2 x^2 + m^4} - x^2 - a^2 (4a^2 x^2 + m^4) \sqrt{4a^2 x^2 + m^4}}$$

$$= -\frac{4a^4 x^2 - 4a^2 x^2 \sqrt{4a^2 x^2 + m^4} + x^2 (4a^2 x^2 + m^4)}{(\sqrt{4a^2 x^2 + m^4} - x^2 - a^2) \sqrt{4a^2 x^2 + m^4} - x^2 - a^2 (4a^2 x^2 + m^4)}$$

$$+ \frac{2a^2 m^4 - (4a^2 x^2 + m^4) \sqrt{4a^2 x^2 + m^4}}{\sqrt{4a^2 x^2 + m^4} - x^2 - a^2 (4a^2 x^2 + m^4) \sqrt{4a^2 x^2 + m^4}}$$

$$= -\frac{(4a^4 x^2 + 4a^2 x^2 + m^4 x^2) \sqrt{4a^2 x^2 + m^4} + 4a^2 x^2 (4a^2 x^2 + m^4) + 2a^2 m^4 \sqrt{4a^2 x^2 + m^4}}{\{\sqrt{4a^2 x^2 + m^4} - x^2 - a^2\}^{\frac{3}{2}} (4a^2 x^2 + m^4)^{\frac{3}{2}}}$$

$$= \frac{-(4a^4 x^2 + m^4)^2 - 2a^2 m^4 x^2 - 2a^4 m^4 x^2 - 2a^4 m^4 + (x^2 + a^2) (4a^2 x^2 + m^4)^{\frac{3}{2}}}{\{\sqrt{4a^2 x^2 + m^4} - x^2 - a^2\}^{\frac{3}{2}} (4a^2 x^2 + m^4)^{\frac{3}{2}}}$$

(i) $x = 0$ の時

$$\frac{d^2 y}{dx^2} = \frac{2a^2 m^6 - m^8 - 2a^4 m^4 + a^2 m^6}{(m^2 - a^2)^{\frac{3}{2}} m^6} = \frac{m^4 (m^2 - a^2) (2a^2 - m^4)}{(m^2 - a^2)^{\frac{3}{2}} m^6} = \frac{2a^2 - m^4}{(\sqrt{m^2 - a^2}) m^2}$$

さて、 $\sqrt{2a} > m > a$ の時上に凹即ち極小、又 $\sqrt{2a} < m$ の時上に凸即ち極大。

(ii) $x = \frac{\sqrt{4a^4 - m^4}}{2a}$ の時 (但し $\sqrt{2a} > m$)

$$\frac{d^2 y}{dx^2} = \frac{-4a^4(4a^4 - m^4) + 4a^4(4a^4 - m^4) + 4a^4 m^4 - 16a^8 - \frac{m^4(4a^4 - m^4)}{2}}{\frac{m^6}{8a^3}(4a^4)^{\frac{3}{2}}}$$

$$\frac{-2a^4 m^4 + 2a^4(8a^4 - m^4)}{\frac{m^6}{8a^3} \times 8a^6} = \frac{\frac{1}{2} m^4 (m^4 - 4a^4)}{\frac{m^6}{a^3}} = -\frac{1}{2} \frac{m^4 (4a^4 - m^4)}{m^6 a^3}$$

$\sqrt{2} a > m$ なる時 $x = \frac{\sqrt{4a^4 - m^4}}{2a}$ にて上に凸即ち極大。

以上の如く P 點の表す曲線は、 $\sqrt{2} a = m$ を境界にして、縊を生ずるゝ生ぜざる事になる。即ち $\sqrt{2} a > m > a$ の時縊を生じ、 $\sqrt{2} a < m$ の時縊れを生じない。

3. Oval of CASSINI の長軸及び短軸の長さ

$$(x^2 + y^2 + a^2)^2 - 4a^2 x^2 = m^4$$

$$y=0 \text{ と置く } (x^2 + a^2)^2 - 4a^2 x^2 - m^4 = 0$$

$$x^2 = x \text{ と置き } x^2 + 2a^2 x + a^4 - 4a^2 x - m^4 = 0 \quad \therefore x^2 - 2a^2 x + a^4 - m^4 = 0$$

$$\therefore x^2 - 2a^2 x + a^4 - m^4 = 0$$

$$\therefore (x - a^2)^2 - m^4 = 0$$

$$\therefore (x - a^2 + m^2)(x - a^2 - m^2) = 0$$

しかるに x は正。又 $m > a$ ($m < a$ は直接蘭形と關係なしと思考せらるるにより替く) 依つて $x - a^2 + m^2 \neq 0$

$$\therefore x - a^2 - m^2 = 0 \quad \therefore x = a^2 + m^2 \text{ 即ち } x = \pm \sqrt{m^2 + a^2}$$

$$\text{依つて長軸の長さ } AA' = |2x| = |2\sqrt{m^2 + a^2}|$$

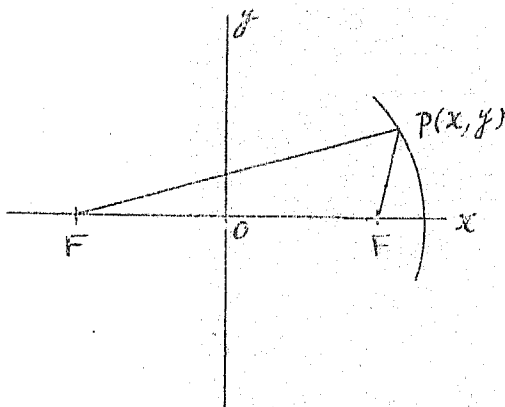
次に $x = 0$ と置く、

$$(y^2 + a^2)^2 - m^4 = 0 \quad (y^2 + a^2 + m^2)(y^2 + a^2 - m^2) = 0$$

$$\text{然るに } y^2 + a^2 + m^2 \neq 0 \quad \therefore y = \pm \sqrt{m^2 - a^2}$$

$$\text{依つて短軸の長さ } BB' = |2y| = |2\sqrt{m^2 - a^2}|$$

4. Oval of CASSINI の追跡と蘭形



二定點を F, F' とし FF' を x 軸、FF' の中點 O より x 軸に垂線を立て、之を y 軸とす。

$$\begin{aligned} \text{今 } \frac{\text{短軸の長さ}}{\text{長軸の長さ}} \times 100 &= \frac{2\sqrt{m^2 - a^2}}{2\sqrt{m^2 + a^2}} \times 100 \\ &= \frac{\sqrt{m^2 - a^2}}{\sqrt{m^2 + a^2}} \times 100 \end{aligned}$$

を標準型番號 (Standard form no.) と名付く。

卵形線の長軸を一様に 8 裡に採り、標準型番號を 84, 82, 80, ……28, 26 迄與へることにより m^2 , m , a^2 , a の値は次表の如くなる。

但し

$$m^2 = FP, F'P \quad a = OF = F'O$$

例へば 84 型に於ては、

$$\sqrt{m^2 + a^2} = 4 \dots\dots (1) \quad \sqrt{m^2 - a^2} = 4 \times .84 = 3.36 \dots\dots (2) \quad (1), (2) \text{ より } m^2 = 13.65,$$

$$m = 3.69; \quad a^2 = 2.36, \quad a = 1.53$$

Table I

| Standard form no. S | m ² | m | a ² | a | Standard form no. S | m ² | m | a ² | a |
|------------------------|----------------|------|----------------|------|------------------------|----------------|------|----------------|------|
| 84 | 13,65 | 3,69 | 2,36 | 1,53 | 54 | 10,34 | 3,22 | 5,67 | 2,38 |
| 82 | 13,38 | 3,66 | 2,62 | 1,62 | 52 | 10,17 | 3,19 | 5,84 | 2,42 |
| 80 | 13,12 | 3,62 | 2,88 | 1,70 | 50 | 10,00 | 3,16 | 6,00 | 2,45 |
| 78 | 12,87 | 3,59 | 3,14 | 1,77 | 48 | 9,85 | 3,14 | 6,16 | 2,48 |
| 76 | 12,62 | 3,55 | 3,38 | 1,84 | 46 | 9,70 | 3,11 | 6,31 | 2,51 |
| 74 | 12,38 | 3,52 | 3,62 | 1,90 | 44 | 9,55 | 3,09 | 6,45 | 2,54 |
| 72 | 12,15 | 3,49 | 3,86 | 1,96 | 42 | 9,41 | 3,07 | 6,59 | 2,57 |
| 70 | 11,92 | 3,45 | 4,08 | 2,02 | 40 | 9,28 | 3,05 | 6,72 | 2,59 |
| 68 | 11,70 | 3,42 | 4,30 | 2,07 | 38 | 9,16 | 3,03 | 6,85 | 2,62 |
| 66 | 11,49 | 3,39 | 4,52 | 2,13 | 36 | 9,04 | 3,01 | 6,97 | 2,64 |
| 64 | 11,28 | 3,36 | 4,73 | 2,17 | 34 | 8,93 | 2,99 | 7,08 | 2,66 |
| 62 | 11,08 | 3,33 | 4,93 | 2,22 | 32 | 8,82 | 2,97 | 7,18 | 2,68 |
| 60 | 10,88 | 3,30 | 5,12 | 2,26 | 30 | 8,72 | 2,95 | 7,28 | 2,70 |
| 58 | 10,69 | 3,27 | 5,31 | 2,30 | 28 | 8,63 | 2,94 | 7,37 | 2,71 |
| 56 | 10,51 | 3,24 | 5,49 | 2,34 | 26 | 8,54 | 2,92 | 7,46 | 2,73 |

上の表より曲線を描くのであるが、例へば84型に於ては、

$$FP \cdot F'P = m^2 = 13.65 \quad FA = 4 - a = 4 - 1.53 = 2.47$$

$$FP \quad 2.47(FA) \quad 2.55 \quad 2.65 \quad 2.80 \quad 3.00 \quad 3.30 \quad 3.69 (=m)$$

$$F'P \quad 5.53 \quad 5.35 \quad 5.15 \quad 4.88 \quad 4.55 \quad 4.14 \quad 3.69$$

新しくして描いたP点の軌跡は Fig 1. (a)となる。

Fig 1. (a)に見る如く、58型を境にして上方へ、楕圓形、圓形、下方へ俵形となる。

次に Oval of CASSINI と實際の繭形とを比較せんために、Fig 1. (a) の原圖上に實物繭の陰影を投じ長軸を總て8種に擴大した。寫真引伸器 (Luckey) の原版を置く位置に供試繭を置き 100watt の電球にて投射した。その結果は驚く程良く合致するを見た。Fig 1. (b) は實物繭である。

Fig. 1 (b) Actual Cocoons

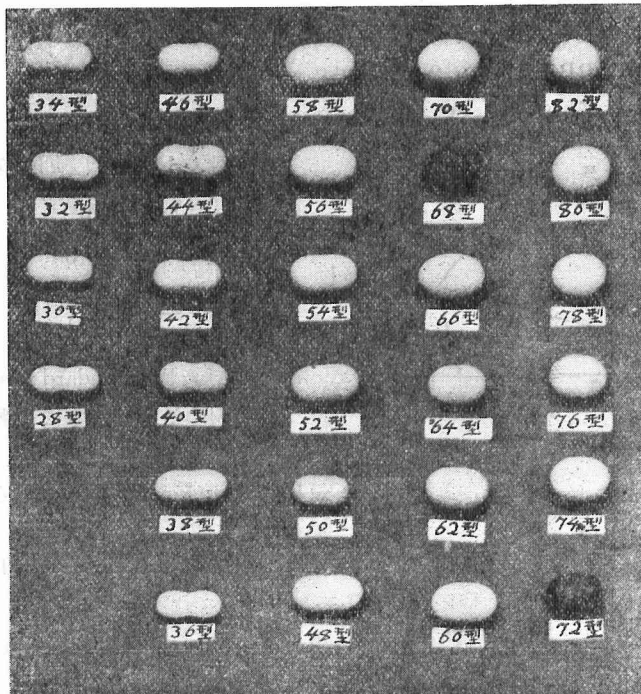
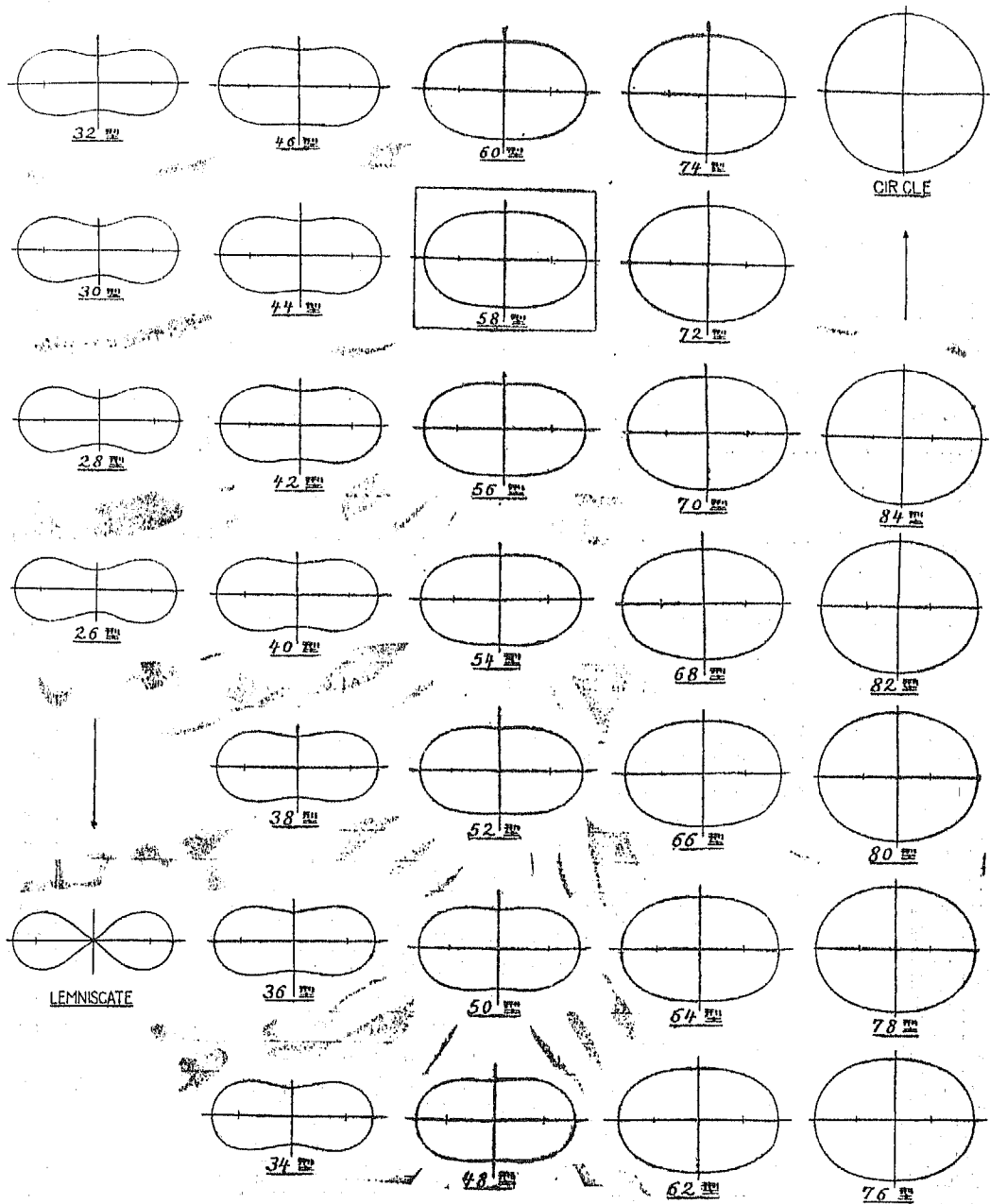


Fig. 1 (a) Curve Tracing of Oval of CASSINI



供試繭を品種別に列記すれば下の如し。

上田蠶絲専門學校産

國蠶日1號、日9號、日10號、日11號、日113號、日114號、日115號。支16號、支20號、支106號、支108號、支109號、支110號、支111號。歐19號、歐20號。褐色卵、伴性油蠶、宮城諸村、專二、K種、大正白。日9號×支108號、改中巢×滿月、日108號×支114號、浙江二龍華仙。支107號×支106號、支110號×日112號、支108號×日115號、日111號×支107號。日115號×支108號、日113號×支111號、支109號×日115號、歐18號×支107號。歐19號×支107號、支111號×日113號、歐19號×支18號、支108號×日114號。日111號×支112號、日9號×支108號、支20號×支109號、歐16號×支16號。支17號×支106號、K×P、日112號×支110號、支108號×日114號、歐19號×支17號。

長野縣南佐久農蠶學校産

支111號、日113號、日114號

供試繭頭數1600 中約1300は標準型に一致せり。品種別に見て、全然一致せざるものに伴性油蠶あり、尙交雜種に縊深きもの相當あり。日本種に兩端細きものあり。されど實驗品種中伴性油蠶を除き、選繭により標準型に合致せしめ得。繭影を投射し標準型と比較するに當り、目測にて大體長軸に對し對稱なる側より光を當て、又供試繭中互ひに直角なる短軸の長さに相當の差異あるものありたり。

例へば上田蠶絲専門學校産の本年春繭中日114號の如きは、短軸中長い方向より光を當つれば縊深きも、短軸中短い軸の方向より光を當てて驗すれば合致するものありたるが如し。

5. 品種別による繭の標準型番號決定

實驗に於て頭數割合に多きもの、品種別による標準型比較に都合良きもの約20種に就き表にして下に掲ぐ。

Table II

| 品 種 名 | 標準型番號該當頭數 | | | | | | | | | | | | | | | | | | | | | | | | | | | | 供 試 繭 數 |
|-------------|-----------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|------------------|
| | 28 | 30 | 32 | 34 | 36 | 38 | 40 | 42 | 44 | 46 | 48 | 50 | 52 | 54 | 56 | 58 | 60 | 62 | 64 | 66 | 68 | 70 | 72 | 74 | 76 | 78 | 80 | 82 | |
| 大正白 | 1 | 2 | 2 | 7 | 5 | 2 | 1 | | | | | | | | | | | | | | | | | | | | | | 20 |
| 日115號 | | | 2 | 0 | 1 | 9 | 12 | 7 | 6 | 2 | 1 | | | | | | | | | | | | | | | | | | 40 |
| 日114號 | | | | 1 | 1 | 3 | 13 | 14 | 5 | 1 | 2 | | | | | | | | | | | | | | | | | | 40 |
| 日11號 | | | | | | 1 | 4 | 5 | 5 | 2 | 2 | 1 | | | | | | | | | | | | | | | | | 20 |
| 日10號 | | | | | | | 2 | 1 | 5 | 8 | 4 | 1 | | | | | | | | | | | | | | | | | 21 |
| 歐20號 | | | | | | | 2 | 0 | 2 | 15 | 43 | 9 | 5 | 2 | 1 | 1 | | | | | | | | | | | | | 50 |
| 歐19號 | | | | | | | 1 | 1 | 6 | 17 | 7 | 5 | 3 | | | | | | | | | | | | | | | | 40 |
| ×日115號 | | | | | | | | | | | | 3 | 2 | 5 | 49 | 40 | 29 | 13 | 7 | 1 | | | | | | | | | 149 |
| ×日112號 | | | | | | | | | | | | | 2 | 1 | 6 | 15 | 20 | 22 | 18 | 9 | 4 | 2 | 1 | | | | | | 100 |
| ×日111號 | | | | | | | | | | | | | 2 | 16 | 14 | 8 | 8 | 0 | 1 | | | | | | | | | | 49 |
| ×日108號 | | | | | | | | | | | | | 1 | 5 | 8 | 10 | 9 | 11 | 6 | | | | | | | | | | 50 |
| ×日107號 | | | | | | | | | | | | | 1 | 1 | 4 | 10 | 10 | 7 | 9 | 5 | 2 | 1 | | | | | | | 50 |
| 支111號 | | | | | | | | | | | | | | | 2 | 2 | 8 | 10 | 9 | 9 | 5 | 3 | 1 | 1 | | | | | 50 |
| 支110號 | | | | | | | | | | | | | | | 1 | 2 | 3 | 12 | 2 | | | | | | | | | | 20 |
| 支108號 | | | | | | | | | | | | | | | | 1 | 1 | 9 | 5 | 4 | 20 | 5 | 4 | 1 | | | | | 50 |
| 支109號 | | | | | | | | | | | | | | | | 1 | 5 | 10 | 6 | 6 | 4 | 4 | 8 | | | | | | 44 |
| 支16號 | | | | | | | | | | | | | | | | | 5 | 7 | 4 | 4 | 8 | 8 | 1 | 2 | 1 | 1 | | | 41 |
| 支20號 | | | | | | | | | | | | | | | | | | 1 | 4 | 0 | 13 | 6 | 5 | 7 | 2 | 1 | 0 | 1 | 40 |
| K×P. | | | | | | | | | | | | | 3 | 5 | 10 | 11 | 13 | 27 | 21 | 10 | | | | | | | | | 100 |
| K. | | | | | | | | | | | | | | | | | | | | | 1 | 2 | 5 | 2 | 5 | 2 | 0 | 1 | 18 |

尙茲には變異の計算を缺けども、その方面に對し少しにても參考となり研究方法の手助けもなれば幸である。

6. Oval of CASSINI による繭の體積計算及び長球體積との比較

繰絲張力、繭絲量測定、繰絲機械其他の合理的研究又は製作上、繭の體積、表面積、斷面積等の必要な場合が相當あり、今後も益々必要になると思考せらるる故以下順次記載せんとする。

〔註〕 同考察に關しては筆者のみならず、多年研究を積まれる服部達吉氏の衣笠蠶報 —347— 掲載の「繭形の特徴の數量的表現」の意見を參考され度い。

方程式 $(x^2 + y^2 + a^2)^2 - 4a^2 x^2 = m^4$ より

$$y^2 = (4a^2 x^2 + m^4)^{\frac{1}{2}} - x^2 - a^2$$

今體積を v として、

$$v = 2\pi \int_0^{\sqrt{m^2+a^2}} y^2 dx = 2\pi \int_0^{\sqrt{m^2+a^2}} \{(4a^2 x^2 + m^4)^{\frac{1}{2}} - x^2 - a^2\} dx$$

$$x = \frac{m^2}{2a} \tan \theta \quad \text{と置く} \quad dx = \frac{m^2}{2a} \sec^2 \theta d\theta$$

$$\therefore v = 2\pi \int_0^{\tan^{-1} \frac{2a\sqrt{m^2+a^2}}{m^2}} \left(m^2 \sec \theta - \frac{m^4}{4a^2} \tan^2 \theta - a^2 \right) \frac{m^2}{2a} \sec^2 \theta d\theta$$

$$= \frac{\pi m^2}{a} \int_0^{\tan^{-1} \frac{2a\sqrt{m^2+a^2}}{m^2}} \left(m^2 \sec^3 \theta - \frac{m^4}{4a^2} \tan^2 \theta \sec^3 \theta - a^2 \sec^3 \theta \right) d\theta$$

$$= \frac{\pi m^2}{a} \left[\frac{m^2}{2} \{ \tan \theta \sec \theta + \log (\tan \theta + \sec \theta) \} - \frac{m^4}{12a^2} \tan^3 \theta - a^2 \tan \theta \right]_0^{\tan^{-1} \frac{2a\sqrt{m^2+a^2}}{m^2}}$$

$$= \frac{\pi m^4}{2a} \left[\tan \theta \sec \theta + \frac{\log_{10} (\tan \theta + \sec \theta)}{0.434294} - \frac{\tan^3 \theta}{6 \left(\frac{a}{m} \right)^2} - 2 \left(\frac{a}{m} \right)^2 \tan \theta \right]_0^{\tan^{-1} 2 \frac{a}{m} \sqrt{1 + \frac{a^2}{m^2}}}$$

