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An Explicit Formula of the Solution of Constant Coefficients Partial Differential Equation with the Meromorphic Cauchy Data

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The analytic Cauch problem with the meromorphic data has been studied dy Hamade ([2] cf. [3]). In this note we give an explicit formula of the solution of constant coefficients equation with the meromorphic data by means of the extended Borel transformation ([1]).

In [1], we give an explicit formula of the solution of constant coefficients partial differential equation $P\left(\frac{\partial}{\partial\zeta}\right)u=0$ with the Cauchy data

$$\frac{\partial^k u}{\partial{\zeta_1}^k}(0,\ \zeta_2,\cdots,\zeta_n)=g_{k+1}(\zeta_2,\cdots,\zeta_n),\quad 0\leqq k\leqq m-1\quad ,$$

in the form

$$u(\zeta)=\mathscr{B}[<(1-z_1\sigma(z_2^{-1},\cdots,z_n^{-1}))^{-r},\ T\binom{r_1,\cdots r_s}{\sigma_1,\cdots,\sigma_s}^{-1}\mathscr{B}^{-1}[g]>](\zeta)\quad , \tag{1}$$

where $P(z)=\Pi_{i=1}^{s}(z_1-\sigma_i(z_2,\cdots,z_n))^{r_i}$ and $(1-z_1\sigma(z_2^{-1},\cdots,z_n^{-1}))^{-r}$ and $\mathscr{B}^{-1}[g]$ are vectors such that

$$\begin{aligned}&(1-z_1\sigma(z_2^{-1},\cdots,z_n^{-1}))^{-r}\\ &=((1-z_1\sigma_1(z_2^{-1},\cdots,z_n^{-1}))^{-1},\ (1-z_1\sigma_1(z_2^{-1},\cdots,z_n^{-1}))^{-2},\cdots,\\ &\quad(1-z_1\sigma_1(z_2^{-1},\cdots,z_n^{-1}))^{r_1},\ (1-z_1\sigma_2(z_2^{-1},\cdots,z_n^{-1}))^{-1},\cdots,\\ &\quad(1-z_1\sigma_s(z_2^{-1},\cdots,z_n^{-1}))^{-r_s}),\\ &\quad\mathscr{B}^{-1}[g]=(\mathscr{B}^{-1}[g_1],\cdots,\ \mathscr{B}^{-1}[g_m])\ ,\end{aligned}$$

and $<F,\ G>=\sum F_iG_i,\ \ F=(F_1,\cdots,F_m),\ \ G=(G_1,\cdots,G_m)$.

On the other hand, in [1], we also show that to define

(2) $$\mathscr{B}[\log z](\zeta) = \log\zeta + \gamma, \quad \gamma \textit{ is Euler's constant},$$

$\mathscr{B}[\log z]$ is well defined and most of the properties of Borel transformation is preserved. Espescially, by (2), we get

(3) $$\mathscr{B}[z^t](\zeta) = \frac{1}{\Gamma(1+t)}\zeta^t, \quad t \neq \textit{negative integer},$$

(4) $$\mathscr{B}[z^{-n}\log z](\zeta) = (-1)^{n-1}(n-1)!\,\zeta^{-n}, \quad n \geqq 1 .$$

By (4), we get

(5) $$\zeta_{i_1}^{-m_1}\cdots\zeta_{i_s}^{-m_s}\zeta_{j_1}^{\alpha_1}\cdots\zeta_{j_{n-s}}^{\alpha_{n-s}}$$

$$= \mathscr{B}\Big[\frac{(-1)^{m_1+\cdots+m_s-s}}{(m_1-1)!\cdots(m_s-1)!}\Gamma(1+\alpha_1)\cdots\Gamma(1+\alpha_{n-s})z_{i_1}{}^{-m_1}\log z_{i_1}\cdots$$

$$\cdot z_{i_s}{}^{-m_s}\log z_{i_s}\cdot z_{j_1}{}^{\alpha_1}\cdots z_{i_{n-s}}{}^{\alpha_{n-s}}\Big](\zeta),$$

$m_1 \geqq 1, \cdots, m_s \geqq 1, \alpha_1, \cdots, \alpha_{n-s}$ *are not negative integers.*

Hence, since any element of $\widetilde{\mathscr{M}}^n$ can be expressed as a Puiseax series (cf. [1]), $\mathscr{B}^{-1}$ is defined on $\widetilde{\mathscr{M}}^n$. Therefore, (1) also express the solution of $P\left(\frac{\partial}{\partial\zeta}\right)u = 0$ with the meromorphic Cauchy data.

Similarly, we obtain the solution of $P\left(\frac{\partial}{\partial\zeta}\right)u = f$ for $f \in \widetilde{\mathscr{M}}^n$. For example, for $f = 1/\zeta_1\cdots\zeta_n$, we get

(6) $$u(\zeta) = \Big[\frac{1}{P(z_1{}^{-1}, \cdots, z_n{}^{-1})}\,\frac{\log z_1\cdots\log z_n}{z_1\cdots z_n}\Big](\zeta) .$$

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