Utilities of Scalar Masses in Chain Breaking Scenarios

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We discuss the utilities of scalar masses based on chain breaking scenarios consistent with the present LEP data within supersymmetric grand unified theories.

§ 1. Introduction

The supersymmetric grand unified theory (SUSY-GUT)\(^1\) has shown promise as a realistic attempt to go beyond the standard model. In fact, the precision measurements at LEP\(^2\) have shown that the gauge coupling constants \(g_3, g_2\), and \(g_1\) of the 'standard model gauge group' \(G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y\) meet at about \(10^{16}\) GeV within the framework of the minimal supersymmetric standard model (MSSM).\(^3\) SUSY SU(5) GUT is the simplest unification scenario and predicts the long lifetime of nucleons consistent with the present data.\(^4\)

However, various unification scenarios consistent with the LEP data have been known within SUSY-GUTs, for example, the direct breaking of the larger group down to \(G_{SM}\) and the models with extra heavy generations. Non-trivial examples are the models of SUSY SO(10) GUT with chain breaking.\(^5\)-\(^7\) The physics at intermediate scales has recently generated a great deal of interest. For example, the energy scale around \(10^{12}\) GeV is interesting for the neutrino masses\(^8\) since an interesting solution for the solar neutrino problem\(^9\) and the dark matter problem\(^10\) exist here. So it is important to check the existence of scale by experiment.

It is difficult to distinguish the gauge coupling unification scenarios and/or to show the existence of the intermediate scale by the use of the precise measurements of gauge couplings alone. It has been pointed out that the scalar mass spectrum is useful to select gauge symmetry breaking patterns.\(^6\),\(^11\) But there are some cases in which the previous analyses do not necessarily give powerful results. (1) For models with relatively small groups such as \(SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}\), there is no non-trivial scalar mass relation which is useful to check the breaking pattern. (2) The same type of scalar mass relations hold for the different chain breaking patterns and are not useful enough for the complete selection. These originate from the fact that we specified only the assignments of usual matter and weak Higgs doublets under the nearest gauge group \(G_{SB}\) above the weak scale and used the group theoretical method. Hence we need some new constraints on the physics above an intermediate scale.

In this paper, we take up the above cases (1) and (2), and discuss the utilities of scalar masses by adopting a kind of gauge coupling unification scenario as a constraint.
§ 2. Unification scenarios and scalar masses

Now we shall give a brief review of the ordinary unification scenario. The renormalization group equations (RGEs) of gauge couplings \( g_i \) are given as

\[
\frac{d}{d\mu} \frac{d}{d\mu} a_i^{-1}(\mu) = -\frac{b_i}{2\pi}, \quad a_i \equiv \frac{g_i^2}{4\pi}
\]

at the one-loop level. Here the \( b_i \)'s are the coefficients of the beta function. If the particle contents of MSSM and the precise measurements at LEP are used, the structure constants \( a_5, a_2 \) and \( a_1 \) \((=5/3a_Y)\) of \( G_{SM} \) meet at the scale

\[ M_X \sim 2.1 \times 10^{16} \text{GeV} \]

and the value of the unified structure constant at \( M_X \) is

\[ a_{GUT} \approx a_i(M_X) \sim \frac{1}{24.6}. \quad (i=1, 2, 3) \]

This fact suggests that \( G_{SM} \) can be unified in the \( SU(5) \) gauge group economically. We call the above scenario the 'minimal unification scenario'.

However, there exist various unification scenarios which cannot be distinguished by the use of the LEP data if the concept of 'simplicity' is put aside. The scalar mass spectrum is useful to select gauge symmetry breaking patterns, but the previous analyses were not powerful enough to specify SUSY-GUTs completely, because only the assignments of matter and Higgs doublets under \( G_{SM} \) were specified. We shall discuss cases (1) and (2) in order.

2.1. First case

First we study the first case by taking SUSY \( SO(10) \) model with the following breaking pattern:

\[
SO(10) \xrightarrow{M_U} G_{3211} \equiv SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \xrightarrow{M_R} G_{SM}
\]

as an example. This case has been examined by Deshpande et al.\(^5\) They assumed that the physics below \( M_R \) is described by MSSM, and impose the relation

\[ b_{sc} = b_3, \quad b_{2L} = b_2, \quad b_1 = b_1, \]

where \( b_1 \equiv (2/5)b_{B-L}+(3/5)b_{2R} \). Then the unification scale \( M_U \) and the unified structure constant \( a_U \) agree with \( M_X \) and \( a_{GUT} \) in the 'minimal unification scenario'. An example of the particle contents is