然るに $\frac{a}{m}$ は Fig.1 の如く各繭形標準型に依つて一定値である。何故なら (4) 項より標準型

$$\text{番號} N = \frac{\sqrt{m^2 - a^2}}{\sqrt{m^2 + a^2}} \times 100 = \frac{\sqrt{1 - \left(\frac{a}{m} \right)^2}}{\sqrt{1 + \left(\frac{a}{m} \right)^2}} \times 100 \text{ であるからである。今上式で}$$

$$\tan^{-1} 2 \frac{a}{m} \sqrt{1 + \left(\frac{a}{m} \right)^2} = \theta' \text{ と置く、}$$

$$V = \frac{\pi}{2} \cdot \frac{m^4}{a} \left\{ \tan \theta' \sec \theta' + \frac{\log_{10} (\tan \theta' + \sec \theta')}{0.434294} - \frac{\tan^3 \theta'}{6 \left(\frac{a}{m} \right)^2} - 2 \left(\frac{a}{m} \right)^2 \tan \theta' \right\}$$

便宜上 $A = \tan \theta' \sec \theta' + \frac{\log_{10} (\tan \theta' + \sec \theta')}{0.434294} - \frac{\tan^3 \theta'}{6 \left(\frac{a}{m} \right)^2} - 2 \left(\frac{a}{m} \right)^2 \tan \theta'$ を繭の標準型體積常數

(Volmic coefficient of standard form of cocoon) と名付けることを許されるならば、

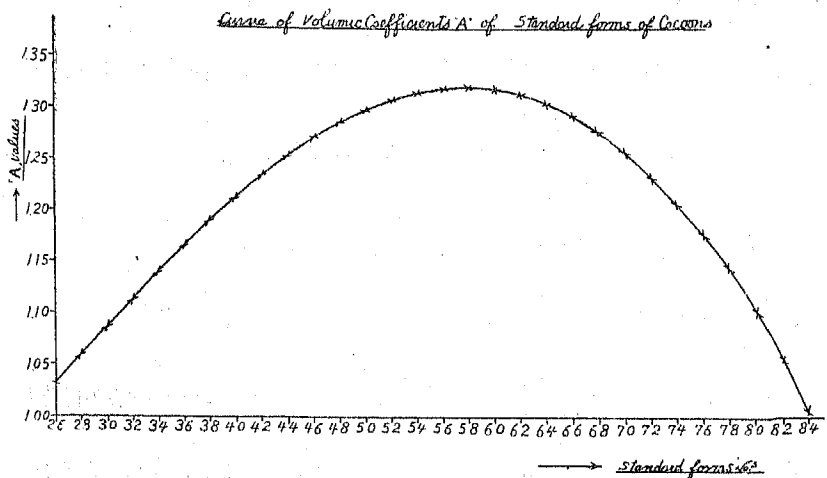
$$V = \frac{\pi}{2} \cdot \frac{m^4}{a} \cdot A \quad A \text{ の値を求むれば次の如くである。}$$

Table III

| 標準型 番 號 | $\left(\frac{a}{m}\right)^2$ | $\tan \theta'$ | $\sec \theta'$ | $\tan \theta' \sec \theta'$ | $\frac{\log_{10} (\tan \theta' + \sec \theta')}{0.434294}$ | $\frac{\tan \theta'}{6 \left(\frac{a}{m}\right)^2}$ | $2 \left(\frac{a}{m}\right)^2 \tan \theta'$ | A |
|------------|------------------------------|----------------|----------------|-----------------------------|--|---|---|---------|
| 26 | 0,87337 | 2,55823 | 2,74673 | 7,02677 | 1,66864 | 3,19498 | 4,46856 | 1,03187 |
| 28 | 0,85460 | 2,51787 | 2,70918 | 6,82136 | 1,65381 | 3,11304 | 4,30354 | 1,05809 |
| 30 | 0,83486 | 2,47537 | 2,66973 | 6,60857 | 1,63801 | 3,02797 | 4,13322 | 1,08539 |
| 32 | 0,81422 | 2,43078 | 2,62843 | 6,38914 | 1,62118 | 2,93990 | 3,95840 | 1,11200 |
| 34 | 0,79276 | 2,38430 | 2,58550 | 6,16458 | 1,60336 | 2,84959 | 3,78040 | 1,13814 |
| 36 | 0,77054 | 2,33601 | 2,54105 | 5,93590 | 1,58449 | 2,75721 | 3,60000 | 1,16319 |
| 38 | 0,74764 | 2,28513 | 2,49527 | 5,70452 | 1,56472 | 2,66350 | 3,41840 | 1,18730 |
| 40 | 0,72414 | 2,23470 | 2,44823 | 5,47110 | 1,54590 | 2,56854 | 3,23650 | 1,20996 |
| 42 | 0,70010 | 2,18196 | 2,40019 | 5,23710 | 1,52215 | 2,47302 | 3,05518 | 1,23105 |
| 44 | 0,67560 | 2,12797 | 2,35122 | 5,00333 | 1,49940 | 2,37713 | 2,87533 | 1,25027 |
| 46 | 0,65071 | 2,07278 | 2,30139 | 4,77028 | 1,47570 | 2,28098 | 2,69755 | 1,26745 |
| 48 | 0,62549 | 2,01662 | 2,25094 | 4,53929 | 1,45102 | 2,18526 | 2,52274 | 1,28231 |
| 50 | 0,60000 | 1,95957 | 2,19993 | 4,31101 | 1,42539 | 2,09016 | 2,35148 | 1,29476 |
| 52 | 0,57431 | 1,90171 | 2,14860 | 4,08601 | 1,39879 | 1,99589 | 2,18433 | 1,30457 |
| 54 | 0,54847 | 1,84311 | 2,09691 | 3,86484 | 1,37117 | 1,90262 | 2,02177 | 1,31162 |
| 56 | 0,52253 | 1,78402 | 2,04517 | 3,64862 | 1,34264 | 1,81106 | 1,86443 | 1,31577 |
| 58 | 0,49656 | 1,72438 | 1,99320 | 3,43644 | 1,31298 | 1,72008 | 1,71221 | 1,31713 |
| 60 | 0,47059 | 1,66376 | 1,94116 | 3,22962 | 1,28257 | 1,63109 | 1,56589 | 1,31521 |
| 62 | 0,44467 | 1,60298 | 1,88932 | 3,02854 | 1,25054 | 1,54382 | 1,42559 | 1,30967 |
| 64 | 0,418842 | 1,54178 | 1,83768 | 2,83330 | 1,21770 | 1,45837 | 1,29152 | 1,30111 |
| 66 | 0,393155 | 1,48014 | 1,78628 | 2,64390 | 1,18360 | 1,37476 | 1,16380 | 1,28890 |
| 68 | 0,36762 | 1,41810 | 1,73522 | 2,46072 | 1,14842 | 1,29293 | 1,04263 | 1,27358 |
| 70 | 0,34228 | 1,35564 | 1,68456 | 2,28366 | 1,11197 | 1,21310 | 0,92802 | 1,25451 |
| 72 | 0,31718 | 1,29270 | 1,63434 | 2,11271 | 1,07398 | 1,13512 | 0,82002 | 1,23155 |
| 74 | 0,29232 | 1,22925 | 1,58462 | 1,94789 | 1,03456 | 1,05900 | 0,71768 | 1,20477 |
| 76 | 0,26775 | 1,16521 | 1,53549 | 1,78917 | 0,99348 | 0,98477 | 0,62397 | 1,17391 |
| 78 | 0,24347 | 1,10044 | 1,48693 | 1,63628 | 0,95063 | 0,91223 | 0,53585 | 1,13883 |
| 80 | 0,21951 | 1,03476 | 1,43900 | 1,48902 | 0,90373 | 0,84123 | 0,45428 | 1,09924 |
| 82 | 0,19589 | 0,96800 | 1,39176 | 1,34722 | 0,85855 | 0,77346 | 0,37924 | 1,05307 |
| 84 | 0,17261 | 0,89978 | 1,34521 | 1,21039 | 0,80870 | 0,70318 | 0,31062 | 1,00529 |

Table III の A のグラフは Fig 2 の如く 58 型が最大となる。

Fig. 2



(註) 26型—84型を計算せるは供試繭1600顆中それ以外のものは無かつた故である。

次に長軸の長さの半分を $\sqrt{m^2 + a^2} = 1$ とする。短軸の長さの半分 $\sqrt{m^2 - a^2} = \frac{N}{100}$

従つて $m^2 = 1^2 \left\{ 1 + \left(\frac{N}{100} \right)^2 \right\} \div 2$; $a^2 = 1^2 \left\{ 1 - \left(\frac{N}{100} \right)^2 \right\} \div 2$

是等により體積 $V = \frac{\pi}{2} \frac{m^4}{a} \Lambda = 1.5708 \times \frac{m^4}{a} \times \Lambda$ を求むれば下の如し。

Table III

| 長軸の 長さ(2l) | 體 | | | | 積 | | | | | |
|---------------|---------------------------------|-------|---------------------------------|-------|---------------------------------|-------|---------------------------------|-------|---------------------------------|-------|
| | 2 6 型 | | 2 8 型 | | 3 0 型 | | 3 2 型 | | 3 4 型 | |
| | $\frac{\pi}{2} \Lambda=1,62086$ | | $\frac{\pi}{2} \Lambda=1,66283$ | | $\frac{\pi}{2} \Lambda=1,70493$ | | $\frac{\pi}{2} \Lambda=1,74673$ | | $\frac{\pi}{2} \Lambda=1,78756$ | |
| cm | m ⁴ /a | V | m ⁴ /a | V | m ⁴ /a | V | m ⁴ /a | V | m ⁴ /a | V |
| 4,2 | 3,86480 | 6,264 | 3,96647 | 6,596 | 4,07803 | 6,953 | 4,20010 | 7,336 | 4,33317 | 7,745 |
| 4,0 | 3,33856 | 5,411 | 3,42638 | 5,697 | 3,52275 | 6,006 | 3,62814 | 6,337 | 3,74315 | 6,691 |
| 3,8 | 2,86241 | 4,640 | 2,93767 | 4,885 | 3,02331 | 5,149 | 3,11067 | 5,434 | 3,20930 | 5,737 |
| 3,6 | 2,43381 | 3,945 | 2,49782 | 4,153 | 2,56839 | 4,378 | 2,64491 | 4,620 | 2,72876 | 4,878 |
| 3,4 | 2,05029 | 3,323 | 2,10423 | 3,499 | 2,16340 | 3,688 | 2,22815 | 3,892 | 2,29877 | 4,109 |
| 3,2 | 1,70935 | 2,771 | 1,75430 | 2,917 | 1,80364 | 3,075 | 1,85761 | 3,245 | 1,91652 | 3,426 |
| 3,0 | 1,40845 | 2,283 | 1,44549 | 2,404 | 1,48615 | 2,534 | 1,53063 | 2,674 | 1,57914 | 2,823 |
| 2,8 | 1,14513 | 1,856 | 1,17524 | 1,934 | 1,20830 | 2,060 | 1,24445 | 2,174 | 1,28390 | 2,295 |
| 2,6 | 0,91685 | 1,486 | 0,94097 | 1,565 | 0,96743 | 1,649 | 0,99638 | 1,740 | 1,02797 | 1,838 |
| 2,4 | 0,72112 | 1,169 | 0,74010 | 1,231 | 0,76091 | 1,297 | 0,78369 | 1,369 | 0,80853 | 1,445 |
| 2,2 | 0,55546 | 0,900 | 0,57005 | 0,948 | 0,58639 | 0,999 | 0,60363 | 1,054 | 0,62277 | 1,113 |
| 2,0 | 0,41732 | 0,676 | 0,42330 | 0,712 | 0,44035 | 0,751 | 0,45352 | 0,792 | 0,46789 | 0,836 |

| 長 軸 の 長さ(2l) | 體 | | | | | | 積 | | | |
|-----------------|---------------------------|-------|---------------------------|-------|---------------------------|-------|---------------------------|-------|---------------------------|--------|
| | 3 6 型 | | 3 8 型 | | 4 0 型 | | 4 2 型 | | 4 4 型 | |
| | $\frac{\pi}{2} A=1,82715$ | | $\frac{\pi}{2} A=1,86501$ | | $\frac{\pi}{2} A=1,90061$ | | $\frac{\pi}{2} A=1,93373$ | | $\frac{\pi}{2} A=1,96392$ | |
| cm | m ⁴ /a | V | m ⁴ /a | V | m ⁴ /a | V | m ⁴ /a | V | m ⁴ /a | V |
| 4,2 | 4,47823 | 8,182 | 4,63591 | 8,645 | 4,83718 | 9,157 | 4,99308 | 9,635 | 5,19463 | 10,202 |
| 4,0 | 3,86844 | 7,063 | 4,00465 | 7,459 | 4,15263 | 7,893 | 4,31318 | 8,341 | 4,48732 | 8,813 |
| 3,8 | 3,31671 | 6,060 | 3,43349 | 6,433 | 3,56335 | 6,767 | 3,69800 | 7,151 | 3,84733 | 7,556 |
| 3,6 | 2,82709 | 5,153 | 2,91942 | 5,445 | 3,02760 | 5,754 | 3,14433 | 6,080 | 3,27126 | 6,424 |
| 3,4 | 2,37570 | 4,341 | 2,45936 | 4,587 | 2,55024 | 4,847 | 2,64985 | 5,122 | 2,75378 | 5,412 |
| 3,2 | 1,98065 | 3,619 | 2,05039 | 3,824 | 2,12615 | 4,041 | 2,20835 | 4,270 | 2,29753 | 4,512 |
| 3,0 | 1,63200 | 2,982 | 1,63948 | 3,151 | 1,75189 | 3,330 | 1,81963 | 3,519 | 1,91317 | 3,757 |
| 2,8 | 1,32688 | 2,424 | 1,37358 | 2,562 | 1,42434 | 2,707 | 1,47942 | 2,861 | 1,53916 | 3,023 |
| 2,6 | 1,06237 | 1,941 | 1,09979 | 2,051 | 1,14041 | 2,167 | 1,18452 | 2,291 | 1,23232 | 2,420 |
| 2,4 | 0,83558 | 1,527 | 0,855022 | 1,595 | 0,87698 | 1,705 | 0,93165 | 1,802 | 0,96926 | 1,904 |
| 2,2 | 0,64362 | 1,176 | 0,665259 | 1,243 | 0,69089 | 1,313 | 0,71761 | 1,338 | 0,74559 | 1,466 |
| 2,0 | 0,48356 | 0,884 | 0,50059 | 0,934 | 0,51938 | 0,987 | 0,53915 | 1,043 | 0,56092 | 1,102 |

| 長 軸 の 長 さ (2l) | 體 | | | | 積 | | | | | |
|-------------------|---------------------------|--------|---------------------------|--------|---------------------------|--------|---------------------------|--------|---------------------------|--------|
| | 4 6 型 | | 4 8 型 | | 5 0 型 | | 5 2 型 | | 5 4 型 | |
| | $\frac{\pi}{2} A=1,99091$ | | $\frac{\pi}{2} A=2,01425$ | | $\frac{\pi}{2} A=2,03381$ | | $\frac{\pi}{2} A=2,04922$ | | $\frac{\pi}{2} A=2,06029$ | |
| cm | m ⁴ /a | V | m ⁴ /a | V | m ⁴ /a | V | m ⁴ /a | V | m ⁴ /a | V |
| 4,2 | 5,41470 | 10,780 | 5,65035 | 11,381 | 5,90748 | 12,015 | 6,18664 | 12,678 | 6,48988 | 13,371 |
| 4,0 | 4,67619 | 9,310 | 4,88098 | 9,832 | 5,10312 | 10,379 | 5,34422 | 10,951 | 5,63566 | 11,549 |
| 3,8 | 4,00924 | 7,982 | 4,18484 | 8,429 | 4,37530 | 8,899 | 4,58200 | 9,390 | 4,83659 | 9,903 |
| 3,6 | 3,40893 | 6,787 | 3,55826 | 7,167 | 3,72017 | 7,566 | 3,89596 | 7,984 | 4,08687 | 8,420 |
| 3,4 | 2,87175 | 5,717 | 2,99761 | 6,038 | 3,13395 | 6,374 | 3,28562 | 6,733 | 3,44284 | 7,093 |
| 3,2 | 2,39422 | 4,767 | 2,49905 | 5,034 | 2,61280 | 5,314 | 2,73625 | 5,607 | 2,87035 | 5,914 |
| 3,0 | 1,97276 | 3,928 | 2,05916 | 4,148 | 2,15289 | 4,379 | 2,25458 | 4,620 | 2,36507 | 4,873 |
| 2,8 | 1,60394 | 3,193 | 1,67416 | 3,372 | 1,75037 | 3,560 | 1,83306 | 3,756 | 1,92291 | 3,962 |
| 2,6 | 1,28419 | 2,557 | 1,34046 | 2,700 | 1,40144 | 2,850 | 1,46766 | 3,008 | 1,53957 | 3,172 |
| 2,4 | 1,01004 | 2,011 | 1,05430 | 2,124 | 1,10228 | 2,242 | 1,15436 | 2,366 | 1,21093 | 2,495 |
| 2,2 | 0,77801 | 1,549 | 0,81207 | 1,636 | 0,84932 | 1,727 | 0,88914 | 1,822 | 0,93273 | 1,922 |
| 2,0 | 0,58451 | 1,164 | 0,61012 | 1,229 | 0,63790 | 1,297 | 0,66804 | 1,369 | 0,70078 | 1,444 |

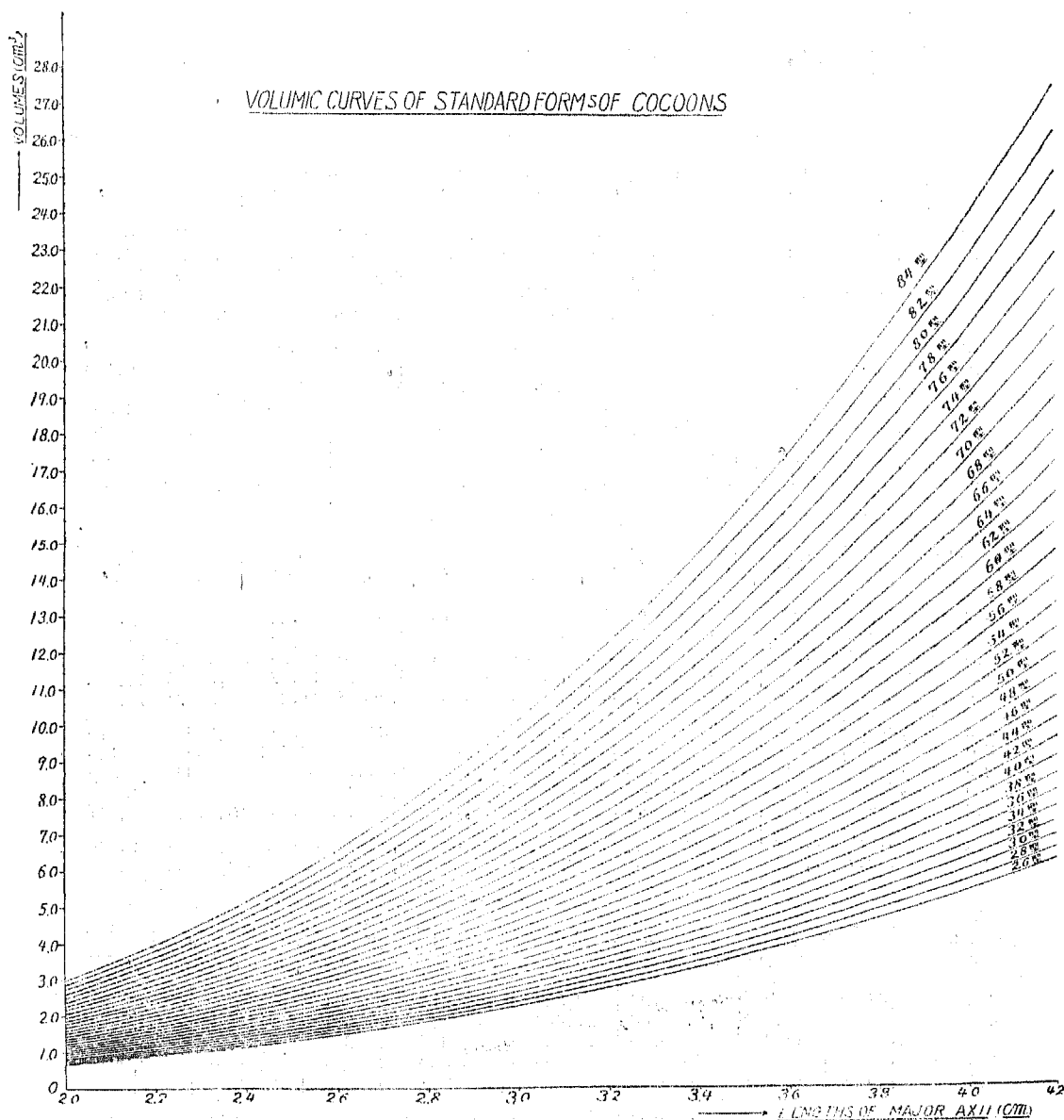
| 長軸の 長さ(2l) | 體 | | | | | | | | | | 積 | | | | | | | | | |
|---------------|---------------------------------|--------|---------|--------|---------------------------------|--------|---------|--------|---------------------------------|--------|---|--|---------------------------------|--|---|--|---------------------------------|--|---|--|
| | 5 6 型 | | | | 5 8 型 | | | | 6 0 型 | | | | 6 2 型 | | | | 6 4 型 | | | |
| | $\frac{\pi}{2} \Lambda=2,06681$ | | | | $\frac{\pi}{2} \Lambda=2,06895$ | | | | $\frac{\pi}{2} \Lambda=2,06593$ | | | | $\frac{\pi}{2} \Lambda=2,05723$ | | | | $\frac{\pi}{2} \Lambda=2,04378$ | | | |
| cm | m ⁴ /a | | V | | m ⁴ /a | | V | | m ⁴ /a | | V | | m ⁴ /a | | V | | m ⁴ /a | | V | |
| 4,2 | 6,81951 | 14,095 | 7,17850 | 14,852 | 7,57014 | 15,659 | 7,99820 | 16,454 | 8,46710 | 17,305 | | | | | | | | | | |
| 4,0 | 5,89124 | 12,176 | 6,20105 | 12,830 | 6,53942 | 13,510 | 6,90908 | 14,214 | 7,31385 | 14,948 | | | | | | | | | | |
| 3,8 | 5,05074 | 10,439 | 5,31661 | 11,000 | 5,60666 | 11,583 | 5,92367 | 12,186 | 6,27100 | 12,817 | | | | | | | | | | |
| 3,6 | 4,29446 | 8,876 | 4,52070 | 9,353 | 4,76718 | 9,849 | 5,03674 | 10,362 | 5,33201 | 10,897 | | | | | | | | | | |
| 3,4 | 3,61776 | 7,477 | 3,80825 | 7,579 | 4,01598 | 8,297 | 4,24305 | 8,729 | 4,49179 | 9,180 | | | | | | | | | | |
| 3,2 | 3,01616 | 6,234 | 3,17494 | 6,569 | 3,34815 | 6,917 | 3,53745 | 7,277 | 3,74487 | 7,654 | | | | | | | | | | |
| 3,0 | 2,48525 | 5,137 | 2,61606 | 5,412 | 2,75880 | 5,699 | 2,91478 | 5,996 | 3,08567 | 6,306 | | | | | | | | | | |
| 2,8 | 2,0206 | 4,176 | 2,12695 | 4,401 | 2,24302 | 4,634 | 2,36981 | 4,875 | 2,50876 | 5,127 | | | | | | | | | | |
| 2,6 | 1,61779 | 3,344 | 1,70298 | 3,523 | 1,79586 | 3,710 | 1,89742 | 3,903 | 2,00863 | 4,105 | | | | | | | | | | |
| 2,4 | 1,27243 | 2,630 | 1,33944 | 2,771 | 1,41247 | 2,918 | 1,49237 | 3,070 | 1,57986 | 3,229 | | | | | | | | | | |
| 2,2 | 0,98012 | 2,026 | 1,03169 | 2,135 | 1,08798 | 2,248 | 1,14950 | 2,365 | 1,21690 | 2,487 | | | | | | | | | | |
| 2,0 | 0,73637 | 1,522 | 0,77513 | 1,604 | 0,81742 | 1,689 | 0,86364 | 1,777 | 0,91427 | 1,869 | | | | | | | | | | |

