Gauge Symmetry Reduction from the Extra Space S^1/Z_2

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We study a mechanism of symmetry transition upon compactification of a 5-dimensional field theory on S^1/Z_2. The transition occurs unless all components in a multiplet of a symmetry group have a common Z_2 parity on S^1/Z_2. This mechanism is applied to a reduction of $SU(5)$ gauge symmetry in grand unified theory, and phenomenological implications are discussed.

§1. Introduction

The study of physics beyond 4-dimensional (4D) space-time traces back to the work by Kaluza and Klein.[1] The exsitence of an extra space is essential in super-string theory[2),3)] and M-theory.[4] Recently, there is a lot of interesting research of the phenomenological and theoretical implications of an extra space with large radii based on several motivations.[5]

As a feature in theories with an extra compact space K, symmetries in the system can change after compactification. Here we give typical examples, where K is an n-dimensional torus T^n. Supersymmetry (SUSY) is, in general, enhanced by a simple dimensional reduction.[6] Also, SUSY can be broken by the choice of different boundary conditions for bosons and fermions in supermultiplets. This mechanism is called the Scherk-Schwarz mechanism.[7] The other example is a gauge symmetry breaking by the Hosotani mechanism.[8),9)] Gauge symmetry is broken by the appearance of non-integrable phase factors which are dynamical degrees of freedom and are related to Wilson loops of gauge fields along compactified directions.

The phenomenon of symmetry transition leads to the idea that symmetries in 4D low-energy theory are derived from a high-energy theory with extra dimensions. It is meaningful to study the relation between symmetries and a compact space more carefully, because the origin of symmetries in the Standard Model is not yet known. As a compact space, the one-dimensional orbifold S^1/Z_2 has attracted our attention since the study of heterotic M-theory.[4] Starting from 5-dimensional (5D) SUSY model, 4D theory with $N = 1$ SUSY is derived through compactification on S^1/Z_2.[10),11)] Bulk fields have a Z_2 parity in a compact direction. The reduction of SUSY originates from a non-universal Z_2 parity assignment among component fields in supermultiplets.

In this paper, we focus on 5D field theories and study a symmetry transition upon compactification on S^1/Z_2. The above-mentioned SUSY reduction mechanism is generalized as follows. *Unless all components in a multiplet of some symmetry*

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group G have a common Z_2 parity on S^1/Z_2, the symmetry is not preserved after the integration of the fifth dimension because zero modes, in general, do not form a full multiplet of G. We apply this mechanism to a reduction of $SU(5)$ gauge symmetry in the grand unified theory (GUT).

This paper is organized as follows. In the next section, we explain the mechanism of symmetry change on S^1/Z_2. For 5D $SU(5)$ GUT with minimal particle content, we discuss the reduction of gauge symmetry, the mass spectrum in 4D theory, and its phenomenological implications in §3. Section 4 is devoted to conclusions and discussion.

§2. Symmetry transition upon compactification

The space-time is assumed to be factorized into a product of 4D Minkowski space-time M^4 and the orbifold S^1/Z_2, whose coordinates are denoted by x^μ ($\mu = 0, 1, 2, 3$) and $y(= x^5)$, respectively. The 5D notation x^M ($M = 0, 1, 2, 3, 5$) is also used. The orbifold S^1/Z_2 is obtained by dividing a circle S^1 with radius R with a Z_2 transformation which acts on S^1 by $y \rightarrow -y$. This compact space is regarded as an interval with a distance of πR. There are two 4D walls placed at fixed points $y = 0$ and $y = \pi R$ on S^1/Z_2.

An intrinsic Z_2 parity of the 5D bulk field $\phi(x^\mu, y)$ is defined by the transformation

$$\phi(x^\mu, y) \rightarrow \phi(x^\mu, -y) = P\phi(x^\mu, y). \tag{2·1}$$

The Lagrangian should be invariant under the Z_2 transformation. By definition, P equals 1 or -1. We denote the field with $P = 1$ ($P = -1$) by ϕ_+ (ϕ_-). The fields ϕ_+ and ϕ_- are Fourier expanded as

$$\phi_+(x^\mu, y) = \frac{1}{\sqrt{\pi R}} \sum_{n=0}^{\infty} \phi_+^{(n)}(x^\mu) \cos \frac{ny}{R}, \tag{2·2}$$

$$\phi_-(x^\mu, y) = \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \phi_-^{(n)}(x^\mu) \sin \frac{ny}{R}, \tag{2·3}$$

where n is an integer, and the fields $\phi_\pm^{(n)}(x^\mu)$ acquire mass n/R upon compactification. Note that 4D massless fields are absent in $\phi_-(x^\mu, y)$.

