On Improving the Tradeoff between Symbol Rate and Diversity Gain Using Quasi-Orthogonal Space-Time Block Codes with Linear Receivers

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SUMMARY In this letter, the tradeoff between symbol rate and diversity gain of Space-Time Block Codes (STBCs) with linear receivers is considered. It is known that Group Orthogonal-Toeplitz Codes (GOTCs) can achieve a good tradeoff with linear receivers. However, the symbol rate of GOTCs is limited to that of the base Orthogonal Space-Time Block Codes (OSTBCs). We propose to simply change the GOTC base codes from OSTBCs to Quasi-Orthogonal Space-Time Block Codes (Q-OSTBCs). Q-OSTBCs can improve the symbol rate of GOTCs at the expense of diversity gain. Simulation results show that Q-OSTBC based GOTCs can improve the tradeoff between symbol rate and diversity gain over that of the original GOTCs.

key words: Orthogonal Space-Time Block Codes, Quasi-Orthogonal Space-Time Block Codes, Group Orthogonal-Toeplitz Codes

1. Introduction

Recently, Multi Input Multi Output (MIMO) systems have received much attention in mobile communication system research. MIMO systems can increase channel capacity by sending parallel data on multiple paths between the transmitter and receiver [1]. Transmitting data in parallel to increase channel capacity is called spatial multiplexing. On the other hand, spatial diversity achieves coding gain by sending the same data via multiple antennae and combining them at the receiver [2]–[5].

There are tradeoffs between spatial multiplexing and spatial diversity. Zheng and Tse [6] studied the optimal tradeoff between spatial multiplexing and spatial diversity and showed an optimal tradeoff curve. Also, Gamal extended Linear Dispersion (LD) codes [7] and proposed Lattice Space Time (LAST) codes [8] which can achieve an optimal tradeoff. However, these techniques use a non-linear Maximum Likelihood (ML) receiver. In mobile terminals, linear receivers such as Zero-Forcing (ZF) and Minimum Mean Square Error (MMSE) are preferred because of battery life limitations. In this letter, we consider the tradeoff between symbol rate and diversity gain of Space Time Block Codes (STBCs) with linear receivers.

Orthogonal Space-Time Block Codes (OSTBCs) were originally proposed by Alamouti [2] and extended to the general case by Tarokh et al. [3]–[5]. OSTBCs can achieve full diversity and can be decoded using maximum likelihood detection with linear receivers by using the orthogonality of the code. OSTBCs have such a good property but their symbol rate is limited to \((k + 1)/2k\), where \(2k - 1\) or \(2k\) represents the number of transmit antennae [9]. Also, OSTBC can achieve full rate and full diversity at the same time only when the number of transmit antennae is two [10].

So, STBCs which have a good tradeoff between symbol rate and diversity gain with linear receivers have received much attention [11]–[13]. Liu et al. [11] proposed Toeplitz Codes which improve the symbol rate of STBC with linear receivers. Shang and Xia [12] improved the symbol rate and diversity gain by combining Toeplitz Codes and Alamouti’s OSTBC. Also, Wang et al. [13] extended their approach and proposed a construction method of STBCs combining Toeplitz Codes and general OSTBCs to improve the tradeoff. The codes proposed in [13] are called Group Orthogonal-Toeplitz Codes (GOTCs) including OSTBC [2]–[5], Toeplitz Codes [11], and Overlapped Alamouti Codes (OACs) [12] as special cases. However, the symbol rate of GOTCs is limited to that of the base OSTBCs which is used when it is generated. In this letter, we propose to simply change the GOTC base codes from OSTBCs to Quasi-Orthogonal Space-Time Block Codes (Q-OSTBCs) [14] to improve the symbol rate of GOTCs.

This letter is organized as follows. In Sect. 2, we apply Quasi-Orthogonal Space-Time Block Codes [14] to GOTCs. Simulation results and some comments about the tradeoff between symbol rate and diversity gain are presented in Sect. 3. Finally, we conclude this letter in Sect. 4.

