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Split Multiplets, Coupling Unification and an Extra Dimension

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We study a gauge coupling unification scenario based on a non-supersymmetric 5dimensional model. Through an orbifold compactification, we obtain the Standard Model with split multiplets on a 4-dimensional wall, which is compatible with a grand unification.

The Standard Model (SM) has been established as an effective theory below the weak scale. One of intriguing investigations beyond the SM is to unify gauge interactions under a simple group, such as SU(5). ¹⁾ This scenario is very attractive, ²⁾ but it suffers from several problems in the simplest version. The first problem is that gauge coupling constants do not meet at a high-energy scale, based on the desert hypothesis. ³⁾ The second problem is that a dangerous proton decay is induced by an exchange of X and Y gauge bosons. ⁴⁾ The last problem is that the weak scale is not stabilized by quantum corrections (the gauge hierarchy problem). ⁵⁾

The introduction of supersymmetry (SUSY) solves the first $^{6)}$ and the third problems. $^{7),**}$ A supersymmetric grand unified theory (SUSY GUT) is an attractive possibility as a high-energy theory, $^{8)}$ but proton stability is threatened due to a contribution from the dimension 5 operator in the minimal SUSY SU(5) GUT. $^{9), 10)}$ Recently, stronger constraints have been obtained from analysis including a Higgsino dressing diagram with right-handed matter fields. $^{11)}$

A new possibility 12 has been proposed to solve the above problems. Starting from a 5-dimensional (5D) SUSY SU(5) model, we have obtained a low-energy theory with particles of the minimal supersymmetric standard model (MSSM) on a 4D wall through compactification upon $S^1/(Z_2 \times Z_2')$. In this theory, proton stability is guaranteed due to the presence of a suppression factor in the coupling to the Kaluza-Klein modes if our 4D wall fluctuates flexibly.

In this paper, we propose another possibility to solve the first and second problems based on a 5D model without SUSY. Here the gauge coupling unification is realized by the introduction of extra multiplets that split after an orbifold compactification.***) The splitting originates from a non-universal parity assignment on a compact space among components in each multiplet. The proton decay is sufficiently suppressed by a suppression factor in the coupling to the Kaluza-Klein

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^{**)} The gauge hierarchy problem is solved partially in the sense that the hierarchy is stabilized against radiative corrections perturbatively, as described by a non-renormalization theorem, although the origin at the tree level is not understood.

^{***)} There are several works treating SU(5) grand unification under the assumption that there are split multiplets $^{13)-15}$ and SU(N) $(N \ge 6)$ one with a radiative splitting mechanism. $^{16)}$

excitations if our 4D wall fluctuates pliantly. In the following, we derive a model discussed in Ref. 15) as an example of a low-energy theory from a 5D SU(5) model through compactification upon $S^1/(Z_2 \times Z_2')$.

The space-time is considered to be factorized into a product of 4D Minkowski space-time M^4 and the orbifold $S^1/(Z_2 \times Z_2')$,*) whose coordinates are denoted by x^{μ} ($\mu = 0, 1, 2, 3$) and $y = x^5$, respectively. The 5D notation x^M (M = 0, 1, 2, 3, 5) is also used. The orbifold is regarded as an interval with a distance of $\pi R/2$. There are two 4D walls placed at fixed points $y = -\pi R/2$ and y = 0 (or y' = 0 and $y' = \pi R/2$) on $S^1/(Z_2 \times Z_2')$, where $y' \equiv y + \pi R/2$.

We assume that the 5D gauge boson $A_M(x^\mu,y)$ and four kinds of scalar fields $\Phi_{\boldsymbol{R}}(x^\mu,y)$ ($\boldsymbol{R}=\!5,\,\overline{\bf 5},\,{\bf 10},\,\overline{\bf 10}$) exist in the bulk $M^4\times S^1/(Z_2\times Z_2')$. The fields A_M and $\Phi_{\boldsymbol{R}}$ form an adjoint representation ${\bf 24}$ and a representation \boldsymbol{R} of SU(5), respectively. We assume that our visible world is a 4D wall fixed at y=0 (We call it "wall I".) and that three families of quarks and leptons, $3\{\psi_{\overline{\bf 5}}+\psi_{{\bf 10}}\}$, are located on wall I. (Here and hereafter we suppress the family index.) The representations of $\psi_{\overline{\bf 5}}$ and $\psi_{{\bf 10}}$ are $\overline{\bf 5}$ and ${\bf 10}$ of SU(5), respectively. Note that matter fields contain no excited states along the $S^1/(Z_2\times Z_2')$ direction.

