# Split Multiplets, Coupling Unification and an Extra Dimension 

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(Received December 27, 2000)


#### Abstract

We study a gauge coupling unification scenario based on a non-supersymmetric 5dimensional model. Through an orbifold compactification, we obtain the Standard Model with split multiplets on a 4 -dimensional wall, which is compatible with a grand unification.


The Standard Model (SM) has been established as an effective theory below the weak scale. One of intriguing investigations beyond the SM is to unify gauge interactions under a simple group, such as $S U(5) .{ }^{1)}$ This scenario is very attractive, ${ }^{2)}$ but it suffers from several problems in the simplest version. The first problem is that gauge coupling constants do not meet at a high-energy scale, based on the desert hypothesis. ${ }^{3)}$ The second problem is that a dangerous proton decay is induced by an exchange of $X$ and $Y$ gauge bosons. ${ }^{4)}$ The last problem is that the weak scale is not stabilized by quantum corrections (the gauge hierarchy problem). ${ }^{5)}$

The introduction of supersymmetry (SUSY) solves the first ${ }^{6}$ ) and the third problems. ${ }^{7), * *)}$ A supersymmetric grand unified theory (SUSY GUT) is an attractive possibility as a high-energy theory, ${ }^{8)}$ but proton stability is threatened due to a contribution from the dimension 5 operator in the minimal SUSY $S U(5)$ GUT. ${ }^{9), 10 \text { ) }}$ Recently, stronger constraints have been obtained from analysis including a Higgsino dressing diagram with right-handed matter fields. ${ }^{11)}$

A new possibility ${ }^{12)}$ has been proposed to solve the above problems. Starting from a 5-dimensional (5D) SUSY $S U(5)$ model, we have obtained a low-energy theory with particles of the minimal supersymmetric standard model (MSSM) on a 4 D wall through compactification upon $S^{1} /\left(Z_{2} \times Z_{2}^{\prime}\right)$. In this theory, proton stability is guaranteed due to the presence of a suppression factor in the coupling to the Kaluza-Klein modes if our 4D wall fluctuates flexibly.

In this paper, we propose another possibility to solve the first and second problems based on a 5D model without SUSY. Here the gauge coupling unification is realized by the introduction of extra multiplets that split after an orbifold compactification. ${ }^{* * *)}$ The splitting originates from a non-universal parity assignment on a compact space among components in each multiplet. The proton decay is sufficiently suppressed by a suppression factor in the coupling to the Kaluza-Klein

[^0]excitations if our 4D wall fluctuates pliantly. In the following, we derive a model discussed in Ref. 15) as an example of a low-energy theory from a 5D $S U(5)$ model through compactification upon $S^{1} /\left(Z_{2} \times Z_{2}^{\prime}\right)$.

The space-time is considered to be factorized into a product of 4D Minkowski space-time $M^{4}$ and the orbifold $S^{1} /\left(Z_{2} \times Z_{2}^{\prime}\right),{ }^{*}$ whose coordinates are denoted by $x^{\mu}$ $(\mu=0,1,2,3)$ and $y\left(=x^{5}\right)$, respectively. The 5 D notation $x^{M}(M=0,1,2,3,5)$ is also used. The orbifold is regarded as an interval with a distance of $\pi R / 2$. There are two 4D walls placed at fixed points $y=-\pi R / 2$ and $y=0$ (or $y^{\prime}=0$ and $y^{\prime}=\pi R / 2$ ) on $S^{1} /\left(Z_{2} \times Z_{2}^{\prime}\right)$, where $y^{\prime} \equiv y+\pi R / 2$.

