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# Generic formula of soft scalar masses in string models

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We derive a formula of soft supersymmetry-breaking scalar masses from four-dimensional string models within a more generic framework. We consider the effects of extra gauge symmetry breaking including an anomalous U(1) breaking through flat directions, that is, $D$-term and $F$-term contributions, particle mixing effects, and heavy-light mass mixing effects. Some phenomenological implications are discussed based on our mass formula. [S0556-2821(97)03117-2]

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## I. INTRODUCTION

Superstring theories (SSTs) are powerful candidates for the unification theory of all forces including gravity. There are various approaches to explore four-dimensional (4D) string models, for example, the compactification on Calabi-Yau manifolds [1], the construction of orbifold models [2,3], and so on. Effective supergravity (SUGRA) theories have been derived based on the above approaches [4–6]. The structure of SUGRA theory [7] is constrained by considering field theoretical nonperturbative effects such as a gaugino condensation [8] and stringy symmetries such as duality [9] in addition to perturbative results.

Effective theories, however, have several problems. First, there are thousands of effective theories corresponding to 4D string models. They have, in general, large gauge groups and many matter multiplets compared with those of the minimal supersymmetric standard model (MSSM). We do not know how to select a realistic model among them from stringy theoretical point of view yet. Another serious problem is that the mechanism of supersymmetry (SUSY) breaking is unknown. To solve these problems, nonperturbative effects in SSTs and SUSY field theories should be fully understood.[1]

At the present circumstances, the following approaches and/or standpoints have been taken. For the first problem, study on flat directions is important [12], because effective theories have, in general, flat directions in the SUSY limit. Large gauge symmetries can break into smaller ones and extra matter fields can get massive through flat directions. Further, flat directions could relate different models in string vacua. Actually, some models with realistic gauge groups and matter contents have been constructed based on $Z_3$ orbifold models [13]. Recently, generic features of flat directions in $Z_{2n}$ orbifold models have been also investigated [14].

The flat directions based on $Z_3$ orbifold models have been analyzed considering the existence of anomalous U(1) symmetry $[\mathrm{U}(1)_A]$ because 4D string models, in general, have the $\mathrm{U}(1)_A$ symmetry. Some interesting features are pointed out in those models. For example, Fayet-Iliopoulos $D$ term [15] is induced at one-loop level[2] for $\mathrm{U}(1)_A$ [16]. As a result, some scalar fields necessarily develop vacuum expectation values (VEVs) and some gauge symmetries can break down [12,13].

For the second problem, some researches have been done from the standpoint that the origin of SUSY breaking is unspecified. That is, soft SUSY-breaking terms have been derived under the assumption that SUSY is broken by $F$-term condensations of the dilaton field $S$ and/or moduli fields $T$ [18–20]. Some phenomenologically interesting features are predicted from the structure of soft SUSY-breaking terms which are parametrized by a few number of parameters, for example, only two parameters such as a goldstino angle $\theta$ and the gravitino mass $m_{3/2}$ in the case with the overall moduli and the vanishing vacuum energy [21]. The cases with multimoduli fields are also discussed in Ref. [22]. Recently, study on soft scalar masses has been extended in the presence of an anomalous U(1) symmetry [23–26].

This strategy for string phenomenology is quite interesting since the soft SUSY-breaking parameters can be powerful probes for physics beyond the MSSM such as SUSY-grand unified theories, SUGRAs, and SSTs. We give two examples. The pattern of gauge symmetry breakdown can be specified by checking certain sum rules among scalar masses. The specific mass relations are derived for SO(10) breakings [27,28] and for $E_6$ breakings [29]. String models with the SUSY breaking due to the dilaton $F$ term lead to the highly restricted pattern in the absence of $\mathrm{U}(1)_A$ such as [19,21]

$$|A| = |M_{1/2}| = \sqrt{3}|m_{3/2}|, \tag{1}$$

where $A$ is a universal $A$ parameter, and gauginos and scalar fields get masses with common values $M_{1/2}$ and $m_{3/2}$, re-

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[1]Recently, there have been various remarkable developments in study on nonperturbative aspects of SSTs and SUSY models [10,11].

[2]Some conditions for absence of anomalous U(1) are discussed in Ref. [17].