The gauge invariant action is given by

$$S = \int d^{5}x \left(-\frac{1}{2} \operatorname{tr} F_{MN}^{2} + \sum_{\mathbf{R}} |D_{M} \Phi_{\mathbf{R}}|^{2} - V(\Phi_{\mathbf{R}}) \right)$$

$$+ \frac{1}{2} \int d^{5}x \delta(y) \sum_{3 \text{families}} (i \overline{\psi}_{\mathbf{10}} \gamma^{M} D_{M} \psi_{\mathbf{10}} + i \overline{\psi}_{\overline{\mathbf{5}}} \gamma^{M} D_{M} \psi_{\overline{\mathbf{5}}}$$

$$+ f_{U(5)} \Phi_{\mathbf{5}} \psi_{\mathbf{10}} \psi_{\mathbf{10}} + f_{D(5)} \Phi_{\overline{\mathbf{5}}} \psi_{\mathbf{10}} \psi_{\overline{\mathbf{5}}} + f_{Q(5)} \Phi_{\mathbf{10}} \psi_{\overline{\mathbf{5}}} \psi_{\overline{\mathbf{5}}} + \text{h.c.})$$

$$+ (\text{terms from a wall fixed at } y = -\pi R), \tag{1}$$

where $D_M \equiv \partial_M - i g_{(5)} A_M(x^\mu, y)$, $g_{(5)}$ is a 5D gauge coupling constant, and $f_{U(5)}$, $f_{D(5)}$ and $f_{Q(5)}$ are 5D Yukawa coupling matrices. In the 4D action, the bulk fields A_M and Φ_R are replaced by fields including the Nambu-Goldstone boson $\phi(x^\mu)$ at wall I such that they acquire the functional dependences $A_M(x^\mu, \phi(x^\mu))$ and $\Phi_R(x^\mu, \phi(x^\mu))$. The Lagrangian is invariant under the $Z_2 \times Z_2'$ transformation

$$A_{\mu}(x^{\mu}, y) \to A_{\mu}(x^{\mu}, -y) = PA_{\mu}(x^{\mu}, y)P^{-1},$$

$$A_{5}(x^{\mu}, y) \to A_{5}(x^{\mu}, -y) = -PA_{5}(x^{\mu}, y)P^{-1},$$

$$\Phi_{R}(x^{\mu}, y) \to \Phi_{R}(x^{\mu}, -y) = P\Phi_{R}(x^{\mu}, y), \qquad (R = 5, \overline{5})$$

$$\Phi_{R}(x^{\mu}, y) \to \Phi_{R}(x^{\mu}, -y) = P\Phi_{R}(x^{\mu}, y)P^{-1}, \qquad (R = 10, \overline{10}) \quad (2a)$$

$$A_{\mu}(x^{\mu}, y') \to A_{\mu}(x^{\mu}, -y') = P'A_{\mu}(x^{\mu}, y')P'^{-1},$$

$$A_{5}(x^{\mu}, y') \to A_{5}(x^{\mu}, -y') = -P'A_{5}(x^{\mu}, y')P'^{-1},$$

$$\Phi_{R}(x^{\mu}, y') \to \Phi_{R}(x^{\mu}, -y') = P'\Phi_{R}(x^{\mu}, y'), \qquad (R = 5, \overline{5})$$

^{*)} Recently, Barbieri, Hall and Nomura have constructed a constrained standard model upon a compactification of a 5D SUSY model on the orbifold $S^1/(Z_2\times Z_2')$. They used a $Z_2\times Z_2'$ parity to reduce SUSY. There are also several works on model building through a reduction of SUSY $^{18)-21}$ or a gauge symmetry $^{22)}$ by the use of a Z_2 parity. Attempts to construct GUT have been made through dimensional reduction over coset space. $^{23)}$

$$\Phi_{R}(x^{\mu}, y') \to \Phi_{R}(x^{\mu}, -y') = P'\Phi_{R}(x^{\mu}, y')P'^{-1}, \quad (R = 10, \overline{10})$$
 (2b)

where P and P' are 5×5 matrices that satisfy $P^2 = P'^2 = I$ where I is the unit matrix. Here A_M and $\Phi_{\mathbf{10}(\overline{\mathbf{10}})}$ are expressed by 5×5 symmetric and anti-symmetric matrices, respectively. The intrinsic $Z_2 \times Z_2'$ parity of each component is given by an eigenvalue of P and P'.

