Progress of Theoretical Physics, Vol. 105, No. 6, June 2001

Triplet-Doublet Splitting, Proton Stability and an Extra Dimension

Yoshiharu Kawamura*)

Department of Physics, Shinshu University, Matsumoto 390-8621, Japan

(Received December 11, 2000)

We propose a new possibility to reconcile the coupling unification scenario with the triplet-doublet mass splitting based on a 5-dimensional supersymmetric model with $SU(5)$ gauge symmetry. It is shown that the minimal supersymmetric standard model is derived on a 4-dimensional wall through compactification on $S^1/(Z_2 \times Z_2')$.

§1. Introduction

The minimal supersymmetric standard model (MSSM) is the most promising model to describe physics beyond the Standard Model (SM). An advantageous feature of the MSSM is that the gauge coupling constants meet at $M_X = 2.1 \times 10^{16}$ GeV if the superpartners and Higgs particles exist below or around $O(1)$ TeV. [1] This fact leads to the possibility that gauge interactions in the MSSM are unified under a simple gauge group, such as $SU(5)$. If this is indeed the case, then the theory can be described as a supersymmetric grand unified theory (SUSY GUT). [2] This scenario is very attractive, but, in general, it suffers from problems related to Higgs multiplets. For example, in the minimal SUSY $SU(5)$ GUT, a fine tuning is required to obtain the $SU(2)_L$ doublet Higgs multiplets with the weak scale mass such that the colored Higgs multiplets remain sufficiently heavy to suppress dangerous nucleon decay (the triplet-doublet splitting problem). There have been several interesting proposals to solve this problem through the extension of the model. [3-8]

In this paper, we propose a new possibility to reconcile the coupling unification scenario with the triplet-doublet mass splitting. The coupling unification originates from the existence of a unified gauge symmetry, G, in a high-energy theory on a higher-dimensional space-time. The full symmetry is not realized in the low-energy physics, where light particles play an essential role. The symmetry G is reduced by the presence of non-universal values of the intrinsic parity on a compact space among components in each multiplet of G. We show that the SUSY part of the Lagrangian in the MSSM is derived on a 4-dimensional (4D) wall through compactification upon the orbifold $S^1/(Z_2 \times Z_2')$, with a suitable assignment of $Z_2 \times Z_2'$ parity, from a 5D SUSY model based on $G = SU(5)$. No particles appear other than the MSSM particles in the massless state, and the triplet-doublet mass splitting is realized by the $Z_2 \times Z_2'$ projection. We also discuss proton stability in our model.

This paper is organized as follows. In the next section, we explain the construction of the orbifold $S^1/(Z_2 \times Z_2')$ and describe an intrinsic parity on this compact

*) E-mail: haru@azusa.shinshu-u.ac.jp

space. Starting from 5D SUSY $SU(5)$ GUT with minimal particle content, we derive the SUSY part of the Lagrangian in the 4D theory and discuss the mass spectrum and its phenomenological implications in §3. Section 4 is devoted to conclusions and discussion.

§2. $S^1/(Z_2 \times Z_2')$ and parity

The space-time is assumed to be factorized into a product of 4D Minkowski space-time M^4 and the orbifold $S^1/(Z_2 \times Z_2')$,*) whose coordinates are denoted by x^μ ($\mu = 0, 1, 2, 3$) and y ($= x^5$), respectively. The 5D notation x^M ($M = 0, 1, 2, 3, 5$) is also used. We first construct the orbifold S^1/Z_2 by dividing a circle S^1 of radius R with a Z_2 transformation which acts on S^1 according to $y \to -y$. This compact space is regarded as the interval $[-\pi R, 0]$ with a length of πR. The orbifold $S^1/(Z_2 \times Z_2')$ is obtained by dividing S^1/Z_2 with another Z_2 transformation, denoted by Z_2', which acts on S^1/Z_2 according to $y' \to -y'$, where $y' \equiv y + \frac{\pi R}{2}$. This compact space is regarded as the interval $[0, \frac{\pi R}{2}]$ with a length of $\frac{\pi R}{2}$. There are two 4D walls placed at the fixed points $y' = 0$ and $y' = \frac{\pi R}{2}$ (or $y = -\frac{\pi R}{2}$ and $y = 0$) on $S^1/(Z_2 \times Z_2')$.