Let us study the case in which a field $\Phi(x^\mu, y)$ is an N-plet under some symmetry group G. Each component of Φ is denoted by ϕ_k, i.e., $\Phi = (\phi_1, \phi_2, ..., \phi_N)^T$. The Z_2 transformation of Φ is given by the same form as (2·1), but in this case P is an $N \times N$ matrix[*] which satisfies $P^2 = I$, where I is the unit matrix. The Z_2 invariance of the Lagrangian does not necessarily require that P be I or $-I$. Unless all components of Φ have common Z_2 parity (i.e., if $P \neq \pm I$), a symmetry transition occurs upon compactification because of the lack of zero modes in components with odd parity.

Here we give two simple examples of a symmetry transition.
(a) Scalar field with N_f flavors

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[*] P is a unitary and hermitian matrix.

We consider a 5D Lagrangian density given by

$$\mathcal{L}_S^{(5)} \equiv |\partial_M \Phi|^2 = \sum_{k=1}^{N_f} |\partial_M \phi_k|^2, \tag{2.4}$$

where $\Phi(x^\mu, y)$ is a 5D complex scalar field with N_f components. The Lagrangian is invariant under the transformation $\Phi(x^\mu, y) \to \Phi(x^\mu, -y) = P\Phi(x^\mu, y)$, with $P^2 = I$. If we take $P = \mathrm{diag}(1, \cdots, 1, -1, \cdots, -1)$ where the first M_f elements equal 1, we derive the following 4D Lagrangian density after integrating out the fifth dimension and rescaling the 4D fields $\phi_k^{(n)}(x^\mu)$:

$$\mathcal{L}_S^{(4)} = \sum_{k=1}^{M_f} |\partial_\mu \phi_k^{(0)}|^2 + \sum_{n=1}^{\infty} \sum_{k=1}^{N_f} \left(|\partial_\mu \phi_k^{(n)}|^2 + \frac{n^2}{R^2} |\phi_k^{(n)}|^2 \right). \tag{2.5}$$

The global symmetry $U(N_f)$ in 5D theory is reduced to its subgroup $U(M_f)$ upon compactification. Note that the Kaluza-Klein excitations $\phi_k^{(n)}(x^\mu)$ form a full multiplet of $U(N_f)$.

(b) Dirac fermion with N_f flavors

We consider a 5D Lagrangian density given by

$$\mathcal{L}_D^{(5)} = i\bar{\Psi}\gamma^M \partial_M \Psi = \sum_{k=1}^{N_f} i\bar{\psi}_k \gamma^M \partial_M \psi_k, \tag{2.6}$$

where $\Psi = (\Psi_L, \Psi_R)^T$ is a 5D Dirac fermion with N_f components denoted by ψ_k. The Lagrangian is invariant under the transformation $\Psi_L(x^\mu, y) \to \Psi_L(x^\mu, -y) = P\Psi_L(x^\mu, y)$ and $\Psi_R(x^\mu, y) \to \Psi_R(x^\mu, -y) = -P\Psi_R(x^\mu, y)$, with $P^2 = I$. When we take $P = \mathrm{diag}(1, \cdots, 1, -1, \cdots, -1)$, where the first M_f elements equal 1, we obtain the 4D Lagrangian density

$$\mathcal{L}_D^{(4)} = \sum_{k=1}^{M_f} i\psi_{Lk}^{(0)\dagger} \sigma^\mu \partial_\mu \psi_{Lk}^{(0)} + \sum_{k=1}^{N_f - M_f} i\psi_{Rk}^{(0)\dagger} \bar{\sigma}^\mu \partial_\mu \psi_{Rk}^{(0)}$$
$$+ \sum_{n=1}^{\infty} \sum_{k=1}^{N_f} \left(i\bar{\psi}_k^{(n)} \gamma^\mu \partial_\mu \psi_k^{(n)} - \frac{n}{R} \bar{\psi}_k^{(n)} \psi_k^{(n)} \right), \tag{2.7}$$

after integrating out the fifth dimension and rescaling the 4D fields $\psi_k^{(n)}(x^\mu)$. Here the components of Ψ_L and Ψ_R are denoted by ψ_{Lk} and ψ_{Rk}, respectively. The 5D theory has the global symmetry $SU(N_f)_V \times U(1)_V$, but a symmetry in 4D theory turns out to be $SU(M_f)_L \times SU(N_f - M_f)_R \times U(1)_V \times U(1)_A$ in the decoupling limit of Kaluza-Klein modes, which form a full multiplet of $SU(N_f)_V \times U(1)_V$.