 

spectively. In this way, soft SUSY-breaking parameters can play important roles to probe a new physics.

The above two approaches are attractive to explore particle phenomenology beyond the standard model based on SST. Hence, it is important to examine what features soft SUSY-breaking terms can show at low energy when we construct a realistic model through flat direction breaking starting from 4D string models with extra gauge symmetries including $\mathrm{U}(1)_A$.

In this paper, we derive a formula of soft SUSY-breaking scalar masses from 4D string models within a more generic framework. We consider effects of extra gauge symmetry breaking, that is, $D$-term and $F$-term contributions, particle mixing effects, and heavy-light mass mixing effects. Some phenomenological implications are discussed based on our mass formula. In particular, we study the degeneracy and the positivity of squared scalar masses in special cases. The effects of $D$-term contributions are discussed mainly at the tree level. In addition, we calculate soft scalar masses explicitly and derive specific relations among them by taking a $Z_3$ orbifold model as an example. They can be useful as a starting point on the analysis of low-energy physics after including quantum corrections.

This paper is organized as follows. In the next section, we explain our starting point reviewing the structure of effective SUGRA derived from SST in a field theory limit. In Sec. III, we derive a formula of soft SUSY-breaking scalar masses and discuss its phenomenological implications. In Sec. III A, the scalar potential is discussed. In Sec. III B, we discuss classification of scalar fields. In Sec. III C, we examine the existence of heavy-light mass mixing. In Sec. III D, a generic formula of soft scalar masses is given. In Sec. III E, the degeneracy and the positivity of squared scalar masses are discussed in special cases. In Sec. IV, the results of Sec. III are applied to an explicit model. In Sec. V, we remark on some extensions. Section VI is devoted to conclusions and discussions. In the Appendix, formulas of the Kähler metric and its inverse are summarized.

## II. EFFECTIVE SUGRA AS A FIELD THEORY LIMIT OF STRING MODELS

Effective SUGRAs are derived from $Z_N$ orbifold models taking a field theory limit. Here, we assume the existence of a realistic effective SUGRA, that is, our starting theory has the following excellent features.

The gauge group is $G=G'_{\mathrm{SM}}\times \mathrm{U}(1)^n\times \mathrm{U}(1)_A\times H'$ where $G'_{\mathrm{SM}}$ is a group which contains the standard model gauge group $G_{\mathrm{SM}}=\mathrm{SU}(3)_C\times \mathrm{SU}(2)_L\times \mathrm{U}(1)_Y$ as a subgroup, $\mathrm{U}(1)^n$ are nonanomalous U(1) symmetries, $\mathrm{U}(1)_A$ is an anomalous $U(1)$ symmetry, and $H'$ is a direct product of some non-Abelian symmetries. The anomalies related to $\mathrm{U}(1)_A$ are canceled by the Green-Schwarz mechanism [30]. When gauginos of $H'$ condense, they can trigger SUSY breaking [8]. Or, $H'$ might be broken by VEVs of some scalar fields at a higher energy scale. We take a standpoint that an origin of SUSY breaking is unspecified.

Chiral multiplets $\Phi^I$ are classified into two categories. One is a set of chiral multiplets whose scalar components $\phi^i$ have large VEVs of $O(M)$. Here, $M$ is the gravitational scale defined as $M\equiv M_{\mathrm{Pl}}/\sqrt{8\pi}$ and $M_{\mathrm{Pl}}$ is the Planck scale. The dilaton field $S$ and the moduli fields $T_{ij}$ belong to $\{\Phi^i\}$. For the present, we treat only the overall moduli field $T$ ($T=T_1=T_2=T_3$, $T_{ij}=0$ for $i\neq j$) and also neglect moduli fields $U_i$ corresponding to complex structure. Further, we neglect effects of threshold corrections and an $S$-$T$ mixing. Later, we will discuss the case with several moduli fields $T_i$ and $U_i$ and the case that Kähler potential has an $S$-$T$ mixing term. The other is a set of matter multiplets denoted as $\Phi^\kappa$ which contains the MSSM matter multiplets and Higgs multiplets. Some of them have nonzero $\mathrm{U}(1)_A$ $[U(1)^n,\ H']$ charges and can induce to the $\mathrm{U}(1)_A$ $[\mathrm{U}(1)^n,\ H']$ breaking at high-energy scales by getting VEVs. We denote the above two types of multiplets together as $\Phi^I$. The matter multiplets are $G$ nonsinglets and correspond to string states one to one.