The intrinsic $Z_2 \times Z_2'$ parity of the 5D bulk field $\phi(x^\mu, y)$ is defined by the transformation

$$\phi(x^\mu, y) \to \phi(x^\mu, -y) = P\phi(x^\mu, y), \tag{2·1}$$

$$\phi(x^\mu, y') \to \phi(x^\mu, -y') = P'\phi(x^\mu, y'). \tag{2·2}$$

The Lagrangian should be invariant under the $Z_2 \times Z_2'$ transformation. By definition, P and P' possess only the eigenvalues 1 and -1. We denote the fields that are simultaneous eigenfunctions of these operators as ϕ_{++}, ϕ_{+-}, ϕ_{-+} and ϕ_{--}, where the first subscript corresponds to the eigenvalue of P and the second to P'. The fields ϕ_{++}, ϕ_{+-}, $\phi_{-+}$and ϕ_{--} are Fourier expanded as

$$\phi_{++}(x^\mu, y) = \sqrt{\frac{2}{\pi R}} \sum_{n=0}^{\infty} \phi_{++}^{(2n)}(x^\mu) \cos\frac{2ny}{R}, \tag{2·3}$$

$$\phi_{+-}(x^\mu, y) = \sqrt{\frac{2}{\pi R}} \sum_{n=0}^{\infty} \phi_{+-}^{(2n+1)}(x^\mu) \cos\frac{(2n+1)y}{R}, \tag{2·4}$$

$$\phi_{-+}(x^\mu, y) = \sqrt{\frac{2}{\pi R}} \sum_{n=0}^{\infty} \phi_{-+}^{(2n+1)}(x^\mu) \sin\frac{(2n+1)y}{R}, \tag{2·5}$$

$$\phi_{--}(x^\mu, y) = \sqrt{\frac{2}{\pi R}} \sum_{n=0}^{\infty} \phi_{--}^{(2n+2)}(x^\mu) \sin\frac{(2n+2)y}{R}, \tag{2·6}$$

where n is an integer, and each field $\phi_{++}^{(2n)}$, $\phi_{+-}^{(2n+1)}$, $\phi_{-+}^{(2n+1)}$ and $\phi_{--}^{(2n+2)}$ acquire a mass $\frac{2n}{R}$, $\frac{2n+1}{R}$, $\frac{2n+1}{R}$ and $\frac{2n+2}{R}$ upon compactification. Note that 4D massless fields

*) Recently, Barbieri, Hall and Nomura have constructed a constrained standard model upon a compactification of a 5D SUSY model on the orbifold $S^1/(Z_2 \times Z_2')$.[9] They used $Z_2 \times Z_2'$ parity to reduce SUSY. There are also several works on model building through a reduction of SUSY[10)-13)] and a gauge symmetry[14] by the use of Z_2 parity.

appear only in $\phi_{++}(x^\mu, y)$. We find that some fields vanish on the wall, for example, $\phi_{+-}(x^\mu, -\frac{\pi R}{2}) = \phi_{--}(x^\mu, -\frac{\pi R}{2}) = 0$ on the wall placed at $y = -\frac{\pi R}{2}$, and $\phi_{-+}(x^\mu, 0) = \phi_{--}(x^\mu, 0) = 0$ on the other wall placed at $y = 0$.

Let us study the case in which a field $\Phi(x^\mu, y)$ is an N-plet under some symmetry group G. The components of Φ are denoted by ϕ_k as $\Phi = (\phi_1, \phi_2, \cdots, \phi_N)^T$. The Z_2 transformation of Φ takes the same form as (2·1), but in this case P is an $N \times N$ matrix*) that satisfies $P^2 = I$, where I is the unit matrix. The Z_2 invariance of the Lagrangian does not necessarily require that P be I or $-I$. Unless all components of Φ have a common Z_2 parity (i.e., if $P \neq \pm I$), a symmetry reduction occurs upon compactification, because of the lack of zero modes in components with odd parity.[14] The same property holds in the case with Z_2' parity.