We assume that the 5D gauge boson $A_{M}\left(x^{\mu}, y\right)$ and four kinds of scalar fields $\Phi_{\boldsymbol{R}}\left(x^{\mu}, y\right)(\boldsymbol{R}=\mathbf{5}, \overline{\mathbf{5}}, \mathbf{1 0}, \overline{\mathbf{1 0}})$ exist in the bulk $M^{4} \times S^{1} /\left(Z_{2} \times Z_{2}^{\prime}\right)$. The fields $A_{M}$ and $\Phi_{\boldsymbol{R}}$ form an adjoint representation 24 and a representation $\boldsymbol{R}$ of $S U(5)$, respectively. We assume that our visible world is a 4D wall fixed at $y=0$ (We call it "wall I".) and that three families of quarks and leptons, $3\left\{\psi_{\overline{5}}+\psi_{\mathbf{1 0}}\right\}$, are located on wall I. (Here and hereafter we suppress the family index.) The representations of $\psi_{\overline{5}}$ and $\psi_{10}$ are $\overline{\mathbf{5}}$ and $\mathbf{1 0}$ of $S U(5)$, respectively. Note that matter fields contain no excited states along the $S^{1} /\left(Z_{2} \times Z_{2}^{\prime}\right)$ direction.

The gauge invariant action is given by

$$
\begin{align*}
S= & \int d^{5} x\left(-\frac{1}{2} \operatorname{tr} F_{M N}^{2}+\sum_{\boldsymbol{R}}\left|D_{M} \Phi_{\boldsymbol{R}}\right|^{2}-V\left(\Phi_{\boldsymbol{R}}\right)\right) \\
& +\frac{1}{2} \int d^{5} x \delta(y) \sum_{\text {3families }}\left(i \bar{\psi}_{\mathbf{1 0}} \gamma^{M} D_{M} \psi_{\mathbf{1 0}}+i \bar{\psi}_{\overline{\mathbf{5}}} \gamma^{M} D_{M} \psi_{\overline{\mathbf{5}}}\right. \\
& \left.+f_{U(5)} \Phi_{\mathbf{5}} \psi_{\mathbf{1 0}} \psi_{\mathbf{1 0}}+f_{D(5)} \Phi_{\overline{\mathbf{5}}} \psi_{\mathbf{1 0}} \psi_{\overline{\mathbf{5}}}+f_{Q(5)} \Phi_{\mathbf{1 0}} \psi_{\overline{\mathbf{5}}} \psi_{\overline{\mathbf{5}}}+\text { h.c. }\right) \\
& +(\text { terms from a wall fixed at } y=-\pi R), \tag{1}
\end{align*}
$$

where $D_{M} \equiv \partial_{M}-i g_{(5)} A_{M}\left(x^{\mu}, y\right), g_{(5)}$ is a 5 D gauge coupling constant, and $f_{U(5)}$, $f_{D(5)}$ and $f_{Q(5)}$ are 5D Yukawa coupling matrices. In the 4 D action, the bulk fields $A_{M}$ and $\Phi_{\boldsymbol{R}}$ are replaced by fields including the Nambu-Goldstone boson $\phi\left(x^{\mu}\right)$ at wall I such that they acquire the functional dependences $A_{M}\left(x^{\mu}, \phi\left(x^{\mu}\right)\right)$ and $\Phi_{\boldsymbol{R}}\left(x^{\mu}, \phi\left(x^{\mu}\right)\right)$. The Lagrangian is invariant under the $Z_{2} \times Z_{2}^{\prime}$ transformation

