

The Theory of Refractive Index in the Presence of a Constant Magnetic Field

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The theory of refractive index in the presence of a constant magnetic field is developed on the basis of the first principle, i.e. of a standpoint of the microscopic Maxwell equations. The formula for refractive index is expressed in terms of a correlation function of the spatial Fourier components of total electric current by no use of the dielectric constant and the magnetic permeability explicitly. The lowest-order term in the wave-number of light of the refractive index is independent of the constant magnetic field and has similar features with the leading term of the Faraday rotational angle as far as one considers the terms up to the first order in the constant magnetic field. Effects of the constant magnetic field on the refractive index are caused by the first-order terms in the wave-number of light.

1. Introduction

Refractive index is one of the familiar optical constants and a number of theoretical investigations are done in various substances.¹⁾ In the presence of a constant magnetic field, however, a few investigations of the refractive index and the dielectric constant are also done.²⁾ In the conventional formulations, the theories have been on the basis of the Maxwell equations in terms of the electric field strength, the electric flux density, the magnetic field strength and the magnetic flux density. Then, the dielectric constant and the magnetic permeability have been discussed. On the other hand we have developed the general theory of the Faraday effect on the basis of the microscopic Maxwell equations constantly in terms of the electric field strength and the magnetic flux density.³⁾ In this formulation, the Faraday rotation including the natural optical rotation is derived directly by no use of the dielectric constant and the magnetic permeability.

The formulation of refractive index in the presence of the constant magnetic field may be carried out by the similar fashion to that of the Faraday rotation in the previous paper.³⁾ By this formulation one may discuss not only the refractive index of substance in the constant magnetic field but also the relations between the refractive index and the Faraday rotational angle. Although the phenomena of the refractive

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index in the presence of the constant magnetic field, the Faraday effect and the natural optical activity have different features between them, these phenomena can be discussed from the same theoretical point of view by our unified theory of optical constants.³⁾

In the present paper, we develop the theory of refractive index in the presence of a constant magnetic field and discuss the relations between the refractive index and the Faraday rotational angle.

In sec. 2 a formula for the refractive index in the presence of the constant magnetic field is derived and is expressed in terms of the Green functions. In sec. 3 Hamiltonian of the system and the total electric current are given. In sec. 4 the lowest-order terms in the wave-number of light are derived and the equations for the Green functions are obtained. Relations between the refractive index and the Faraday rotational angle are also presented. In sec. 5 the effects of the first order in the wave-number on the refractive index are discussed. Summary and discussion are given in the last section.

2. Formulation of Refractive Index

In a previous paper³⁾ a general theory of the Faraday effect has been developed on the basis of a standpoint of the microscopic Maxwell equations and of the first principle by no use of the conventional formulae. The general formula for the Faraday rotation encompasses that for the natural optical rotation and, furthermore, a formula for refractive index in the presence of a constant magnetic field has been able to derive by our similar method³⁾ from the same theoretical point of view.

Let us take the direction of propagation of an incident monochromatic light with the angular frequency ω to be parallel to the z -axis in medium. The refractive index $n(\omega)$ in the presence of a constant magnetic field \mathbf{H} and the Faraday rotational angle $\phi(\omega)$ of the plane of polarized light per unit length* (i.e. the magneto-optical rotatory power) are expressed in terms of a correlation function of the spatial Fourier components of total electric current, that is³⁾

$$n(\omega) = \left\{ 1 - \frac{1}{\omega^2} Q_D(q_0, \omega) \right\}^{1/2} \quad (1)$$

and

$$\phi(\omega) = -\frac{i}{2c\omega n(\omega)} Q_N(q_0, \omega), \quad (2)$$

where $q_0 = \omega/c$, c is the speed of light in vacuum,

$$Q_D(q, \omega) = \frac{4\pi i \omega}{V} \int_{-\infty}^{\infty} dt e^{-i\omega t} \theta(t) \int_0^\beta d\lambda \langle \mathcal{J}_x(q, t) \mathcal{J}_x(-q, i\hbar\lambda) \rangle, \quad (3)$$

$$Q_N(q, \omega) = \frac{4\pi i \omega}{V} \int_{-\infty}^{\infty} dt e^{-i\omega t} \theta(t) \int_0^\beta d\lambda \langle \mathcal{J}_x(q, t) \mathcal{J}_y(-q, i\hbar\lambda) \rangle. \quad (4)$$

* The sense of rotation is defined so that positive ϕ corresponds to counterclockwise rotation as seen by an observer against the z direction of propagation of the incident light.