§3. A model with $SU(5)$ gauge symmetry

We apply the symmetry transition mechanism to 5D $SU(5)$ GUT with minimal particle content. We assume that the 5D gauge boson $A_M(x^\mu, y)$ and the Higgs

boson $\Phi(x^\mu, y)$ exist in the bulk $M^4 \times S^1/Z_2$. The fields A_M and Φ form an adjoint representation **24** and a fundamental representation **5** of $SU(5)$, respectively. We assume that our visible world is one of 4D walls (we choose the wall fixed at $y = 0$ as the visible one and call it wall I) and that three families of quarks and leptons, $3\{\psi_{\bar{5}} + \psi_{10}\}$, are located on wall I. That is, matter fields contain no excited states along the S^1/Z_2 direction.

The gauge invariant action is given by

$$S = \int \mathcal{L}^{(5)} d^5 x + \int \mathcal{L}^{(4)} d^4 x, \tag{3·1}$$

$$\mathcal{L}^{(5)} \equiv -\frac{1}{2}\mathrm{tr}F_{MN}^2 + |D_M\Phi|^2 - V(\Phi), \tag{3·2}$$

$$\mathcal{L}^{(4)} \equiv \sum_{3\,\mathrm{families}} (i\bar{\psi}_{10}\gamma^\mu D_\mu\psi_{10} + i\bar{\psi}_{\bar{5}}\gamma^\mu D_\mu\psi_{\bar{5}}$$
$$+ f_{U(5)}\Phi\psi_{10}\psi_{10} + f_{D(5)}\Phi^\dagger\psi_{10}\psi_{\bar{5}} + \mathrm{h.c.}), \tag{3·3}$$

where $D_M \equiv \partial_M - ig_{(5)}A_M(x^\mu, y)$, $g_{(5)}$ is a 5D gauge coupling constant, and $f_{U(5)}$ and $f_{D(5)}$ are 5D Yukawa coupling matrices. The representations of $\psi_{\bar{5}}$ and ψ_{10} are $\bar{5}$ and **10** under $SU(5)$, respectively. In $\mathcal{L}^{(4)}$, the bulk fields A_μ and Φ are replaced by fields with values at wall I, $A_\mu(x^\mu, 0)$ and $\Phi(x^\mu, 0)$. The Lagrangian is invariant under the Z_2 transformation

$$A_\mu(x^\mu, y) \to A_\mu(x^\mu, -y) = PA_\mu(x^\mu, y)P^{-1},$$
$$A_5(x^\mu, y) \to A_5(x^\mu, -y) = -PA_5(x^\mu, y)P^{-1},$$
$$\Phi(x^\mu, y) \to \Phi(x^\mu, -y) = P\Phi(x^\mu, y). \tag{3·4}$$

When we take $P = \mathrm{diag}(-1, -1, -1, 1, 1)$, the $SU(5)$ gauge symmetry is reduced to that of the Standard Model, $G_{SM} \equiv SU(3) \times SU(2) \times U(1)$, in 4D theory.[*] This is because the boundary conditions on S^1/Z_2 given in (3·4) do not respect $SU(5)$ symmetry, as we see from the relations for the gauge generators T^α ($\alpha = 1, 2, \cdots, 24$),

$$PT^a P^{-1} = T^a, \quad PT^{\hat{a}} P^{-1} = -T^{\hat{a}}. \tag{3·5}$$

The T^as are gauge generators of G_{SM} and the $T^{\hat{a}}$s are other gauge generators.[**] After integrating out the fifth dimension, we obtain the 4D lagrangian density

$$\mathcal{L}_{\mathrm{eff}}^{(4)} = \mathcal{L}_B^{(4)} + \mathcal{L}^{(4)}, \tag{3·6}$$

$$\mathcal{L}_B^{(4)} \equiv -\frac{1}{4}\sum_a F_{\mu\nu}^{a(0)2} + |D_\mu A_5^{\hat{a}(0)}|^2$$
$$+ |D_\mu\phi_W^{(0)}|^2 + g_U^2|A_5^{\hat{a}(0)}\phi_W^{(0)}|^2 - V(\phi_W^{(0)}) + \cdots, \tag{3·7}$$

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[*] Our symmetry reduction mechanism is different from the Hosotani mechanism. In fact, the Hosotani mechanism does not work in our case, because $A_5^a(x^\mu, y)$ has odd parity, as given in (3·4), and its VEV should vanish.