§3. A model with $SU(5)$ gauge symmetry

We now study 5D SUSY $SU(5)$ GUT with minimal particle content and non-universal parity assignment. We assume that the vector supermultiplet V and two kinds of hypermultiplets H^s $(s = 1, 2)$ exist in the bulk $M^4 \times S^1/(Z_2 \times Z_2')$. The vector multiplet V consists of a vector boson A_M, two bispinors λ_L^i $(i = 1, 2)$, and a real scalar Σ, which together form an adjoint representation **24** of $SU(5)$, and the hypermultiplets H^s consist of two complex scalar fields and two Dirac fermions $\psi^s = (\psi_L^s, \psi_R^s)^T$, which are equivalent to four sets of chiral supermultiplets: $H^1 = \{H_{\mathbf{5}} \equiv (H_1^1, \psi_L^1),\ \hat{H}_{\bar{\mathbf{5}}} \equiv (H_2^1, \bar{\psi}_R^1)\}$ and $H^2 = \{\hat{H}_{\mathbf{5}} \equiv (H_1^2, \psi_L^2),\ H_{\bar{\mathbf{5}}} \equiv (H_2^2, \bar{\psi}_R^2)\}$. The hypermultiplets $H_{\mathbf{5}}$ and $\hat{H}_{\mathbf{5}}$ ($\hat{H}_{\bar{\mathbf{5}}}$ and $H_{\bar{\mathbf{5}}}$) form a fundamental representation **5** ($\bar{\mathbf{5}}$). We assume that our visible world is a 4D wall fixed at $y = 0$ (we refer to as "wall I") and that three families of quark and lepton chiral supermultiplets, $3\{\Phi_{\bar{\mathbf{5}}} + \Phi_{\mathbf{10}}\}$, are located on this wall. (Here and hereafter the family index does not appear.) That is, matter fields contain no excited states along the $S^1/(Z_2 \times Z_2')$ direction.

The gauge invariant action is given by

$$S = \int \mathcal{L}^{(5)} d^5x + \frac{1}{2}\int \delta(y)\mathcal{L}^{(4)} d^5x$$
$$+(\text{terms from a brane fixed at } y = -\pi R), \qquad (3\cdot1)$$

$$\mathcal{L}^{(5)} = \mathcal{L}^{(5)}_{YM} + \mathcal{L}^{(5)}_{H}, \qquad (3\cdot2)$$

$$\mathcal{L}^{(5)}_{\mathrm{YM}} = -\frac{1}{2}\mathrm{Tr}F_{MN}^2 + \mathrm{Tr}|D_M\Sigma|^2 + \mathrm{Tr}(i\bar{\lambda}_i\gamma^M D_M\lambda^i) - \mathrm{Tr}(\bar{\lambda}_i[\Sigma, \lambda^i]), \qquad (3\cdot3)$$

$$\mathcal{L}^{(5)}_{\mathrm{H}} = |D_M H_i^s|^2 + i\bar{\psi}_s\gamma^M D_M\psi^s - (i\sqrt{2}g_{(5)}\bar{\psi}_s\lambda^i H_i^s + \mathrm{h.c.})$$
$$-\bar{\psi}_s\Sigma\psi^s - H_s^{\dagger i}\Sigma^2 H_i^s - \frac{g_{(5)}^2}{2}\sum_{m,A}\left(H_s^{\dagger i}(\sigma^m)_i^j T^A H_j^s\right)^2, \qquad (3\cdot4)$$

$$\mathcal{L}^{(4)} \equiv \sum_{\text{3families}} \int d^2\bar{\theta} d^2\theta (\Phi^\dagger_{\bar{\mathbf{5}}} e^{2g_{(5)}V^A T^A}\Phi_{\bar{\mathbf{5}}} + \Phi^\dagger_{\mathbf{10}} e^{2g_{(5)}V^A T^A}\Phi_{\mathbf{10}})$$

---

*) P is a unitary and hermitian matrix.