The symbol $\theta(t)$ is defined by

$$\theta(t) = \begin{cases} 1 & \text{for } t > 0 \\ 0 & \text{for } t < 0 \end{cases}$$

and

$$\beta = \frac{1}{kT}.$$

Expression $\mathcal{J}(q, t)$ is the spatial Fourier component of total electric current operator at time t in the presence of \mathbf{H} , k Boltzmann's constant, T the absolute temperature and V is the volume of the system. The triangular brackets denote the canonical ensemble average under the total Hamiltonian.

As far as the Faraday rotation, the natural optical rotation and the refractive index are discussed, it is enough to consider terms up to the first order in the wave-number q of the light. The formula (3) is expanded up to the first order in q and is rewritten in the terms of the Green function as

$$Q_b(q, \omega) = -\frac{4\pi i \omega}{V} \gamma(q) \{G_{xx}(q, \omega) + G_{yy}(q, \omega)\}, \quad (5)$$

where $\gamma(q)$ is an operator which has the value of $1/2$ and 1 when it operates to the zeroth-order term and the first-order term in q , respectively. The Fourier component $G_{\mu\mu}(q, \omega)$ of the Green function $G_{\mu\mu}(q, t)$ is defined by

$$G_{\mu\mu}(q, \omega) = \int_{-\infty}^{\infty} dt e^{-i\omega t} G_{\mu\mu}(q, t), \quad (6)$$

where

$$G_{\mu\mu}(q, t) = -\frac{i}{\hbar} \theta(t) \langle [\mathcal{J}_\mu(q, t), \mu_\mu(0, 0)] \rangle \quad (7)$$

with the spatial Fourier component $\mu(q, t)$ of the electric dipole moment operator at time t .

The system under consideration is a polymer composed of similar monomers in the presence of the constant magnetic field. The spatial Fourier component of the total electric current operator at time t is expressed in the form

$$\mathcal{J}(q, t) = \sum_n \sum_i \frac{1}{2} \{ \exp(i\mathbf{q} \cdot \mathbf{r}_{in}(t)) \mathcal{J}(\mathbf{r}_{in}, t) + \mathcal{J}(\mathbf{r}_{in}, t) \exp(i\mathbf{q} \cdot \mathbf{r}_{in}(t)) \}, \quad (8)$$

where $\mathbf{r}_{in}(t)$ is the co-ordinate operator of the i th electron in the n th monomer in the polymer at time t . When eq. (8) is expanded up to the first order in q , the expression $\mathcal{J}(q, t)$ becomes

$$\mathcal{J}(q, t) = \mathcal{J}(t) + \frac{i}{2} q \sum_n \sum_i \{ z_{in}(t) \mathcal{J}(\mathbf{r}_{in}, t) + \mathcal{J}(\mathbf{r}_{in}, t) z_{in}(t) \}. \quad (9)$$

By making use of eqs. (6), (7), (9) the formula $Q_b(q, \omega)$ is expressed in terms of the Green functions, that is

$$Q_D(q, \omega) = -\frac{2\pi i \omega}{V} \int_{-\infty}^{\infty} dt e^{-i\omega t} \{ [G_{xx}(t) + G_{yy}(t)] + 2iq[\mathcal{G}_{xx}(t) + \mathcal{G}_{yy}(t)] \} \quad (10)$$

$$= -\frac{2\pi i \omega}{V} \{ [G_{xx}(\omega) + G_{yy}(\omega)] + 2iq[\mathcal{G}_{xx}(\omega) + \mathcal{G}_{yy}(\omega)] \}, \quad (11)$$

where

$$G_{\mu\mu}(t) = -\frac{i}{\hbar} \theta(t) \langle [\mathcal{J}_\mu(t), \mu_\mu(0)] \rangle, \quad (12)$$

$$\mathcal{G}_{\mu\mu}(t) = -\frac{i}{\hbar} \theta(t) \langle [\sum_n \sum_i z_{in}(t) \mathcal{J}_\mu(\mathbf{r}_{in}(t)), \mu_\mu(0)] \rangle, \quad (13)$$

$$\boldsymbol{\mu}(t) = \sum_n \sum_i e \mathbf{r}_{in}(t) \quad (14)$$

and e is the charge of an electron.