When we use P = diag(1, 1, 1, 1, 1) and P' = diag(-1, -1, -1, 1),* the SU(5) gauge symmetry is reduced to that of the SM, $G_{\text{SM}} \equiv SU(3) \times SU(2) \times U(1)$, in the 4D theory.** This is because the boundary conditions on $S^1/(Z_2 \times Z_2')$ given in (2b) do not respect SU(5) symmetry, as we see from the relations for the gauge generators T^A (A = 1, 2, ..., 24),

$$P'T^aP'^{-1} = T^a, \quad P'T^{\hat{a}}P'^{-1} = -T^{\hat{a}}.$$
 (3)

The T^a are gauge generators of $G_{\rm SM}$, and the $T^{\hat{a}}$ are the other gauge generators. The parity assignment and mass spectrum after compactification are given in Table I. The scalar fields Φ_R are broken up into several pieces as

$$\Phi_{\mathbf{5}} = \phi_C + \phi_W, \quad \Phi_{\overline{\mathbf{5}}} = \overline{\phi}_C + \overline{\phi}_W,
\Phi_{\mathbf{10}} = Q + \overline{U} + \overline{E}, \quad \Phi_{\overline{\mathbf{10}}} = \overline{Q} + U + E.$$
(4)

In the second column, we give $SU(3) \times SU(2)$ quantum numbers of 4D fields. In the third column, (\pm, \pm) and (\pm, \mp) denote the eigenvalues $(\pm 1, \pm 1)$ and $(\pm 1, \mp 1)$ of $Z_2 \times Z_2'$ parity. The fields $\phi_{\pm\pm}(x^{\mu}, y)$ and $\phi_{\pm\mp}(x^{\mu}, y)$, whose values of intrinsic parity are $(\pm 1, \pm 1)$ and $(\pm 1, \mp 1)$, are Fourier expanded as

$$\phi_{++}(x^{\mu}, y) = \sqrt{\frac{2}{\pi R}} \sum_{n=0}^{\infty} \phi_{++}^{(2n)}(x^{\mu}) \cos \frac{2ny}{R}, \tag{5a}$$

$$\phi_{+-}(x^{\mu}, y) = \sqrt{\frac{2}{\pi R}} \sum_{n=0}^{\infty} \phi_{+-}^{(2n+1)}(x^{\mu}) \cos \frac{(2n+1)y}{R}, \tag{5b}$$

$$\phi_{-+}(x^{\mu}, y) = \sqrt{\frac{2}{\pi R}} \sum_{n=0}^{\infty} \phi_{-+}^{(2n+1)}(x^{\mu}) \sin \frac{(2n+1)y}{R},$$
 (5c)

$$\phi_{--}(x^{\mu}, y) = \sqrt{\frac{2}{\pi R}} \sum_{n=0}^{\infty} \phi_{--}^{(2n+2)}(x^{\mu}) \sin \frac{(2n+2)y}{R}, \tag{5d}$$

where n is zero or a positive integer, and the fields $\phi_{++}^{(2n)}(x^{\mu})$, $\phi_{\pm\mp}^{(2n+1)}(x^{\mu})$ and $\phi_{--}^{(2n+2)}(x^{\mu})$ acquire masses $\frac{2n}{R}$, $\frac{2n+1}{R}$ and $\frac{2n+2}{R}$ upon compactification. Note that 4D massless fields appear only from components with even parity (+1,+1). The contribution from the potential $V(\Phi_R)$ is not considered in the fourth column. In the low-energy spectrum, there are a pair of lepto-quark bosons, $Q^{(0)}$ and $\overline{Q}^{(0)}$, which

^{*)} The exchange of P for P' is equivalent to the exchange of two walls.