| 長軸の 長さ(2l) | 體 | | | | | | | | | | 積 | | | | | | | | | |
|---------------|---------------------------------|--------|---------|--------|---------------------------------|--------|---------|--------|---------------------------------|--------|---|--|---------------------------------|--|---|--|---------------------------------|--|---|--|
| | 6 6 型 | | | | 6 8 型 | | | | 7 0 型 | | | | 7 2 型 | | | | 7 4 型 | | | |
| | $\frac{\pi}{2} \Lambda=2,02460$ | | | | $\frac{\pi}{2} \Lambda=2,00054$ | | | | $\frac{\pi}{2} \Lambda=1,97058$ | | | | $\frac{\pi}{2} \Lambda=1,93452$ | | | | $\frac{\pi}{2} \Lambda=1,89245$ | | | |
| cm | m ⁴ /a | | V | | m ⁴ /a | | V | | m ⁴ /a | | V | | m ⁴ /a | | V | | m ⁴ /a | | V | |
| 4,2 | 8,98240 | 18,186 | 9,55040 | 19,106 | 10,17900 | 20,059 | 10,8789 | 21,045 | 11,6593 | 22,065 | | | | | | | | | | |
| 4,0 | 7,75922 | 15,709 | 8,24987 | 16,504 | 8,79334 | 17,328 | 9,39674 | 18,178 | 10,0717 | 19,060 | | | | | | | | | | |
| 3,8 | 6,65262 | 13,469 | 7,07325 | 14,150 | 7,53920 | 14,857 | 8,05651 | 15,586 | 8,63529 | 16,342 | | | | | | | | | | |
| 3,6 | 5,65649 | 11,452 | 6,01422 | 12,032 | 6,41006 | 12,632 | 6,85032 | 13,252 | 7,34230 | 13,895 | | | | | | | | | | |
| 3,4 | 4,76513 | 9,647 | 5,06557 | 10,136 | 5,39998 | 10,641 | 5,77076 | 11,164 | 6,18531 | 11,705 | | | | | | | | | | |
| 3,2 | 3,97277 | 8,043 | 4,22395 | 8,450 | 4,50302 | 8,872 | 4,81110 | 9,307 | 5,15675 | 9,759 | | | | | | | | | | |
| 3,0 | 3,27345 | 6,627 | 3,48045 | 6,963 | 3,70951 | 7,310 | 3,96422 | 7,669 | 4,24903 | 8,041 | | | | | | | | | | |
| 2,8 | 2,66154 | 5,389 | 2,82971 | 5,661 | 3,01598 | 5,943 | 3,22304 | 6,235 | 3,45462 | 6,538 | | | | | | | | | | |
| 2,6 | 2,13087 | 4,314 | 2,26365 | 4,533 | 2,41478 | 4,759 | 2,58060 | 4,992 | 2,76594 | 5,234 | | | | | | | | | | |
| 2,4 | 1,67598 | 3,393 | 1,78198 | 3,565 | 1,89930 | 3,743 | 2,02971 | 3,927 | 2,17549 | 4,117 | | | | | | | | | | |
| 2,2 | 1,29096 | 2,614 | 1,37257 | 2,746 | 1,46292 | 2,883 | 1,56337 | 3,024 | 1,67570 | 3,171 | | | | | | | | | | |
| 2,0 | 0,96992 | 1,964 | 1,03124 | 2,063 | 1,09913 | 2,166 | 1,17453 | 2,272 | 1,25897 | 2,383 | | | | | | | | | | |

| 長軸の 長さ(2l) | 體 | | | | | | | | | | 積 | | | | | | | | | |
|---------------|---------------------------------|--------|---------|--------|---------------------------------|--------|---------|--------|---------------------------------|--------|---|--|---------------------------------|--|---|--|---------------------------------|--|---|--|
| | 7 6 型 | | | | 7 8 型 | | | | 8 0 型 | | | | 8 2 型 | | | | 8 4 型 | | | |
| | $\frac{\pi}{2} \Lambda=1,84398$ | | | | $\frac{\pi}{2} \Lambda=1,76887$ | | | | $\frac{\pi}{2} \Lambda=1,72669$ | | | | $\frac{\pi}{2} \Lambda=1,65416$ | | | | $\frac{\pi}{2} \Lambda=1,57911$ | | | |
| cm | m ⁴ /a | | V | | m ⁴ /a | | V | | m ⁴ /a | | V | | m ⁴ /a | | V | | m ⁴ /a | | V | |
| 4,2 | 12,53850 | 23,121 | 13,5357 | 24,214 | 14,6775 | 25,343 | 16,0001 | 26,467 | 17,5551 | 27,721 | | | | | | | | | | |
| 4,0 | 10,83133 | 19,973 | 11,6927 | 20,917 | 12,6790 | 21,593 | 13,8215 | 22,863 | 15,1646 | 23,947 | | | | | | | | | | |
| 3,8 | 9,28655 | 17,125 | 10,0250 | 17,933 | 10,8707 | 18,770 | 11,8501 | 19,602 | 13,0018 | 20,531 | | | | | | | | | | |
| 3,6 | 7,89630 | 14,560 | 8,52400 | 15,249 | 9,24298 | 15,960 | 10,0759 | 16,657 | 11,0549 | 17,457 | | | | | | | | | | |
| 3,4 | 6,65178 | 12,265 | 7,18078 | 12,845 | 7,78562 | 13,443 | 8,49808 | 14,057 | 9,31297 | 14,706 | | | | | | | | | | |
| 3,2 | 5,54561 | 10,226 | 5,95669 | 10,709 | 6,49161 | 11,209 | 7,07709 | 11,707 | 7,76436 | 12,261 | | | | | | | | | | |
| 3,0 | 4,56947 | 8,426 | 4,93282 | 8,824 | 5,34897 | 9,236 | 5,83094 | 9,645 | 6,39766 | 10,103 | | | | | | | | | | |
| 2,8 | 3,71517 | 6,851 | 4,01057 | 7,174 | 4,34593 | 7,509 | 4,74075 | 7,842 | 5,20149 | 8,214 | | | | | | | | | | |
| 2,6 | 2,97454 | 5,485 | 3,21109 | 5,744 | 3,48196 | 6,012 | 3,79579 | 6,279 | 4,16461 | 6,576 | | | | | | | | | | |
| 2,4 | 2,33956 | 4,314 | 2,52562 | 4,518 | 2,73870 | 4,729 | 2,98548 | 4,938 | 3,27554 | 5,172 | | | | | | | | | | |
| 2,2 | 1,80204 | 3,323 | 1,94537 | 3,480 | 2,10945 | 3,642 | 2,29955 | 3,304 | 2,52304 | 3,984 | | | | | | | | | | |
| 2,0 | 1,35392 | 2,497 | 1,46159 | 2,615 | 1,58487 | 2,737 | 1,72768 | 2,858 | 1,89561 | 2,993 | | | | | | | | | | |

體積の graph は次の如し。

Fig. 3



一般に所謂橢圓形繭は、數學式 $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ にて表はされる橢圓曲線、即ち長球より幾分膨み大であるを謂はれてゐる。以下長球の體積を求めんとする。

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \therefore y^2 = \frac{b^2}{a^2} (a^2 - x^2) \quad \text{但し } 2a \text{ は長軸の長さ、} 2b \text{ は短軸の長さ。}$$

依つて長球の體積

$$V' = 2\pi \int_0^a y^2 dx = 2\pi \times \frac{b^2}{a^2} \int_0^a (a^2 - x^2) dx = 2\pi \cdot \frac{b^2}{a^2} \left[a^2 x - \frac{x^3}{3} \right]_0^a$$

$$= 2\pi \cdot \frac{b^2}{a^2} \left(a^3 - \frac{a^3}{3} \right) = \frac{4}{3} \pi ab^2$$

上述を比較するため、 $\frac{b}{a} \times 100 = N$ を假りに何々型と呼ぶことにする。

Table. V

| 長軸の 長さ(2l) (cm) | 體 積 | | | | | | | | | |
|-----------------------|-----------------|--------|-----------------|--------|-----------------|--------|-----------------|--------|-----------------|--------|
| | 5 8 型 | | 6 0 型 | | 6 2 型 | | 6 4 型 | | 6 6 型 | |
| | ab ² | V' | ab ² | V' | ab ² | V' | ab ² | V' | ab ² | V' |
| 4,2 | 3,11539 | 13,050 | 3,33395 | 13,965 | 3,57088 | 14,958 | 3,79331 | 15,889 | 4,03410 | 16,898 |
| 4,0 | 2,69120 | 11,273 | 2,83300 | 12,054 | 3,07520 | 12,881 | 3,27680 | 13,726 | 3,48480 | 14,597 |
| 3,8 | 2,30736 | 9,665 | 2,45924 | 10,343 | 2,63659 | 11,044 | 2,83945 | 11,768 | 2,98779 | 12,515 |
| 3,6 | 1,96189 | 8,218 | 2,09952 | 8,794 | 2,24133 | 9,391 | 2,38878 | 10,006 | 2,54041 | 10,641 |
| 3,4 | 1,65274 | 6,923 | 1,76863 | 7,409 | 1,83856 | 7,911 | 2,01236 | 8,429 | 2,14010 | 8,964 |
| 3,2 | 1,37789 | 5,772 | 1,47456 | 6,177 | 1,57450 | 6,595 | 1,67773 | 7,028 | 1,78422 | 7,474 |
| 3,0 | 1,13535 | 4,755 | 1,21500 | 5,082 | 1,29735 | 5,434 | 1,38240 | 5,791 | 1,47015 | 6,153 |
| 2,8 | 0,92308 | 3,857 | 0,98734 | 4,138 | 1,05479 | 4,418 | 1,12395 | 4,708 | 1,19529 | 5,007 |
| 2,6 | 0,73908 | 3,096 | 0,79092 | 3,313 | 0,84453 | 3,538 | 0,89989 | 3,769 | 0,95701 | 4,009 |
| 2,4 | 0,58130 | 2,435 | 0,62208 | 2,606 | 0,66425 | 2,782 | 0,70778 | 2,965 | 0,75271 | 3,153 |
| 2,2 | 0,44774 | 1,875 | 0,47916 | 2,007 | 0,51163 | 2,143 | 0,54518 | 2,284 | 0,57979 | 2,429 |
| 2,0 | 0,33640 | 1,439 | 0,36000 | 1,508 | 0,38440 | 1,610 | 0,40960 | 1,716 | 0,43560 | 1,825 |

| 長軸の 長さ(2l) (cm) | 體 積 | | | | | | | | | |
|-----------------------|-----------------|--------|-----------------|--------|-----------------|--------|-----------------|--------|-----------------|--------|
| | 6 8 型 | | 7 0 型 | | 7 2 型 | | 7 4 型 | | 7 6 型 | |
| | ab ² | V' | ab ² | V' | ab ² | V' | ab ² | V' | ab ² | V' |
| 4,2 | 4,28223 | 17,938 | 4,53789 | 19,078 | 4,81089 | 20,110 | 5,07133 | 21,243 | 5,34916 | 22,407 |
| 4,0 | 3,69920 | 15,435 | 3,92000 | 16,420 | 4,14720 | 17,372 | 4,38080 | 18,350 | 4,62080 | 19,356 |
| 3,8 | 3,17159 | 13,285 | 3,35071 | 14,078 | 3,55570 | 14,894 | 3,75600 | 15,733 | 3,95395 | 16,604 |
| 3,6 | 2,69672 | 11,295 | 2,85768 | 11,970 | 3,02332 | 12,664 | 3,19360 | 13,377 | 3,36856 | 14,110 |
| 3,4 | 2,27178 | 9,516 | 2,40737 | 10,034 | 2,54691 | 10,668 | 2,69035 | 11,269 | 2,83774 | 11,887 |
| 3,2 | 1,89398 | 7,933 | 2,00704 | 8,407 | 2,12336 | 8,894 | 2,24298 | 9,395 | 2,35586 | 9,910 |
| 3,0 | 1,56060 | 6,537 | 1,65375 | 6,927 | 1,74760 | 7,329 | 1,84815 | 7,742 | 1,94940 | 8,166 |
| 2,8 | 1,26882 | 5,315 | 1,34456 | 5,632 | 1,42243 | 5,958 | 1,50262 | 6,294 | 1,58494 | 6,639 |
| 2,6 | 1,05590 | 4,255 | 1,07653 | 4,509 | 1,13893 | 4,771 | 1,20307 | 5,039 | 1,26900 | 5,316 |
| 2,4 | 0,79903 | 3,347 | 0,84672 | 3,547 | 0,89568 | 3,752 | 0,94625 | 3,964 | 0,99809 | 4,181 |
| 2,2 | 0,61545 | 2,578 | 0,65219 | 2,732 | 0,68999 | 2,890 | 0,72836 | 3,053 | 0,76879 | 3,220 |
| 2,0 | 0,46240 | 1,937 | 0,49000 | 2,053 | 0,51841 | 2,171 | 0,54760 | 2,294 | 0,57760 | 2,419 |

| 長軸の 長さ(2l) (cm) | 體 積 | | | | | | | | | |
|-----------------------|-----------------|--------|-----------------|--------|-----------------|--------|-----------------|--------|-----------------|--------|
| | 7 8 型 | | 8 0 型 | | 8 2 型 | | 8 4 型 | | 8 6 型 | |
| | ab ² | V' | ab ² | V' | ab ² | V' | ab ² | V' | ab ² | V' |
| 4,2 | 5,63438 | 23,601 | 5,92704 | 24,827 | 6,22709 | 26,084 | 6,53457 | 27,372 | 6,84880 | 28,685 |
| 4,0 | 4,86720 | 20,388 | 5,12000 | 21,447 | 5,37920 | 22,532 | 5,64480 | 23,645 | 5,91920 | 24,783 |
| 3,8 | 4,17301 | 17,480 | 4,38976 | 18,388 | 4,61198 | 19,319 | 4,87972 | 20,273 | 5,15005 | 21,237 |
| 3,6 | 3,54820 | 14,863 | 3,73248 | 15,635 | 3,92144 | 16,426 | 4,11505 | 17,237 | 4,31505 | 18,061 |
| 3,4 | 2,98908 | 12,521 | 3,14432 | 13,171 | 3,30351 | 13,838 | 3,46661 | 14,521 | 3,63661 | 15,261 |
| 3,2 | 2,49200 | 10,438 | 2,62144 | 10,981 | 2,75414 | 11,957 | 2,89014 | 12,106 | 3,03614 | 12,975 |
| 3,0 | 2,05335 | 8,601 | 2,16000 | 9,048 | 2,26935 | 9,506 | 2,38140 | 9,975 | 2,49917 | 10,461 |
| 2,8 | 1,66944 | 6,993 | 1,75616 | 7,356 | 1,84506 | 7,729 | 1,93617 | 8,110 | 2,03117 | 8,603 |
| 2,6 | 1,33666 | 5,599 | 1,40608 | 5,890 | 1,47727 | 6,188 | 1,55020 | 6,493 | 1,62927 | 6,817 |
| 2,4 | 1,05132 | 4,404 | 1,10592 | 4,632 | 1,16196 | 4,867 | 1,21927 | 5,107 | 1,27927 | 5,361 |
| 2,2 | 0,809780 | 3,392 | 0,85184 | 3,563 | 0,89496 | 3,749 | 0,93916 | 3,934 | 0,98416 | 4,134 |
| 2,0 | 0,60840 | 2,548 | 0,64000 | 2,681 | 0,67240 | 2,817 | 0,70560 | 2,956 | 0,73960 | 3,101 |

Oval of CASSINI の體積は長球の體積より大にして、之は Table I に見る如く主に支那種蘭に於て比較さるべきものである。

Fig. 4

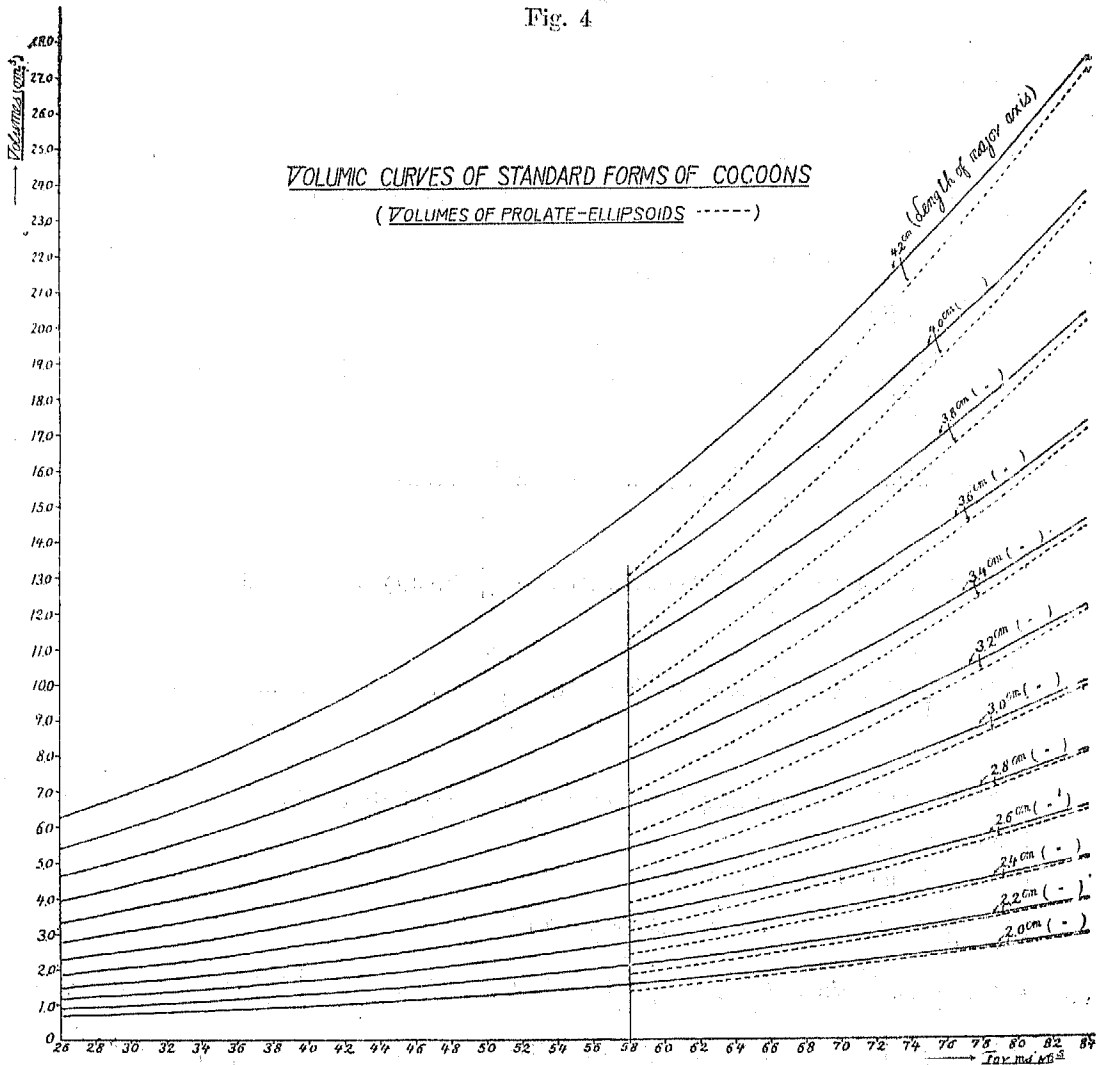


Fig.3. Fig.4. の graph が出来上つた上は、長軸の長さが 2 種と 4.2 種の間の如何なる値に於ても體積を求むることが出来る。

7. Oval of CASSINI に依る繭の表面積計算と長球の表面積との比較

體積に次いで表面積計算を行ふ。

Oval of CASSINI の式 $(x^2 + y^2 + a^2)^2 - 4a^2 x^2 = m^4$

に於て $x = r \cos \theta$, $y = r \sin \theta$ と置くこゝ、

$$(\gamma^2 + a^2)^2 - 4a^2 \gamma^2 \cos^2 \theta = m^4$$

$$\therefore \gamma^4 - 2a^2 \cos 2\theta \cdot \gamma^2 - m^4 + a^4 = 0 \dots\dots(1)$$

$$\therefore \gamma^2 = a^2 \cos 2\theta \pm \sqrt{a^4 \cos^2 2\theta + m^4 - a^4} = a^2 \cos 2\theta \pm \sqrt{m^4 - a^4 \sin^2 2\theta}$$