Thus a formula for the refractive index $n(\omega)$ is found to be

$$\{n(\omega)\}^2 = \left\{ 1 + \frac{2\pi i}{V\omega} [G_{xx}(\omega) + G_{yy}(\omega)] \right\} + \left\{ -\frac{4\pi}{VC} [\mathcal{G}_{xx}(\omega) + \mathcal{G}_{yy}(\omega)] \right\}. \quad (15)$$

The first and the second curly-bracket terms on the right-hand side of eq. (15) are derived from the zeroth and the first order in the wave-number q of light, respectively.

3. Hamiltonian

The Hamiltonian \mathcal{H} of the system is expressed as

$$\begin{aligned} \mathcal{H} = & \sum_n \sum_i \left\{ \frac{1}{2m} (\mathbf{p}_{in} - \frac{e}{c} \mathbf{A}(\mathbf{r}_{in}))^2 + v_n(\mathbf{r}_{in}) \right\} \\ & + \frac{1}{2} \sum_n \sum_{i \neq j} v'_n(\mathbf{r}_{in} - \mathbf{r}_{jn}) + \frac{1}{2} \sum_{n \neq m} \sum_i \sum_j V_{nm}(\mathbf{r}_{in} - \mathbf{r}_{jm}), \end{aligned} \quad (16)$$

where m is the mass of the electron, \mathbf{p}_{in} the momentum operator of the i th electron in the n th monomer in the polymer and $\mathbf{A}(\mathbf{r}_{in})$ is the vector potential satisfying the equation

$$\text{rot}_{in} \mathbf{A}(\mathbf{r}_{in}) = \mathbf{H}. \quad (17)$$

The expression $v_n(\mathbf{r}_{in})$ is the interaction between the i th electron in the n th monomer and the nucleus in the n th monomer, $v'_n(\mathbf{r}_{in} - \mathbf{r}_{jn})$ the interaction between the i th and the j th electrons in the same n th monomer and $V_{nm}(\mathbf{r}_{in} - \mathbf{r}_{jm})$ is the interaction between the i th electron in the n th monomer and the j th electron in the m th monomer.

We may assume without loss of generality that the x , y and z components of the constant magnetic field \mathbf{H} are $H \sin \alpha$, 0 , $H \cos \alpha$, respectively, by an introduction of an angle α between the directions of \mathbf{H} and the z -axis. The expression \mathcal{J} can be written as

$$\mathcal{J}_x = \frac{e}{m} p_x + \frac{e}{2} \omega_z y, \quad (18)$$

$$\mathcal{J}_y = \frac{e}{m} p_y - \frac{e}{2}(\omega_{zx} - \omega_{xz}), \quad (19)$$

$$\mathcal{J}_z = \frac{e}{m} p_z - \frac{e}{2}\omega_{xy}, \quad (20)$$

where

$$\omega_x = \frac{eH\sin\alpha}{mc} \quad \text{and} \quad \omega_z = \frac{eH\cos\alpha}{mc}. \quad (21)$$

4. Refractive Index Caused by the Lowest Order in the Wave-Number

As far as one investigates only the lowest-order terms in the wave-number q of the light in eq. (15), the expression for the refractive index $n_0(\omega)$ takes the form

$$\{n_0(\omega)\}^2 = 1 + \frac{2\pi i}{V\omega} \{G_{xx}(\omega) + G_{yy}(\omega)\}. \quad (22)$$

A successive calculation of the Green function $G_{xx}(t)$ defined by eq. (12) leads the coupled equations for the Green functions of various types of the form

$$-\frac{\hbar}{i} \frac{dG_{xx}(t)}{dt} = \frac{N\hbar e^2}{im} \delta(t) - \frac{\hbar\omega_z}{i} G_{yx}(t) + \frac{e}{m} G_{xx}^1(t), \quad (23)$$

$$-\frac{\hbar}{i} \frac{dG_{yx}(t)}{dt} = \frac{\hbar\omega_z}{i} G_{xx}(t) - \frac{\hbar\omega_x}{i} G_{zx}(t) + \frac{e}{m} G_{yx}^1(t), \quad (24)$$