\[
3\{(3, 2, 1, -\frac{1}{2\sqrt{6}})+(1, 2, 1, \frac{3}{2\sqrt{6}})+(\bar{3}, 1, 2, \frac{1}{2\sqrt{6}})+(4, 1, 2, 0)\}
\]

for matter multiplets,

\[
(8, 1, 1, 0)+(1, 3, 1, 0)+(1, 1, 3, 0)+(1, 1, 1, 0)
\]

for gauge multiplets,
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\[(1, 2, 2, 0) + 3\left(\left(1, 1, 2, \frac{3}{2\sqrt{6}}\right) + \left(1, 1, 2, -\frac{3}{2\sqrt{6}}\right)\right) \text{ for Higgs multiplets}\]

under \(G_{321}\). The particles not included in MSSM are assumed to have the mass of \(O(M_R)\). Note that the breaking scale \(M_R\) remains a free parameter in their scenario.

As a sufficient condition on the gauge coupling unification, we impose the more general condition than (5)

\[\Delta b = b_{3c} - b_3 = b_{2L} - b_2 = \hat{b}_1 - b_1 . \tag{6}\]

Bando et al.\(^7\) have analyzed SUSY SO(10) model by demanding that the intermediate scale \(M_R\) is approximately \(10^{12}\) GeV. Their result is summarized as follows. Only the models with the breaking pattern (4) are possible candidates when assuming that there is at least one Higgs multiplet to give right-handed neutrinos the masses of \(O(M_R)\) under some reasonable conditions. Their solutions (i) and (ii) satisfy condition (6). In their scenario, \(M_R\) is also a free parameter, and it is treated as an input parameter.

It is important to check the existence of \(M_R\) and to know its experimental value because this information can be important in regard to neutrino physics. The precise measurements of gauge couplings and gaugino masses will not give any useful information. In addition, the sfermion masses have been studied,\(^6\) but no useful relation has been derived. We shall re-examine the question of whether or not the scalar masses are useful. This time we use the Higgs doublet masses as well as sfermion masses and impose the relation (6). Then all \(m^2_{l(a)}\) are not necessarily independent quantities. (For the RGEs of \(m^2_{l(a)}\), see the Appendix.) Hence we expect that there is a non-trivial relation.

Using Eqs. (35) and (36), the scalar masses at \(M_R\) are calculated as follows,*

\[m_q^2 = m_{q_6}^2 - \frac{4}{3} \tilde{M}^2_{3c} - \frac{3}{4} \tilde{M}^2_{2L} - \frac{1}{24} \tilde{M}^2_{3-L} + g^2_{3-L} D , \tag{7}\]

\[m_l^2 = m_{l_6}^2 - \frac{3}{4} \tilde{M}^2_{2L} - \frac{3}{8} \tilde{M}^2_{3-L} - 3 g^2_{3-L} D , \tag{8}\]

\[m_d^2 = m_{d_6}^2 - \frac{4}{3} \tilde{M}^2_{3c} - \frac{3}{4} \tilde{M}^2_{2R} - \frac{1}{24} \tilde{M}^2_{3-L} - (g^2_{3-L} + 2 g^2_{2R}) D , \tag{9}\]

\[m_u^2 = m_{u_6}^2 - \frac{4}{3} \tilde{M}^2_{3c} - \frac{3}{4} \tilde{M}^2_{2R} - \frac{1}{24} \tilde{M}^2_{3-L} - (g^2_{3-L} - 2 g^2_{2R}) D , \tag{10}\]

\[m_e^2 = m_{e_6}^2 - \frac{3}{4} \tilde{M}^2_{2R} - \frac{3}{8} \tilde{M}^2_{3-L} + (3 g^2_{3-L} - 2 g^2_{2R}) D , \tag{11}\]

\[m_{l_1}^2 = m_{l_1(1,2)}^2 - 2 g^2_{2R} D , \tag{12}\]

\[m_{l_2}^2 = m_{l_1(1,2)}^2 + 2 g^2_{2R} D . \tag{13}\]