$$+ \sum_{\text{3families}} \int d^2\theta (f_{U(5)} H_{\mathbf{5}} \Phi_{\mathbf{10}} \Phi_{\mathbf{10}} + \hat{f}_{U(5)} \hat{H}_{\mathbf{5}} \Phi_{\mathbf{10}} \Phi_{\mathbf{10}}$$
$$+ f_{D(5)} H_{\bar{\mathbf{5}}} \Phi_{\mathbf{10}} \Phi_{\bar{\mathbf{5}}} + \hat{f}_{D(5)} \hat{H}_{\bar{\mathbf{5}}} \Phi_{\mathbf{10}} \Phi_{\bar{\mathbf{5}}}) + \text{h.c.}, \qquad (3\cdot5)$$

where $\lambda^i \equiv (\lambda_L^i, \epsilon^{ij} \bar{\lambda}_{Lj})^T$, $D_M \equiv \partial_M - i g_{(5)} A_M(x^\mu, y)$, $g_{(5)}$ is a 5D gauge coupling constant, the σ^m are Pauli matrices, the T^A are $SU(5)$ gauge generators, $V^A T^A$ is an $SU(5)$ vector supermultiplet, and $f_{U(5)}$, $f_{D(5)}$, $\hat{f}_{U(5)}$ and $\hat{f}_{D(5)}$ are 5D Yukawa coupling matrices. If we impose Z_2 invariance*) under $H_{\mathbf{5}} \leftrightarrow \hat{H}_{\mathbf{5}}$ and $H_{\bar{\mathbf{5}}} \leftrightarrow \hat{H}_{\bar{\mathbf{5}}}$ on $\mathcal{L}^{(4)}$, the relations $f_{U(5)} = \hat{f}_{U(5)}$ and $f_{D(5)} = \hat{f}_{D(5)}$ are derived. The representations of $\Phi_{\bar{\mathbf{5}}}$ and $\Phi_{\mathbf{10}}$ are $\bar{\mathbf{5}}$ and $\mathbf{10}$, respectively. In $\mathcal{L}^{(4)}$, the bulk fields are replaced by fields including the Nambu-Goldstone boson $\phi(x^\mu)$ at the wall I, $V^A(x^\mu, \theta, \bar{\theta}, y = \phi(x^\mu))$ and $H^s(x^\mu, \theta, y = \phi(x^\mu))$.[15] In the above action, we assume that there is a symmetry such as R parity to forbid the term $\Phi_{\bar{\mathbf{5}}} \Phi_{\bar{\mathbf{5}}} \Phi_{\mathbf{10}}$ from appearing in the superpotential, which induces rapid proton decay. The Lagrangian is invariant under the Z_2 transformation

$$\begin{aligned}
&A_\mu(x^\mu, y) \to A_\mu(x^\mu, -y) = P A_\mu(x^\mu, y) P^{-1}, \\
&A_5(x^\mu, y) \to A_5(x^\mu, -y) = -P A_5(x^\mu, y) P^{-1}, \\
&\lambda_L^1(x^\mu, y) \to \lambda_L^1(x^\mu, -y) = -P \lambda_L^1(x^\mu, y) P^{-1}, \\
&\lambda_L^2(x^\mu, y) \to \lambda_L^2(x^\mu, -y) = P \lambda_L^2(x^\mu, y) P^{-1}, \\
&\Sigma(x^\mu, y) \to \Sigma(x^\mu, -y) = -P \Sigma(x^\mu, y) P^{-1}, \\
&H_{\mathbf{5}}(x^\mu, y) \to H_{\mathbf{5}}(x^\mu, -y) = P H_{\mathbf{5}}(x^\mu, y), \\
&\hat{H}_{\bar{\mathbf{5}}}(x^\mu, y) \to \hat{H}_{\bar{\mathbf{5}}}(x^\mu, -y) = -P \hat{H}_{\bar{\mathbf{5}}}(x^\mu, y), \\
&\hat{H}_{\mathbf{5}}(x^\mu, y) \to \hat{H}_{\mathbf{5}}(x^\mu, -y) = -P \hat{H}_{\mathbf{5}}(x^\mu, y), \\
&H_{\bar{\mathbf{5}}}(x^\mu, y) \to H_{\bar{\mathbf{5}}}(x^\mu, -y) = P H_{\bar{\mathbf{5}}}(x^\mu, y)
\end{aligned} \qquad (3\cdot6)$$

and under the Z_2' transformation, obtained by replacing y and P by y' and P' in the above.