We suppose the following situations related to an extra gauge symmetry breaking.

(1) The $\mathrm{U}(1)_A$ symmetry is broken at $M$ by VEVs of $S$ and some chiral matter multiplets.

(2) Some parts of $\mathrm{U}(1)^n$ and $H'$ are broken at much higher energy scales than the weak scale by VEVs of some chiral matter multiplets. Those VEVs are smaller than those of $S$ and $T$, i.e.,

$$\langle \phi^\kappa\rangle \ll \langle S\rangle, \langle T\rangle = O(M). \tag{2}$$

This condition is justified from the fact that a $D$-term condensation of $\mathrm{U}(1)_A$ vanishes up to $O(m_{3/2}^2)$ as will be shown. Here, $m_{3/2}$ is the gravitino mass defined later.

(3) The rest extra gauge symmetries are broken spontaneously or radiatively by the SUSY breaking effects at some lower scales.

It is straightforward to apply our method to more complicated situations.

We give a comment here. Such symmetry breaking generates a scale $M_I$, which is defined as the magnitude of VEVs of scalar fields, below the Planck scale $M_{\mathrm{Pl}}$. Using the ratio $M_I/M_{\mathrm{Pl}}$, higher-dimensional couplings could explain hierarchical structures in particle physics like the fermion masses and their mixing angles. Recently, much attention has been paid to such a study on the fermion mass matrices [31,32]. In Ref. [31], U(1) symmetries are used to generate realistic fermion mass matrices and some of them are anomalous, while stringy selection rules on nonrenormalizable couplings are used in Ref. [32].

Next, let us explain the three constituents, the Kähler potential $K$, the superpotential $W$, and the gauge kinetic function $f_{\alpha\beta}$, in effective SUGRAs derived from SSTs. Orbifold models lead to the following Kähler potential $K$ [4–6]:

$$K=-\ln(S+S^*+\delta_{\mathrm{GS}}^A V_A)-3\ln(T+T^*) + \sum_\kappa (T+T^*)^{n_\kappa}|\phi^\kappa|^2+\cdots, \tag{3}$$

where $\delta_{\mathrm{GS}}^A$ is a coefficient of the Green-Schwarz mechanism to cancel the $\mathrm{U}(1)_A$ anomaly and $V_A$ is a vector superfield of $\mathrm{U}(1)_A$. Here and hereafter, we take $M=1$ according to circumstances. The dilaton field $S$ transforms nontrivially as

$S \rightarrow S - i\delta^A_{\rm GS}\theta(x)$ under $\mathrm{U}(1)_A$ with the transformation parameter $\theta(x)$. The coefficient $\delta^A_{\rm GS}$ is given as

$$k_A^{1/2}\delta^A_{\rm GS} = \frac{1}{96\pi^2}\mathrm{Tr}Q^A, \tag{4}$$

where $k_A$ is Kac-Moody level of $\mathrm{U}(1)_A$ and $Q^A$ is a $\mathrm{U}(1)_A$ charge operator. Further, $n_\kappa$'s are modular weights of matter fields $\phi^\kappa$. The formulas of $n_\kappa$ are given in Refs. [6,18]. The same Kähler potential is derived from Calabi-Yau models with the large $T$ limit up to twisted sector field's contributions.

The superpotential $W$ consists of the following two parts:

$$W = W_{\rm NP} + W_{\rm pert}. \tag{5}$$

Here, $W_{\rm NP}$ is a superpotential induced by some nonperturbative effects, and it is expected that VEVs of $S$ and $T$ are fixed and SUSY is broken owing to this part. The other part $W_{\rm pert}$ is a superpotential at the tree level and starts from trilinear couplings for massless fields

$$W_{\rm pert} = \sum_{\kappa,\lambda,\mu} f_{\kappa\lambda\mu}\phi^\kappa\phi^\lambda\phi^\mu + \cdots, \tag{6}$$