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* We refer to the chiral multiplets as \(q\) for left-handed quarks, \(l\) left-handed leptons, \(u\) right-handed ups, \(d\) right-handed downs and \(e\) for right-handed charged leptons. The tilde represents their scalar components. \(m_q^2\) and \(m_l^2\) stand for the soft SUSY breaking mass terms of the Higgs bosons with hypercharge \(-1/2\) and \(+1/2\), respectively.
where $\tilde{M}_3^2(\mu) = (2/b_1)(M_{SO(10)} - M_3^2(\mu))$, and $M_{SO(10)}$ is an $SO(10)$ gaugino mass. Here we used the relation $S_{B-L} = 0$ (See Eq. (34) for the definition of $S_{B-L}$) and the assumption that the Higgs doublets belong to the representation $(1, 2, 2, 0)$ under $G_{221}$. The last terms of the above equations represent the $D$-term contribution to scalar masses on the symmetry breaking of $G_R$. One can show that sizable $D$-term contributions generally exist when the soft SUSY breaking terms in the scalar potential are non-universal, and the rank of the group is reduced due to the gauge symmetry breakings. As we impose the relation (6), we have six unknown parameters, for example, $m_{16}$, $m_{(1,2,2)}$, $b_{3c}$, $b_{B-L}$, $M_R$ and $D$. Because we have seven observables, there must exist one non-trivial relation. In fact, the following four relations are derived:

$$m_1^2 - m_2^2 = m_a^2 - m_a^2,$$

$$g_{B-L}^2 (m_1^2 - m_d^2 + m_d^2 - m_e^2) = 2 g_{B-L}^2 (m_d^2 - m_a^2),$$

$$m_1^2 - m_a^2 = m_d^2 - m_e^2 + 8/3 \tilde{M}_{3c} - 2/3 \tilde{M}_{B-L}^2,$$

$$m_1^2 - m_d^2 + m_d^2 - m_e^2 = 2(m_a^2 - m_d^2) - 3 \tilde{M}_{3c}^2 + 3 \tilde{M}_{B-L}^2,$$

by eliminating the parameters $m_{16}$, $m_{(1,2,2)}$ and $D$. The relation (14) can be used to determine the breaking scale $M_R$. And we can check the scenario by examining whether the remaining relations are consistent or not at the scale $M_R$. So we can check the scenario by the precise measurements of scalar masses.

### 2.2. Second case

Next we study the second case by taking SUSY $E_8$ model. For the following three breaking patterns:

(1) $E_8 \xrightarrow{M_U} SU(5)_F \times U(1)_2 \times U(1)_1 \xrightarrow{M_{SB}} G_{SM},$ \hspace{1cm} (18)

(II) $E_8 \xrightarrow{M_U} SU(6) \times SU(2)_I \xrightarrow{M_{SB}} G_{SM}$ \hspace{1cm} (19)

and

(III) $E_8 \xrightarrow{M_U} SU(5)_F \times U(1)_Y \times SU(2)_I \xrightarrow{M_{SB}} G_{SM},$ \hspace{1cm} (20)

the same type of scalar mass relation

$$m_a^2(M_{SB}) - m_a^2(M_{SB}) = m_a^2(M_{SB}) - m_i^2(M_{SB})$$

is derived. Here the gauge group $SU(5)_F \times U(1)_2$ corresponds to that of the flipped $SU(5)$ model.\(^{15}\) The gauge group $SU(5)_F \times U(1)_Y \times SU(2)_I$ is one of subgroups of $SU(6) \times SU(2)_I$. We shall show that they can be discriminated by imposing a kind of condition on the $b_i$'s.

For the breaking pattern (I), it turns out that the breaking scale $M_{SB}$ agrees with the unification scale $M_X$ if we postulate that the physics below $M_{SB}$ is described by

\(^{15}\) Historically, it was demonstrated that the $D$-term contribution occurs when the gauge symmetry is broken at an intermediate scale due to the soft SUSY breaking terms in Refs. 13, and its existence in a more general situation was suggested in Ref. 14.)
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MSSM. There exists a relation among the structure constants at $M_X$ such as

$$\frac{25}{\alpha_i} = \frac{1}{\alpha_{SF}} + \frac{24}{\alpha_{i(2)}},$$

(22)

where the structure constants of $SU(5)_F$ and $U(1)_2$ are denoted as $\alpha_{SF}$ and $\alpha_{i(2)}$, respectively. From the LEP data, the relation $\alpha_{SF}(M_X) = \alpha_i(M_X)$ holds and so the accidental relation $\alpha_{SF}(M_X) = \alpha_{i(2)}(M_X)$ is derived. On the other hand, the relation $\alpha_{\text{GUT}} = \alpha_{SF}(M_U) = \alpha_{i(2)}(M_U)$ holds because of $E_6$ gauge symmetry. We can take a scenario with the relation $\alpha_{SF}(\mu) = \alpha_{i(2)}(\mu)$ from $M_U$ to $M_X$ as the simplest one. In this case, we have a condition $b_5 = b_{1(2)}$ from $M_U$ to $M_X$. It is a difficult problem whether there is a realistic model which satisfies the condition $b_5 = b_{1(2)}$ or not. We need to check that only MSSM particles have masses of $O(1)$ TeV in this breaking. We will not go into this issue which is beyond the scope of this paper.