When we use $P = \mathrm{diag}(1,1,1,1,1)$ and $P' = \mathrm{diag}(-1,-1,-1,1,1)$,**) the $SU(5)$ gauge symmetry is reduced to that of the Standard Model, $G_{\mathrm{SM}} \equiv SU(3) \times SU(2) \times U(1)$, in the 4D theory.***) This is because the boundary conditions on $S^1/(Z_2 \times Z_2')$ do not respect $SU(5)$ symmetry, as we see from the relations for the gauge generators T^A $(A = 1, 2, \cdots, 24)$,

$$P' T^a P'^{-1} = T^a, \quad P' T^{\hat{a}} P'^{-1} = -T^{\hat{a}}. \qquad (3\cdot7)$$

*) This Z_2 is a discrete subgroup of $SU(2)_H$, which is one of the global symmetries of $\mathcal{L}^{(5)}$. The Higgs bosons transform as $\mathbf{2}$ under $SU(2)_H$.

**) The exchange of P and P' is equivalent to the exchange of two walls. The origin of this specific $Z_2 \times Z_2'$ parity assignment is unknown, and we believe that it will be explained in terms of some yet to be constructed underlying theory.

***) Our symmetry reduction mechanism is different from the Hosotani mechanism.[16] In fact, the Hosotani mechanism does not work in our case, because $A_5^a(x^\mu, y)$ has odd parity, as seen from (3·6), and its VEV should vanish.

Table I. Parity and mass spectrum at the tree level.

4D fields	Quantum numbers	$Z_2 \times Z_2'$ parity	Mass
$A_\mu^{a(2n)}$, $\lambda^{2a(2n)}$	$(\mathbf{8},\mathbf{1})+(\mathbf{1},\mathbf{3})+(\mathbf{1},\mathbf{1})$	$(+,+)$	$\frac{2n}{R}$
$A_\mu^{\hat{a}(2n+1)}$, $\lambda^{2\hat{a}(2n+1)}$	$(\mathbf{3},\mathbf{2})+(\bar{\mathbf{3}},\mathbf{2})$	$(+,-)$	$\frac{2n+1}{R}$
$A_5^{a(2n+2)}$, $\Sigma^{a(2n+2)}$, $\lambda^{1a(2n+2)}$	$(\mathbf{8},\mathbf{1})+(\mathbf{1},\mathbf{3})+(\mathbf{1},\mathbf{1})$	$(-,-)$	$\frac{2n+2}{R}$
$A_5^{\hat{a}(2n+1)}$, $\Sigma^{\hat{a}(2n+1)}$, $\lambda^{1\hat{a}(2n+1)}$	$(\mathbf{3},\mathbf{2})+(\bar{\mathbf{3}},\mathbf{2})$	$(-,+)$	$\frac{2n+1}{R}$
$H_C^{(2n+1)}$	$(\mathbf{3},\mathbf{1})$	$(+,-)$	$\frac{2n+1}{R}$
$H_u^{(2n)}$	$(\mathbf{1},\mathbf{2})$	$(+,+)$	$\frac{2n}{R}$
$\hat{H}_{\bar{C}}^{(2n+1)}$	$(\bar{\mathbf{3}},\mathbf{1})$	$(-,+)$	$\frac{2n+1}{R}$
$\hat{H}_d^{(2n+2)}$	$(\mathbf{1},\mathbf{2})$	$(-,-)$	$\frac{2n+2}{R}$
$\hat{H}_C^{(2n+1)}$	$(\mathbf{3},\mathbf{1})$	$(-,+)$	$\frac{2n+1}{R}$
$\hat{H}_u^{(2n)}$	$(\mathbf{1},\mathbf{2})$	$(-,-)$	$\frac{2n+2}{R}$
$H_{\bar{C}}^{(2n+1)}$	$(\bar{\mathbf{3}},\mathbf{1})$	$(+,-)$	$\frac{2n+1}{R}$
$H_d^{(2n)}$	$(\mathbf{1},\mathbf{2})$	$(+,+)$	$\frac{2n}{R}$