さて $m \geq a$ として Lemniscate を除き一般に $m > a$ なる關係のみを考へるこゝ複號は正を取り

$$\gamma^2 = a^2 \cos 2\theta + \sqrt{m^4 - a^4 \sin^2 2\theta} \dots\dots(2)$$

(1) を θ に關し微分して

$$4\gamma^3 \frac{d\gamma}{d\theta} - 2a^2 (-\sin 2\theta \cdot 2) \gamma^2 - 2a^2 \cos 2\theta \cdot 2\gamma \cdot \frac{d\gamma}{d\theta} = 0$$

4rで割つて

$$(r^2 - a^2 \cos 2\theta) \frac{dr}{d\theta} = -a^2 \sin 2\theta \cdot r$$

$$\left(\frac{dr}{d\theta}\right)^2 = \frac{a^4 r^2 \sin^2 2\theta}{(r^2 - a^2 \cos 2\theta)^2}$$

$$\begin{aligned} \therefore r^2 + \left(\frac{dr}{d\theta}\right)^2 &= a^2 \cos 2\theta + \sqrt{m^4 - a^4 \sin^2 2\theta} + \frac{a^4 \sin^2 2\theta (a^2 \cos 2\theta + \sqrt{m^4 - a^4 \sin^2 2\theta})}{m^4 - a^4 \sin^2 2\theta} \\ &= \frac{a^2 m^4 \cos 2\theta + m^4 \sqrt{m^4 - a^4 \sin^2 2\theta}}{m^4 - a^4 \sin^2 2\theta} \end{aligned}$$

依つて全表面積

$$\begin{aligned} K &= 2\pi \int_0^\pi r \sin \theta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta = 4\pi \int_0^{\frac{\pi}{2}} \frac{m^2 \sin \theta (a^2 \cos 2\theta + \sqrt{m^4 - a^4 \sin^2 2\theta})}{\sqrt{m^4 - a^4 \sin^2 2\theta}} d\theta \\ &= 4\pi a^2 m^2 \int_0^{\frac{\pi}{2}} \frac{\sin \theta \cos 2\theta}{\sqrt{m^4 - a^4 \sin^2 2\theta}} d\theta + 4\pi m^2 \int_0^{\frac{\pi}{2}} \sin \theta d\theta \\ &= 4\pi m^2 + 4\pi a^2 \int_0^{\frac{\pi}{2}} (\sin \theta - 2 \sin^3 \theta) \left(1 - \frac{a^4}{m^4} \sin^2 2\theta\right)^{-\frac{1}{2}} d\theta \dots \dots (3) \end{aligned}$$

$$\text{さて } I_1 = \int_0^{\frac{\pi}{2}} \sin \theta \left(1 - \frac{a^4}{m^4} \sin^2 2\theta\right)^{-\frac{1}{2}} d\theta \quad I_2 = \int_0^{\frac{\pi}{2}} 2 \sin^3 \theta \left(1 - \frac{a^4}{m^4} \sin^2 2\theta\right)^{-\frac{1}{2}} d\theta$$

を置く

共に第二種の楕圓積分 (Elliptische Integral Zweitung Gattung; Elliptic integral of 2nd Kind) である。

依つて Binominal theorem により

$$\begin{aligned} \left(1 - \frac{a^4}{m^4} \sin^2 2\theta\right)^{-\frac{1}{2}} &= \left(1 - 4 \frac{a^4}{m^4} \sin^2 \theta \cos^2 \theta\right)^{-\frac{1}{2}} \\ &= 1 + \frac{1}{2} \left(4 \frac{a^4}{m^4} \sin^2 \theta \cos^2 \theta\right) + \frac{1 \cdot 3}{2 \cdot 4} \left(4 \frac{a^4}{m^4} \sin^2 \theta \cos^2 \theta\right)^2 + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \left(4 \frac{a^4}{m^4} \sin^2 \theta \cos^2 \theta\right)^3 + \dots \\ &= 1 + 2 \frac{a^4}{m^4} \sin^2 \theta \cos^2 \theta + 6 \frac{a^8}{m^8} \sin^4 \theta \cos^4 \theta + 20 \left(\frac{a}{m}\right)^{12} \sin^6 \theta \cos^6 \theta + 70 \left(\frac{a}{m}\right)^{16} \sin^8 \theta \cos^8 \theta \\ &\quad + 252 \left(\frac{a}{m}\right)^{20} \sin^{10} \theta \cos^{10} \theta + 924 \left(\frac{a}{m}\right)^{24} \sin^{12} \theta \cos^{12} \theta + 3432 \left(\frac{a}{m}\right)^{28} \sin^{14} \theta \cos^{14} \theta \\ &\quad + 12870 \left(\frac{a}{m}\right)^{32} \sin^{16} \theta \cos^{16} \theta + \dots \end{aligned}$$

Reduction formula に依つて次の結果を得る。

$$\begin{aligned} \int_0^{\frac{\pi}{2}} (\sin \theta - 2 \sin^3 \theta) \left(1 - \frac{a^4}{m^4} \sin^2 2\theta\right)^{-\frac{1}{2}} d\theta &= -\left\{ \frac{1}{3} + \frac{4}{105} \frac{a^4}{m^4} + \frac{16}{1155} \frac{a^8}{m^8} + \frac{64}{9009} \frac{a^{12}}{m^{12}} \right. \\ &\quad \left. + \frac{1792}{415701} \frac{a^{16}}{m^{16}} + \frac{3072}{1062347} \frac{a^{20}}{m^{20}} + \frac{45056}{21729825} \frac{a^{24}}{m^{24}} + \frac{2342912}{1502700975} \frac{a^{28}}{m^{28}} + \frac{851968}{701260455} \frac{a^{32}}{m^{32}} + \dots \right\} \\ &= -(0.33333 + 0.03810 \frac{a^4}{m^4} + 0.01385 \frac{a^8}{m^8} + 0.00710 \frac{a^{12}}{m^{12}} + 0.00431 \frac{a^{16}}{m^{16}} \\ &\quad + 0.00289 \frac{a^{20}}{m^{20}} + 0.00207 \frac{a^{24}}{m^{24}} + 0.00156 \frac{a^{28}}{m^{28}} + 0.00121 \frac{a^{32}}{m^{32}} + \dots) \end{aligned}$$

この結果は $\left(\frac{a}{m}\right)^{4n}$ なる形の項のみを有し、定まつた標準型番號に對し一定の値を有する故に

宜上その絶対値をBで表し、之を標準型表面積常數 (Surface area coefficient of Standard form) と呼ぶことを許さるるならば、

$$K = 4\pi m^2 - 4\pi a^2 B$$

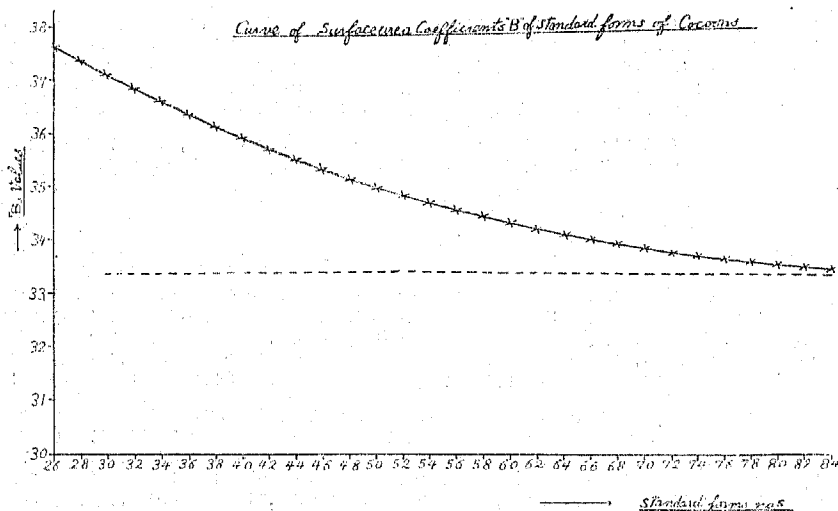
Bの値を求めれば次の如し。

Table VI

| 型番號 | $\left(\frac{n}{m}\right)^2$ | 0,33333 | $\times \left(\frac{n}{m}\right)^1$ | $\times \left(\frac{n}{m}\right)^8$ | $\times \left(\frac{n}{m}\right)^{12}$ | $\left(\frac{n}{m}\right)^{16}$ | $\times \left(\frac{n}{m}\right)^{20}$ | $\times \left(\frac{n}{m}\right)^{24}$ | $\times \left(\frac{n}{m}\right)^{28}$ | $\times \left(\frac{n}{m}\right)^{32}$ | B |
|-----|------------------------------|---------|-------------------------------------|-------------------------------------|--|---------------------------------|--|--|--|--|---------|
| 26 | 0,87337 | 0,33333 | 0,03906 | 0,00806 | 0,00315 | 0,00146 | 0,00075 | 0,00041 | 0,00023 | 0,00014 | 0,57659 |
| 28 | 0,554599 | 0,33333 | 0,02783 | 0,00739 | 0,00277 | 0,00123 | 0,00060 | 0,00031 | 0,00017 | 0,00010 | 0,57373 |
| 30 | 0,33486 | 0,33333 | 0,02659 | 0,00673 | 0,00240 | 0,00102 | 0,00048 | 0,00024 | 0,00012 | 0,00007 | 0,57095 |
| 32 | 0,214300 | 0,33333 | 0,02526 | 0,00609 | 0,00207 | 0,00083 | 0,00037 | 0,00018 | 0,00009 | 0,00005 | 0,56827 |
| 34 | 0,122757 | 0,33333 | 0,02394 | 0,00547 | 0,00176 | 0,00067 | 0,00028 | 0,00013 | 0,00006 | 0,00003 | 0,56567 |
| 36 | 0,077038 | 0,33333 | 0,02262 | 0,00488 | 0,00149 | 0,00054 | 0,00021 | 0,00009 | 0,00004 | 0,00002 | 0,56322 |
| 38 | 0,474640 | 0,33333 | 0,02130 | 0,00433 | 0,00124 | 0,00042 | 0,00016 | 0,00006 | 0,00003 | 0,00001 | 0,56088 |
| 40 | 0,224137 | 0,33333 | 0,01998 | 0,00381 | 0,00102 | 0,00033 | 0,00011 | 0,00004 | 0,00002 | 0,00001 | 0,55864 |
| 42 | 0,120102 | 0,33333 | 0,01867 | 0,00333 | 0,00084 | 0,00025 | 0,00008 | 0,00003 | 0,00001 | 0,00001 | 0,55654 |
| 44 | 0,675603 | 0,33333 | 0,01739 | 0,00289 | 0,00068 | 0,00019 | 0,00006 | 0,00002 | 0,00001 | 0,00001 | 0,55456 |
| 46 | 0,650709 | 0,33333 | 0,01613 | 0,00248 | 0,00054 | 0,00014 | 0,00004 | 0,00001 | 0,00001 | 0,00001 | 0,55267 |
| 48 | 0,625487 | 0,33333 | 0,01491 | 0,00212 | 0,00043 | 0,00010 | 0,00003 | 0,00001 | 0,00001 | 0,00001 | 0,55092 |
| 50 | 0,600000 | 0,33333 | 0,01372 | 0,00179 | 0,00033 | 0,00007 | 0,00002 | 0,00001 | 0,00001 | 0,00001 | 0,54924 |
| 52 | 0,574307 | 0,33333 | 0,01257 | 0,00151 | 0,00025 | 0,00005 | 0,00001 | 0,00001 | 0,00001 | 0,00001 | 0,54771 |
| 54 | 0,548467 | 0,33333 | 0,01146 | 0,00125 | 0,00019 | 0,00004 | 0,00001 | 0,00001 | 0,00001 | 0,00001 | 0,54527 |
| 56 | 0,522533 | 0,33333 | 0,01040 | 0,00103 | 0,00014 | 0,00002 | 0,00001 | 0,00001 | 0,00001 | 0,00001 | 0,54292 |
| 58 | 0,496558 | 0,33333 | 0,00939 | 0,00084 | 0,00011 | 0,00002 | 0,00001 | 0,00001 | 0,00001 | 0,00001 | 0,54069 |
| 60 | 0,470589 | 0,33333 | 0,00841 | 0,00068 | 0,00008 | 0,00001 | 0,00001 | 0,00001 | 0,00001 | 0,00001 | 0,53853 |
| 62 | 0,444669 | 0,33333 | 0,00753 | 0,00054 | 0,00005 | 0,00001 | 0,00001 | 0,00001 | 0,00001 | 0,00001 | 0,53645 |
| 64 | 0,418842 | 0,33333 | 0,00668 | 0,00043 | 0,00004 | 0,00001 | 0,00001 | 0,00001 | 0,00001 | 0,00001 | 0,53448 |
| 66 | 0,393145 | 0,33333 | 0,00589 | 0,00033 | 0,00003 | 0,00001 | 0,00001 | 0,00001 | 0,00001 | 0,00001 | 0,53255 |
| 68 | 0,367615 | 0,33333 | 0,00515 | 0,00025 | 0,00002 | 0,00001 | 0,00001 | 0,00001 | 0,00001 | 0,00001 | 0,53073 |
| 70 | 0,342282 | 0,33333 | 0,00446 | 0,00019 | 0,00001 | 0,00001 | 0,00001 | 0,00001 | 0,00001 | 0,00001 | 0,52898 |
| 72 | 0,317175 | 0,33333 | 0,00383 | 0,00014 | 0,00001 | 0,00001 | 0,00001 | 0,00001 | 0,00001 | 0,00001 | 0,52730 |
| 74 | 0,292323 | 0,33333 | 0,00326 | 0,00010 | 0,00001 | 0,00001 | 0,00001 | 0,00001 | 0,00001 | 0,00001 | 0,52569 |
| 76 | 0,267748 | 0,33333 | 0,00273 | 0,00007 | 0,00001 | 0,00001 | 0,00001 | 0,00001 | 0,00001 | 0,00001 | 0,52413 |
| 78 | 0,24347 | 0,33333 | 0,00222 | 0,00005 | 0,00001 | 0,00001 | 0,00001 | 0,00001 | 0,00001 | 0,00001 | 0,52264 |
| 80 | 0,21951 | 0,33333 | 0,00184 | 0,00003 | 0,00001 | 0,00001 | 0,00001 | 0,00001 | 0,00001 | 0,00001 | 0,52120 |
| 82 | 0,19589 | 0,33333 | 0,00146 | 0,00002 | 0,00001 | 0,00001 | 0,00001 | 0,00001 | 0,00001 | 0,00001 | 0,51981 |
| 84 | 0,17261 | 0,33333 | 0,00114 | 0,00001 | 0,00001 | 0,00001 | 0,00001 | 0,00001 | 0,00001 | 0,00001 | 0,51847 |

Bの graph は Fig. 5 の如し。

Fig. 5



この結果を用ひて表面積を計算すれば次の如し。

Table VII 表面積 $K = 4\pi (m^2 - a^2 B)$

| 長軸の 長さ(2l) (cm) | 2 6 型 | | | 2 8 型 | | | 3 0 型 | | | 3 2 型 | | |
|-----------------------|----------------|----------------|--------|----------------|----------------|--------|----------------|----------------|--------|----------------|----------------|--------|
| | B=0,37659 | | | B=0,37373 | | | B=0,37095 | | | B=0,36827 | | |
| | m ² | a ² | k | m ² | a ² | k | m ² | a ² | k | m ² | a ² | k |
| 4,2 | 2,35406 | 2,05594 | 19,853 | 2,37787 | 2,03213 | 20,337 | 2,40045 | 2,06555 | 20,849 | 2,43079 | 1,97921 | 21,387 |
| 4,0 | 2,13570 | 1,86430 | 18,007 | 2,15610 | 1,84320 | 18,447 | 2,18000 | 1,82000 | 18,911 | 2,20480 | 1,79520 | 19,399 |
| 3,8 | 1,92702 | 1,68298 | 16,251 | 1,94651 | 1,66349 | 16,648 | 1,96745 | 1,64255 | 17,067 | 1,98983 | 1,62017 | 17,507 |
| 3,6 | 1,72951 | 1,51049 | 14,585 | 1,74701 | 1,49299 | 14,942 | 1,76580 | 1,47420 | 15,318 | 1,78589 | 1,45411 | 15,713 |
| 3,4 | 1,54268 | 1,34732 | 13,010 | 1,55829 | 1,33171 | 13,328 | 1,57305 | 1,31495 | 13,663 | 1,59297 | 1,29703 | 14,015 |
| 3,2 | 1,36653 | 1,19347 | 11,524 | 1,38035 | 1,17965 | 11,806 | 1,39520 | 1,16480 | 12,103 | 1,41107 | 1,14893 | 12,415 |
| 3,0 | 1,20102 | 1,04895 | 10,129 | 1,21320 | 1,03680 | 10,376 | 1,22625 | 1,02375 | 10,637 | 1,24020 | 1,00980 | 10,912 |
| 2,8 | 1,04625 | 0,91375 | 8,823 | 1,05683 | 0,90317 | 9,039 | 1,06820 | 0,89180 | 9,266 | 1,08035 | 0,87965 | 9,505 |
| 2,6 | 0,90212 | 0,78788 | 7,608 | 0,91125 | 0,77875 | 7,794 | 0,92105 | 0,76895 | 7,990 | 0,93153 | 0,75847 | 8,196 |
| 2,4 | 0,76867 | 0,67133 | 6,482 | 0,77645 | 0,66355 | 6,641 | 0,78480 | 0,65520 | 6,808 | 0,79373 | 0,64627 | 6,984 |
| 2,2 | 0,64550 | 0,56410 | 5,447 | 0,65243 | 0,55757 | 5,580 | 0,65945 | 0,55055 | 5,720 | 0,66695 | 0,54305 | 5,868 |
| 2,0 | 0,53380 | 0,46620 | 4,502 | 0,53920 | 0,46080 | 4,612 | 0,54500 | 0,45500 | 4,728 | 0,55120 | 0,44880 | 4,850 |

| 長軸の 長さ(2l) (cm) | 3 4 型 | | | 3 6 型 | | | 3 8 型 | | | 4 0 型 | | |
|-----------------------|----------------|----------------|--------|----------------|----------------|--------|----------------|----------------|--------|----------------|----------------|--------|
| | B=0,36567 | | | B=0,36322 | | | B=0,36088 | | | B=0,35864 | | |
| | m ² | a ² | k | m ² | a ² | k | m ² | a ² | k | m ² | a ² | k |
| 4,2 | 2,45990 | 1,95010 | 21,951 | 2,49077 | 1,91923 | 22,540 | 2,52340 | 1,88660 | 23,154 | 2,55780 | 1,85220 | 23,795 |
| 4,0 | 2,23120 | 1,76880 | 19,910 | 2,25920 | 1,74080 | 20,444 | 2,28860 | 1,71120 | 21,002 | 2,32000 | 1,68000 | 21,583 |
| 3,8 | 2,01366 | 1,59634 | 17,969 | 2,03893 | 1,57107 | 18,451 | 2,06564 | 1,54466 | 18,954 | 2,09380 | 1,51620 | 19,478 |
| 3,6 | 1,80727 | 1,43273 | 16,127 | 1,82995 | 1,41005 | 16,560 | 1,85393 | 1,38607 | 17,012 | 1,87920 | 1,36060 | 17,482 |
| 3,4 | 1,61204 | 1,27796 | 14,385 | 1,63227 | 1,25773 | 14,771 | 1,65366 | 1,23634 | 15,174 | 1,67620 | 1,21380 | 15,593 |
| 3,2 | 1,42797 | 1,13203 | 12,743 | 1,44589 | 1,11411 | 13,084 | 1,46483 | 1,09517 | 13,441 | 1,48480 | 1,07520 | 13,813 |
| 3,0 | 1,25505 | 0,99495 | 11,200 | 1,27080 | 0,97920 | 11,500 | 1,28475 | 0,96255 | 11,813 | 1,30000 | 0,94500 | 12,140 |
| 2,8 | 1,09329 | 0,86671 | 9,756 | 1,10701 | 0,85299 | 10,018 | 1,12151 | 0,83849 | 10,291 | 1,13680 | 0,82320 | 10,576 |
| 2,6 | 0,94268 | 0,74732 | 8,412 | 0,95451 | 0,73549 | 8,638 | 0,96702 | 0,72258 | 8,873 | 0,98020 | 0,70980 | 9,119 |
| 2,4 | 0,80323 | 0,63677 | 7,168 | 0,81331 | 0,62669 | 7,360 | 0,82397 | 0,61603 | 7,561 | 0,83520 | 0,60480 | 7,770 |
| 2,2 | 0,67494 | 0,53506 | 6,023 | 0,68341 | 0,52659 | 6,184 | 0,69236 | 0,51764 | 6,353 | 0,70180 | 0,50820 | 6,529 |
| 2,0 | 0,55780 | 0,44220 | 4,978 | 0,56480 | 0,43520 | 5,111 | 0,57220 | 0,42780 | 5,250 | 0,58000 | 0,42000 | 5,396 |