$$-\frac{\hbar}{i} \frac{dG_{zx}(t)}{dt} = \frac{\hbar\omega_x}{i} G_{yx}(t) + \frac{e}{m} G_{zx}^1(t), \quad (25)$$

$$-\frac{\hbar}{i} \frac{dG_{\mu\nu}^1(t)}{dt} = -\frac{1}{2m} G_{\mu\nu}^{21}(t) - \frac{1}{e} G_{\mu\nu}^{22}(t), \quad (26)$$

$$-\frac{\hbar}{i} \frac{dG_{\mu\nu}^{21}(t)}{dt} = -\frac{1}{2m} G_{\mu\nu}^{311}(t) - \frac{1}{e} G_{\mu\nu}^{312}(t), \quad (27)$$

$$-\frac{\hbar}{i} \frac{dG_{\mu\nu}^{22}(t)}{dt} = \frac{N\hbar e^2}{im} C_{\mu\nu} \delta(t) - \frac{1}{e} G_{\mu\nu}^{321}(t) - G_{\mu\nu}^{322}(t) + \frac{e}{m} G_{\mu\nu}^{323}(t) \\ + \frac{e}{2m} G_{\mu\nu}^{324}(t) + \frac{e}{2m} G_{\mu\nu}^{325}(t), \quad (28)$$

$$(\mu = x, y, z, \quad \nu = x, y)$$

where N is the number of electrons in the system and

$$G_{\mu\nu}(t) = -\frac{i}{\hbar} \theta(t) \langle [\mathcal{J}_\mu(t), \mu_\nu(0)] \rangle, \quad (29)$$

$$G_{\mu\nu}^1(t) = -\frac{i}{\hbar} \theta(t) \langle [\sum_n \sum_i (\hat{p}_{\mu in} v_n(\mathbf{r}_{in}(t))), \mu_\nu(0)] \rangle, \quad (30)$$

$$G_{\mu\nu}^{21}(t) = -\frac{i}{\hbar} \theta(t) \langle [\sum_n \sum_i \sum_{\mu'} (\hat{p}_{\mu in} \hat{p}_{\mu' in} \hat{p}_{\mu' in} v_n(\mathbf{r}_{in}(t))), \mu_\nu(0)] \rangle, \quad (31)$$

$$G_{\mu\nu}^{22}(t) = -\frac{i}{\hbar} \theta(t) \langle [\sum_n \sum_i \sum_{\mu'} (\hat{p}_{\mu in} \hat{p}_{\mu' in} v_n(\mathbf{r}_{in}(t))) \mathcal{J}_{\mu' in}(t), \mu_\nu(0)] \rangle, \quad (32)$$

$$G_{\mu\nu}^{311}(t) = -\frac{i}{\hbar} \theta(t) \langle [\sum_n \sum_i \sum_{\mu'} \sum_{\nu'} (\hat{p}_{\mu in} \hat{p}_{\mu' in} \hat{p}_{\mu' in} \hat{p}_{\nu' in} v_n(\mathbf{r}_{in}(t))), \mu_\nu(0)] \rangle, \quad (33)$$

$$G_{\mu\nu}^{312}(t) = -\frac{i}{\hbar}\theta(t)\langle[\sum_n\sum_i\sum_{\mu'}\sum_{\nu'}(\rho_{\mu in}\rho_{\mu' in}\rho_{\nu' in}\rho_{\nu in}V_n(\mathbf{r}_{in}(t)))\mathcal{J}_{\mu' in}(t), \mu_\nu(0)]\rangle, \quad (34)$$

$$C_{\mu\nu} = \langle\frac{1}{N}\sum_n\sum_i(\rho_{\mu in}\rho_{\nu in}V_n(\mathbf{r}_{in}))\rangle, \quad (35)$$

$$G_{\mu\nu}^{321}(t) = -\frac{i}{\hbar}\theta(t)\langle[\sum_n\sum_i\sum_{\mu'}\sum_{\nu'}(\rho_{\mu in}\rho_{\mu' in}\rho_{\nu' in}\rho_{\nu in}V_n(\mathbf{r}_{in}(t)))\mathcal{J}_{\nu' in}(t)\mathcal{J}_{\mu' in}(t), \mu_\nu(0)]\rangle, \quad (36)$$