$$
\begin{array}{rll}
A_{\mu}\left(x^{\mu}, y\right) \rightarrow A_{\mu}\left(x^{\mu},-y\right) & =P A_{\mu}\left(x^{\mu}, y\right) P^{-1}, & \\
A_{5}\left(x^{\mu}, y\right) \rightarrow A_{5}\left(x^{\mu},-y\right) & =-P A_{5}\left(x^{\mu}, y\right) P^{-1}, & \\
\Phi_{\boldsymbol{R}}\left(x^{\mu}, y\right) \rightarrow \Phi_{\boldsymbol{R}}\left(x^{\mu},-y\right) & =P \Phi_{\boldsymbol{R}}\left(x^{\mu}, y\right), & (\boldsymbol{R}=\mathbf{5}, \overline{\mathbf{5}}) \\
\Phi_{\boldsymbol{R}}\left(x^{\mu}, y\right) \rightarrow \Phi_{\boldsymbol{R}}\left(x^{\mu},-y\right) & =P \Phi_{\boldsymbol{R}}\left(x^{\mu}, y\right) P^{-1}, \quad(\boldsymbol{R}=\mathbf{1 0}, \overline{\mathbf{1 0}}) \\
A_{\mu}\left(x^{\mu}, y^{\prime}\right) \rightarrow A_{\mu}\left(x^{\mu},-y^{\prime}\right) & =P^{\prime} A_{\mu}\left(x^{\mu}, y^{\prime}\right) P^{\prime-1}, \\
A_{5}\left(x^{\mu}, y^{\prime}\right) \rightarrow A_{5}\left(x^{\mu},-y^{\prime}\right) & =-P^{\prime} A_{5}\left(x^{\mu}, y^{\prime}\right) P^{\prime-1}, \\
\Phi_{\boldsymbol{R}}\left(x^{\mu}, y^{\prime}\right) \rightarrow \Phi_{\boldsymbol{R}}\left(x^{\mu},-y^{\prime}\right) & =P^{\prime} \Phi_{\boldsymbol{R}}\left(x^{\mu}, y^{\prime}\right), \quad(\boldsymbol{R}=\mathbf{5}, \overline{\mathbf{5}})
\end{array}
$$

[^1]\[

$$
\begin{equation*}
\Phi_{\boldsymbol{R}}\left(x^{\mu}, y^{\prime}\right) \rightarrow \Phi_{\boldsymbol{R}}\left(x^{\mu},-y^{\prime}\right) \quad=\quad P^{\prime} \Phi_{\boldsymbol{R}}\left(x^{\mu}, y^{\prime}\right) P^{\prime-1}, \quad(\boldsymbol{R}=\mathbf{1 0}, \overline{\mathbf{1 0}}) \tag{2b}
\end{equation*}
$$

\]

where $P$ and $P^{\prime}$ are $5 \times 5$ matrices that satisfy $P^{2}=P^{\prime 2}=I$ where $I$ is the unit matrix. Here $A_{M}$ and $\Phi_{\mathbf{1 0}(\overline{\mathbf{1 0}})}$ are expressed by $5 \times 5$ symmetric and anti-symmetric matrices, respectively. The intrinsic $Z_{2} \times Z_{2}^{\prime}$ parity of each component is given by an eigenvalue of $P$ and $P^{\prime}$.

When we use $P=\operatorname{diag}(1,1,1,1,1)$ and $P^{\prime}=\operatorname{diag}(-1,-1,-1,1,1),{ }^{*)}$ the $S U(5)$ gauge symmetry is reduced to that of the $\mathrm{SM}, G_{\mathrm{SM}} \equiv S U(3) \times S U(2) \times U(1)$, in the 4 D theory.**) This is because the boundary conditions on $S^{1} /\left(Z_{2} \times Z_{2}^{\prime}\right)$ given in (2b) do not respect $S U(5)$ symmetry, as we see from the relations for the gauge generators $T^{A}(A=1,2, \ldots, 24)$,

$$
\begin{equation*}
P^{\prime} T^{a} P^{\prime-1}=T^{a}, \quad P^{\prime} T^{\hat{a}} P^{\prime-1}=-T^{\hat{a}} . \tag{3}
\end{equation*}
$$