By the use of Eqs. (35) and (36), the scalar masses at $M_X$ are given as

$$m_{u'}^2 = m_{\tilde{u}'}^2 + \frac{12}{5} \tilde{M}_{u'}^2 + \frac{9}{40} \tilde{M}_{i(2)}^2 + \frac{1}{24} \tilde{M}_{i(1)}^2 + D_1 + (8 g_F^2 - 3 g_{i(2)}^2) D_F,$$

(23)

$$m_{t'}^2 = m_{\tilde{t}'}^2 + \frac{12}{5} \tilde{M}_{t'}^2 + \frac{9}{40} \tilde{M}_{i(2)}^2 + \frac{1}{24} \tilde{M}_{i(1)}^2 + D_1 - (12 g_F^2 + 3 g_{i(2)}^2) D_F,$$

(24)

$$m_{q'}^2 = m_{\tilde{q}'}^2 + \frac{18}{5} \tilde{M}_{q'}^2 + \frac{1}{40} \tilde{M}_{i(2)}^2 + \frac{1}{24} \tilde{M}_{i(1)}^2 + D_1 + (4 g_F^2 + g_{i(2)}^2) D_F,$$

(25)

$$m_{d'}^2 = m_{\tilde{d}'}^2 + \frac{18}{5} \tilde{M}_{d'}^2 + \frac{1}{40} \tilde{M}_{i(2)}^2 + \frac{1}{24} \tilde{M}_{i(1)}^2 + D_1 - (16 g_F^2 - g_{i(2)}^2) D_F,$$

(26)

$$m_{s'}^2 = m_{\tilde{s}'}^2 + \frac{5}{8} \tilde{M}_{s'}^2 + \frac{1}{24} \tilde{M}_{i(1)}^2 + 5 g_{i(2)}^2 D_F,$$

(27)

$$m_{l'}^2 = m_{\tilde{l}'}^2 - 2 D_1 - (12 g_F^2 - 2 g_{i(2)}^2) D_F,$$

(28)

$$m_{e'}^2 = m_{\tilde{e}'}^2 - 2 D_1 - (12 g_F^2 + 2 g_{i(2)}^2) D_F,$$

(29)

where $\tilde{M}_{i(\mu)} = (2/b_i)(M_{s_{\mu}} - M_{i(\mu)})$ and $M_{s_{\mu}}$ is an $E_6$ gaugino mass. Here we also used the relation $S_{i(2)}(M_U) = S_{i(2)}(M_X) = 0$ and the assumption that the Higgs doublets with hypercharge 1/2 and $-1/2$ belong to 5 and $\bar{5}$ under $SU(5)_F$, respectively. By eliminating the unknown parameters, we get the following relations,

$$m_{\tilde{q}'}^2 - m_{\tilde{d}'}^2 = m_{\tilde{u}'}^2 - m_{t'}^2$$

(30)

and

$$2(m_{\tilde{d}'}^2 - m_{t'}^2) = m_{\tilde{u}'}^2 - m_{\tilde{e}'}^2.$$

Equation (31) is a new relation.

In the same way, we can obtain specific relations among 1st and 2nd generation sfermion masses at $M_{SB}$ in the breaking patterns (II) and (III). The results are

* Here we ignore the effects of higher dimensional operators which split the values of $\alpha_{SF}(M_U)$ and $\alpha_{i(2)}(M_U)$ on the $E_6$ breaking.

** We assume that the decouplings of particles occur keeping the relation $S_{i(2)} = 0$. 
Table I. Sfermion mass relations for three $E_6$ breaking patterns (I)∼(III). (N) represents the new scalar mass relation derived based on our unification scenarios.