$$G_{\mu\nu}^{322}(t) = -\frac{i}{\hbar}\theta(t)\langle[\sum_n\sum_i\left\{\frac{1}{2m}\sum_{\mu'}(\rho_{\mu in}\rho_{\mu' in}\rho_{\mu' in}\rho_{\mu in}V_n(\mathbf{r}_{in}(t)))\right. \\ \left.+\frac{\hbar\boldsymbol{\omega}}{i}\times(\rho_{\mu in}\rho_{in}V_n(\mathbf{r}_{in}(t)))\right\}\cdot\mathcal{J}_{in}(t), \mu_\nu(0)]\rangle, \quad (37)$$

$$G_{\mu\nu}^{323}(t) = -\frac{i}{\hbar}\theta(t)\langle[\sum_n\sum_i\sum_{\mu'}(\rho_{\mu in}\rho_{\mu' in}V_n(\mathbf{r}_{in}(t)))(\rho_{\mu' in}V_n(\mathbf{r}_{in}(t))), \mu_\nu(0)]\rangle, \quad (38)$$

$$G_{\mu\nu}^{324}(t) = -\frac{i}{\hbar}\theta(t)\langle[\sum_n\sum_i\sum_{\mu'}\sum_{\mu''}(\rho_{\mu in}\rho_{\mu' in}V_n(\mathbf{r}_{in}(t)) - \rho_{\mu in}\rho_{\mu'' in}V_n(\mathbf{r}_{in}(t))) \\ \times(\rho_{\mu' in}V_n'(\mathbf{r}_{in}(t) - \mathbf{r}_{jn}(t))), \mu_\nu(0)]\rangle, \quad (39)$$

$$G_{\mu\nu}^{325}(t) = -\frac{i}{\hbar}\theta(t)\langle[\sum_{n\neq m}\sum_i\sum_j\sum_{\mu'}\sum_{\mu''}(\rho_{\mu in}\rho_{\mu' in}V_n(\mathbf{r}_{in}(t)) - \rho_{\mu in}\rho_{\mu'' in}V_n(\mathbf{r}_{in}(t))) \\ \times(\rho_{\mu' in}V_{nm}(\mathbf{r}_{in}(t) - \mathbf{r}_{jm}(t))), \mu_\nu(0)]\rangle. \quad (40)$$

The equations for the Green function $G_{yy}(t)$ are also obtained by a similar fashion as is demonstrated above, that is

$$-\frac{\hbar}{i}\frac{dG_{yy}(t)}{dt} = \frac{N\hbar e^2}{im}\delta(t) + \frac{\hbar\omega_z}{i}G_{xy}(t) - \frac{\hbar\omega_x}{i}G_{zy}(t) + \frac{e}{m}G_{yy}^1(t), \quad (41)$$

$$-\frac{\hbar}{i}\frac{dG_{xy}(t)}{dt} = -\frac{\hbar\omega_z}{i}G_{yy}(t) + \frac{e}{m}G_{xy}^1(t), \quad (42)$$

$$-\frac{\hbar}{i}\frac{dG_{zy}(t)}{dt} = \frac{\hbar\omega_x}{i}G_{yy}(t) + \frac{e}{m}G_{zy}^1(t) \quad (43)$$

and eqs. (26)–(28). As it is seen in eqs. (23)–(28), effects of the electron-electron interaction appear for the first time only in $G_{\mu x}^{324}(t)$ and $G_{\mu x}^{325}(t)$ on the right-hand side of eq. (28), which are defined by eqs. (39) and (40) with terms of the form $(\rho_{\mu' in}V_n'(\mathbf{r}_{in} - \mathbf{r}_{jn}))$ and $(\rho_{\mu' in}V_{nm}(\mathbf{r}_{in} - \mathbf{r}_{jm}))$, respectively. Such a fact can be found also in a set of equations for the Green functions in the case of the Faraday effect and equations (26)–(28) are completely the same ones as those derived for the Faraday effect.⁴⁾ This implies that there is hardly effects of the electron-electron interaction on the refractive index and these effects on the refractive index are the same as those on the Faraday rotational angle.