| 長軸の 長さ(2l) (cm) | 4 2 型 | | | 4 4 型 | | | 4 6 型 | | | 4 8 型 | | |
|-----------------------|----------------|----------------|--------|----------------|----------------|--------|----------------|----------------|--------|----------------|----------------|--------|
| | B=0,35654 | | | B=0,35456 | | | B=0,35267 | | | B=0,35092 | | |
| | m ² | a ² | k | m ² | a ² | k | m ² | a ² | k | m ² | a ² | k |
| 4,2 | 2,59396 | 1,81604 | 24,460 | 2,63189 | 1,77811 | 25,151 | 2,67158 | 1,73842 | 25,868 | 2,71303 | 1,69697 | 26,610 |
| 4,0 | 2,35280 | 1,64720 | 22,186 | 2,38720 | 1,61280 | 22,813 | 2,42320 | 1,57680 | 23,463 | 2,46080 | 1,53920 | 24,136 |
| 3,8 | 2,12340 | 1,48660 | 20,023 | 2,15445 | 1,45555 | 20,588 | 2,18694 | 1,42306 | 21,175 | 2,22087 | 1,38913 | 21,783 |
| 3,6 | 1,90577 | 1,33423 | 17,971 | 1,93363 | 1,30637 | 18,478 | 1,96279 | 1,27721 | 19,005 | 1,99325 | 1,24675 | 19,550 |
| 3,4 | 1,69990 | 1,19010 | 16,029 | 1,72475 | 1,16525 | 16,482 | 1,75076 | 1,13924 | 16,952 | 1,77793 | 1,11207 | 17,438 |
| 3,2 | 1,50579 | 1,05421 | 14,199 | 1,52781 | 1,03219 | 14,600 | 1,55085 | 1,00915 | 15,016 | 1,57491 | 0,98509 | 15,447 |
| 3,0 | 1,32345 | 0,92655 | 12,480 | 1,34280 | 0,92720 | 12,832 | 1,36305 | 0,88695 | 13,198 | 1,38420 | 0,86580 | 13,576 |
| 2,8 | 1,15287 | 0,80713 | 10,871 | 1,16973 | 0,79027 | 11,171 | 1,18737 | 0,77263 | 11,497 | 1,20579 | 0,75421 | 11,828 |
| 2,6 | 0,99406 | 0,69594 | 9,374 | 1,00859 | 0,68141 | 9,638 | 1,02380 | 0,66620 | 9,913 | 1,03969 | 0,65031 | 10,197 |
| 2,4 | 0,84701 | 0,59299 | 7,987 | 0,85939 | 0,58061 | 8,213 | 0,87235 | 0,56765 | 8,447 | 0,88589 | 0,55411 | 8,689 |
| 2,2 | 0,71172 | 0,49828 | 6,711 | 0,72213 | 0,48787 | 6,901 | 0,73302 | 0,47698 | 7,098 | 0,74439 | 0,46561 | 7,301 |
| 2,0 | 0,58820 | 0,41180 | 5,547 | 0,59680 | 0,40320 | 5,703 | 0,60580 | 0,39420 | 5,866 | 0,61520 | 0,38480 | 6,034 |

| 長軸の 長さ(2l) (cm) | 5 0 型 | | | 5 2 型 | | | 5 4 型 | | | 5 6 型 | | |
|-----------------------|----------------|----------------|--------|----------------|----------------|--------|----------------|----------------|--------|----------------|----------------|--------|
| | B=0,34924 | | | B=0,34771 | | | B=0,34627 | | | B=0,34492 | | |
| | m ² | a ² | k | m ² | a ² | k | m ² | a ² | k | m ² | a ² | k |
| 4,2 | 2,75625 | 1,65375 | 27,378 | 2,80123 | 1,60877 | 28,172 | 2,84758 | 1,56202 | 28,992 | 2,89649 | 1,51351 | 29,838 |
| 4,0 | 2,50000 | 1,50000 | 24,833 | 2,54080 | 1,45920 | 25,553 | 2,58320 | 1,41680 | 26,296 | 2,62720 | 1,37280 | 27,064 |
| 3,8 | 2,25625 | 1,35375 | 22,412 | 2,29707 | 1,31693 | 23,061 | 2,33124 | 1,27666 | 23,733 | 2,37105 | 1,23895 | 24,425 |
| 3,6 | 2,03500 | 1,21500 | 20,115 | 2,05805 | 1,18195 | 20,698 | 2,09239 | 1,14761 | 21,300 | 2,12803 | 1,11197 | |
| 3,4 | 1,80625 | 1,08375 | 17,942 | 1,83573 | 1,05427 | 18,462 | 1,86636 | 1,02364 | 18,999 | 1,89815 | 0,99185 | 19,554 |
| 3,2 | 1,60000 | 0,96000 | 15,693 | 1,62611 | 0,93389 | 16,354 | 1,65325 | 0,90675 | 16,830 | 1,68141 | 0,87859 | 17,391 |
| 3,0 | 1,40625 | 0,84375 | 13,969 | 1,42920 | 0,82080 | 14,373 | 1,45305 | 0,79695 | 14,792 | 1,47780 | 0,77220 | 15,224 |
| 2,8 | 1,22500 | 0,73500 | 12,168 | 1,24499 | 0,71501 | 12,521 | 1,26577 | 0,69423 | 12,885 | 1,28733 | 0,67267 | 13,261 |
| 2,6 | 1,05625 | 0,63375 | 10,492 | 1,07349 | 0,61651 | 10,796 | 1,09140 | 0,59860 | 11,100 | 1,10999 | 0,58001 | 11,435 |
| 2,4 | 0,90000 | 0,54000 | 8,940 | 0,91459 | 0,52531 | 9,199 | 0,92995 | 0,51005 | 9,467 | 0,94579 | 0,49421 | 9,743 |
| 2,2 | 0,75625 | 0,45375 | 7,512 | 0,76859 | 0,44141 | 7,730 | 0,78142 | 0,42858 | 7,955 | 0,79473 | 0,41527 | 8,187 |
| 2,0 | 0,62500 | 0,37500 | 6,208 | 0,63520 | 0,36480 | 6,388 | 0,64580 | 0,35420 | 6,574 | 0,65680 | 0,34320 | 6,766 |

| 長軸の長さ(2l) (cm) | 5 8 型 | | | 6 0 型 | | | 6 2 型 | | | 6 4 型 | | |
|-------------------|----------------|----------------|--------|----------------|----------------|--------|----------------|----------------|--------|----------------|----------------|--------|
| | B=0,34369 | | | B=0,34253 | | | B=0,34145 | | | B=0,34048 | | |
| | m ² | a ² | k | m ² | a ² | k | m ² | a ² | k | m ² | a ² | k |
| 4,2 | 2,94676 | 1,46324 | 30,711 | 2,99880 | 1,41120 | 31,610 | 3,05260 | 1,35740 | 32,536 | 3,10817 | 1,30183 | 33,488 |
| 4,0 | 2,67280 | 1,32720 | 27,855 | 2,72000 | 1,28000 | 28,671 | 2,76880 | 1,23120 | 29,511 | 2,81920 | 1,18080 | 30,375 |
| 3,8 | 2,41220 | 1,19780 | 25,139 | 2,45480 | 1,15520 | 25,876 | 2,49884 | 1,11162 | 26,634 | 2,54433 | 1,06567 | 27,413 |
| 3,6 | 2,16495 | 1,07503 | 22,563 | 2,20320 | 1,03660 | 23,223 | 2,24273 | 0,99729 | 23,904 | 2,28355 | 0,95645 | 24,604 |
| 3,4 | 1,93110 | 0,95890 | 20,126 | 1,96520 | 0,92480 | 20,715 | 2,00046 | 0,88954 | 21,322 | 2,03687 | 0,85313 | 21,946 |
| 3,2 | 1,71059 | 0,84941 | 17,827 | 1,74060 | 0,81920 | 18,349 | 1,77203 | 0,78797 | 18,887 | 1,80429 | 0,75571 | 19,440 |
| 3,0 | 1,50345 | 0,74655 | 15,669 | 1,53000 | 0,72000 | 16,127 | 1,55745 | 0,69255 | 16,600 | 1,58580 | 0,66420 | 17,066 |
| 2,8 | 1,30967 | 0,65033 | 13,649 | 1,33280 | 0,62720 | 14,049 | 1,35671 | 0,60324 | 14,460 | 1,38141 | 0,57859 | 14,884 |
| 2,6 | 1,12926 | 0,56074 | 11,769 | 1,14920 | 0,54080 | 12,114 | 1,16932 | 0,52018 | 12,468 | 1,19111 | 0,49489 | 12,833 |
| 2,4 | 0,96221 | 0,47779 | 10,028 | 0,97920 | 0,46080 | 10,322 | 0,99677 | 0,44323 | 10,624 | 1,01491 | 0,42052 | 10,935 |
| 2,2 | 0,80852 | 0,40148 | 8,426 | 0,82280 | 0,38720 | 8,673 | 0,83756 | 0,37244 | 8,927 | 0,85281 | 0,35719 | 9,188 |
| 2,0 | 0,66820 | 0,33180 | 6,964 | 0,68000 | 0,32000 | 7,169 | 0,69220 | 0,30700 | 7,378 | 0,70480 | 0,29520 | 7,594 |

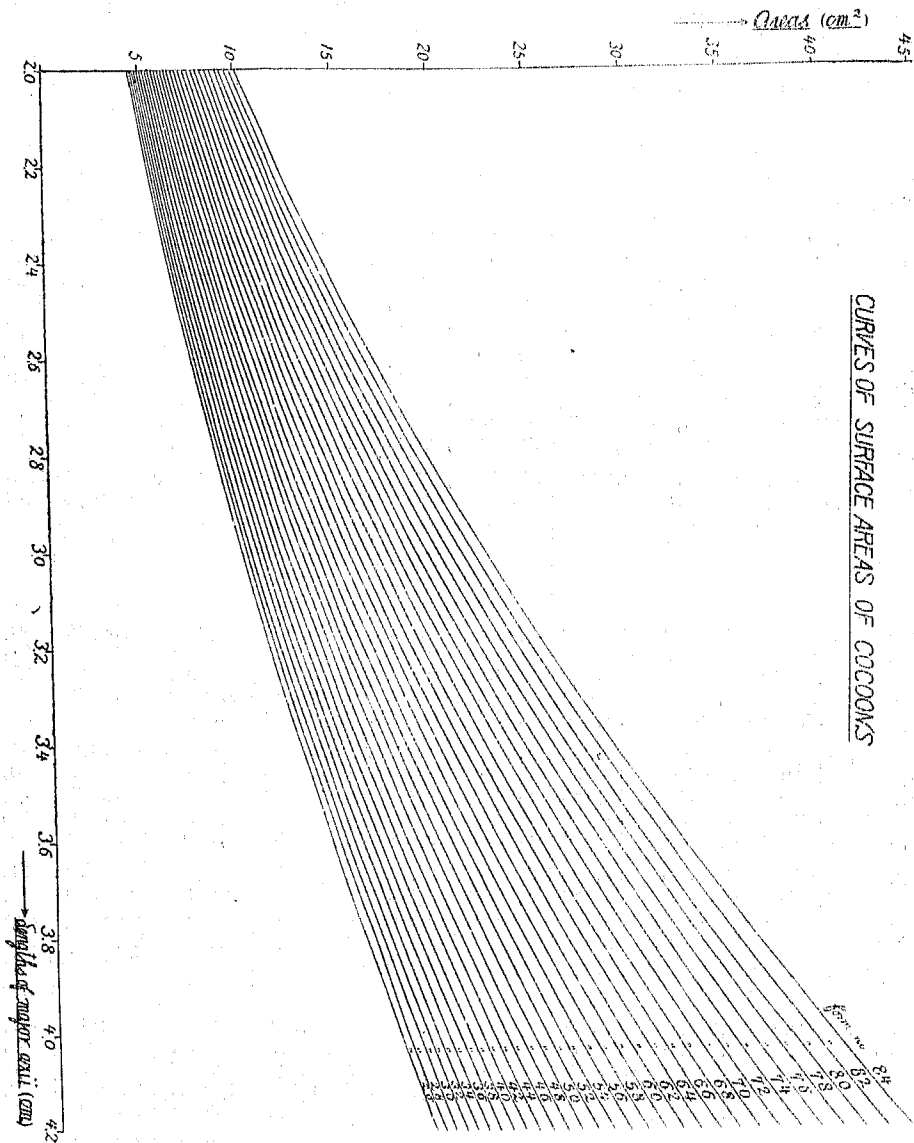
| 長軸の長さ(2l) (cm) | 6 6 型 | | | 6 8 型 | | | 7 0 型 | | | 7 2 型 | | |
|-------------------|----------------|----------------|--------|----------------|----------------|--------|----------------|----------------|--------|----------------|----------------|--------|
| | B=0,33955 | | | B=0,33873 | | | B=0,33798 | | | B=0,33730 | | |
| | m ² | a ² | k | m ² | a ² | k | m ² | a ² | k | m ² | a ² | k |
| 4,2 | 3,16550 | 1,24450 | 34,469 | 3,22459 | 1,18541 | 35,476 | 3,28545 | 1,12455 | 36,510 | 3,34807 | 1,06193 | 37,572 |
| 4,0 | 2,87120 | 1,12880 | 31,264 | 2,92480 | 1,07520 | 32,178 | 2,98000 | 1,02000 | 33,116 | 3,03660 | 0,96320 | 34,079 |
| 3,8 | 2,59126 | 1,01874 | 28,216 | 2,63963 | 0,97037 | 29,040 | 2,68945 | 0,92055 | 29,887 | 2,74071 | 0,86929 | 30,756 |
| 3,6 | 2,32567 | 0,91433 | 25,324 | 2,36909 | 0,87091 | 26,064 | 2,41380 | 0,82620 | 26,824 | 2,45981 | 0,78019 | 27,604 |
| 3,4 | 2,07444 | 0,81556 | 22,588 | 2,11317 | 0,77683 | 23,248 | 2,15305 | 0,73695 | 23,926 | 2,19409 | 0,69591 | 24,622 |
| 3,2 | 1,83757 | 0,72243 | 20,009 | 1,87187 | 0,68813 | 20,594 | 1,90720 | 0,65280 | 21,194 | 1,94355 | 0,61645 | 21,810 |
| 3,0 | 1,61505 | 0,63493 | 17,586 | 1,64520 | 0,60480 | 18,100 | 1,67625 | 0,57375 | 18,628 | 1,70820 | 0,54180 | 19,169 |
| 2,8 | 1,40689 | 0,55311 | 15,319 | 1,43315 | 0,52685 | 15,767 | 1,46020 | 0,49980 | 16,227 | 1,48803 | 0,47197 | 16,699 |
| 2,6 | 1,21308 | 0,47692 | 13,209 | 1,23573 | 0,45427 | 13,595 | 1,25905 | 0,43095 | 13,991 | 1,28305 | 0,40695 | 14,398 |
| 2,4 | 1,03368 | 0,40637 | 11,255 | 1,05293 | 0,38707 | 11,584 | 1,07260 | 0,36720 | 11,922 | 1,09325 | 0,34675 | 12,268 |
| 2,2 | 0,86854 | 0,34146 | 9,457 | 0,88475 | 0,32525 | 9,737 | 0,90145 | 0,30855 | 10,018 | 0,91863 | 0,29137 | 10,309 |
| 2,0 | 0,71780 | 0,28220 | 7,816 | 0,73120 | 0,26880 | 8,044 | 0,74500 | 0,25500 | 8,279 | 0,75920 | 0,24080 | 8,520 |

| 長軸の長さ(2l) (cm) | 7 4 型 | | | 7 6 型 | | | 7 8 型 | | |
|-------------------|----------------|----------------|--------|----------------|----------------|--------|----------------|----------------|--------|
| | B=0,33669 | | | B=0,33613 | | | B=0,33564 | | |
| | m ² | a ² | k | m ² | a ² | k | m ² | a ² | k |
| 4,2 | 3,41246 | 0,99754 | 38,662 | 3,47861 | 0,93139 | 39,779 | 3,54652 | 0,86348 | 40,925 |
| 4,0 | 3,09520 | 0,90480 | 35,067 | 3,15520 | 0,84480 | 36,081 | 3,21680 | 0,78320 | 37,120 |
| 3,8 | 2,79342 | 0,81658 | 31,648 | 2,84737 | 0,76243 | 32,563 | 2,90316 | 0,70684 | 33,501 |
| 3,6 | 2,50711 | 0,73289 | 28,404 | 2,55571 | 0,68429 | 29,226 | 2,60561 | 0,64839 | 30,067 |
| 3,4 | 2,23628 | 0,65372 | 25,336 | 2,27963 | 0,61037 | 26,069 | 2,32414 | 0,56566 | 26,819 |
| 3,2 | 1,98093 | 0,57907 | 22,443 | 2,01933 | 0,54067 | 23,092 | 2,05875 | 0,50125 | 23,757 |
| 3,0 | 1,74105 | 0,50895 | 19,725 | 1,77480 | 0,47520 | 20,296 | 1,80945 | 0,44055 | 20,880 |
| 2,8 | 1,51665 | 0,44335 | 17,183 | 1,54605 | 0,41395 | 17,680 | 1,57623 | 0,38377 | 18,189 |
| 2,6 | 1,30772 | 0,38228 | 14,816 | 1,33307 | 0,35693 | 15,244 | 1,35910 | 0,33090 | 15,683 |
| 2,4 | 1,11427 | 0,32573 | 12,624 | 1,13587 | 0,30413 | 12,989 | 1,15605 | 0,28195 | 13,363 |
| 2,2 | 0,93630 | 0,27370 | 10,608 | 0,95445 | 0,25555 | 10,915 | 0,97308 | 0,23692 | 11,229 |
| 2,0 | 0,77380 | 0,22920 | 8,767 | 0,78880 | 0,21120 | 9,020 | 0,80420 | 0,19580 | 9,280 |

| 長軸の長さ(2l) (cm) | 8 0 型 | | | 8 2 型 | | | 8 4 型 | | |
|-------------------|----------------|----------------|--------|----------------|----------------|--------|----------------|----------------|--------|
| | B=0,33520 | | | B=0,33481 | | | B=0,33448 | | |
| | m ² | a ² | k | m ² | a ² | k | m ² | a ² | k |
| 4,2 | 3,61620 | 0,79380 | 42,099 | 3,68764 | 0,72236 | 43,301 | 3,76085 | 0,64915 | 44,532 |
| 4,0 | 3,28000 | 0,72000 | 38,185 | 3,34480 | 0,65520 | 39,275 | 3,41120 | 0,58880 | 40,392 |
| 3,8 | 2,96020 | 0,64980 | 34,462 | 3,01868 | 0,59132 | 35,445 | 3,07861 | 0,53139 | 36,453 |
| 3,6 | 2,65680 | 0,58320 | 30,930 | 2,70929 | 0,53071 | 31,813 | 2,76307 | 0,47693 | 32,717 |
| 3,4 | 2,36980 | 0,52020 | 27,589 | 2,4166 | 0,47228 | 28,381 | 2,46459 | 0,42541 | 29,183 |
| 3,2 | 2,09920 | 0,46080 | 24,438 | 2,14067 | 0,41933 | 25,136 | 2,18317 | 0,37653 | 25,851 |
| 3,0 | 1,84500 | 0,40500 | 21,479 | 1,88145 | 0,36855 | 22,092 | 1,91880 | 0,33120 | 22,720 |
| 2,8 | 1,60720 | 0,35280 | 18,711 | 1,63895 | 0,32105 | 19,245 | 1,67149 | 0,28851 | 19,792 |
| 2,6 | 1,38580 | 0,30420 | 16,133 | 1,41318 | 0,27682 | 16,594 | 1,44123 | 0,24877 | 17,065 |
| 2,4 | 1,18080 | 0,25920 | 13,747 | 1,20413 | 0,23587 | 14,139 | 1,22803 | 0,21197 | 14,541 |
| 2,2 | 0,99220 | 0,21780 | 11,551 | 1,01180 | 0,19820 | 11,881 | 1,03189 | 0,17611 | 12,219 |
| 2,0 | 0,82000 | 0,18000 | 9,546 | 0,83640 | 0,16380 | 9,819 | 0,85280 | 0,14720 | 10,098 |

表面積を graph にて表せば次の如し。

Fig. 6



尙之を長球の表面積と比較せんとする。

楕圓の方程式

$$\begin{cases} x = a \cos \theta \\ y = b \sin \theta \end{cases} \quad \text{但し } 0 \leq \theta \leq \pi, a, b \text{ は 次々長、短軸の長さの半分。}$$

$$dx = -a \sin \theta d\theta \quad dy = b \cos \theta d\theta$$

$$\sqrt{dx^2 + dy^2} = \sqrt{a^2 \sin^2 \theta + b^2 \cos^2 \theta} d\theta = a \sqrt{1 - \left(1 - \frac{b^2}{a^2}\right) \cos^2 \theta} d\theta.$$

依つて表面積を K' とすれば

$$K' = 2 \pi \int_0^\pi b \sin \theta \left(a \sqrt{1 - \left(1 - \frac{b^2}{a^2}\right) \cos^2 \theta} \right) d\theta$$

$\cos \theta = t$ とおく

$$\begin{aligned} K' &= -2 \pi ab \int_1^{-1} \sqrt{1 - \left(1 - \frac{b^2}{a^2}\right) t^2} dt = 4 \pi ab \sqrt{1 - \frac{b^2}{a^2}} \int_0^1 \sqrt{1 - \frac{t^2}{\frac{b^2}{a^2}}} dt \\ &= 4 \pi ab \cdot \frac{1}{2} \cdot \frac{1}{1 - \frac{b^2}{a^2}} \left(\sqrt{1 - \frac{b^2}{a^2}} \sqrt{\frac{b^2}{a^2}} + \sin^{-1} \sqrt{1 - \frac{b^2}{a^2}} \right) = 2 \pi a^2 \left\{ \frac{b^2}{a^2} + \frac{b}{a} \frac{\sin^{-1} \sqrt{1 - \frac{b^2}{a^2}}}{\sqrt{1 - \frac{b^2}{a^2}}} \right\} \end{aligned}$$

この括弧内は標準型番號が定まれば一定の値を取る。之を B' にて表せば、

$$K' = 2 \pi a^2 B'$$

B' の値を求むれば次の如し。

Table VIII

| forms' No. ⁸ | $\frac{b^2}{a^2}$ | $\sqrt{1 - \frac{b^2}{a^2}}$ | $\frac{\sin^{-1} \sqrt{1 - \frac{b^2}{a^2}}}{\sqrt{1 - \frac{b^2}{a^2}}}$ | B' | forms' No. ⁸ | $\frac{b^2}{a^2}$ | $\sqrt{1 - \frac{b^2}{a^2}}$ | $\frac{\sin^{-1} \sqrt{1 - \frac{b^2}{a^2}}}{\sqrt{1 - \frac{b^2}{a^2}}}$ | B' |
|-------------------------|-------------------|------------------------------|---|---------|-------------------------|-------------------|------------------------------|---|---------|
| 58 | 0,33640 | 0,81462 | 0,95208 | 1,01427 | 72 | 0,51840 | 0,69397 | 0,76707 | 1,31424 |
| 60 | 0,36000 | 0,80000 | 0,92735 | 1,05551 | 74 | 0,54760 | 0,67261 | 0,73769 | 1,35920 |
| 62 | 0,38440 | 0,78460 | 0,90205 | 1,09722 | 76 | 0,57760 | 0,64992 | 0,70744 | 1,40486 |
| 64 | 0,40960 | 0,76837 | 0,87273 | 1,13652 | 78 | 0,60840 | 0,62577 | 0,67603 | 1,45104 |
| 66 | 0,43560 | 0,75126 | 0,84998 | 1,18232 | 80 | 0,64000 | 0,60000 | 0,64345 | 1,49790 |
| 68 | 0,46240 | 0,73321 | 0,82321 | 1,22587 | 82 | 0,67240 | 0,57236 | 0,60941 | 1,54548 |
| 70 | 0,49000 | 0,71414 | 0,79479 | 1,26905 | 84 | 0,70560 | 0,54258 | 0,57363 | 1,59367 |