The T^a are gauge generators of $G_{\rm SM}$, and the $T^{\hat{a}}$ are the other gauge generators. The parity assignment and mass spectrum after compactification are given in Table I. Each Higgs multiplet is divided into two pieces: $H_{\mathbf{5}}$ ($\hat{H}_{\bar{\mathbf{5}}}$, $\hat{H}_{\mathbf{5}}$, $H_{\bar{\mathbf{5}}}$) is divided into the colored triplet piece, H_C ($\hat{H}_{\bar{C}}$, $\hat{H}_C$, $H_{\bar{C}}$), and the $SU(2)$ doublet piece, H_u ($\hat{H}_d$, $\hat{H}_u$, H_d). In the second column, we give the $SU(3)\times SU(2)$ quantum numbers of the 4D fields. In the third column, $(\pm,\pm)$ and $(\pm,\mp)$ denote the eigenvalues $(\pm 1,\pm 1)$ and $(\pm 1,\mp 1)$ of $Z_2\times Z_2'$ parity, respectively. In the fourth column, n represents 0 or a positive integer. The massless fields are ($A_\mu^{a(0)}$, $\lambda^{2a(0)}$) and ($H_u^{(0)}$, $H_d^{(0)}$), which are equivalent to the gauge multiplets and the weak $SU(2)$ doublet Higgs multiplets in the MSSM, respectively. We find that the triplet-doublet mass splitting of the Higgs multiplets is realized by projecting out zero modes of the colored components.

After integrating out the fifth dimension, we obtain the 4D Lagrangian density,

$$\mathcal{L}_{\rm eff}^{(4)} = \mathcal{L}_B^{(4)} + \mathcal{L}^{(4)}, \tag{3·8}$$

$$\mathcal{L}_B^{(4)} \equiv -\frac{1}{4}\int d^2\theta W^{a(0)}W^{a(0)} + \text{h.c.} + \int d^2\bar{\theta}d^2\theta (H_u^{\dagger(0)} e^{2g_U V^{a(0)}T^a} H_u^{(0)} + H_d^{\dagger(0)} e^{2g_U V^{a(0)}T^a} H_d^{(0)}) + \cdots, \tag{3·9}$$

$$\mathcal{L}^{(4)} \equiv \sum_{\rm 3families}\int d^2\bar{\theta}d^2\theta \Phi^\dagger e^{2g_U V^{a(0)}T^a}\Phi + \sum_{\rm 3families}\int d^2\theta (f_U H_u^{(0)} Q U^c + f_D H_d^{(0)} Q D^c + f_D H_d^{(0)} L E^c) + \text{h.c.} + \cdots, \tag{3·10}$$

where $V^{a(0)}$ is a vector multiplet of the MSSM gauge bosons and gauginos, and the dots represent terms including Kaluza-Klein modes. In this equation, g_U $\left(\equiv \sqrt{\frac{2}{\pi R}} g_{(5)}\right)$ is a 4D gauge coupling constant, f_U $\left(\equiv \sqrt{\frac{2}{\pi R}} f_{U(5)}\right)$ and f_D $\left(\equiv \sqrt{\frac{2}{\pi R}} f_{D(5)}\right)$ are 4D Yukawa coupling matrices, Q, U^c and D^c are quark chiral supermultiplets, and L and E^c are lepton chiral supermultiplets. We denote these matter multiplets as Φ generically in the kinetic term. With our assignment of $Z_2 \times Z_2'$ parity, the SUSY part of the Lagrangian density in the MSSM is obtained, with the exception of the μ term.

The theory predicts that coupling constants of $G_{\rm SM}$ are unified around the compactification scale $M_C(\equiv 1/R)$ to zero-th order approximation, as in the ordinary $SU(5)$ GUT: [17]

$$g_3 = g_2 = g_1 = g_U, \quad f_d = f_e = f_D \ , \tag{3·11}$$

where f_d and f_e are Yukawa coupling matrices on down-type quarks and electron-type leptons, respectively. From the precise measurements at LEP, [18] it is natural to identify M_C with $M_X = 2.1 \times 10^{16}$ GeV. In this case, the masses of the Kaluza-Klein excitations are quantized in units of M_X.