When a few approximations for the simultaneous equations (23)–(28) and (41)–(43) can be made, it becomes possible to solve the equations for the Green functions. Let us make some approximations for $G_{\mu\nu}^{21}(t)$ and $G_{\mu\nu}^{22}(t)$ defined by eqs. (31) and (32), respectively, as follows

$$G_{\mu\nu}^{21}(t) \approx \langle \sum_{\mu'} p_{\mu' in} p_{\mu' in} \rangle G_{\mu\nu}^1(t) \quad (44)$$

and

$$G_{\mu\nu}^{22}(t) \approx \sum_{\mu'} \langle (p_{\mu in} p_{\mu' in} v_n(\mathbf{r}_{in})) \rangle G_{\mu\nu}(t). \quad (45)$$

This approximation suggests that the electron-electron interaction can be neglected.

From eq. (22), in the presence of \mathbf{H} , the refractive index $n_0(\omega)$ caused by the lowest order in q becomes

$$\{n_0(\omega)\}^2 = 1 - \frac{4\pi N \hbar^2 e^2}{Vm} \frac{\hbar\omega - \frac{\langle p^2 \rangle}{2m}}{\hbar\omega} \frac{\left[\left(\hbar\omega - \frac{\langle p^2 \rangle}{4m} \right)^2 - D \right]^2 - E - H_N}{\left[\left(\hbar\omega - \frac{\langle p^2 \rangle}{4m} \right)^6 + A \left(\hbar\omega - \frac{\langle p^2 \rangle}{4m} \right)^4 \right.}, \quad (46)$$

$$\left. + B \left(\hbar\omega - \frac{\langle p^2 \rangle}{4m} \right)^2 + C + H_b \right]}$$

where

$$A = -3 \left(\frac{\langle p^2 \rangle}{4m} \right)^2 + \frac{\langle p^2 v \rangle}{m}, \quad (47)$$

$$B = 3 \left(\frac{\langle p^2 \rangle}{4m} \right)^4 - 2 \left(\frac{\langle p^2 \rangle}{4m} \right)^2 \frac{\langle p^2 v \rangle}{m} + \frac{1}{3} \left(\frac{\langle p^2 v \rangle}{m} \right)^2 - \left\{ \left(\frac{\langle p_y p_z v \rangle}{m} \right)^2 + \left(\frac{\langle p_z p_x v \rangle}{m} \right)^2 + \left(\frac{\langle p_x p_y v \rangle}{m} \right)^2 \right\}, \quad (48)$$

$$C = - \left(\frac{\langle p^2 \rangle}{4m} \right)^6 + \left(\frac{\langle p^2 \rangle}{4m} \right)^4 \frac{\langle p^2 v \rangle}{m} - \left(\frac{\langle p^2 \rangle}{4m} \right)^2 \left\{ \frac{1}{3} \left(\frac{\langle p^2 v \rangle}{m} \right)^2 - \left(\left(\frac{\langle p_y p_z v \rangle}{m} \right)^2 + \left(\frac{\langle p_z p_x v \rangle}{m} \right)^2 + \left(\frac{\langle p_x p_y v \rangle}{m} \right)^2 \right\} + \frac{1}{27} \left(\frac{\langle p^2 v \rangle}{m} \right)^3 + 2 \frac{\langle p_y p_z v \rangle}{m} \frac{\langle p_z p_x v \rangle}{m} \frac{\langle p_x p_y v \rangle}{m} - \frac{1}{3} \frac{\langle p^2 v \rangle}{m} \left(\left(\frac{\langle p_y p_z v \rangle}{m} \right)^2 + \left(\frac{\langle p_z p_x v \rangle}{m} \right)^2 + \left(\frac{\langle p_x p_y v \rangle}{m} \right)^2 \right), \quad (49)$$

$$D = \left(\frac{\langle p^2 \rangle}{4m} \right)^2 - \frac{\langle p^2 v \rangle}{4m}, \quad (50)$$

$$E = \frac{\langle p_y p_z v \rangle}{m} + \frac{\langle p_z p_x v \rangle}{m}, \quad (51)$$

$$H_N = \frac{2N \hbar e^2}{im} \left(\frac{eH \sin \alpha}{mc} \right)^2 \left(\hbar\omega - \frac{\langle p^2 \rangle}{2m} \right)^3, \quad (52)$$

$$H_D = \left(\frac{eH}{mc} \right)^2 \left(\hbar\omega - \frac{\langle p^2 \rangle}{2m} \right)^2 \left\{ \hbar\omega \left(\hbar\omega - \frac{\langle p^2 \rangle}{2m} \right) + \frac{\langle p_z^2 v \rangle}{m} \cos^2 \alpha + \frac{\langle p_x^2 v \rangle}{m} \sin^2 \alpha \right\}. \quad (53)$$