^{**)} Our symmetry reduction mechanism is different from the Hosotani mechanism. ²⁴⁾ In fact, the Hosotani mechanism does not work in our case, because $A_5^a(x^{\mu}, y)$ has odd parity, as given in (2a), and its VEV should vanish.

| Table 1. Parity and mass spectrum. | | | | |
|------------------------------------|---|--------------------------|------------------|--|
| 4D fields | Quantum numbers | $Z_2 \times Z_2'$ parity | Mass | |
| $A_{\mu}^{a(2n)}$ | (8 , 1) + (1 , 3) + (1 , 1) | (+,+) | $\frac{2n}{R}$ | |
| $A_{\mu}^{\hat{a}(2n+1)}$ | $({f 3},{f 2})+({f \overline 3},{f 2})$ | (+, -) | $\frac{2n+1}{R}$ | |
| $A_5^{a(2n+2)}$ | (8 , 1) + (1 , 3) + (1 , 1) | (-, -) | $\frac{2n+2}{R}$ | |
| $A_5^{\hat{a}(2n+1)}$ | $({f 3},{f 2})+({f \overline 3},{f 2})$ | (-, +) | $\frac{2n+1}{R}$ | |
| $\phi_C^{(2n+1)}$ | (3,1) | (+, -) | $\frac{2n+1}{R}$ | |
| $\phi_W^{(2n)}$ | (1 , 2) | (+,+) | $\frac{2n}{R}$ | |
| $\overline{\phi}_C^{(2n+1)}$ | $(\overline{3},1)$ | (+, -) | $\frac{2n+1}{R}$ | |
| $\overline{\phi}_W^{(2n)}$ | (1 , 2) | (+,+) | $\frac{2n}{R}$ | |
| $Q^{(2n)}$ | (3, 2) | (+,+) | $\frac{2n}{R}$ | |
| $\overline{U}^{(2n+1)}$ | $(\overline{3},1)$ | (+, -) | $\frac{2n+1}{R}$ | |
| $\overline{E}^{(2n+1)}$ | (1, 1) | (+, -) | $\frac{2n+1}{R}$ | |
| $\overline{Q}^{(2n)}$ | $(\overline{f 3},{f 2})$ | (+,+) | $\frac{2n}{R}$ | |
| $U^{(2n+1)}$ | (3, 1) | (+, -) | $\frac{2n+1}{R}$ | |
| r_{2n+1} | (4.4) | () | 2n+1 | |

Table I. Parity and mass spectrum.

have both color and weak charge. The SM gauge bosons and the weak Higgs doublet are equivalent to $A_{\mu}^{a(0)}$ and $\phi_W^{(0)}$ (or $\overline{\phi}_W^{(0)}$), respectively. The mass split of the bosons is realized by the $Z_2 \times Z_2'$ projection.

After integrating out the fifth dimension, we obtain the 4D Lagrangian density

(+, -)

$$\mathcal{L}_{\text{eff}}^{(4)} = -\frac{1}{4} \sum_{a} F_{\mu\nu}^{a(0)^{2}} + |D_{\mu}\phi_{W}^{(0)}|^{2} + |D_{\mu}\overline{\phi}_{W}^{(0)}|^{2}
+ |D_{\mu}Q^{(0)}|^{2} + |D_{\mu}\overline{Q}^{(0)}|^{2} - V(\phi_{W}^{(0)}, \overline{\phi}_{W}^{(0)}, Q^{(0)}, \overline{Q}^{(0)})
+ \sum_{3\text{families}} (i\overline{\psi}_{\mathbf{10}}\gamma^{\mu}D_{\mu}\psi_{\mathbf{10}} + i\overline{\psi}_{\overline{\mathbf{5}}}\gamma^{\mu}D_{\mu}\psi_{\overline{\mathbf{5}}}
+ f_{U}\phi_{W}^{(0)}q\overline{u} + f_{D}\phi_{\overline{W}}^{(0)}q\overline{d} + f_{D}\phi_{\overline{W}}^{(0)}l\overline{e} + f_{Q}Ql\overline{d} + \text{h.c.}) + \cdots,$$
(6)