2. Proposed Method

The system model, assumptions, definitions and notations used here are the same as [13]. To construct GOTCs, \(N_t\) transmit antennae are divided into \(K\) groups, and the number in group \(k\) is \(n_k\). Hence, \(\sum_{k=1}^{K} n_k = N_t\) is satisfied. Also, the \(L\) transmitted symbols are divided into groups of \(K^*\) symbols. Then, as mentioned in [13], the symbol rate of GOTCs is described as follows.

\[
R_s = \frac{L}{T_o(L/K^* + n_1 - 1)}
\]
where, $T_o$ is the length of the time slot of the base OSTBC, $O_{K',K}$. Here, $O_{K',K}$ represents an OSTBC which sends $K$ symbols using $K$ transmit antennas. From Eq. (1), $R_s = K/T_o$, when $L$ is sufficiently large. This is the same as the symbol rate of the base OSTBC, $O_{K',K}$. Hence, the symbol rate of GOTCs is limited to the symbol rate of the base OSTBC. In this letter, we denote a GOTC as GOTC($N_{K,K}$, $O_{K,K}$. Hence, the symbol rate of GOTCs is limited to the symbol rate of the base OSTBC. For example, to divide $N_t = 8$ transmit antennas into $K = 4$ groups, we can use grouping schemes such that $N_{8,4} = (2, 2, 2, 2)$, $N_{8,4} = (1, 2, 2, 3)$ and so on. The detailed construction method of GOTCs is omitted here for want of space, but is described in the original paper [13].

The main idea of this letter is to replace the base OSTBCs with Quasi-Orthogonal Space-Time Block Codes (Q-OSTBCs) to improve the symbol rate of the original GOTCs. Q-OSTBCs can improve the symbol rate at the expense of the orthogonality of the code [14]. It is pointed out in [13] that Q-OSTBC also has Group Orthogonality but it cannot achieve full diversity. However, the possibility to improve the tradeoff at the expense of diversity gain is not considered in [13]. The purpose of this letter is to improve the symbol rate using Q-OSTBCs at the expense of diversity gain and determine the parameters which achieve a better tradeoff. In this letter, we represent Q-OSTBCs which transmit $K$ symbols with $K$ transmit antennas as $Q_{K,K}$. We also represent GOTCs generated by $Q_{K,K}$ as GOTC($N_{K,K}$, $Q_{K,K}$). A Q-OSTBC which transmits 4 symbols in 4 time slots with 4 transmit antennas is described as follows [14].

$$Q_{4,4} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ -x_3^* & -x_4^* & x_1^* & x_2^* \\ x_4 & -x_3 & -x_2 & x_1 \end{bmatrix}. \quad (2)$$

Then, $\tilde{X}_{N_t,4} = \text{GOTC}(N_{8,4}, Q_{4,4})$ becomes as follows instead of Eq. (11) in [13].

$$\tilde{X}_{N_t,4} = \begin{bmatrix} S_{r,1}^{(1)} & S_{r,2}^{(2)} & S_{r,3}^{(3)} & S_{r,4}^{(4)} \\ -S_{r,2}^{(1)} & S_{r,1}^{(2)} & -S_{r,3}^{(3)} & S_{r,4}^{(4)} \\ -S_{r,3}^{(1)} & -S_{r,2}^{(2)} & S_{r,1}^{(3)} & S_{r,4}^{(4)} \\ S_{r,4}^{(1)} & -S_{r,3}^{(2)} & -S_{r,2}^{(3)} & S_{r,1}^{(4)} \end{bmatrix}. \quad (3)$$

Furthermore, when the grouping scheme is $N_{8,4} = (2, 2, 2, 2)$, the code becomes

$$\tilde{X}_{N_t,4} = \begin{bmatrix} s_1 & 0 & s_3 & 0 & s_5 & 0 & s_7 & 0 \\ s_2 & s_1 & s_4 & s_3 & s_6 & s_5 & s_8 & s_7 \\ 0 & s_2 & 0 & s_4 & 0 & s_6 & 0 & s_8 \\ -s_4 & s_2 & -s_8 & 0 & s_8 & s_0 & 0 & s_0 \\ -s_3 & -s_5 & s_1 & -s_2 & -s_7 & s_8 & s_5 & s_6 \\ 0 & -s_3 & 0 & s_1 & 0 & -s_2 & 0 & s_8^* \\ -s_4 & -s_8^* & 0 & -s_7 & 0 & s_8 & s_5 & s_0 \\ -s_5 & s_8^* & -s_6 & -s_4 & -s_3 & s_7 & s_8 & s_2^* \\ 0 & -s_5 & 0 & -s_7 & 0 & s_3 & 0 & s_2^* \\ s_7 & 0 & -s_5 & 0 & -s_3 & 0 & s_1 & 0 \\ s_8 & s_7 & -s_6 & -s_4 & -s_3 & s_2 & s_1 & 0 \\ 0 & s_8 & 0 & -s_6 & 0 & -s_4 & 0 & s_2 \end{bmatrix}. \quad (4)$$