<table>
<thead>
<tr>
<th>$G_{SB}$</th>
<th>Condition among $b_i$</th>
<th>Sfermion Masses</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SU(5)_F \times U(1)_2 \times U(1)_1$</td>
<td>$b_{5F} = b_{1(2)}$</td>
<td>$m_{\tilde{e}}^2 - m_{\tilde{d}}^2 = m_{\tilde{u}}^2 - m_{\tilde{t}}^2$, $2(m_{\tilde{e}}^2 - m_{\tilde{t}}^2) = m_{\tilde{u}}^2 - m_{\tilde{e}}^2$ (N)</td>
</tr>
<tr>
<td>$SU(6) \times SU(2)_L$</td>
<td>$b_u = b_t$</td>
<td>$m_{\tilde{e}}^2 - m_{\tilde{u}}^2 = m_{\tilde{d}}^2 - m_{\tilde{t}}^2$, $m_{\tilde{e}}^2 = m_{\tilde{t}}^2$ (N)</td>
</tr>
<tr>
<td>$SU(5)_F \times U(1)_F \times SU(2)_L$</td>
<td>$b_{5F} = \frac{3}{8} b_{1(2)} + \frac{5}{8} b_{21}$</td>
<td>$m_{\tilde{e}}^2 - m_{\tilde{d}}^2 = m_{\tilde{u}}^2 - m_{\tilde{t}}^2$</td>
</tr>
</tbody>
</table>

summarized in Table I. Thus three types of breaking patterns (I)∼(III) can be discriminated by the scalar mass relations obtained.

§ 3. Conclusions

In conclusion, we have re-examined the scalar masses based on the chain breaking scenarios consistent with the present LEP data. For the model with chain breaking such as $SO(10) \rightarrow G_{3Z2} \rightarrow G_{SM}$, we have shown that there exists one non-trivial scalar mass relation specific to the breaking pattern. The intermediate scale $M_R$ can be determined by using this. A kind of gauge coupling unification condition among the $b_i$'s has been given for three types of $E_6$ breakings (I)∼(III), where the same type of scalar mass relation was derived in previous analyses. We have derived a new type of relations which are useful to discriminate three breaking patterns under this condition. It is believed that the measurements of soft masses can be helpful in the exploration of physics at the Planck scale, but they can also be useful in probing physics at the intermediate scale or the GUT scale if SUSY-GUT is realized in nature. Hence it is expected that the measurements of the scalar masses will be influential in the study of high energy physics in the future.

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Appendix

--- RGEs of Scalar Masses ---

We write down the RGEs$^{16}$ of scalar masses $m_{R(a)}$ above $M_{SB}$,

$$\mu \frac{d}{d\mu} m_{R(a)}(\mu) = -\frac{2}{\pi} \sum S_c(R_i^{a}) \alpha_i(\mu) M_{R_i}^2(\mu) + \frac{1}{2\pi} \sum Q_{R_i} \alpha_i(\mu) S_i(\mu), \quad (32)$$

$$\mu \frac{d}{d\mu} S_i(\mu) = \frac{b_i}{2\pi} \alpha_i(\mu) S_i(\mu), \quad (33)$$
where \(i\) runs over all the gauge groups, but \(j\) runs over only \(U(1)\) gauge groups whose charges are \(Q_{R(a)}\), the \(C_2(R^a)\)'s are the second order Casimir invariants, the \(M_{i}(\mu)\)'s are gaugino masses and \(\eta_{R(a)}\) is the multiplicity. Here we neglect the effects of Yukawa couplings. This approximation can be applied only for 1st and 2nd generation fields. It is straightforward to generalize our results to 3rd generation fields and Higgs doublets by the incorporation of the effects of Yukawa couplings. In deriving Eq. (33), we used the anomaly cancellation condition \(\sum_{R(a)} C_2(R^a) Q_R Q_{R(a)} \eta_{R(a)} = 0\,^*\) and the relation of orthgonality \(\sum_{R(a)} Q_{R(a)}^2 = b_{ij} \delta_{ij}\). The solutions of the above RGEs are given as

\[
m_{R(a)}^2(\mu) = m_T^2(\mu_0) - \frac{2}{b_1} C_2(R^a) (M_T^2(\mu) - M_T^2(\mu_0)) + \frac{1}{b_1} \sum_{R(a)} Q_{R(a)} (S_J(\mu) - S_J(\mu_0)),
\]

\[
S_J(\mu) = \frac{a_J(\mu)}{a_J(\mu_0)} S_J(\mu_0).
\]

References

12. For a review, see M. Fukugita and T. Yanagida, in Physics and Astrophysics of Neutrinos (Springer-Verlag, Tokyo, 1994).
16. For a review, see M. Fukugita and T. Yanagida, in Physics and Astrophysics of Neutrinos (Springer-Verlag, Tokyo, 1994).

\[^*\] It is supposed that the particle contents consist of the anomaly free set.