where $D_{\mu} \equiv \partial_{\mu} - ig_{U}A_{\mu}^{(0)}$ and the dots represent terms including Kaluza-Klein modes. In this equation, $g_{U} \ (\equiv \sqrt{2/\pi R}g_{(5)})$ is a 4D gauge coupling constant, $f_{U} \ (\equiv \sqrt{2/\pi R}f_{U(5)})$, $f_{D} \ (\equiv \sqrt{2/\pi R}f_{D(5)})$ and $f_{Q} \ (\equiv \sqrt{2/\pi R}f_{Q(5)})$ are 4D Yukawa coupling matrices, q, \overline{u} and \overline{d} are quarks, and l and \overline{e} are leptons. With our parity assignment, we have obtained an extension of the SM with two Higgs doublets, $\phi_{W}^{(0)}$ and $\overline{\phi}_{W}^{(0)}$, and extra lepto-quark bosons $Q^{(0)}$ and $\overline{Q}^{(0)}$.

The theory predicts that coupling constants are unified around the compactification scale $M_C(\equiv 1/R)$ to zero-th order approximation, as in the ordinary SU(5) GUT, $^{(1)}$

$$g_3 = g_2 = g_1 = g_U, \quad f_d = f_e = f_D,$$
 (7)

where f_d and f_e are Yukawa coupling matrices on down-type quarks and electrontype leptons, respectively. As shown in Ref. 15), this type of extension of the SM can survive with the precision measurements at LEP.³⁾

It is known that there is a significant contribution to the proton decay, due to the X and Y gauge bosons in the minimal SU(5) GUT. ⁴⁾ In our model, we have diagrams similar to those in the minimal SU(5) GUT, because a quark and lepton couple to the Kaluza-Klein modes of extra vector bosons at the tree level. However, we expect that proton stability is guaranteed if our 4D wall fluctuates flexibly. This is due to the fact that there is an exponential suppression factor in the coupling to the Kaluza-Klein excitations by the brane recoil effect. ²⁵⁾

We have obtained the simplest extension of the SM compatible with SU(5) grand unification. It would be possible to construct more complex models by increasing the number of extra multiplets. For reference, the pattern of split due to Z'_2 parity is

Table II. Split due to Z'_2 parity.

| R | Quantum numbers | Z_2' parity | |
|----|---|---------------|--|
| 5 | (3,1) | P_{5} | |
| | (1,2) | $-P_5$ | |
| 10 | (3, 2) | P_{10} | |
| | $({f \overline{3}},{f 1})+({f 1},{f 1})$ | $-P_{10}$ | |
| 15 | (3, 2) | P_{15} | |
| | (6 , 1)+(1 , 3) | $-P_{15}$ | |
| 24 | (8,1) + (1,3) + (1,1) | P_{24} | |
| | $(3,2)+(\overline{3},2)$ | $-P_{24}$ | |
| 45 | $(8,2)+(\overline{3},2)+(1,2)$ | P_{45} | |
| | $(\overline{f 6},{f 1})+({f 3},{f 1})+(\overline{f 3},{f 1})$ | $-P_{45}$ | |
| | +(3,3) | | |
| 75 | $({f 6},{f 2})+({f \overline{6}},{f 2})+({f 3},{f 2})$ | P_{75} | |
| | $+(\overline{f 3},{f 2})$ | | |
| | $(3,1) + (\overline{3},1) + (1,1)$ | $-P_{75}$ | |
| | +(8,3)+(8,1) | | |

given in Table II for several low dimensional representations of SU(5). In the second column, we give $SU(3) \times SU(2)$ quantum numbers for split multiplets. In the third column, P_R is an eigenvalue of Z_2' parity, i.e., $P_R = 1$ or -1. The table includes components that can induce a rapid nucleon decay when they couple to quarks and leptons.

Our grand unification scenario is phenomenologically interesting because it suggests the existence of extra split multiplets at the weak scale. However, there is a problem of determining how to break the electro-weak symmetry naturally and how to stabilize the weak scale; that is, our model suffers from a naturalness problem. ²⁶⁾ There is an alternative description in which the extra space has a large radius. ²⁷⁾ In this case, the low-energy gauge coupling unification is expected to be realized by a power-law correction. ²⁸⁾

¹⁾ H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32 (1974), 438.