where Yukawa couplings $f_{\kappa\lambda\mu}$ generally depend on the moduli fields $T$ and the ellipsis stands for terms of higher orders in $\phi^\kappa$. Note that if the above superpotential includes mass terms such as $m_{\kappa\lambda}\phi^\kappa\phi^\lambda$, a natural order of these masses is of $O(M)$. Thus, we do not include these fields with mass terms at the tree level.[3] The total Kähler potential $G$ is defined as $G \equiv K + \ln|W|^2$. The gauge kinetic function $f_{\alpha\beta}$ is given as $f_{\alpha\beta} = S\delta_{\alpha\beta}$. For simplicity, here we assume that Kac-Moody levels satisfy $k_\alpha = 1$ because our results on soft terms are independent of a value of $k_\alpha$. The scalar potential is given as

$$V = V^{(F)} + V^{(D)},$$
$$V^{(F)} \equiv e^G[G^I(G^{-1})_I^J G_J - 3], \tag{7}$$

$$\begin{aligned} V^{(D)} &\equiv \tfrac{1}{2}(\mathrm{Re}f^{-1})_{\alpha\beta}D^\alpha D^\beta \\ &= \frac{1}{S+S^*}[K_\kappa(T^a\phi)^\kappa]^2 \\ &\quad + \frac{1}{S+S^*}\left(\frac{\delta^A_{\rm GS}}{S+S^*} + K_\kappa(Q^A\phi)^\kappa\right)^2 \\ &\quad + \frac{1}{S+S^*}[K_\kappa(Q^B\phi)^\kappa]^2 + \frac{1}{S+S^*}[K_\kappa(T^C\phi)^\kappa]^2, \end{aligned} \tag{8}$$

where $G_I = \partial G/\partial\phi^I$ and $G^J = \partial G/\partial\phi^*_J$, and $(\mathrm{Re}f^{-1})_{\alpha\beta}$ and $(G^{-1})_I^J$ are the inverse matrices of $\mathrm{Re}f_{\alpha\beta}$ and $G_I^J$, respectively. The indices $I, J, \ldots$ run all scalar species, the index $a$ $(B, C)$ runs generators of the $G'_{\rm SM}$ $[\mathrm{U}(1)^n, H']$ gauge group, and $Q^B$'s are $\mathrm{U}(1)^n$ charge operators. Note that the Fayet-Iliopoulos $D$ term appears in $V^{(D)}$ for $\mathrm{U}(1)_A$ if we replace $S$ by its VEV [16,12,13].

By the use of the Kähler potential (3), $D$ terms for $\mathrm{U}(1)_A$ and $\mathrm{U}(1)^n$ are given as

$$D^A = \frac{\delta^A_{\rm GS}}{S+S^*} + \sum_\kappa (T+T^*)^{n_\kappa}q^A_\kappa|\phi^\kappa|^2 \tag{9}$$

and

$$D^B = \sum_\kappa (T+T^*)^{n_\kappa}q^B_\kappa|\phi^\kappa|^2, \tag{10}$$

where $q_\kappa^{A(B)}$ is the $\mathrm{U}(1)_A[\mathrm{U}(1)^n]$ charge of the scalar field $\phi^\kappa$ and we neglect the contributions from higher order terms [which is denoted as the ellipsis in Eq. (3)] in $K$.

Finally, let us give our assumption on the SUSY breaking. The gravitino mass $m_{3/2}$ is given by

$$m_{3/2} = \left\langle e^{K/2M^2}\frac{W}{M^2}\right\rangle, \tag{11}$$

where the angular brackets denote the VEV of the quantity. In the next section, it will be often taken to be real as a phase convention. The $F$-auxiliary fields of the chiral multiplets $\Phi^I$ are defined as

$$F^I \equiv Me^{G/2M^2}(G^{-1})^I_J G^J. \tag{12}$$

It is assumed that SUSY is broken by the $F$-term condensations of $\phi^i$ such that[4]

$$\langle F^i\rangle = O(m_{3/2}M). \tag{13}$$

In this case, stationary conditions of $V$ by $\phi^I$ require that VEVs of $D$-auxilary fields should be very small, i.e., $\langle D^\alpha\rangle \leq O(m^2_{3/2})$ and $\langle V^{(D)}\rangle$ should vanish up to $O(m^4_{3/2})$, i.e., $\langle V^{(D)}\rangle = O(m^4_{3/2})$ [34,35].