長球の表面積は次の如し。

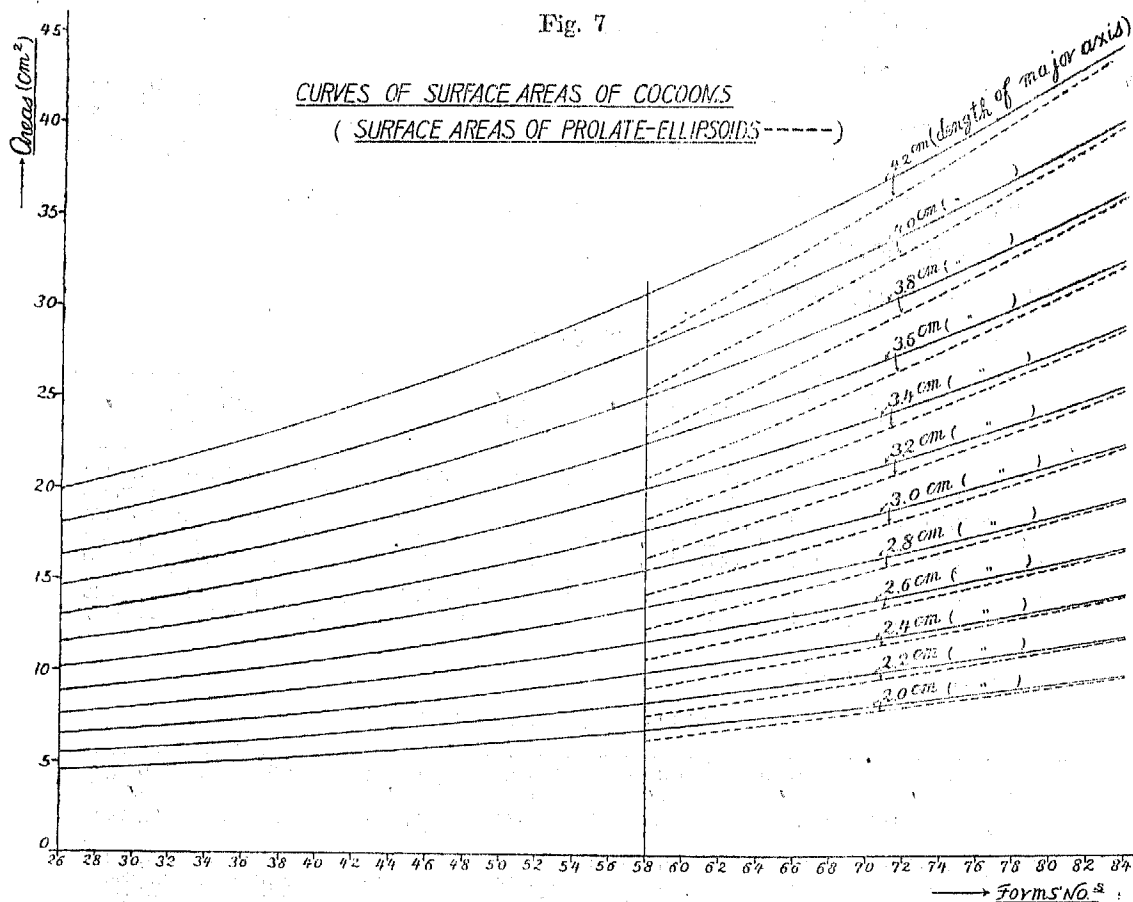
Table IX $K' = 2 \pi a^2 B'$

| 長軸の長さ(2a) (cm) | 58型 B' =1,01427 | 60型 B' =1,05551 | 62型 B' =1,09722 | 64型 B' =1,13652 | 66型 B' =1,18232 | 68型 B' =1,22587 | 70型 B' =1,26905 |
|-------------------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|
| 4,2 | 23,104 | 29,247 | 30,403 | 31,492 | 32,761 | 33,967 | 35,164 |
| 4,0 | 25,491 | 26,528 | 27,576 | 28,564 | 29,715 | 30,809 | 31,895 |
| 3,8 | 23,006 | 23,941 | 24,887 | 25,779 | 26,818 | 27,806 | 28,785 |
| 3,6 | 20,648 | 21,488 | 22,337 | 23,137 | 24,069 | 24,956 | 25,835 |
| 3,4 | 18,418 | 19,166 | 19,924 | 20,637 | 21,469 | 22,266 | 23,044 |
| 3,2 | 16,314 | 16,978 | 17,649 | 18,281 | 19,016 | 19,718 | 20,413 |
| 3,0 | 14,339 | 14,922 | 15,512 | 16,067 | 16,715 | 17,330 | 17,941 |
| 2,8 | 12,491 | 12,999 | 13,512 | 13,996 | 14,560 | 15,097 | 15,628 |
| 2,6 | 10,770 | 11,208 | 11,651 | 12,068 | 12,555 | 13,018 | 13,476 |
| 2,4 | 9,177 | 9,550 | 9,927 | 10,283 | 10,697 | 11,091 | 11,482 |
| 2,2 | 7,711 | 8,025 | 8,342 | 8,641 | 8,989 | 9,320 | 9,648 |
| 2,0 | 6,373 | 6,632 | 6,984 | 7,141 | 7,429 | 7,702 | 7,974 |

| 長軸の 長さ(2a) (cm) | 7 2 型 | 7 4 型 | 7 6 型 | 7 8 型 | 8 0 型 | 8 2 型 | 8 4 型 |
|-----------------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| | B' =1,31424 | B' =1,35920 | B' =1,40486 | B' =1,45104 | B' =1,49793 | B' =1,54548 | B' =1,59367 |
| 4,2 | 36,416 | 37,662 | 38,927 | 40,207 | 41,506 | 42,823 | 44,159 |
| 4,0 | 33,030 | 34,160 | 35,308 | 36,469 | 37,647 | 38,842 | 40,053 |
| 3,8 | 29,810 | 30,830 | 31,865 | 32,913 | 33,976 | 35,055 | 36,148 |
| 3,6 | 26,755 | 27,670 | 28,599 | 29,540 | 30,494 | 31,462 | 32,443 |
| 3,4 | 23,864 | 24,681 | 25,510 | 26,349 | 27,200 | 28,063 | 28,938 |
| 3,2 | 21,139 | 21,863 | 22,597 | 23,340 | 24,094 | 24,859 | 25,634 |
| 3,0 | 18,580 | 19,215 | 19,961 | 20,514 | 21,176 | 21,849 | 22,530 |
| 2,8 | 16,185 | 16,739 | 17,301 | 17,870 | 18,447 | 19,033 | 19,626 |
| 2,6 | 13,955 | 14,433 | 14,918 | 15,408 | 15,906 | 16,411 | 16,922 |
| 2,4 | 11,891 | 12,298 | 12,711 | 13,129 | 13,553 | 13,983 | 14,419 |
| 2,2 | 9,992 | 10,334 | 10,681 | 11,032 | 11,388 | 11,750 | 12,116 |
| 2,0 | 8,258 | 8,540 | 8,827 | 9,117 | 9,412 | 9,710 | 10,013 |

之をOval of CASSINI のそれと比較するに次の如し。

Fig. 7



上の表面積計算に於て、標準型番號 N の値の小なるものは、小數點以下 3 位に相當の誤差を生ずる。計算が割合簡單なので上の方法に依つたのであるが、展開に於て收斂が甚だ緩慢である。より正確なる結果を求むるには次の算式による方がよい。

$$\text{表面積} \quad K = 4\pi m^2 + 4\pi a^2 \int_0^{\frac{\pi}{2}} (\sin \theta - 2 \sin^3 \theta) \left(1 - \frac{a^4}{m^4} \sin^2 2\theta\right)^{-\frac{1}{2}} d\theta$$

に於て $\left(1 - \frac{a^4}{m^4} \sin^2 2\theta\right)^{-\frac{1}{2}}$ を直接展開せずに

$$\frac{a^2}{m^2} = \sin^2 \alpha \text{ と置き}$$

$\sin^2 \alpha + \cos^2 \alpha = 1$ を平方して

$$\cos^4 \alpha + \sin^4 \alpha = 1 - 2 \sin^2 \alpha \cos^2 \alpha$$

さて

$$(\cos^2 \alpha + e^{i\theta} \sin^2 \alpha)(\cos^2 \alpha + e^{-i\theta} \sin^2 \alpha) = \cos^4 \alpha + \sin^4 \alpha + \sin^2 \alpha \cos^2 \alpha (e^{i\theta} + e^{-i\theta})$$

しかるに

$$e^{\pm i\theta} = \cos \theta \pm i \sin \theta \text{ なるより}$$

$$(\cos^2 \alpha + e^{i\theta} \sin^2 \alpha)(\cos^2 \alpha + e^{-i\theta} \sin^2 \alpha) = \cos^4 \alpha + \sin^4 \alpha + \sin^2 \alpha \cos^2 \alpha (2 \cos \theta)$$

$$= 1 - 2 \sin^2 \alpha \cos^2 \alpha + \sin^2 \alpha \cos^2 \alpha (2 \cos \theta) = 1 - 2 \sin^2 \alpha \cos^2 \alpha (1 - \cos \theta)$$

$$= 1 - \sin^2 2\alpha \sin^2 \theta$$

$$\text{依つて } 1 - \sin^2 2\alpha \sin^2 \theta = \cos^4 \alpha (1 + \tan^2 \alpha, e^{i\theta}) (1 + \tan^2 \alpha, e^{-i\theta})$$

$$\text{しかるに } \frac{a^2}{m^2} = \sin^2 \alpha < 1 \text{ なるにより } \alpha < \frac{\pi}{4}$$

従つて $\tan^2 \alpha < 1$ 尙又 $e^{\pm i\theta} = \cos \theta \pm i \sin \theta$ の絶対値は $\cos^2 \theta + \sin^2 \theta = 1$ なるに依り $(1 + \tan^2 \alpha, e^{i\theta})^{-\frac{1}{2}}$ 及び $(1 + \tan^2 \alpha, e^{-i\theta})^{-\frac{1}{2}}$ は共に Binominal theorem に依りて展開するこゝを得。即ち

$$(1 + \tan^2 \alpha, e^{i\theta})^{-\frac{1}{2}} = 1 - \frac{1}{2} \tan^2 \alpha, e^{i\theta} + \frac{-\frac{1}{2} \cdot -\frac{3}{2}}{2!} \tan^4 \alpha, e^{2i\theta} + \frac{-\frac{1}{2} \cdot -\frac{3}{2} \cdot -\frac{5}{2}}{3!} \tan^6 \alpha, e^{3i\theta} + \dots$$

$$(1 + \tan^2 \alpha, e^{-i\theta})^{-\frac{1}{2}} = 1 - \frac{1}{2} \tan^2 \alpha, e^{-i\theta} + \frac{-\frac{1}{2} \cdot -\frac{3}{2}}{2!} \tan^4 \alpha, e^{-2i\theta} + \frac{-\frac{1}{2} \cdot -\frac{3}{2} \cdot -\frac{5}{2}}{3!} \tan^6 \alpha, e^{-3i\theta} + \dots$$

$$\therefore (1 + \tan^2 \alpha, e^{i\theta})^{-\frac{1}{2}} (1 + \tan^2 \alpha, e^{-i\theta})^{-\frac{1}{2}}$$

$$= 1 + \left(-\frac{1}{2}\right) \tan^2 \alpha, 2 \cos(4\theta) + \tan^4 \alpha \left\{ \frac{-\frac{1}{2} \cdot -\frac{3}{2}}{2!} \cdot 2 \cos(8\theta) + \left(-\frac{1}{2}\right)^2 \right\}$$

$$+ \tan^6 \alpha \left\{ \frac{-\frac{1}{2} \cdot -\frac{3}{2} \cdot -\frac{5}{2}}{3!} \cdot 2 \cos(12\theta) + \left(-\frac{1}{2}\right) \frac{-\frac{1}{2} \cdot -\frac{3}{2}}{2!} \cdot 2 \cos(4\theta) \right\}$$

$$+ \tan^8 \alpha \left\{ \frac{-\frac{1}{2} \cdot -\frac{3}{2} \cdot -\frac{5}{2} \cdot -\frac{7}{2}}{4!} \cdot 2 \cos(16\theta) + \left(-\frac{1}{2}\right) \frac{-\frac{1}{2} \cdot -\frac{3}{2} \cdot -\frac{5}{2}}{3!} \cdot 2 \cos(8\theta) + \left(\frac{-\frac{1}{2} \cdot -\frac{3}{2}}{2!}\right)^2 \right\}$$

$$+ \tan^{10} \alpha \left\{ \frac{-\frac{1}{2} \cdot -\frac{3}{2} \cdot -\frac{5}{2} \cdot -\frac{7}{2} \cdot -\frac{9}{2}}{5!} \cdot 2 \cos(20\theta) + \left(-\frac{1}{2}\right) \frac{-\frac{1}{2} \cdot -\frac{3}{2} \cdot -\frac{5}{2} \cdot -\frac{7}{2}}{4!} \cdot 2 \cos(12\theta) \right.$$

$$\left. + \frac{-\frac{1}{2} \cdot -\frac{3}{2}}{2!} \cdot \frac{-\frac{1}{2} \cdot -\frac{3}{2} \cdot -\frac{5}{2}}{3!} \cdot 2 \cos(4\theta) \right\} + \dots$$

$$= 1 - \frac{1}{2} \tan^2 \alpha, 2 \cos(4\theta) + \tan^4 \alpha \left(\frac{1 \cdot 3}{2 \cdot 4} \cdot 2 \cos(8\theta) + \frac{1}{2^2} \right)$$

$$+ \tan^6 \alpha \left\{ \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \cdot 2 \cos(12\theta) - \frac{1 \cdot 3}{2 \cdot 4} \cos(4\theta) \right\} + \tan^8 \alpha \left\{ \frac{1 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 4 \cdot 6 \cdot 8} \cdot 2 \cos(16\theta) + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \cdot 2 \cos(8\theta) \right.$$

$$\left. + \frac{9}{2^3} \right\} + \tan^{10} \alpha \left\{ -\frac{1 \cdot 3 \cdot 5 \cdot 7 \cdot 9}{2 \cdot 4 \cdot 6 \cdot 8 \cdot 10} \cdot 2 \cos(20\theta) - \frac{1 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 4 \cdot 6 \cdot 8} \cos(12\theta) - \frac{1 \cdot 3}{2 \cdot 4} \cdot \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \cdot 2 \cos(4\theta) \right\} + \dots$$

之を $\cos(4n\theta)$ を因数に有する項を夫々一つに纏めて整頓するこゝ

$$(1 - \sin^2 2\alpha, \sin^2 \theta)^{-\frac{1}{2}} = A_0 + 2A_1 \cos(4\theta) + 2A_2 \cos(8\theta) + 2A_3 \cos(12\theta) + \dots \text{なる形となる。}$$

こゝで

$$A_0 = \frac{1}{\cos^2 \alpha} \left[1 + \left(\frac{-\frac{1}{2}}{1!}\right)^2 \tan^4 \alpha + \left(\frac{-\frac{1}{2} \cdot -\frac{3}{2}}{2!}\right)^2 \tan^6 \alpha + \left(\frac{-\frac{1}{2} \cdot -\frac{3}{2} \cdot -\frac{5}{2}}{3!}\right)^2 \tan^{10} \alpha + \dots \right]$$

$$A_1 = \frac{1}{\cos^2 \alpha} \left[\frac{-\frac{1}{2}}{1!} \tan^2 \alpha + \frac{-\frac{1}{2} \cdot -\frac{3}{2}}{2!} \tan^4 \alpha + \frac{-\frac{1}{2} \cdot -\frac{3}{2} \cdot -\frac{5}{2}}{3!} \tan^6 \alpha + \dots \right]$$

$$A_2 = \frac{1}{\cos^2 \alpha} \left[\frac{-\frac{1}{2} \cdot -\frac{3}{2}}{2!} \tan^4 \alpha + \frac{-\frac{1}{2} \cdot -\frac{3}{2} \cdot -\frac{5}{2}}{3!} \tan^6 \alpha + \frac{-\frac{1}{2} \cdot -\frac{3}{2} \cdot -\frac{5}{2} \cdot -\frac{7}{2}}{4!} \tan^{10} \alpha + \dots \right]$$

一般に

$$A_r = \frac{1}{\cos^2 \alpha} \left[\binom{m}{r} \tan^{2r} \alpha + \binom{m}{1} \binom{m}{r+1} \tan^{2r+2} \alpha + \binom{m}{2} \binom{m}{r+2} \tan^{2r+4} \alpha + \dots \right]$$

よつて

$$\begin{aligned} A_0 &= \frac{1}{\cos^2 \alpha} \left[1 + \frac{1}{4} \tan^2 \alpha + \frac{9}{64} \tan^4 \alpha + \frac{25}{256} \tan^6 \alpha + \frac{1225}{16384} \tan^8 \alpha + \dots \right] \\ &= \frac{1}{\cos^2 \alpha} \left[1 + 0.25 \tan^2 \alpha + 0.14063 \tan^4 \alpha + 0.09766 \tan^6 \alpha + 0.07477 \tan^8 \alpha + \dots \right] \end{aligned}$$

$$\begin{aligned} A_1 &= \frac{1}{\cos^2 \alpha} \left[-\frac{1}{2} \tan^2 \alpha - \frac{3}{16} \tan^4 \alpha - \frac{15}{128} \tan^6 \alpha - \frac{175}{2048} \tan^8 \alpha - \dots \right] \\ &= -\frac{1}{\cos^2 \alpha} \left[0.50000 \tan^2 \alpha + 0.18750 \tan^4 \alpha + 0.11719 \tan^6 \alpha + 0.08545 \tan^8 \alpha + \dots \right] \end{aligned}$$

$$\begin{aligned} A_2 &= \frac{1}{\cos^2 \alpha} \left[\frac{3}{8} \tan^4 \alpha + \frac{5}{32} \tan^6 \alpha + \frac{105}{1024} \tan^8 \alpha + \frac{315}{4096} \tan^{10} \alpha + \dots \right] \\ &= \frac{1}{\cos^2 \alpha} \left[0.37500 \tan^4 \alpha + 0.15625 \tan^6 \alpha + 0.10254 \tan^8 \alpha + 0.076904 \tan^{10} \alpha + \dots \right] \end{aligned}$$

以下同様にして

$$A_3 = -\frac{1}{\cos^2 \alpha} \left[0.31250 \tan^6 \alpha + 0.13672 \tan^8 \alpha + 0.092235 \tan^{10} \alpha + \dots \right]$$

$$A_4 = \frac{1}{\cos^2 \alpha} \left[0.273437 \tan^8 \alpha + 0.12305 \tan^{10} \alpha + 0.08459 \tan^{12} \alpha + \dots \right]$$

$$A_5 = -\frac{1}{\cos^2 \alpha} \left[0.24609 \tan^{10} \alpha + 0.11279 \tan^{12} \alpha + \dots \right]$$

$$A_6 = \frac{1}{\cos^2 \alpha} \left[0.22559 \tan^{12} \alpha + 0.10474 \tan^{14} \alpha + \dots \right]$$

$$A_7 = -\frac{1}{\cos^2 \alpha} \left[0.20947 \tan^{14} \alpha + \dots \right]$$

$$A_8 = \frac{1}{\cos^2 \alpha} \left[0.19638 \tan^{16} \alpha + \dots \right]$$

$$\text{さて } \tan \alpha = \frac{1 - \sqrt{1 - \frac{a^4}{m^4}}}{\frac{a^2}{m^2}} = \frac{\frac{a^4}{m^4}}{\frac{a^2}{m^2} \left(1 + \sqrt{1 - \frac{a^4}{m^4}} \right)} = \frac{\frac{a^2}{m^2}}{1 + \sqrt{1 - \frac{a^4}{m^4}}}$$

依つて $\tan \alpha$ の値は $\frac{a^2}{m^2}$ の値の大なる程大なり。

しかるに $\frac{a^2}{m^2}$ は標準型番號の小なる程大であつて、 A の收斂は緩慢なり。

依つて26型に就き A の値を求めんに

$$\tan \alpha = \frac{1 - \sqrt{1 - \frac{a^4}{m^4}}}{\frac{a^2}{m^2}} = \frac{1 - \sqrt{0.237225}}{0.87337} = \frac{0.51295}{0.87337} = 0.58732$$

$$\tan^2 \alpha = 0.344945, \quad \tan^4 \alpha = 0.118987, \quad \tan^6 \alpha = 0.041044$$

$$\tan^8 \alpha = 0.014153, \quad \tan^{10} \alpha = 0.004884, \quad \tan^{12} \alpha = 0.001685$$

$$\tan^{14} \alpha = 0.000581, \quad \tan^{16} \alpha = 0.000200$$

依つて

$$A_0 = \frac{1}{\cos^2 \alpha} \left[1 + 0.02975 + 0.00199 + 0.00016 + 0.00001 + \dots \right] = 1.03191 \times \frac{1}{\cos^2 \alpha}$$

$$A_1 = -\frac{1}{\cos^2 \alpha} \left[0.17247 + 0.00770 + 0.00057 + 0.00005 + \dots \right] = -0.18079 \times \frac{1}{\cos^2 \alpha}$$

$$A_2 = \frac{1}{\cos^2 \alpha} \left[0.04462 + 0.00221 + 0.00017 + 0.00002 + \dots \right] = 0.04702 \times \frac{1}{\cos^2 \alpha}$$

$$A_3 = -\frac{1}{\cos^2 a} [0.01283 + 0.00067 + 0.00005 + \dots] = -0.01355 \times \frac{1}{\cos^2 a}$$

$$A_4 = \frac{1}{\cos^2 a} [0.00387 + 0.00021 + 0.00002 + \dots] = 0.00410 \times \frac{1}{\cos^2 a}$$

$$A_5 = -\frac{1}{\cos^2 a} [0.00120 + 0.00006 + \dots] = -0.00126 \times \frac{1}{\cos^2 a}$$

$$A_6 = \frac{1}{\cos^2 a} [0.00038 + 0.00002 + \dots] = 0.00040 \times \frac{1}{\cos^2 a}$$

$$A_7 = -\frac{1}{\cos^2 a} [0.00012 + \dots] = -0.00012 \times \frac{1}{\cos^2 a}$$

$$A_8 = 0.00004 \times \frac{1}{\cos^2 a}$$

依つて

$$\begin{aligned} (1 - \sin^2 a \sin^2 \theta)^{-\frac{1}{2}} &= \frac{1}{\cos^2 a} [1.03191 - 0.36158 \cos(4\theta) + 0.09404 \cos(8\theta) \\ &- 0.02710 \cos(12\theta) + 0.00820 \cos(16\theta) - 0.00252 \cos(20\theta) + 0.00080 \cos(24\theta) \\ &- 0.00012 \cos(28\theta) + 0.00004 \cos(32\theta) \dots] \end{aligned}$$

$$\text{又 } \int_0^{\frac{\pi}{2}} (\sin \theta - 2 \sin^3 \theta) (1 - \sin^2 a \sin^2 \theta)^{-\frac{1}{2}} d\theta \text{ に於て}$$

$$\sin \theta - 2 \sin^3 \theta = \frac{1}{2} (\sin 3\theta - \sin \theta) \text{ であるから}$$