The $T^{a}$ are gauge generators of $G_{S M}$, and the $T^{\hat{a}}$ are the other gauge generators. The parity assignment and mass spectrum after compactification are given in Table I. The scalar fields $\Phi_{\boldsymbol{R}}$ are broken up into several pieces as

$$
\begin{align*}
\Phi_{\mathbf{5}} & =\phi_{C}+\phi_{W}, \quad \Phi_{\overline{\mathbf{5}}}=\bar{\phi}_{C}+\bar{\phi}_{W} \\
\Phi_{\mathbf{1 0}} & =Q+\bar{U}+\bar{E}, \quad \Phi_{\overline{\mathbf{1 0}}}=\bar{Q}+U+E . \tag{4}
\end{align*}
$$

In the second column, we give $S U(3) \times S U(2)$ quantum numbers of 4 D fields. In the third column, $( \pm, \pm)$ and $( \pm, \mp)$ denote the eigenvalues $( \pm 1, \pm 1)$ and $( \pm 1, \mp 1)$ of $Z_{2} \times Z_{2}^{\prime}$ parity. The fields $\phi_{ \pm \pm}\left(x^{\mu}, y\right)$ and $\phi_{ \pm \mp}\left(x^{\mu}, y\right)$, whose values of intrinsic parity are $( \pm 1, \pm 1)$ and $( \pm 1, \mp 1)$, are Fourier expanded as

$$
\begin{align*}
\phi_{++}\left(x^{\mu}, y\right) & =\sqrt{\frac{2}{\pi R}} \sum_{n=0}^{\infty} \phi_{++}^{(2 n)}\left(x^{\mu}\right) \cos \frac{2 n y}{R}  \tag{5a}\\
\phi_{+-}\left(x^{\mu}, y\right) & =\sqrt{\frac{2}{\pi R}} \sum_{n=0}^{\infty} \phi_{+-}^{(2 n+1)}\left(x^{\mu}\right) \cos \frac{(2 n+1) y}{R}  \tag{5b}\\
\phi_{-+}\left(x^{\mu}, y\right) & =\sqrt{\frac{2}{\pi R}} \sum_{n=0}^{\infty} \phi_{-+}^{(2 n+1)}\left(x^{\mu}\right) \sin \frac{(2 n+1) y}{R}  \tag{5c}\\
\phi_{--}\left(x^{\mu}, y\right) & =\sqrt{\frac{2}{\pi R}} \sum_{n=0}^{\infty} \phi_{--}^{(2 n+2)}\left(x^{\mu}\right) \sin \frac{(2 n+2) y}{R} \tag{5d}
\end{align*}
$$

where $n$ is zero or a positive integer, and the fields $\phi_{++}^{(2 n)}\left(x^{\mu}\right), \phi_{ \pm \mp}^{(2 n+1)}\left(x^{\mu}\right)$ and $\phi_{--}^{(2 n+2)}\left(x^{\mu}\right)$ acquire masses $\frac{2 n}{R}, \frac{2 n+1}{R}$ and $\frac{2 n+2}{R}$ upon compactification. Note that 4D massless fields appear only from components with even parity $(+1,+1)$. The contribution from the potential $V\left(\Phi_{\boldsymbol{R}}\right)$ is not considered in the fourth column. In the low-energy spectrum, there are a pair of lepto-quark bosons, $Q^{(0)}$ and $\bar{Q}^{(0)}$, which

[^2]Table I. Parity and mass spectrum.