## III. DERIVATION OF SOFT SCALAR MASS FORMULA

### A. On scalar potential

The effective theories derived from SSTs have, in general, flat directions in the SUSY limit, which can be a source to break gauge symmetries [12]. In this subsection, we discuss the VEVs of scalar fields in the framework of SUGRA with $\mathrm{U}(1)_A$. The reasons are as follows. First, we should specify the symmetry-breaking mechanism including $\mathrm{U}(1)_A$ and its scale. It is known that the breaking scale is fixed from

[3]The electroweak symmetry breaking requires the Higgsino mass of $O(m_{3/2})$. This mass term called the $\mu$ term can be generated through nonperturbative effects and/or nonrenormalizable interactions [33]. The $W_{\rm NP}$ includes mass terms generated nonperturbatively.

[4]It is also applicable to the case of SUSY breaking by gaugino condensations [8] because the dynamics are effectively described by a nonperturbative superpotential for $\phi^i$ after integrating out gauginos. However, it is required to extend our discussion for the SUSY-breaking scenario due to matter fields with $\mathrm{U}(1)_A$ [26].

$D$-vanishing condition of $U(1)_A$ in the SUSY limit [12]. Second, we should classify scalar fields in a well-defined manner to derive low-energy effective theory. That is, we need to specify light fields which appear in a low-energy spectrum.

Using the notation

$$f(a_\kappa)\equiv\sum_\kappa a_\kappa(T+T^*)^{n_\kappa}|\phi^\kappa|^2 \tag{14}$$

and the formulas in the Appendix, we can write $V$ as

$$V=V^{(F)}+V^{(D)}, \tag{15}$$

$$V^{(F)}\equiv e^G\left[\left|1-(S+S^*)\frac{W_S}{W}\right|^2+3\left(\left|1-(T+T^*)\frac{W_T}{W}\right|^2-1\right)\right.$$
$$\left.+\left|1-(T+T^*)\frac{W_T}{W}\right|^2 f(n_\lambda)+f(1)\right], \tag{16}$$

$$V^{(D)}\equiv\frac{1}{S+S^*}\sum_\alpha\left(\frac{\delta^A_{\rm GS}}{S+S^*}\delta^{A\alpha}+f(q^\alpha_\kappa)\right)^2, \tag{17}$$

where we neglect the higher order terms of $f(a_\kappa)$.

Using the stationary condition of $\phi^\kappa$, the magnitudes of $\delta^A_{\rm GS}/(S+S^*)\cdot\delta^{A\alpha}+f(q^\alpha_\kappa)$ are estimated as $O(m^2_{3/2})$. Under the assumption that $\langle\phi^\kappa\rangle\ll\langle\phi^i\rangle=O(M)$, the VEVs of $\phi^I$ are derived iteratively in the following step. First, we obtain the solution of the stationary conditions $\partial\tilde{V}^{(F)}/\partial\phi^i=0$ (we denote it as $\tilde{\phi}^i$), where $\tilde{V}^{(F)}$ is the scalar potential in $V^{(F)}$ including only hidden fields.[5] Second, the VEVs of matter fields are determined by the following conditions that the SUSY is not spontaneously broken in the observable sector:

$$\frac{\partial W}{\partial\phi^\kappa}=0,\quad D^\alpha=0, \tag{18}$$

$$W_{\rm pert}=0, \tag{19}$$

where $\phi^i$'s are replaced by $\tilde{\phi}^i$'s in $W$ and $D^\alpha$. Next, we solve the condition $\partial V^{(F)}/\partial\phi^i=0$. The effect of matter fields is introduced through the third and fourth terms in Eq. (16) and then the VEVs of $\phi^i$ receive corrections of $O(M_I^2/M)$. We can obtain the next order solutions of $\phi^\kappa$ from the conditions (18) and (19) where $\phi^i$'s are replaced by the improved values. The solutions are denoted as $\phi^\kappa_0$.