The symbol rate of GOTC($N_{8,4}$, $Q_{4,4}$) can be described as Eq. (1) which is the same as that of GOTC($N_{8,4}$, $O_{4,4}$). So, if $L$ is the same, the symbol rate of GOTC($N_{8,4}$, $Q_{4,4}$) becomes higher than that of GOTC($N_{8,4}$, $O_{4,4}$), because $K'$ is larger than that of $O_{4,4}$. Similarly, GOTC($N_{8,4}$, $Q_{4,4}$) needs a smaller value of $L$ to achieve the same symbol rate as GOTC($N_{8,4}$, $O_{4,4}$). This means the decoding delay is small and it has tolerance to channel conditions. A comparison of the symbol rate for $O_{3,4}$, $Q_{4,4}$, Toelplitz Codes [11], and Overlapped Alamouti Codes (OAC) [12] are described in Table 1. Table 1 shows that a higher symbol rate can be achieved if we use Q-OSTBC as the base code. However, we must consider the loss in diversity gain caused by improving the symbol rate, because there is a tradeoff between symbol rate and diversity gain. In the next section, we evaluate the performance for the same frequency efficiency by computer simulations and show that GOTCs with Q-OSTBCs can improve the symbol rate without much loss in performance.

### 3. Simulation Results

In this section, we compare the performance of Q-OSTBC based GOTCs with original GOTCs. We use GOTC($N_{8,2}$, $O_{2,2}$) instead of OAC, because OAC and GOTC($N_{8,2}$, $O_{2,2}$) are essentially the same as mentioned in [13]. We use Alamouti [2] codes as $O_{2,2}$ and we use Eqs. (7) and (8) in [13] as $O_{3,4}$ and $O_{4,4}$ respectively. Also, we use Eq. (2) in this letter as $Q_{4,4}$. A systematic approach to construct OSTBC with an arbitrary number of transmit antennas is described in [9]. We use $B_8^*$ in [9] as OSTBC with 8 transmit antennas. To evaluate fairly, we compare the performance under the same frequency efficiency, and we consider both low and high frequency efficiency cases. We also assume a quasi-flat fading environment and use gray coding. Finally, a Zero Forcing (ZF) receiver is used as the linear receiver.

Figure 1 shows the bit error probability when the number of transmit antenna and receive antenna are 8 and 1 respectively, and the frequency efficiency is 2.5 bit/s/Hz. Also, Fig. 2 shows the result when the number of transmit and receive antenna are the same as above, and the frequency efficiency is 5.0 bit/s/Hz. The parameters used here are shown in Table 2. For example, the frequency efficiency of OSTBC with 16QAM is 2.5 bit/s/Hz because the symbol rate of OSTBC is 0.625 from Table 2 and 16QAM has 4 bits/symbol. So, the frequency efficiency is $4 \times 0.625 = 2.5$ bit/s/Hz. We represent OSTBC based
Fig. 1 An average bit error rate comparison of the proposed Quasi-OSTBC based GOTCs (GOTC-Q) with OSTBC based GOTCs [13], OSTBC [3] and Toeplitz code [11] at an efficiency of 2.5 bit/s/Hz. The number of Tx/Rx antennae are 8 and 1 respectively.

Fig. 2 An average bit error rate comparison of the proposed Quasi-OSTBC based GOTCs (GOTC-Q) with OSTBC based GOTCs [13], OSTBC [3] and Toeplitz code [11] at an efficiency of 5.0 bit/s/Hz. The number of Tx/Rx antennae are 8 and 1 respectively.

Table 2 Simulation parameters when the frequency efficiency is 2.5 bit/s/Hz and 5.0 bit/s/Hz.