$(\sin \theta - 2 \sin^3 \theta) (1 - \sin^2 a \sin^2 \theta)^{-\frac{1}{2}}$ は總て $\sin \theta \cdot \cos(4n\theta)$ 及び $\sin 3\theta \cdot \cos(4n\theta)$ の項のみとなる。茲に $n=1, 2, 3, \dots$

一般に

$$I_n = \int_0^{\frac{\pi}{2}} \sin \theta \cdot \cos(4n\theta) d\theta, \quad J_n = \int_0^{\frac{\pi}{2}} \sin(3\theta) \cdot \cos(4n\theta) d\theta \quad \text{で}$$

表す可すれば、

$$\begin{aligned} I_1 &= \int_0^{\frac{\pi}{2}} \sin \theta \cdot \cos(4\theta) d\theta = \left[\frac{\sin(4\theta)}{4} \sin \theta \right]_0^{\frac{\pi}{2}} - \frac{1}{4} \int_0^{\frac{\pi}{2}} \sin(4\theta) \cdot \cos \theta d\theta \\ &= -\frac{1}{4} \left\{ -\left[\frac{\cos(4\theta)}{4} \cos \theta \right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \frac{1}{4} \cos(4\theta) \sin \theta d\theta \right\} = \frac{1}{4} \left\{ -\frac{1}{4} + \frac{1}{4} I_1 \right\} \end{aligned}$$

$$\therefore I_1 = \frac{-\frac{1}{4^2}}{1 - \frac{1}{4^2}} = \frac{-1}{4^2 - 1}$$

$$\text{同様にして} \quad I_2 = \frac{-1}{8^2 - 1}$$

$$\text{一般に} \quad I_i = \frac{-1}{(4i)^2 - 1}$$

$$\begin{aligned} \text{次に } J_1 &= \int_0^{\frac{\pi}{2}} \sin(3\theta) \cdot \cos(4\theta) d\theta = \frac{1}{4} \left[\sin(4\theta) \cdot \sin(3\theta) \right]_0^{\frac{\pi}{2}} - \frac{3}{4} \int_0^{\frac{\pi}{2}} \sin(4\theta) \cdot \cos(3\theta) d\theta \\ &= -\frac{3}{4} \left\{ -\left[\frac{\cos(4\theta)}{4} \cos(3\theta) \right]_0^{\frac{\pi}{2}} - \frac{3}{4} \int_0^{\frac{\pi}{2}} \cos(4\theta) \cdot \sin(3\theta) d\theta \right\} \\ &= \frac{3}{4} \left(-\frac{1}{4} + \frac{3}{4} J_1 \right) = -\frac{3}{4^2} + \left(\frac{3}{4} \right)^2 J_1 \end{aligned}$$

$$\therefore J_1 = \frac{-\frac{3}{4^2}}{1 - \left(\frac{3}{4}\right)^2} = \frac{-3}{4^2 - 3^2}$$

$$\text{同様にして} \quad J_2 = \frac{-3}{8^2 - 3^2}$$

$$\text{一般に} \quad J_i = \frac{-3}{(4i)^2 - 3^2}$$

$$\begin{aligned} \therefore \int_0^{\frac{\pi}{2}} (\sin \theta - 2 \sin^3 \theta) (1 - \sin^2 2a \sin^2 2\theta)^{-\frac{1}{2}} d\theta \\ &= \frac{1}{2 \cos^2 a} \left[(0.34397 + \frac{3}{7} \times 0.36158 - 0.09404 \times \frac{3}{55} + 0.02710 \times \frac{3}{135} \right. \\ &\quad \left. - 0.00820 \times \frac{3}{247} + 0.00252 \times \frac{3}{391} - 0.00080 \times \frac{3}{567} + \dots) \right. \\ &\quad \left. + (-1.03191 - 0.36518 \times \frac{1}{15} + 0.09404 \times \frac{1}{63} - 0.02710 \times \frac{1}{143} + 0.00820 \times \frac{1}{255} \right. \\ &\quad \left. - 0.00252 \times \frac{1}{399} + 0.00080 \times \frac{1}{575} - \dots) \right] \\ &= \frac{1}{2 \cos^2 a} \left[(0.34397 + 0.15496 - 0.00513 + 0.00060 - 0.00010 + 0.00002 - \dots) \right. \\ &\quad \left. + (-1.03191 - 0.02411 + 0.00149 - 0.00019 + 0.00003 - 0.00001 + \dots) \right] \\ &= \frac{1}{2 \cos^2 a} (0.50107 - 1.06145) = -\frac{1}{2 \cos^2 a} \times 0.56038 \end{aligned}$$

又 $\cos^2 a$ を求むるに

$$\sin 2a = \frac{a^2}{m^2} = 0.87337 \quad \text{より} \quad \cos^2 a = 0.743525$$

$$\therefore \text{表面積 } K = 4\pi m^2 - 4\pi a^2 \times \frac{0.56038}{1.48705} = 4\pi (m^2 - 0.37684a^2)$$

前に計算した標準型表面積常数 $B = 0.37659$ にして、正しき 0.37684 の差は 0.00025 である。

又前の計算による $2\frac{3}{4}$ 型で、長軸の長さ 4.2 寸の表面積は 19.853 平方寸に對し、此の計算に依れば $K = 4\pi (2.35406 - 0.37684 \times 2.05594) = 4\pi \times 1.57930 = 19.846$ にて、その誤差 0.007 即ち前の計算に依るに、 0.007 立方寸大きな誤差を生ずる。

實際蘭を取扱ふ場合、かゝる程度の誤差は許されるものと考え、且つは計算の繁雜等を考慮に入れ、敢て前の法式により表面積を算出せり。體積計算に於ても、本表面積計算に於ても、尙次項の長軸を含む断面積計算に於ても、graph が出來た上は、長軸の長さは 2 寸と 4.2 寸の間の如何なる寸法にても、各々體積、表面積、断面積は正確に讀み取ることが出来る。

(8) Oval of Cassini に依る蘭の長軸を含む断面積計算と、長球の長軸を含む断面積、即ち橢圓面積との比較

$$(x^2 + y^2 + a^2)^2 - 4a^2 x^2 = m^4$$

polar co-ordinates を用ひて

$$(\gamma^2 + a^2)^2 - 4a^2 \gamma^2 \cos^2 \theta + a^4 - m^4 = 0$$

七項を同様に

$$\gamma^2 = a^2 \cos 2\theta + \sqrt{m^4 - a^4 \sin^2 2\theta}$$

依つて断面積 S は次の様になる。

$$S = \frac{1}{2} \times 4 \int_0^{\frac{\pi}{2}} r^2 d\theta = 2 \int_0^{\frac{\pi}{2}} (a^2 \cos 2\theta + \sqrt{m^4 - a^4 \sin^2 2\theta}) d\theta$$

$$\begin{aligned}
&= 2a^2 \int_0^{\frac{\pi}{2}} \cos 2\theta \, d\theta + 2 \int_0^{\frac{\pi}{2}} \sqrt{m^4 - a^4 \sin^2 2\theta} \, d\theta \\
&= \frac{2a^2}{2} \left[\sin 2\theta \right]_0^{\frac{\pi}{2}} + 2m^2 \int_0^{\frac{\pi}{2}} \sqrt{1 - \frac{a^4}{m^4} \sin^2 2\theta} \, d\theta = 2m^2 \int_0^{\frac{\pi}{2}} \sqrt{1 - \frac{a^4}{m^4} \sin^2 2\theta} \, d\theta
\end{aligned}$$

今 $\frac{a^2}{m^2} = \sin 2\alpha$ と置き前項の終りの展開方法を用ひて

$$\sqrt{1 - \sin^2 2\alpha \sin^2 2\theta} = A_0 + 2A_1 \cos(4\theta) + 2A_2 \cos(8\theta) + 2A_3 \cos(12\theta) + \dots$$

$$\text{しかるに} \quad \int_0^{\frac{\pi}{2}} \cos 4\theta \, d\theta = \int_0^{\frac{\pi}{2}} \cos 8\theta \, d\theta = \dots = 0$$

$$\begin{aligned}
\text{又} \quad A_0 &= \cos^2 \alpha \left[1 + \left(\frac{1}{1}\right)^2 \tan^4 \alpha + \left(\frac{1}{2}\right)^2 \tan^6 \alpha + \left(\frac{1}{3}\right)^2 \tan^{12} \alpha + \dots \right] \\
&= \cos^2 \alpha \left[1 + \frac{1}{4} \tan^4 \alpha + \left(\frac{1}{2 \cdot 4}\right)^2 \tan^6 \alpha + \left(\frac{1 \cdot 3}{2 \cdot 4 \cdot 6}\right)^2 \tan^{12} \alpha + \left(\frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6 \cdot 8}\right)^2 \tan^{16} \alpha + \dots \right] \\
&= \cos^2 \alpha \left(1 + \frac{1}{4} \tan^4 \alpha + \frac{1}{64} \tan^6 \alpha + \frac{1}{256} \tan^{12} \alpha + \frac{25}{16384} \tan^{16} \alpha \right. \\
&\quad \left. + \frac{49}{65536} \tan^{20} \alpha + \frac{441}{1048576} \tan^{24} \alpha + \frac{1089}{4194304} \tan^{28} \alpha + \dots \right)
\end{aligned}$$

$$\therefore S = 2m^2 \int_0^{\frac{\pi}{2}} A_0 \, d\theta = \pi m^2 A_0$$

A_0 の値を長軸を含む斷面積標準型常數 (Sectional area coefficient containing major axis of standard form) C と稱するここを許さるるならば

$$S = \pi m^2 C \quad \text{但し}$$

$$C = \cos^2 \alpha \left(1 + 0.25 \tan^4 \alpha + 0.01563 \tan^6 \alpha + 0.00391 \tan^{12} \alpha + 0.00153 \tan^{16} \alpha + \dots \right)$$

各標準型に就き C を求むれば次の如し。

$$\text{但し} \quad \tan \alpha = \frac{1 - \sqrt{1 - \frac{a^4}{m^4}}}{\frac{a^2}{m^2}}, \quad \cos^2 \alpha = \frac{1 + \sqrt{1 - \frac{a^4}{m^4}}}{2}$$

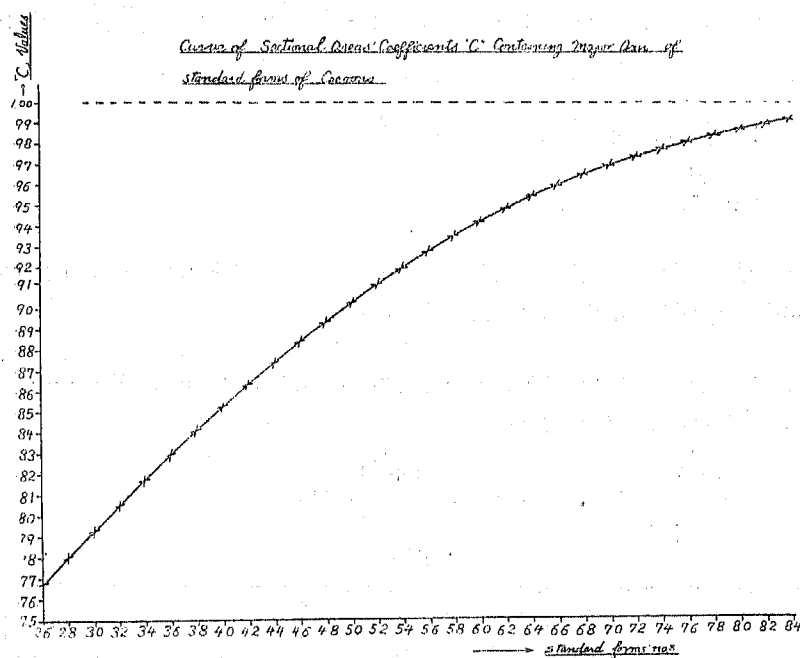
Table X

| 標準型 番號N | $\frac{a^2}{m^2}$ | $\frac{a^4}{m^4}$ | $\sqrt{1 - \frac{a^4}{m^4}}$ | $\cos^2 \alpha$ | $\tan \alpha$ | $\tan^2 \alpha$ | $\tan^4 \alpha$ | $\tan^8 \alpha$ | C |
|------------|-------------------|-------------------|------------------------------|-----------------|---------------|-----------------|-----------------|-----------------|---------|
| 26 | 0,873370 | 0,76278 | 0,48705 | 0,74353 | 0,58732 | 0,34494 | 0,11898 | 0,01416 | 0,76582 |
| 28 | 0,854599 | 0,730339 | 0,51928 | 0,75964 | 0,56251 | 0,31642 | 0,10012 | 0,01002 | 0,77878 |
| 30 | 0,834860 | 0,696991 | 0,55046 | 0,77523 | 0,53846 | 0,28994 | 0,08407 | 0,00707 | 0,79161 |
| 32 | 0,814220 | 0,662954 | 0,58055 | 0,79028 | 0,51516 | 0,26539 | 0,07043 | 0,00496 | 0,80426 |
| 34 | 0,792757 | 0,628464 | 0,60953 | 0,80477 | 0,49255 | 0,24261 | 0,05886 | 0,00346 | 0,81666 |
| 36 | 0,770538 | 0,593729 | 0,63739 | 0,81870 | 0,47059 | 0,22145 | 0,04904 | 0,00240 | 0,82877 |
| 38 | 0,747640 | 0,558966 | 0,66410 | 0,83205 | 0,44928 | 0,20185 | 0,04074 | 0,00166 | 0,84055 |
| 40 | 0,724137 | 0,524374 | 0,68965 | 0,84483 | 0,42853 | 0,18368 | 0,03374 | 0,00114 | 0,85197 |
| 42 | 0,700102 | 0,490143 | 0,71404 | 0,85702 | 0,40846 | 0,16684 | 0,02784 | 0,00077 | 0,86300 |
| 44 | 0,675603 | 0,456439 | 0,73726 | 0,86863 | 0,38890 | 0,15124 | 0,02287 | 0,00052 | 0,87361 |
| 46 | 0,650709 | 0,423422 | 0,75932 | 0,87966 | 0,36987 | 0,13680 | 0,01871 | 0,00035 | 0,88379 |
| 48 | 0,625487 | 0,391234 | 0,78023 | 0,89012 | 0,35136 | 0,12345 | 0,01524 | 0,00021 | 0,89351 |
| 50 | 0,600000 | 0,360000 | 0,80000 | 0,90000 | 0,33333 | 0,11111 | 0,01235 | 0,00014 | 0,90278 |
| 52 | 0,574307 | 0,329829 | 0,81863 | 0,90932 | 0,31581 | 0,09974 | 0,00995 | 0,00009 | 0,91158 |
| 54 | 0,548467 | 0,300816 | 0,83617 | 0,91809 | 0,29870 | 0,08922 | 0,00796 | 0,00006 | 0,91992 |
| 56 | 0,522533 | 0,273041 | 0,85260 | 0,92630 | 0,28209 | 0,07957 | 0,00633 | 0,00004 | 0,92776 |
| 58 | 0,496558 | 0,246370 | 0,86800 | 0,93400 | 0,26583 | 0,07067 | 0,00499 | 0,00003 | 0,93517 |
| 60 | 0,470558 | 0,221453 | 0,88235 | 0,94118 | 0,25002 | 0,06251 | 0,00391 | 0,00002 | 0,94210 |
| 62 | 0,444659 | 0,197731 | 0,89567 | 0,94785 | 0,23458 | 0,05503 | 0,00303 | 0,00001 | 0,94857 |
| 64 | 0,418842 | 0,175429 | 0,90805 | 0,95403 | 0,21953 | 0,04819 | 0,00232 | 0,00001 | 0,95458 |

| 標準型 番號N | $\frac{a^2}{m^2}$ | $\frac{a^4}{m^4}$ | $\sqrt{1 - \frac{a^4}{m^4}}$ | $\cos 2a$ | $\tan a$ | $\tan 2a$ | $\tan 4a$ | $\tan 8a$ | C |
|------------|-------------------|-------------------|------------------------------|-----------|----------|-----------|-----------|-----------|---------|
| 66 | 0,393145 | 0,154563 | 0,91947 | 0,95974 | 0,20483 | 0,04196 | 0,00176 | /// | 0,96016 |
| 68 | 0,367615 | 0,135141 | 0,92997 | 0,96499 | 0,19050 | 0,03629 | 0,00132 | /// | 0,96531 |
| 70 | 0,342282 | 0,117157 | 0,93959 | 0,96980 | 0,17649 | 0,03115 | 0,00097 | /// | 0,97003 |
| 72 | 0,317175 | 0,100600 | 0,94836 | 0,97418 | 0,16281 | 0,02651 | 0,00070 | /// | 0,97432 |
| 74 | 0,292323 | 0,085453 | 0,95631 | 0,97816 | 0,14946 | 0,02234 | 0,00050 | /// | 0,97829 |
| 76 | 0,267748 | 0,071669 | 0,96348 | 0,98174 | 0,13640 | 0,01860 | 0,00035 | /// | 0,98183 |
| 78 | 0,24347 | 0,059278 | 0,96950 | 0,98495 | 0,12363 | 0,01528 | 0,00023 | /// | 0,98501 |
| 80 | 0,21951 | 0,048185 | 0,97561 | 0,98781 | 0,11111 | 0,01235 | 0,00015 | /// | 0,98785 |
| 82 | 0,19589 | 0,038373 | 0,98062 | 0,99031 | 0,09893 | 0,00979 | 0,00010 | /// | 0,99034 |
| 84 | 0,17261 | 0,029794 | 0,98499 | 0,99250 | 0,08696 | 0,00756 | 0,00006 | /// | 0,99252 |

C の graph を描けば次の如し。

Fig. 8



C に依つて断面積を求めれば次の如し。

Table II $S = \pi m^2 C$

| 長軸の 長さ(2l) (cm) | 2 6 型 | | 2 8 型 | | 3 0 型 | | 3 2 型 | | 3 4 型 | |
|-----------------------|----------------|-------|----------------|-------|----------------|-------|----------------|-------|----------------|-------|
| | C=0,76582 | | C=0,77878 | | C=0,79161 | | C=0,80426 | | C=0,81666 | |
| | m ² | S | m ² | S | m ² | S | m ² | S | m ² | S |
| 4,2 | 2,35406 | 5,654 | 2,37790 | 5,818 | 2,40345 | 5,977 | 2,43079 | 6,142 | 2,45990 | 6,311 |
| 4,0 | 2,13520 | 5,137 | 2,15860 | 5,277 | 2,18000 | 5,421 | 2,20480 | 5,571 | 2,23120 | 5,724 |
| 3,8 | 1,92702 | 4,636 | 1,94651 | 4,762 | 1,96745 | 4,893 | 1,98983 | 5,028 | 2,01366 | 5,166 |
| 3,6 | 1,72951 | 4,161 | 1,74701 | 4,274 | 1,76580 | 4,391 | 1,78589 | 4,512 | 1,80727 | 4,637 |
| 3,4 | 1,54268 | 3,712 | 1,55829 | 3,813 | 1,57505 | 3,917 | 1,59297 | 4,025 | 1,61204 | 4,136 |
| 3,2 | 1,36653 | 3,288 | 1,38035 | 3,377 | 1,39520 | 3,470 | 1,41107 | 3,565 | 1,42797 | 3,664 |
| 3,0 | 1,20105 | 2,890 | 1,21320 | 2,968 | 1,22625 | 3,050 | 1,24020 | 3,134 | 1,25505 | 3,220 |
| 2,8 | 1,04625 | 2,517 | 1,05683 | 2,586 | 1,06820 | 2,657 | 1,08135 | 2,730 | 1,09329 | 2,805 |
| 2,6 | 0,90212 | 2,170 | 0,91125 | 2,229 | 0,92105 | 2,291 | 0,93153 | 2,354 | 0,94268 | 2,419 |
| 2,4 | 0,76867 | 1,849 | 0,77645 | 1,900 | 0,78480 | 1,952 | 0,79373 | 2,005 | 0,80323 | 2,061 |
| 2,2 | 0,64590 | 1,554 | 0,65243 | 1,596 | 0,65945 | 1,640 | 0,66695 | 1,685 | 0,67494 | 1,732 |
| 2,0 | 0,53380 | 1,284 | 0,53920 | 1,319 | 0,54500 | 1,355 | 0,55120 | 1,393 | 0,55780 | 1,431 |