| 4 D fields | Quantum numbers | $Z_{2} \times Z_{2}^{\prime}$ parity | Mass |
| :--- | :--- | :--- | :--- |
| $A_{\mu}^{a(2 n)}$ | $(\mathbf{8}, \mathbf{1})+(\mathbf{1}, \mathbf{3})+(\mathbf{1}, \mathbf{1})$ | $(+,+)$ | $\frac{2 n}{R}$ |
| $A_{\mu}^{\hat{a}(2 n+1)}$ | $(\mathbf{3}, \mathbf{2})+(\overline{\mathbf{3}}, \mathbf{2})$ | $(+,-)$ | $\frac{2 n+1}{R}$ |
| $A_{5}^{a(2 n+2)}$ | $(\mathbf{8}, \mathbf{1})+(\mathbf{1}, \mathbf{3})+(\mathbf{1}, \mathbf{1})$ | $(-,-)$ | $\frac{2 n+2}{R}$ |
| $A_{5}^{\hat{a}(2 n+1)}$ | $(\mathbf{3}, \mathbf{2})+(\overline{\mathbf{3}}, \mathbf{2})$ | $(-,+)$ | $\frac{2 n+1}{R}$ |
| $\phi_{C}^{(2 n+1)}$ | $(\mathbf{3}, \mathbf{1})$ | $(+,-)$ | $\frac{2 n+1}{R}$ |
| $\phi_{W}^{(2 n)}$ | $(\mathbf{1}, \mathbf{2})$ | $(+,+)$ | $\frac{2 n}{R}$ |
| $\bar{\phi}_{C}^{(2 n+1)}$ | $(\overline{\mathbf{3}}, \mathbf{1})$ | $(+,-)$ | $\frac{2 n+1}{R}$ |
| $\bar{\phi}_{W}^{(2 n)}$ | $(\mathbf{1}, \mathbf{2})$ | $(+,+)$ | $\frac{2 n}{R}$ |
| $Q^{(2 n)}$ | $(\mathbf{3}, \mathbf{2})$ | $(+,+)$ | $\frac{2 n}{R}$ |
| $\bar{U}^{(2 n+1)}$ | $(\overline{\mathbf{3}}, \mathbf{1})$ | $(+,-)$ | $\frac{2 n+1}{R}$ |
| $\bar{E}^{(2 n+1)}$ | $(\mathbf{1}, \mathbf{1})$ | $(+,-)$ | $\frac{2 n+1}{R}$ |
| $\bar{Q}^{(2 n)}$ | $(\overline{\mathbf{3}}, \mathbf{2})$ | $(+,+)$ | $\frac{2 n}{R}$ |
| $U^{(2 n+1)}$ | $(\mathbf{3}, \mathbf{1})$ | $(+,-)$ | $\frac{2 n+1}{R}$ |
| $E^{(2 n+1)}$ | $(\mathbf{1}, \mathbf{1})$ | $(+,-)$ | $\frac{2 n+1}{R}$ |

have both color and weak charge. The SM gauge bosons and the weak Higgs doublet are equivalent to $A_{\mu}^{a(0)}$ and $\phi_{W}^{(0)}\left(\right.$ or $\left.\bar{\phi}_{W}^{(0)}\right)$, respectively. The mass split of the bosons is realized by the $Z_{2} \times Z_{2}^{\prime}$ projection.

After integrating out the fifth dimension, we obtain the 4D Lagrangian density

$$
\begin{align*}
\mathcal{L}_{\text {eff }}^{(4)}= & -\frac{1}{4} \sum_{a} F_{\mu \nu}^{a(0)^{2}}+\left|D_{\mu} \phi_{W}^{(0)}\right|^{2}+\left|D_{\mu} \bar{\phi}_{W}^{(0)}\right|^{2} \\
& +\left|D_{\mu} Q^{(0)}\right|^{2}+\left|D_{\mu} \bar{Q}^{(0)}\right|^{2}-V\left(\phi_{W}^{(0)}, \bar{\phi}_{W}^{(0)}, Q^{(0)}, \bar{Q}^{(0)}\right) \\
& +\sum_{\text {3families }}\left(i \bar{\psi}_{\mathbf{1 0}} \gamma^{\mu} D_{\mu} \psi_{\mathbf{1 0}}+i \bar{\psi} \overline{\overline{\mathbf{5}}} \gamma^{\mu} D_{\mu} \psi_{\overline{\mathbf{5}}}\right. \\
& \left.+f_{U} \phi_{W}^{(0)} q \bar{u}+f_{D} \phi_{\bar{W}}^{(0)} q \bar{d}+f_{D} \phi_{\bar{W}}^{(0)} l \bar{e}+f_{Q} Q l \bar{d}+\text { h.c. }\right)+\cdots, \tag{6}
\end{align*}
$$