<table>
<thead>
<tr>
<th></th>
<th>L</th>
<th>Rate 2.5 bit/s/Hz</th>
<th>Rate 5.0 bit/s/Hz</th>
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<tbody>
<tr>
<td>OSTBC [3]</td>
<td>-</td>
<td>0.625</td>
<td>16QAM</td>
</tr>
<tr>
<td>Toeplitz [11]</td>
<td>35</td>
<td>0.833</td>
<td>8QAM</td>
</tr>
<tr>
<td>GOTC(N_8,2, O_{2,2}) [13]</td>
<td>30</td>
<td>0.833</td>
<td>8QAM</td>
</tr>
<tr>
<td>GOTC(N_8,3, O_{3,2}) [13]</td>
<td>30</td>
<td>0.625</td>
<td>16QAM</td>
</tr>
<tr>
<td>GOTC(N_8,4, O_{4,2}) [13]</td>
<td>15</td>
<td>0.625</td>
<td>16QAM</td>
</tr>
<tr>
<td>GOTC(N_8,4, Q_{4,2})</td>
<td>20</td>
<td>0.833</td>
<td>8QAM</td>
</tr>
</tbody>
</table>

GOTC(N_{N,K}, O_{K,K}) [13] as GOTC-O(K,K), and the Q-OSTBC based GOTC(N_{N,K}, Q_{K,K}) scheme proposed in this letter as GOTC-Q(K,K) for short in the figures.

Table 3 Simulation parameters for various frequency efficiencies.

<table>
<thead>
<tr>
<th>L</th>
<th>Modulation</th>
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<tbody>
<tr>
<td>14</td>
<td>21 35 77 8QAM/64QAM</td>
</tr>
<tr>
<td>12</td>
<td>18 30 66 8QAM/64QAM</td>
</tr>
<tr>
<td>12</td>
<td>18 30 66 16QAM/256QAM</td>
</tr>
<tr>
<td>6</td>
<td>9 15 33 16QAM/256QAM</td>
</tr>
<tr>
<td>8</td>
<td>12 20 44 8QAM/64QAM</td>
</tr>
</tbody>
</table>

GOTC(N_{8,4}, O_{3,4}) outperforms that of GOTC(N_{8,4}, Q_{4,4}) when the frequency efficiency is low (2.5 bit/s/Hz). From Table 2, to achieve a frequency efficiency of 2.5 bit/s/Hz with GOTC(N_{8,4}, O_{3,4}), 16QAM modulation must be used. On the other hand, for GOTC(N_{8,4}, Q_{4,4}), 8QAM modulation is sufficient to achieve the same 2.5 bit/s/Hz. Generally, the difference in power efficiency between 8QAM and 16QAM is about 2 dB (e.g. Figure 8.13 in [15]). In this case, GOTC(N_{8,4}, O_{3,4}) has better performance, because the loss in diversity gain caused by improving the symbol rate has a larger effect on the performance than the improvement in power efficiency caused by using 8QAM rather than 16QAM. OSTBC has the best performance because the diversity gain has a larger effect on the performance in the low frequency efficiency case.

On the other hand, Fig. 2 shows that the performance of GOTC(N_{8,4}, Q_{4,4}) outperforms that of GOTC(N_{8,4}, O_{3,4}) when the frequency efficiency is high (5.0 bit/s/Hz). Similarly, Table 2 shows that 256QAM modulation must be used to achieve a frequency efficiency of 5.0 bit/s/Hz with GOTC(N_{8,4}, O_{3,4}). On the other hand, for GOTC(N_{8,4}, Q_{4,4}), 64QAM modulation is sufficient to achieve the same 5.0 bit/s/Hz. Generally, the difference in power efficiency between 64QAM and 256QAM is about 5 dB (e.g. Figure 8.13 in [15]). In this case, GOTC(N_{8,4}, Q_{4,4}) has better performance, because the improvement in power efficiency caused by using 64QAM instead of 256QAM has a larger effect than the loss in diversity gain caused by improving the symbol rate.