Here use of abbreviations has been made, i.e.

$$\langle p^2 \rangle = \langle \sum_{\mu'} p_{\mu' in} p_{\mu' in} \rangle, \quad (54)$$

$$\langle p^2 v \rangle = \sum_{\mu'} \langle (p_{\mu' in} p_{\mu' in} v_n(\mathbf{r}_{in})) \rangle, \quad (55)$$

$$\langle p_{\mu} p_{\nu} v \rangle = \langle (p_{\mu in} p_{\nu in} v_n(\mathbf{r}_{in})) \rangle. \quad (56)$$

Expression (46) contains only the second-order terms in H with the first-order terms missing. It should be noted that as far as we consider the terms up to the first order in the constant magnetic field H , the refractive index $n_0(\omega)$ caused by the zeroth order in the wave-number q of light is independent of H and, therefore, the dispersion of refractive index $n_0(\omega)$ is the same one in the absence of H .

The dispersion of the refractive index $n_0(\omega)$ expressed by eq. (46) is very similar to that of the Faraday rotation.⁴⁾ This implies that there is a relation between the refractive index $n_0(\omega)$ and the Faraday rotational angle $\phi(\omega)$. The relations between the Faraday rotation and the refractive index in the absence of H has been proved in general in our previous paper³⁾ in the form

$$\phi(\omega) = \frac{eH}{4mc^2 n_0(\omega)} [\{n_0(\omega, H=0)\}^2 - 1]. \quad (57)$$

This expression is concerned with the Becquerel formula.⁵⁾

5. Effects of the First Order in the Wave-Number

Since the constant magnetic field H is ineffective on the lowest-order terms in the wave-number q of the refractive index, the effects of the constant magnetic field on the refractive index are yielded from the first-order terms in q . The first-order terms in q are expressed in terms of the Fourier components of the Green functions $\mathcal{G}_{xx}(t)$ and $\mathcal{G}_{yy}(t)$ defined by eq. (13), which are similar to the Green functions for the natural optical rotation.⁶⁾

An equation for the Green function $\mathcal{G}_{xx}(t)$ can be calculated by the fashion as is seen in the preceding section and may be written in the form

$$\begin{aligned} -\frac{\hbar}{i} \frac{d\mathcal{G}_{xx}(t)}{dt} &= \frac{N\hbar e^2}{im} \langle z \rangle \delta(t) - \frac{\hbar\omega_z}{i} \mathcal{G}_{xx}(t) - \frac{\hbar}{i} \frac{1}{e} \mathcal{G}_{xx}^1(t) \\ &+ \frac{e}{m} \mathcal{G}_{xx}^2(t) + \frac{1}{2} \mathcal{G}_{xx}^3(t) + \frac{1}{2} \mathcal{G}_{xx}^4(t), \end{aligned} \quad (58)$$

where

$$\langle z \rangle = \langle \frac{1}{N} \sum_n \sum_i z_{in} \rangle, \quad (59)$$

$$\mathcal{G}_{\mu\nu}^1(t) = -\frac{i}{\hbar} \theta(t) \langle [\sum_n \sum_i \mathcal{J}_{\mu in}(t) \mathcal{J}_{\nu in}(t), \mu_{\nu}(0)] \rangle, \quad (60)$$

$$\mathcal{G}_{\mu\nu}^2(t) = -\frac{i}{\hbar} \theta(t) \langle [\sum_n \sum_i z_{in}(t) (p_{\mu in} v_n(\mathbf{r}_{in}(t))), \mu_{\nu}(0)] \rangle, \quad (61)$$

$$\mathcal{G}_{\mu\nu}^3(t) = -\frac{i}{\hbar} \theta(t) \langle [\sum_n \sum_{i \neq j} (z_{in}(t) - z_{jn}(t)) (p_{\mu in} v'_n(\mathbf{r}_{in}(t) - \mathbf{r}_{jn}(t))), \mu_{\nu}(0)] \rangle, \quad (62)$$

$$\mathcal{G}_{\mu\nu}^A(t) = -\frac{i}{\hbar}\theta(t)\langle[\sum_{n\neq m}\sum_i\sum_j(z_{in}(t)-z_{jm}(t))(\hat{p}_{\mu in}V_{nm}(\mathbf{r}_{in}(t)-\mathbf{r}_{jm}(t))), \mu\nu(0)]\rangle. \quad (63)$$