| 長軸の 長さ(2l) (cm) | 3 6 型 | | 3 8 型 | | 4 0 型 | | 4 2 型 | | 4 4 型 | |
|-----------------------|----------------|-------|----------------|-------|----------------|--------|----------------|--------|----------------|--------|
| | C=0,82877 | | C=0,84055 | | C=0,85197 | | C=0,86300 | | C=0,87361 | |
| | m ² | S | m ² | S | m ² | S | m ² | S | m ² | S |
| 4,2 | 2,49077 | 6,485 | 2,52340 | 6,663 | 2,55780 | 6,845 | 2,59396 | 7,033 | 2,63189 | 7,223 |
| 4,0 | 2,25920 | 5,882 | 2,28980 | 6,044 | 2,32000 | 6,210 | 2,35280 | 6,379 | 2,38720 | 6,552 |
| 3,8 | 2,03893 | 5,309 | 2,06564 | 5,455 | 2,09380 | 5,604 | 2,12340 | 5,757 | 2,15445 | 5,913 |
| 3,6 | 1,82995 | 4,765 | 1,85393 | 4,896 | 1,87920 | 5,030 | 1,90577 | 5,167 | 1,93363 | 5,307 |
| 3,4 | 1,63227 | 4,250 | 1,65365 | 4,367 | 1,67620 | 4,485 | 1,69990 | 4,609 | 1,72475 | 4,734 |
| 3,2 | 1,44589 | 3,765 | 1,46483 | 3,868 | 1,48480 | 3,974 | 1,50579 | 4,082 | 1,52781 | 4,193 |
| 3,0 | 1,27080 | 3,309 | 1,28725 | 3,400 | 1,30500 | 3,493 | 1,32345 | 3,588 | 1,34280 | 3,685 |
| 2,8 | 1,10701 | 2,882 | 1,12151 | 2,962 | 1,13680 | 3,043 | 1,15287 | 3,126 | 1,16973 | 3,210 |
| 2,6 | 0,95451 | 2,485 | 0,96702 | 2,554 | 0,98020 | 2,624 | 0,99406 | 2,695 | 1,00859 | 2,768 |
| 2,4 | 0,81331 | 2,118 | 0,82397 | 2,176 | 0,83520 | 2,235 | 0,84701 | 2,296 | 0,85939 | 2,359 |
| 2,2 | 0,68941 | 1,779 | 0,69236 | 1,828 | 0,70180 | 1,878 | 0,71172 | 1,930 | 0,72213 | 1,982 |
| 2,0 | 0,56480 | 1,471 | 0,57220 | 1,511 | 0,58000 | 1,552 | 0,58820 | 1,595 | 0,59680 | 1,638 |
| 長軸の 長さ(2l) (cm) | 4 6 型 | | 4 8 型 | | 5 0 軸 | | 5 2 型 | | 5 4 型 | |
| | C=0,88389 | | C=0,89351 | | C=0,90278 | | C=0,91158 | | C=0,91992 | |
| | m ² | S | m ² | S | m ² | S | m ² | S | m ² | S |
| 4,2 | 2,67158 | 7,418 | 2,71303 | 7,616 | 2,75625 | 7,817 | 2,80123 | 8,022 | 2,84798 | 8,231 |
| 4,0 | 2,42320 | 6,728 | 2,46030 | 6,908 | 2,50000 | 7,090 | 2,54080 | 7,276 | 2,58320 | 7,465 |
| 3,8 | 2,18695 | 6,072 | 2,22387 | 6,234 | 2,25625 | 6,399 | 2,29307 | 6,567 | 2,33134 | 6,738 |
| 3,6 | 1,96279 | 5,450 | 1,99325 | 5,595 | 2,02500 | 5,743 | 2,05805 | 5,894 | 2,09289 | 6,047 |
| 3,4 | 1,74076 | 4,861 | 1,77793 | 4,991 | 1,80625 | 5,123 | 1,83573 | 5,257 | 1,86536 | 5,394 |
| 3,2 | 1,55085 | 4,305 | 1,57491 | 4,421 | 1,60000 | 4,538 | 1,62611 | 4,657 | 1,65325 | 4,778 |
| 3,0 | 1,36305 | 3,785 | 1,38420 | 3,886 | 1,40525 | 3,988 | 1,42920 | 4,093 | 1,45305 | 4,199 |
| 2,8 | 1,18737 | 3,297 | 1,20579 | 3,385 | 1,22500 | 3,474 | 1,24499 | 3,565 | 1,26577 | 3,658 |
| 2,6 | 1,02380 | 2,843 | 1,03959 | 2,918 | 1,05525 | 2,993 | 1,07349 | 3,074 | 1,09149 | 3,154 |
| 2,4 | 0,87235 | 2,422 | 0,88589 | 2,487 | 0,90000 | 2,553 | 0,91459 | 2,619 | 0,92995 | 2,686 |
| 2,2 | 0,73302 | 2,035 | 0,74439 | 2,090 | 0,75625 | 2,145 | 0,76859 | 2,201 | 0,78142 | 2,258 |
| 2,0 | 0,60580 | 1,682 | 0,61520 | 1,727 | 0,62500 | 1,773 | 0,63520 | 1,819 | 0,64580 | 1,866 |
| 長軸の 長さ(2l) (cm) | 5 6 型 | | 5 8 型 | | 6 0 型 | | 6 2 型 | | 6 4 型 | |
| | C=0,92556 | | C=0,93517 | | C=0,94210 | | C=0,94857 | | C=0,95458 | |
| | m ² | S | m ² | S | m ² | S | m ² | S | m ² | S |
| 4,2 | 2,87649 | 8,442 | 2,94576 | 8,657 | 2,99880 | 8,876 | 3,05260 | 9,097 | 3,10817 | 9,321 |
| 4,0 | 2,62720 | 7,637 | 2,67280 | 7,852 | 2,72000 | 8,050 | 2,76880 | 8,251 | 2,81920 | 8,454 |
| 3,8 | 2,37105 | 6,911 | 2,41220 | 7,087 | 2,45480 | 7,255 | 2,49884 | 7,445 | 2,54433 | 7,630 |
| 3,6 | 2,12803 | 6,202 | 2,16497 | 6,361 | 2,20320 | 6,521 | 2,24273 | 6,683 | 2,28355 | 6,848 |
| 3,4 | 1,89815 | 5,532 | 1,93110 | 5,673 | 1,96520 | 5,816 | 2,00045 | 5,961 | 2,03687 | 6,108 |
| 3,2 | 1,68141 | 4,901 | 1,71059 | 5,025 | 1,74080 | 5,152 | 1,77203 | 5,281 | 1,80429 | 5,411 |
| 3,0 | 1,47780 | 4,307 | 1,50345 | 4,417 | 1,53000 | 4,528 | 1,55745 | 4,641 | 1,58580 | 4,756 |
| 2,8 | 1,28733 | 3,752 | 1,30967 | 3,848 | 1,33280 | 3,945 | 1,35671 | 4,043 | 1,38141 | 4,143 |
| 2,6 | 1,10999 | 3,235 | 1,12926 | 3,313 | 1,14920 | 3,401 | 1,16982 | 3,486 | 1,19111 | 3,572 |
| 2,4 | 0,94579 | 2,757 | 0,96221 | 2,827 | 0,97920 | 2,898 | 0,99677 | 2,970 | 1,01491 | 3,044 |
| 2,2 | 0,79473 | 2,316 | 0,80852 | 2,375 | 0,82280 | 2,435 | 0,83756 | 2,496 | 0,85281 | 2,557 |
| 2,0 | 0,65680 | 1,914 | 0,66820 | 1,963 | 0,68000 | 2,013 | 0,69220 | 2,063 | 0,70480 | 2,114 |
| 長軸の 長さ(2l) (cm) | 6 6 型 | | 6 8 型 | | 7 0 型 | | 7 2 型 | | 7 4 型 | |
| | C=0,96016 | | C=0,96531 | | C=0,97003 | | C=0,97432 | | C=0,97829 | |
| | m ² | S | m ² | S | m ² | S | m ² | S | m ² | S |
| 4,2 | 3,16550 | 9,549 | 3,22459 | 9,779 | 3,28545 | 10,012 | 3,34807 | 10,248 | 3,41246 | 10,488 |
| 4,0 | 2,87120 | 8,661 | 2,94480 | 8,870 | 2,98000 | 9,031 | 3,03680 | 9,275 | 3,09520 | 9,513 |
| 3,8 | 2,59126 | 7,816 | 2,63963 | 8,005 | 2,68945 | 8,196 | 2,74071 | 8,389 | 2,79342 | 8,585 |
| 3,6 | 2,32567 | 7,015 | 2,36909 | 7,185 | 2,41380 | 7,356 | 2,45981 | 7,529 | 2,50711 | 7,705 |
| 3,4 | 2,07444 | 6,257 | 2,11317 | 6,408 | 2,15305 | 6,561 | 2,19409 | 6,716 | 2,23628 | 6,873 |
| 3,2 | 1,83757 | 5,543 | 1,87187 | 5,677 | 1,90720 | 5,812 | 1,94355 | 5,949 | 1,98093 | 6,088 |
| 3,0 | 1,61505 | 4,872 | 1,64520 | 4,989 | 1,67625 | 5,108 | 1,70820 | 5,229 | 1,74105 | 5,351 |
| 2,8 | 1,40689 | 4,244 | 1,43315 | 4,346 | 1,46020 | 4,450 | 1,48803 | 4,555 | 1,51665 | 4,661 |
| 2,6 | 1,21308 | 3,659 | 1,23573 | 3,747 | 1,25905 | 3,837 | 1,28305 | 3,927 | 1,30772 | 4,019 |
| 2,4 | 1,03363 | 3,118 | 1,05293 | 3,193 | 1,07280 | 3,269 | 1,09325 | 3,346 | 1,11427 | 3,425 |
| 2,2 | 0,86854 | 2,620 | 0,88475 | 2,683 | 0,90145 | 2,747 | 0,91853 | 2,812 | 0,93630 | 2,878 |
| 2,0 | 0,71780 | 2,165 | 0,73120 | 2,217 | 0,74500 | 2,270 | 0,75920 | 2,324 | 0,77380 | 2,378 |

| 長軸の 長さ(2l) (cm) | 7 6 型 | | 7 8 型 | | 8 0 型 | | 8 2 型 | | 8 4 型 | |
|-----------------------|----------------|--------|----------------|--------|----------------|--------|----------------|--------|----------------|--------|
| | C=0,98183 | | C=0,98501 | | C=0,98785 | | C=0,99034 | | C=0,99252 | |
| | m ² | S | m ² | S | m ² | S | m ² | S | m ² | S |
| 4,2 | 3,47861 | 10,730 | 3,54652 | 10,975 | 3,61620 | 11,223 | 3,68764 | 11,473 | 3,76085 | 11,727 |
| 4,0 | 3,15520 | 9,732 | 3,21680 | 9,954 | 3,28000 | 10,179 | 3,34480 | 10,406 | 3,41120 | 10,636 |
| 3,8 | 2,84757 | 8,783 | 2,90316 | 8,984 | 2,96020 | 9,187 | 3,01868 | 9,392 | 3,07861 | 9,599 |
| 3,6 | 2,55571 | 7,883 | 2,60561 | 8,063 | 2,65680 | 8,245 | 2,70929 | 8,429 | 2,76307 | 8,616 |
| 3,4 | 2,27963 | 7,032 | 2,32414 | 7,192 | 2,36950 | 7,355 | 2,41662 | 7,519 | 2,46459 | 7,685 |
| 3,2 | 2,01933 | 6,229 | 2,05875 | 6,371 | 2,09920 | 6,515 | 2,14067 | 6,660 | 2,18317 | 6,807 |
| 3,0 | 1,77480 | 5,474 | 1,80945 | 5,599 | 1,84500 | 5,726 | 1,88145 | 5,854 | 1,91880 | 5,983 |
| 2,8 | 1,54605 | 4,769 | 1,57633 | 4,878 | 1,60720 | 4,968 | 1,63895 | 5,099 | 1,67149 | 5,212 |
| 2,6 | 1,33307 | 4,112 | 1,35910 | 4,206 | 1,38580 | 4,301 | 1,41318 | 4,397 | 1,44123 | 4,494 |
| 2,4 | 1,13587 | 3,504 | 1,15805 | 3,584 | 1,18050 | 3,665 | 1,20413 | 3,746 | 1,22803 | 3,829 |
| 2,2 | 0,95445 | 2,944 | 0,97308 | 3,011 | 0,99220 | 3,078 | 1,01180 | 3,148 | 1,03189 | 3,218 |
| 2,0 | 0,78880 | 2,433 | 0,80420 | 2,489 | 0,82000 | 2,545 | 0,83620 | 2,602 | 0,85280 | 2,659 |

之を graph にて表せば Fig. 9 の如し。

橢圓の面積を比較するために、橢圓曲線を表す式 $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ に於て $y = \frac{b}{a} \sqrt{a^2 - x^2}$

依つて面積

$$S' = 4 \frac{b}{a} \int_0^a \sqrt{a^2 - x^2} dx = 4 \frac{b}{a} \left[\frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right]_0^a$$

$$= 4 \frac{b}{a} \cdot \frac{\pi a^2}{4} = \pi ab = \pi a^2 \cdot \frac{N}{100}$$

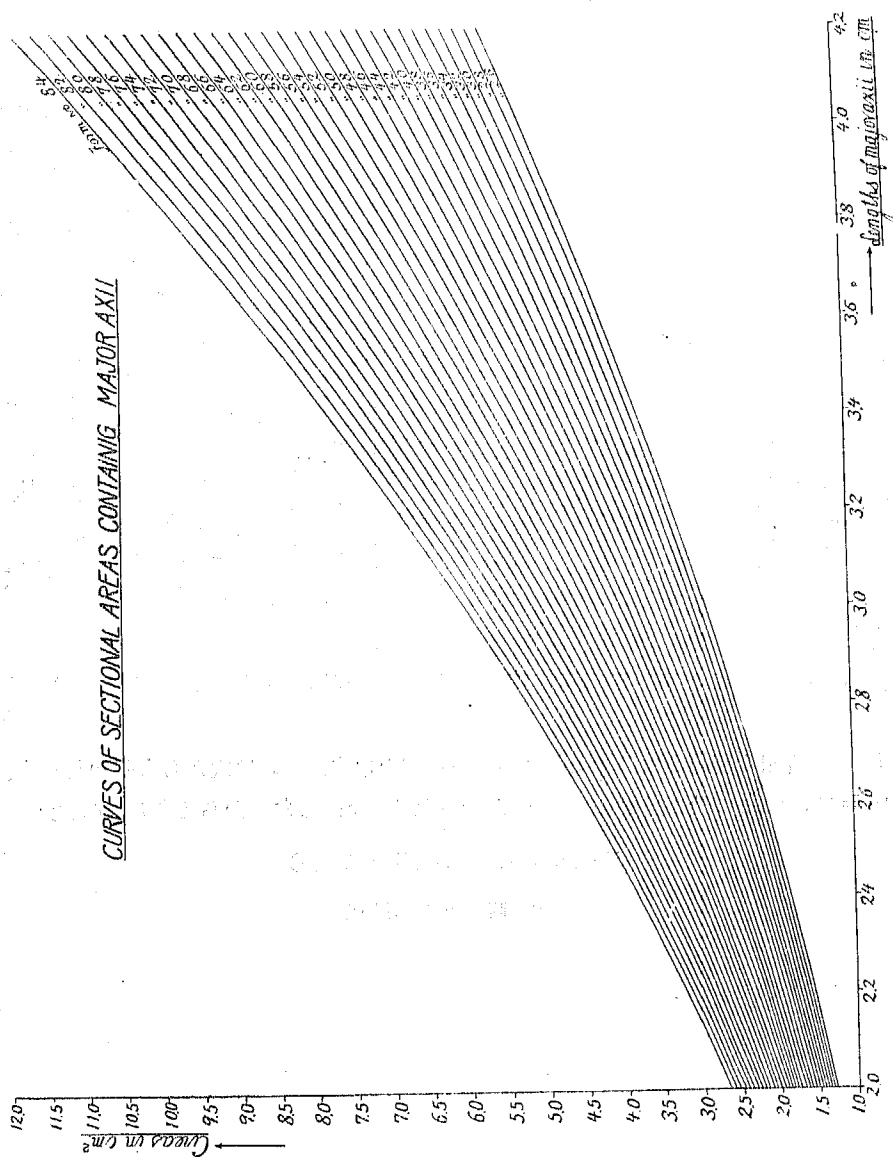
各長軸の長さに對する面積を計算すれば次の如し。

Table XII

| 長軸の 長さ(2a) (cm) | a ² | 5 8 型 | 6 0 型 | 6 2 型 | 6 4 型 | 6 6 型 | 6 8 型 | 7 0 型 |
|-----------------------|----------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| | | $\pi N \div 100$ | $\pi N \div 100$ | $\pi N \div 100$ | $\pi N \div 100$ | $\pi N \div 100$ | $\pi N \div 100$ | $\pi N \div 100$ |
| | | =1,82212 | =1,88495 | =1,94779 | =2,01652 | =2,07345 | =2,13628 | =2,19911 |
| 4,2 | 4,41 | 8,035 | 8,313 | 8,590 | 8,867 | 9,144 | 9,421 | 9,698 |
| 4,0 | 4,00 | 7,288 | 7,543 | 7,791 | 8,042 | 8,294 | 8,545 | 8,796 |
| 3,8 | 3,61 | 6,578 | 6,805 | 7,032 | 7,258 | 7,485 | 7,712 | 7,939 |
| 3,6 | 3,24 | 5,904 | 6,107 | 6,311 | 6,514 | 6,718 | 6,922 | 7,125 |
| 3,4 | 2,89 | 5,266 | 5,448 | 5,629 | 5,811 | 5,992 | 6,174 | 6,355 |
| 3,2 | 2,56 | 4,655 | 4,825 | 4,986 | 5,147 | 5,308 | 5,469 | 5,630 |
| 3,0 | 2,25 | 4,100 | 4,241 | 4,383 | 4,524 | 4,665 | 4,807 | 4,948 |
| 2,8 | 1,96 | 3,571 | 3,695 | 3,818 | 3,941 | 4,064 | 4,187 | 4,310 |
| 2,6 | 1,69 | 3,079 | 3,186 | 3,292 | 3,398 | 3,504 | 3,610 | 3,716 |
| 2,4 | 1,44 | 2,624 | 2,714 | 2,805 | 2,895 | 2,986 | 3,076 | 3,167 |
| 2,2 | 1,21 | 2,205 | 2,281 | 2,357 | 2,433 | 2,509 | 2,585 | 2,661 |
| 2,0 | 1,00 | 1,822 | 1,885 | 1,948 | 2,011 | 2,073 | 2,136 | 2,199 |

| 長軸の 長さ(2a) (cm) | a ² | 7 2 型 | 7 4 型 | 7 6 型 | 7 8 型 | 8 0 型 | 8 2 型 | 8 4 型 |
|-----------------------|----------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| | | $\pi N \div 100$ | $\pi N \div 100$ | $\pi N \div 100$ | $\pi N \div 100$ | $\pi N \div 100$ | $\pi N \div 100$ | $\pi N \div 100$ |
| | | =2,26194 | =2,32478 | =2,38761 | =2,45044 | =2,51327 | =2,57610 | =2,63894 |
| 4,2 | 4,41 | 9,775 | 10,252 | 10,529 | 10,806 | 11,084 | 11,361 | 11,638 |
| 4,0 | 4,00 | 9,048 | 9,299 | 9,550 | 9,802 | 10,053 | 10,304 | 10,556 |
| 3,8 | 3,61 | 8,166 | 8,392 | 8,619 | 8,846 | 9,073 | 9,300 | 9,527 |
| 3,6 | 3,24 | 7,329 | 7,532 | 7,736 | 7,939 | 8,143 | 8,347 | 8,550 |
| 3,4 | 2,89 | 6,537 | 6,719 | 6,900 | 7,082 | 7,263 | 7,445 | 7,627 |
| 3,2 | 2,56 | 5,791 | 5,951 | 6,112 | 6,273 | 6,434 | 6,595 | 6,756 |
| 3,0 | 2,25 | 5,089 | 5,231 | 5,372 | 5,513 | 5,655 | 5,796 | 5,938 |
| 2,8 | 1,96 | 4,433 | 4,557 | 4,680 | 4,803 | 4,926 | 5,049 | 5,172 |
| 2,6 | 1,69 | 3,823 | 3,929 | 4,035 | 4,141 | 4,247 | 4,354 | 4,460 |
| 2,4 | 1,44 | 3,257 | 3,348 | 3,438 | 3,529 | 3,619 | 3,710 | 3,800 |
| 2,2 | 1,21 | 2,737 | 2,813 | 2,889 | 2,965 | 3,041 | 3,117 | 3,193 |
| 2,0 | 1,00 | 2,262 | 2,325 | 2,388 | 2,450 | 2,512 | 2,576 | 2,639 |

Fig. 9



Oval of CASSINI の長軸を含む斷面積と橢圓面積との比較を graph にて表せば次の如し。

Tig. 10

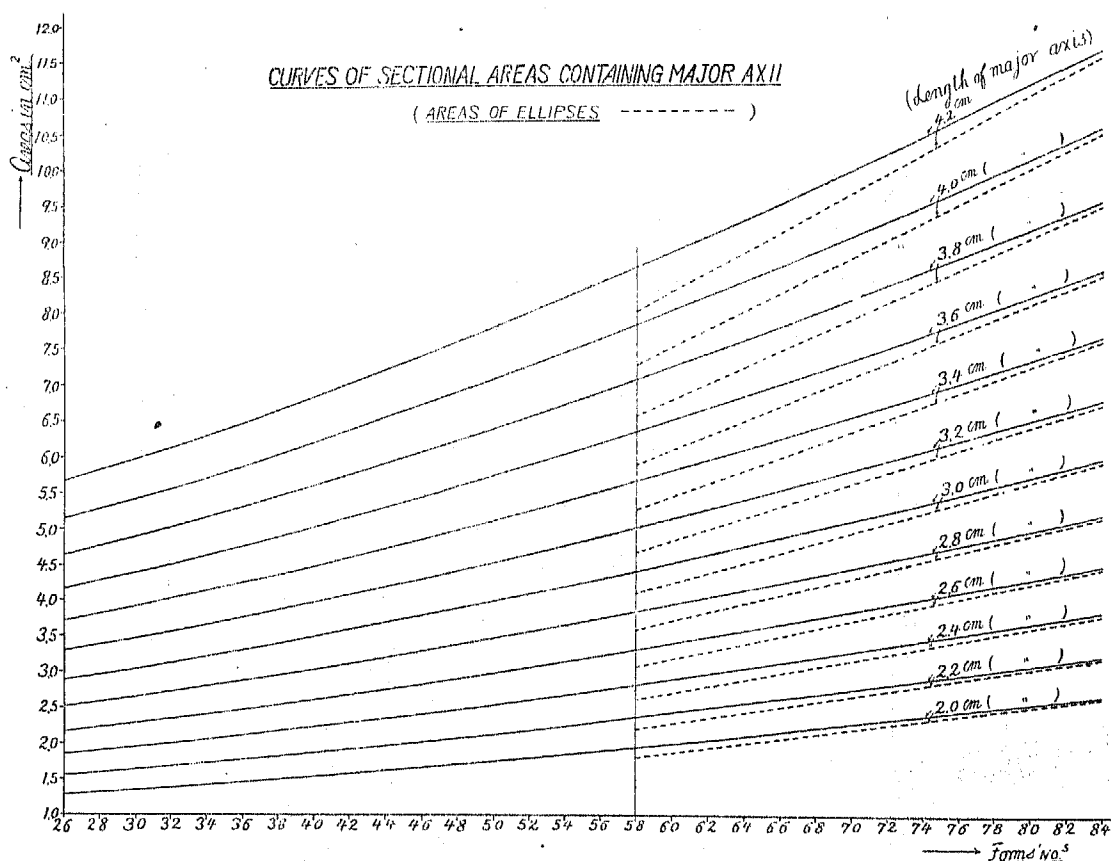


Fig. 10 に見る如く橢圓面積の graph は各々一直線をなす。それは式からも明かな如く、
 $S' = \pi a^2 \cdot \frac{N}{100}$ に於て長軸の長さ $2a$ が一定だま $S' \propto N$ なるからである。(未完)

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