[**] We expect that the specific Z_2 parity given by $P = \mathrm{diag}(-1, -1, -1, 1, 1)$ is determined non-perturbatively in an underlying theory.

$$
\begin{aligned}
\mathcal{L}^{(4)} \equiv \sum_{\text{3 families}} \Bigg[& i\bar{\psi}_{10}\gamma^\mu\Big(\partial_\mu - ig_U \sum_{n=0}^{\infty} A_\mu^{a(n)} T^a(\mathbf{10})\Big)\psi_{10} \\
& + i\bar{\psi}_{\bar{5}}\gamma^\mu\Big(\partial_\mu - ig_U \sum_{n=0}^{\infty} A_\mu^{a(n)} T^a(\bar{\mathbf{5}})\Big)\psi_{\bar{5}} \\
& + f_U \sum_{n=0}^{\infty} \phi_W^{(n)} q\bar{u} + f_D \sum_{n=0}^{\infty} \tilde{\phi}_W^{(n)} q\bar{d} + f_D \sum_{n=0}^{\infty} \tilde{\phi}_W^{(n)} l\bar{e} + \text{h.c.} \\
& + \sum_{n=0}^{\infty} (g_U \bar{\psi}_{10}\gamma^5 A_5^{\hat{a}(n)} T^{\hat{a}}(\mathbf{10})\psi_{10} + g_U \bar{\psi}_{\bar{5}}\gamma^5 A_5^{\hat{a}(n)} T^{\hat{a}}(\bar{\mathbf{5}})\psi_{\bar{5}}) \Bigg], \quad (3\cdot8)
\end{aligned}
$$

where the dots in $\mathcal{L}_B^{(4)}$ represent terms including Kaluza-Klein modes, g_U ($\equiv g_{(5)}/\sqrt{\pi R}$) is a 4D gauge coupling constant, f_U ($\equiv f_{U(5)}/\sqrt{\pi R}$) and f_D ($\equiv f_{D(5)}/\sqrt{\pi R}$) are 4D Yukawa coupling matrices, q, $\bar{u}$ and $\bar{d}$ are quarks, l and $\bar{e}$ are leptons, and ϕ_W ($\tilde{\phi}_W^{(n)} \equiv i\tau_2 \phi_W^{(n)*}$) is a weak Higgs doublet. The massive modes should be rescaled by a factor of $\sqrt{2}$, e.g., $A_\mu^{a(n)} \to \sqrt{2} A_\mu^{a(n)}$ and $\phi_W^{(n)} \to \sqrt{2}\phi_W^{(n)}$ ($n \neq 0$), as a result of the proper normalization of their kinetic terms. The mass spectrum after compactification is given in Table I. In the second column, we give $SU(3) \times SU(2)$ quantum numbers of 4D fields. The triplet-doublet mass splitting of the Higgs boson is realized by projecting out zero modes of colored components in the Higgs boson. There exist extra 4D scalar fields $A_5^{\hat{a}(0)}(x^\mu)$, whose quantum numbers are $(\mathbf{3},\mathbf{2},-5/6) + (\bar{\mathbf{3}},\mathbf{2},5/6)$ under G_{SM}, and they couple to G_{SM} gauge bosons, a weak Higgs doublet, and matter fermions. There is a possibility that $A_5^{\hat{a}(0)}$ becomes superheavy and that the proton decay induced by the exchange of $A_5^{\hat{a}(0)}$ is suppressed, while $\phi_W^{(0)}$ remains in the weak scale spectrum after radiative corrections are received. This is based on the following premises. The field $A_5^{\hat{a}(0)}$ has a vanishing bare mass because of gauge symmetry, but $\phi_W^{(0)}$ has a mass whose square is given by

$$
\mu^2 = \frac{\partial^2 V}{\partial \phi_W^{(0)2}}. \quad (3\cdot9)
$$

Both $A_5^{\hat{a}(0)}$ and $\phi_W^{(0)}$ receive radiative corrections. The renormalized mass of $\phi_W^{(0)}$ is on the order of the weak scale.

Table I. Mass spectrum at the tree level.