²⁾ For a review, see P. Langacker, Phys. Rep. 72 (1981), 185.

- 3) LEP Collaborations, Phys. Lett. **B276** (1992), 247.
- 4) Super-Kamiokande Collaborations, Phys. Rev. Lett. 81 (1998), 3319.
- 5) E. Gildener, Phys. Rev. **D14** (1976), 1667.
- 6) C. Giunti, C. W. Kim and U. W. Lee, Mod. Phys. Lett. A6 (1991), 1745.
 - J. Ellis, S. Kelley and D. V. Nanopoulos, Phys. Lett. **B260** (1991), 131.
 - U. Amaldi, W. de Boer and H. Fürstenau, Phys. Lett. **B260** (1991), 447.
 - P. Langacker and M. Luo, Phys. Rev. **D44** (1991), 817.
- 7) M. J. G. Veltman, Acta Phys. Pol. **B12** (1981), 437.
 - L. Maiani, Gif-sur-Yvette Summer School on Particle Physics, 11th, Gif-sur-Yvette, France, 1979 (Inst. Nat. Phys. Nucl. Phys. Particules, Paris, 1979).
 - E. Witten, Nucl. Phys. **B185** (1981), 513.
 - M. Dine, W. Fischler and M. Srednicki, Nucl. Phys. B189 (1981), 575.
 - S. Dimopoulos and S. Raby, Nucl. Phys. **B192** (1981), 353.
- 8) S. Dimopoulos and H. Georgi, Nucl. Phys. **B193** (1981), 150.
 - N. Sakai, Z. Phys. **C11** (1981), 153.
- 9) N. Sakai and T. Yanagida, Nucl. Phys. **B197** (1982), 533.
 - S. Weinberg, Phys. Rev. **D26** (1982), 287.
- 10) Super-Kamiokande Collaborations, Phys. Rev. Lett. 83 (1999), 1529.
- 11) T. Goto and T. Nihei, Phys. Rev. **D59** (1999), 115009.
- 12) Y. Kawamura, hep-ph/0012125.
- 13) P. H. Frampton and S. L. Glashow, Phys. Lett. 131 (1983), 340.
- 14) K. Hagiwara, F. Halzen and K. Hikasa, Phys. Lett. 141 (1984), 372.
- 15) H. Murayama and T. Yanagida, Mod. Phys. Lett. A7 (1992), 147.
- J. L. Chkareuli, I. G. Gogoladze and A. B. Kobakhidze, Phys. Lett. B340 (1994), 63.
 J. L. Chkareuli, I. G. Gogoladze and A. B. Kobakhidze, Phys. Lett. B376 (1996), 111.
- 17) R. Barbieri, L. J. Hall and Y. Nomura, hep-ph/0011311.
- L. Antoniadis, C. Muñoz and M. Quirós, Phys. Lett. B246 (1990), 377; Nucl. Phys. B397 (1993), 515.
 - L. Antoniadis and K. Benakli, Phys. Lett. **B326** (1994), 69.
- 19) P. Horava and E. Witten, Nucl. Phys. **B460** (1996), 506; **B475** (1996), 94.
- 20) E. A. Mirabelli and M. Peskin, Phys. Rev. D58 (1998), 065002.
- 21) A. Pomarol and M. Quirós, Phys. Lett. **B438** (1998), 255.
- 22) Y. Kawamura, Prog. Theor. Phys. 103 (2000), 613.
- 23) D. Kapetanakis and G. Zoupanos, Phys. Rep. 219 (1992), 1 and references therein.
- 24) Y. Hosotani, Phys. Lett. **B126** (1983), 309; Ann. of Phys. **190** (1989), 233.
- 25) M. Bando, T. Kugo, T. Noguchi and K. Yoshioka, Phys. Rev. Lett. 83 (1999), 3601.
- 26) G. 't Hooft, Cargese Summer Inst. Lectures, 1979 (Plenum Press, New York, 1980).
- 27) L. Antoniadis, Phys. Lett. **B246** (1990), 377.
 - J. Lykken, Phys. Rev. **D54** (1996), 3693.
 - N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. **B429** (1998), 263.
- 28) K. R. Dienes, E. Dudas and T. Gherghetta, Phys. Lett. **B436** (1998), 55.