Finally, we discuss nucleon stability in our model. [19] It is known that there is a significant contribution to proton decay from the dimension 5 operator in the minimal SUSY $SU(5)$ GUT. [20] Stronger constraints on the colored Higgs mass M_{H_C} and the sfermion mass $m_{\tilde{f}}$ have been obtained (e.g. $M_{H_C} > 6.5 \times 10^{16}$ GeV for $m_{\tilde{f}} < 1$ TeV) from analysis including a Higgsino dressing diagram with right-handed matter fields. [21] In our model, we have diagrams similar to those in the minimal SUSY $SU(5)$ GUT, because quark and lepton supermultiplets couple to the Kaluza-Klein modes of extra vector supermultiplets and the colored Higgs triplets at the tree level. Hence the identification $M_C = M_X$ seems to be incompatible with the above constraint, $M_{H_C} > 6.5 \times 10^{16}$ GeV for $m_{\tilde{f}} < 1$ TeV. However there is a natural way to escape this difficulty. It is pointed out that there is an exponential suppression factor in the coupling to the Kaluza-Klein modes resulting from the brane recoil effect. [15] There is a possibility that proton stability is guaranteed if our 4D wall fluctuates pliantly.[*]

§4. Conclusions and discussion

We have proposed a new possibility to reconcile the coupling unification scenario with the triplet-doublet mass splitting. The coupling unification originates from the existence of a unified gauge symmetry $SU(5)$ in 5D space-time. The full symmetry is not realized in low-energy physics, where light particles play an essential role; that is, here this symmetry is reduced by the existence of non-universal values of the intrinsic parity on a compact space among components in each multiplet of $SU(5)$. We have shown that the SUSY part of the Lagrangian in the MSSM is derived, with the exception of the μ term, on a 4D wall through compactification

[*] It is not obvious whether or not the recoil effect exists for a brane at an orbifold fixed point. [22]

upon $S^1/(Z_2 \times Z_2')$ from a 5D SUSY model based on $SU(5)$, under the assumption that our visible world consists of a 4D wall and that matter multiplets live on the wall. In the sector with renormalizable interactions, the theory predicts the coupling unification $g_3 = g_2 = g_1 = g_U$ and $f_d = f_e = f_D$, as in the GUT model. No particles appear other than the MSSM ones in the massless state, and the triplet-doublet mass splitting is realized through the $Z_2 \times Z_2'$ projection. Although the quark and lepton multiplets couple to the Kaluza-Klein modes of extra vector multiplets and the colored Higgs triplets at the tree level, there is a possibility that proton stability is guaranteed by the appearance of a suppression factor in the coupling to the Kaluza-Klein modes if our 4D wall fluctuates flexibly. It is not yet known if the above mechanism works for a brane at an orbifold fixed point.

There are several problems with our model. Here we list some of them. The first three problems are peculiar to the MSSM, and the others are related to the 4D walls. The first one involves the question of how to avoid rapid proton decay that results from the term $\Phi_{\bar{\mathbf{5}}}\Phi_{\bar{\mathbf{5}}}\Phi_{\mathbf{10}}$ in the superpotential. We need a symmetry, such as R parity. The second problem involves the question of how to generate the μ term with a suitable magnitude consistent with the electro-weak symmetry. The third problem regards the origin of SUSY breaking. The fourth problem concerns the necessity of non-universal $Z_2 \times Z_2'$ parity, i.e., whether there is a selection rule that picks out a specific $Z_2 \times Z_2'$ parity to break $SU(5)$ down to G_{SM} and whether it is compatible with brane fluctuations. The last problem regards how matter fields are localized on the 4D wall. We expect that these problems can be solved by a yet unknown underlying theory.