where $D_{\mu} \equiv \partial_{\mu}-i g_{U} A_{\mu}^{(0)}$ and the dots represent terms including Kaluza-Klein modes. In this equation, $g_{U}\left(\equiv \sqrt{2 / \pi R} g_{(5)}\right)$ is a 4 D gauge coupling constant, $f_{U}$ $\left(\equiv \sqrt{2 / \pi R} f_{U(5)}\right), f_{D}\left(\equiv \sqrt{2 / \pi R} f_{D(5)}\right)$ and $f_{Q}\left(\equiv \sqrt{2 / \pi R} f_{Q(5)}\right)$ are 4D Yukawa coupling matrices, $q, \bar{u}$ and $\bar{d}$ are quarks, and $l$ and $\bar{e}$ are leptons. With our parity assignment, we have obtained an extension of the SM with two Higgs doublets, $\phi_{W}^{(0)}$ and $\bar{\phi}_{W}^{(0)}$, and extra lepto-quark bosons $Q^{(0)}$ and $\bar{Q}^{(0)}$.

The theory predicts that coupling constants are unified around the compactification scale $M_{C}(\equiv 1 / R)$ to zero-th order approximation, as in the ordinary $S U(5)$ GUT, ${ }^{1)}$

$$
\begin{equation*}
g_{3}=g_{2}=g_{1}=g_{U}, \quad f_{d}=f_{e}=f_{D} \tag{7}
\end{equation*}
$$

where $f_{d}$ and $f_{e}$ are Yukawa coupling matrices on down-type quarks and electrontype leptons, respectively. As shown in Ref. 15), this type of extension of the SM can survive with the precision measurements at LEP. ${ }^{3)}$

It is known that there is a significant contribution to the proton decay, due to the $X$ and $Y$ gauge bosons in the minimal $S U(5)$ GUT. ${ }^{4)}$ In our model, we have diagrams similar to those in the minimal $S U(5)$ GUT, because a quark and lepton couple to the Kaluza-Klein modes of extra vector bosons at the tree level. However, we expect that proton stability is guaranteed if our 4D wall fluctuates flexibly. This is due to the fact that there is an exponential suppression factor in the coupling to the Kaluza-Klein excitations by the brane recoil effect. ${ }^{25)}$

We have obtained the simplest extension of the SM compatible with $S U(5)$ grand unification. It would be possible to construct more complex models by increasing the number of extra multiplets. For reference, the pattern of split due to $Z_{2}^{\prime}$ parity is given in Table II for several low dimen-

Table II. Split due to $Z_{2}^{\prime}$ parity.