4D fields	Quantum numbers	Mass
$A_\mu^{a(0)}(x^\mu)$	$(\mathbf{8},\mathbf{1}) + (\mathbf{1},\mathbf{3}) + (\mathbf{1},\mathbf{1})$	0
$A_5^{\hat{a}(0)}(x^\mu)$	$(\mathbf{3},\mathbf{2}) + (\bar{\mathbf{3}},\mathbf{2})$	0
$\phi_W^{(0)}(x^\mu)$	$(\mathbf{1},\mathbf{2})$	$\sqrt{\mu^2}$
$A_M^{\alpha(n)}(x^\mu)$ $(n \neq 0)$	$(\mathbf{8},\mathbf{1}) + (\mathbf{1},\mathbf{3}) + (\mathbf{1},\mathbf{1})$ $+ (\mathbf{3},\mathbf{2}) + (\bar{\mathbf{3}},\mathbf{2})$	n/R
$\Phi^{(n)}(x^\mu)$ $(n \neq 0)$	$(\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{2})$	$\sqrt{(n/R)^2 + \mu^2}$
$\psi_{\bar{5}}(x^\mu)$	$(\bar{\mathbf{3}},\mathbf{1}) + (\mathbf{1},\mathbf{2})$	0
$\psi_{10}(x^\mu)$	$(\mathbf{3},\mathbf{2}) + (\bar{\mathbf{3}},\mathbf{1}) + (\mathbf{1},\mathbf{1})$	0

The theory predicts that coupling constants are unified around the compactification scale $M_C(\equiv 1/R)$, as in the ordinary $SU(5)$ GUT,[12]

$$g_3 = g_2 = g_1 = g_U, \quad f_d = f_e = f_D, \tag{3.10}$$

where f_d and f_e are Yukawa coupling matrices on down-type quarks and electron-type leptons, respectively.[*] The other feature is that quarks and leptons couple neither to X and Y gauge bosons $A_\mu^{\hat{a}(n)}$ nor to the colored Higgs triplet $\phi_C^{(n)}$ at the tree level. Hence it is expected that the proton decay process due to X and Y gauge bosons $A_\mu^{\hat{a}(n)}$ is suppressed.[**]

§4.   Conclusions and discussion

We have studied a mechanism of symmetry transition upon compactification of 5D field theory on S^1/Z_2. The transition occurs unless all components in a multiplet of a symmetry group have a common Z_2 parity on S^1/Z_2. This mechanism has been applied to a reduction of gauge symmetry in 5D $SU(5)$ GUT. Under the assumption that our visible world is a 4D wall fixed at $y = 0$ and that quarks and leptons live on the wall, we have derived the same type of action as that in the Standard Model. The triplet-doublet mass splitting on Higgs bosons is realized at the tree level by the Z_2 projection. In the sector with renormalizable interactions, the theory predicts the coupling unification $g_3 = g_2 = g_1 = g_U$ and $f_d = f_e = f_D$. Quarks and leptons couple neither to off-diagonal gauge bosons $A_\mu^{\hat{a}}$ nor to the colored Higgs triplet ϕ_C at the tree level.

On the other hand, there are several problems in our $SU(5)$ model. Here we list some of them. The first one is the existence of 4D scalar fields $A_5^{\hat{a}(0)}(x^\mu)$ with quantum numbers $(\mathbf{3}, \mathbf{2}, -5/6) + (\bar{\mathbf{3}}, \mathbf{2}, 5/6)$ under G_{SM} in the weak scale spectrum at the tree level. Their presence induces a dangerous proton decay because they couple to quarks and leptons at the tree level. To avoid this problem, these scalar fields should be superheavy. They can acquire superheavy masses by radiative corrections. Unless the radiative corrections to $A_5^{\hat{a}(0)}(x^\mu)$ are finite, the renormalizability can be violated because of the lack of mass and self-interaction terms of $A_5^{\hat{a}(0)}(x^\mu)$. This problem is under investigation based on the 5-dimensional calculation discussed in Ref. 15). Otherwise it is necessary to introduce extra Higgs bosons in order to eliminate unwanted particles from the low-energy spectrum. The second problem involves the question of how to break the electro-weak symmetry naturally and how to stabilize the weak scale. Our model suffers from gauge hierarchy problem.[14] The third problem regards the reality of coupling unification, i.e., whether or not our model is consistent with experimental data of gauge couplings and fermion masses. Power-law corrections from the extra space S^1/Z_2 should also be considered.[13] The fourth problem concerns the necessity of non-universal Z_2 parity, i.e., whether or not there is a selection rule which picks out a specific Z_2 parity to break $SU(5)$ down

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[*] The unification conditions are, in general, corrected by non-renormalizable interactions.

[**] If we consider non-renormalizable interactions, there are dangerous interactions including derivatives with respect to the fifth coordinate, which can induce a rapid proton decay.

to G_{SM}. The last problem regards how matter fields are localized on the 4D wall. In spite of these problems, it would be worthwhile to search for a realistic model of grand unification in this direction.[*]

Acknowledgements

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[*] Attempts to construct GUT have been made through the dimensional reduction over coset space. [16]