The electron-electron interactions appear in the Green functions $\mathcal{G}_{\mu\nu}^3(t)$ and $\mathcal{G}_{\mu\nu}^4(t)$ on the right-hand side of eq. (58) immediately. Thus it implies that the electron-electron interaction is effective on the refractive index caused by the first order in q in contrast with that caused by the zeroth order in q . A successive calculation of $\mathcal{G}_{\mu\nu}^1(t)$, $\mathcal{G}_{\mu\nu}^2(t)$, $\mathcal{G}_{\mu\nu}^3(t)$, $\mathcal{G}_{\mu\nu}^4(t)$ makes a set of the equations for $\mathcal{G}_{xx}(t)$, which is fairly complicated than that for $G_{xx}(t)$ calculated in the preceding section. Thus the calculation to solve the simultaneous equations for the Green functions suggests a more complicated behavior even if the approximations are made and this calculation of the refractive index is similar to that of the natural optical rotation.

Furthermore, the dispersion of refractive index caused by the first order in q is very similar to that of the natural optical rotational angle as is seen in our previous paper.⁶⁾ Therefore, it becomes clear that the dispersion of the refractive index caused by the first order in q has a close connection with that of the natural optical rotation.

6. Summary and Discussion

The theory of refractive index in the presence of a constant magnetic field \mathbf{H} has been developed on the basis of the same standpoint of the general theory of the Faraday effect and the natural optical activity formulated, by no use of the conventional formulae, by the method demonstrated in our previous paper.³⁾ It has been proved that the refractive index in the presence of \mathbf{H} is closely connected with the Faraday rotation and natural optical rotation.

As far as we consider only the lowest-order terms in the wave-number q of light, electron-electron interactions have a little effect on the refractive index $n_0(\omega)$ in the presence of \mathbf{H} being analogous to the case of the Faraday rotation. Thus effects of the exciton model, which originates in the electron-electron interaction, are a little ones on $n_0(\omega)$. Furthermore, the expression $n_0(\omega)$ contains only the zeroth- and the second-order terms in H by neglect of the electron-electron interaction and then the refractive index $n_0(\omega)$ given by eq. (46) comes to be independent of H as far as one considers the terms up to the first order in H . Then there is a relation concerned with the Becquerel formula between the refractive index and the Faraday rotation,⁵⁾ which has been proved in general formerly by us.³⁾

The effects of the first-order terms in the wave-number q of light on the refractive index have been also discussed. Since the calculations of Green functions defined by eq. (13) are made by a similar fashion in the case of the natural optical rotation, the electron-electron interaction is effective on the refractive index being analogous to the

case of the natural optical rotation as is shown in the previous paper.⁶⁾ Furthermore, it can be considered that the effects of the constant magnetic field \mathbf{H} on the refractive index originates in the first order in q as far as one considers the terms up to the first order in H .

The electron-electron interaction between an electron in a monomer and the nucleus in the other monomer is neglected in comparison with the interactions $v'_n(\mathbf{r}_{in} - \mathbf{r}_{jn})$ and $V_{nm}(\mathbf{r}_{in} - \mathbf{r}_{jm})$ in the Hamiltonian (16).

In the present paper, the exciton-phonon interaction is also neglected. The exciton-phonon interaction has a little effect on the natural optical rotation and the refractive index in the absence of \mathbf{H} as is proved in the previous paper.⁷⁾ This suggests to hold still that the effects of the electron-phonon interaction is a little on the refractive index in the presence of \mathbf{H} .

In conclusion, it should be noted that the refractive index in the presence of the constant magnetic field can be discussed in similar fashion for the Faraday effect from the same theoretical point of view by the general theory developed formerly by us.³⁾ It has been shown that, although the refractive index has different features from the Faraday effect, however, these are in close connection each other and there is a relation concerned with the Becquerel formula.⁵⁾ Furthermore, the effects of the electron-electron interaction on the refractive index in the presence of the constant magnetic field have been investigated by our general theory³⁾ just now.

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