| $R$ | Quantum numbers | $Z_{2}^{\prime}$ parity |
| ---: | :--- | ---: |
| $\mathbf{5}$ | $(\mathbf{3}, \mathbf{1})$ | $P_{\mathbf{5}}$ |
|  | $(\mathbf{1}, \mathbf{2})$ | $-P_{\mathbf{5}}$ |
| $\mathbf{1 0}$ | $(\mathbf{3}, \mathbf{2})$ | $P_{\mathbf{1 0}}$ |
|  | $(\overline{\mathbf{3}}, \mathbf{1})+(\mathbf{1}, \mathbf{1})$ | $-P_{\mathbf{1 0}}$ |
| $\mathbf{1 5}$ | $(\mathbf{3}, \mathbf{2})$ | $P_{\mathbf{1 5}}$ |
|  | $(\mathbf{6}, \mathbf{1})+(\mathbf{1}, \mathbf{3})$ | $-P_{\mathbf{1 5}}$ |
| $\mathbf{2 4}$ | $(\mathbf{8}, \mathbf{1})+(\mathbf{1}, \mathbf{3})+(\mathbf{1}, \mathbf{1})$ | $P_{\mathbf{2 4}}$ |
|  | $(\mathbf{3}, \mathbf{2})+(\overline{\mathbf{3}}, \mathbf{2})$ | $-P_{\mathbf{2 4}}$ |
| $\mathbf{4 5}$ | $(\mathbf{8}, \mathbf{2})+(\overline{\mathbf{3}}, \mathbf{2})+(\mathbf{1}, \mathbf{2})$ | $P_{\mathbf{4 5}}$ |
|  | $(\overline{\mathbf{6}}, \mathbf{1})+(\mathbf{3}, \mathbf{1})+(\overline{\mathbf{3}}, \mathbf{1})$ | $-P_{\mathbf{4 5}}$ |
|  | $+(\mathbf{3}, \mathbf{3})$ |  |
| $\mathbf{7 5}$ | $(\mathbf{6}, \mathbf{2})+(\overline{\mathbf{6}}, \mathbf{2})+(\mathbf{3}, \mathbf{2})$ | $P_{\mathbf{7 5}}$ |
|  | $+(\overline{\mathbf{3}}, \mathbf{2})$ |  |
|  | $(\mathbf{3}, \mathbf{1})+(\overline{\mathbf{3}}, \mathbf{1})+(\mathbf{1}, \mathbf{1})$ | $-P_{\mathbf{7 5}}$ |
|  | $+(\mathbf{8}, \mathbf{3})+(\mathbf{8}, \mathbf{1})$ |  |

sional representations of $S U(5)$. In the second column, we give $S U(3) \times S U(2)$ quantum numbers for split multiplets. In the third column, $P_{\boldsymbol{R}}$ is an eigenvalue of $Z_{2}^{\prime}$ parity, i.e., $P_{\boldsymbol{R}}=1$ or -1 . The table includes components that can induce a rapid nucleon decay when they couple to quarks and leptons.

Our grand unification scenario is phenomenologically interesting because it suggests the existence of extra split multiplets at the weak scale. However, there is a problem of determining how to break the electro-weak symmetry naturally and how to stabilize the weak scale; that is, our model suffers from a naturalness problem. ${ }^{26)}$ There is an alternative description in which the extra space has a large radius. ${ }^{27)}$ In this case, the low-energy gauge coupling unification is expected to be realized by a power-law correction. ${ }^{28)}$

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    ${ }^{* *)}$ The gauge hierarchy problem is solved partially in the sense that the hierarchy is stabilized against radiative corrections perturbatively, as described by a non-renormalization theorem, although the origin at the tree level is not understood.
    ${ }^{* * *)}$ There are several works treating $S U(5)$ grand unification under the assumption that there are split multiplets ${ }^{13)-15)}$ and $S U(N)(N \geq 6)$ one with a radiative splitting mechanism. ${ }^{16)}$

[^1]:    ${ }^{*)}$ Recently, Barbieri, Hall and Nomura have constructed a constrained standard model upon a compactification of a 5D SUSY model on the orbifold $S^{1} /\left(Z_{2} \times Z_{2}^{\prime}\right) .{ }^{17}$ ) They used a $Z_{2} \times Z_{2}^{\prime}$ parity to reduce SUSY. There are also several works on model building through a reduction of SUSY ${ }^{18)-21)}$ or a gauge symmetry ${ }^{22}$ ) by the use of a $Z_{2}$ parity. Attempts to construct GUT have been made through dimensional reduction over coset space. ${ }^{23}$ )

[^2]:    *) The exchange of $P$ for $P^{\prime}$ is equivalent to the exchange of two walls.
    ${ }^{* *)}$ Our symmetry reduction mechanism is different from the Hosotani mechanism. ${ }^{24)}$ In fact, the Hosotani mechanism does not work in our case, because $A_{5}^{a}\left(x^{\mu}, y\right)$ has odd parity, as given in (2a), and its VEV should vanish.