Also, GOTC(N_{6,4}, Q_{4,4}) has almost the same performance as GOTC(N_{8,2}, O_{2,2}) in both low and high frequency efficiency cases. This is because Q-OSTBC is generated by combining OSTBC(O_{2,2}) [14]. However, Fig. 1 and 2 show that GOTC(N_{8,4}, Q_{4,4}) has slightly better performance. For example, GOTC(N_{8,4}, Q_{4,4}) is superior by 0.13 dB than GOTC(N_{8,2}, O_{2,2}) to achieve BER = 10^{-3} when the frequency efficiency is 5.0 bit/s/Hz. This is because a larger diversity gain can be obtained when Q_{4,4} is combined with O_{2,2} and a smaller L, instead of O_{2,2} with a larger L. Also, Table 2 shows that the value of L required to achieve the same symbol rate is smaller when GOTC(N_{8,4}, Q_{4,4}) is used instead of GOTC(N_{8,2}, O_{2,2}). This means that the decoding delay is smaller when GOTC(N_{8,4}, Q_{4,4}) is used instead of GOTC(N_{8,2}, O_{2,2}).

Figure 3 shows the relationship between frequency efficiency and the SNR which achieves a bit error rate of 10^{-3}. Here, we varied L, and we evaluated various frequency ef-
ficiencies: 2.0, 2.25, 2.5, 2.75 bit/s/Hz, and 4.0, 4.5, 5.0, 5.5 bit/s/Hz. The values of $L$ used here are shown in Table 3. Also, we used $B^{s}_{t}$ in [9] as the OSTBC with 8 transmit antennae. This has the highest symbol rate (0.625) for 8 transmit antennae [9]. We could not find other OSTBCs with 8 transmit antennae in the literature. Furthermore, we used the commonly used modulation schemes 16QAM, 64QAM and 256QAM, so, the frequency efficiencies of the OSTBCs used here are 2.5 bit/s/Hz ($= 0.625 \times 4$ bits), 3.75 bit/s/Hz ($= 0.625 \times 6$ bits), and 5.0 bit/s/Hz ($= 0.625 \times 8$ bits), respectively. We could not find combinations of OSTBCs with 8 transmit antennae and modulation schemes to achieve 2.0, 2.25, 2.75, 4.0, 4.5, and 5.5 bit/s/Hz. Therefore, we show plots for only 2.5, 3.75 and 5.0 bit/s/Hz as a reference.

Figure 3 shows that the highest frequency efficiency 2.5 bit/s/Hz can be achieved when we use OSTBC at an SNR of 9 dB. Also, the highest frequency efficiency 2.75 bit/s/Hz can be achieved when we use GOTC($N_{8,3}, O_{5,3}$) at 11.6 dB. Furthermore, GOTC($N_{8,3}, Q_{4,4}$) has the best performance (5.5 bit/s/Hz) when the SNR is 18.2 dB. In this way, an automatic grouping scheme configuration can be done by measuring the received SNR and selecting the parameters to achieve the highest frequency efficiency. Here, as mentioned in [13], note that OSTBCs correspond to GOTCs with grouping scheme $N_{Nt,Nr}$ and $L = 1$. Also, the OAC and Toeplitz codes correspond to GOTCs with $N_{Nt,2}$ and $N_{Nt,1}$ respectively.

Finally, Fig. 4 shows the results when the frequency efficiency is 5.0 bit/s/Hz and the number of received antennae is changed. Fig. 4 shows that GOTC($N_{8,4}, Q_{4,4}$) is better than GOTC($N_{8,4}, O_{5,4}$), and the difference becomes larger as the number of received antennae increase. This is because the loss in diversity gain caused by improving the symbol rate is complemented by using multiple antennae. The results show that a better tradeoff can be achieved by using GOTC($N_{8,4}, Q_{4,4}$) and multiple antennae.

4. Conclusions

In this letter, the tradeoff between symbol rate and diversity gain of STBCs with linear receivers was considered. The results show that GOTC($N_{8,4}, Q_{4,4}$) has a better tradeoff than GOTC($N_{8,4}, O_{5,4}$). Also, GOTC($N_{8,4}, Q_{4,4}$) with a smaller value of $L$ has almost the same performance as GOTC($N_{8,2}, O_{2,2}$) with a larger value of $L$. This means that the decoding delay is smaller for GOTC($N_{8,4}, Q_{4,4}$) than for GOTC($N_{8,2}, O_{2,2}$). In the future, we will work on the optimal GOTC base code to achieve the best tradeoff with linear receivers.

References