A Note on Right Locally Finite Simple Ring Extensions

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Throughout $A$ will represent an (Artinian) simple ring, $B$ a simple subring of $A$ containing $1$ of $A$, and $V$ the centralizer of $B$ in $A$. A ring extension $A'/B'$ is said to be right locally finite if for any finite subset $F'$ of $A'$ the subring $B'[F']$ is right finite over $B'$. In [1], S. Takamatsu and the second author dealt with a right locally finite extension $A/B$ such that $V$ is simple and $A = BN$ with the normalizer $N$ of $B$ in $A$, and proved that $A/BV$ is right locally finite, which played an important role in the proof of [1, Theorem]. In this note, we shall prove the same without any restriction.

Theorem. If $A/B$ is right locally finite, then so is $A/BV$.

Proof. Let $F$ be an arbitrary finite subset of $A$, and choose an intermediate ring $B'$ of $A/B[F]$ such that $A/B'$ is irreducible and the right rank $[B':B]_R$ is finite. Then by [2, Proposition 5.4 (b)] the centralizer $V'$ of $B'$ in $A$ is a division ring and $m = [V : V']_R \leq [B : B]_R$. Let $\{v_1, v_2, \ldots, v_m\}$ be a right $V'$-basis of $V$ and set $B'' = B[F, v_1, \ldots, v_m] = \sum_{j=1}^{v} b_j B$. Since every element of $V'$ commutes with all the elements of $B[F]$, we see that $B''V' \supset V'B''$, namely, $B''V'$ is a subring of $A$. Hence, $(BV)[F] = B''V' = \sum_{j=1}^{v} b'_j (BV)$, which proves the right local finiteness of $A/BV$.

References