In spite of these problems, it is worthwhile to study the relation between the symmetry in the SM and the characteristics of a compact space for the purpose of constructing a realistic model.[*)]

References

1) C. Giunti, C. W. Kim and U. W. Lee, Mod. Phys. Lett. **A6** (1991), 1745.
   J. Ellis, S. Kelley and D. V. Nanopoulos, Phys. Lett. **B260** (1991), 131.
   U. Amaldi, W. de Boer and H. Fürstenau, Phys. Lett. **B260** (1991), 447.
   P. Langacker and M. Luo, Phys. Rev. **D44** (1991), 817.
2) S. Dimopoulos and H. Georgi, Nucl. Phys. **B193** (1981), 150.
   N. Sakai, Z. Phys. **C11** (1981), 153.
3) E. Witten, Phys. Lett. **B105** (1981), 267.
4) A. Masiero, D. V. Nanopoulos, K. Tamvakis and T. Yanagida, Phys. Lett. **115** (1982), 380.
   B. Grinstein, Nucl. Phys. **B206** (1982), 387.
5) S. Dimopoulos and F. Wilczek, NSF-ITP-82-07.
   M. Srednicki, Nucl. Phys. **B202** (1982), 327.
6) K. Inoue, A. Kakuto and T. Takano, Prog. Theor. Phys. **75** (1986), 664.
7) E. Witten, Nucl. Phys. **B258** (1985), 75.
   I. Antoniadis, J. Ellis, J. S. Hagelin and D. V. Nanopoulos, Phys. Lett. **B194** (1987), 231.
   N. Haba, C. Hattori, M. Matsuda, T. Matsuoka and D. Mochinaga, Prog. Theor. Phys. **94** (1995), 233.
8) T. Yanagida, Phys. Lett. **B344** (1995), 211.

[*)] Attempts to construct GUT have been made through dimensional reduction over coset space.[23)]

9) R. Barbieri, L. J. Hall and Y. Nomura, hep-ph/0011311.
10) L. Antoniadis, Phys. Lett. **B246** (1990), 377.
L. Antoniadis, C. Muñoz and M. Quirós, Nucl. Phys. **B397** (1993), 515.
L. Antoniadis and K. Benakli, Phys. Lett. **B326** (1994), 69.
11) P. Hořava and E. Witten, Nucl. Phys. **B460** (1996), 506; **B475** (1996), 94.
12) E. A. Mirabelli and M. Peskin, Phys. Rev. **D58** (1998), 065002.
13) A. Pomarol and M. Quirós, Phys. Lett. **B438** (1998), 255.
14) Y. Kawamura, Prog. Theor. Phys. **103** (2000), 613.
15) M. Bando, T. Kugo, T. Noguchi and K. Yoshioka, Phys. Rev. Lett. **83** (1999), 3601.
16) Y. Hosotani, Phys. Lett. **B126** (1983), 309; Ann. of Phys. **190** (1989), 233.
17) H. Georgi and S. L. Glashow, Phys. Rev. Lett. **32** (1974), 438.
18) LEP Collaborations, Phys. Lett. **276B** (1992), 247.
19) For other possibilities to suppress a proton decay in theories with extra dimensions, see
N. Arkani-Hamed and M. Schmaltz, Phys. Rev. **D61** (2000), 033005.
L. E. Ibáñez and F. Quevedo, J. High Energy Phys. **10** (1999), 001.
A. Aranda and C. D. Carone, hep-ph/0012092.
20) N. Sakai and T. Yanagida, Nucl. Phys. **B197** (1982), 533.
S. Weinberg, Phys. Rev. **D26** (1982), 287.
21) T. Goto and T. Nihei, Phys. Rev. **D59** (1999), 115009.
22) M. Bando and T. Noguchi, hep-ph/0011374.
23) D. Kapetanakis and G. Zoupanos, Phys. Rep. **219** (1992), 1 and references therein.
24) G. Altarelli and F. Feruglio, hep-ph/0102301.
25) L. J. Hall and Y. Nomura, hep-ph/0103125.

Note added: While revising this manuscript, we found Ref. 24) by G. Altarelli and F. Feruglio and Ref. 25) by L. J. Hall and Y. Nomura in the hep-archive. In the context of our model, proton decay is analyzed in the former paper and several phenomenological features, including gauge coupling unification, are studied in the latter paper.