In this way, the symmetry breaking at a very large scale is induced by $D$-vanishing condition of $U(1)_A$ and the order is given as $O(\langle\delta^A_{\rm GS}/(S+S^*)\rangle^{1/2})$. We denote it by $M_I$ and it is estimated as $\sim10^{-1}M$–$10^{-2}M$ by using explicit models. Other symmetry breaking can occur by the SUSY-breaking effects spontaneously or radiatively at some lower scales $M_{I'}$ than $M_I$. In the following sections, we discuss only the case with two typical symmetry-breaking scales $M_I$ [and $M$ for $U(1)_A$] and $m_{3/2}$ for simplicity. Our method is applied to the case with intermediate-breaking scales.

In Ref. [14], generic flat directions of $Z_{2n}$ orbifold models are discussed. In these flat directions, pairs of fields with $R$ and $\overline{R}$ representations in the same twisted sector, $n_R=n_{\overline{R}}$, develop their VEVs as $\langle R\rangle=\langle\overline{R}\rangle\neq0$. In the case with $U(1)_A$, some fields get the VEVs such as $\langle R\rangle=\langle\overline{R}\rangle=O(M_I)$ and extra symmetries break as a result. A very small difference can be generated between the VEVs of pairs of fields such as $\langle R\rangle=\langle\overline{R}\rangle+O(m^2_{3/2}/M_I)$ by SUSY-breaking effects. It is crucial for the existence of sizable $D$-term condensations of $O(m^2_{3/2})$ [36,28].

It is an important subject to study the absolute minimum of scalar potential and the conditions for the vanishing vacuum energy. Since the nonperturbative superpotential is not fully understood, we cannot give a definite answer at present. On the analysis, quantum corrections to the vacuum

[5] In this first approximation, we neglect the $D$ term of $U(1)_A$ because it is a quantum correction at one-loop level.

TABLE I. The classification of scalar fields and our notation.

$$\phi^I=\begin{cases}\phi^i\\ \phi^\kappa=\begin{cases}\phi^s=\begin{cases}\phi^V\\ \phi^X\end{cases}\\ \phi^P=\begin{cases}\hat{\phi}^K\\ \hat{\phi}^k\\ \hat{\phi}^{\hat{\alpha}(N)}\end{cases}\end{cases}\end{cases}$$

| Fields | Features |
|---|---|
| $\phi^I$ | Massless states in string theory |
| $\phi^i$ | $S,T,\ldots,\langle\phi^i\rangle=O(M)$ |
| $\phi^\kappa$ | Matter fields, $G$ nonsinglets |
| $\phi^s$ | $G_0$ singlets |
| $\phi^P$ | $G_0$ nonsinglets |
| $\phi^V$ | $\phi^V_0=O(M_I)$ |
| $\phi^X$ | $\phi^X_0=0$ |
| $\hat{\phi}^K$ | Heavy complex, $\mu_{KL}=O(M_I)$ |
| $\hat{\phi}^k$ | Light complex, $\mu_{kl}=O(m_{3/2})$ |
| $\hat{\phi}^{\hat{\alpha}(N)}$ | Heavy real, $\mu_{\hat{\alpha}^{(N)}\hat{\beta}^{(N)}}=0$ |
| $\hat{\phi}^{\mathcal{H}}$ | $(\hat{\phi}^H,\hat{\phi}^*_H)^T$ |
| $\hat{\phi}^{\mathcal{L}}$ | $(\hat{\phi}^l,\hat{\phi}^*_l)^T$ |
| $\hat{\phi}^H$ | $(\hat{\phi}^K,\hat{\phi}^{\hat{\alpha}(N)})^T$ |
| **Generators** | **Features** |
| $T^\alpha$ | All gauge generators |
| $T^{\hat{\alpha}(N)}$ | Off diagonal and broken at $M_I$ |
| $T^{\hat{\alpha}(D)}$ | Diagonal and broken at $M_I$ (or $M$) |
| $T^{\hat{\alpha}'}$ | Nonanomalous, diagonal, and broken at $M_I$ |
| $Q^A$ | Anomalous U(1) |
| $Q^B$ | Nonanomalous extra U(1)'s |

energy are also important [37].

### B. Classification of scalar fields

Now, let us explain a procedure to classify scalar fields using their VEVs, quantum numbers, SUSY fermionic masses and $D$ terms. We summarize the classification of scalar fields and our notation in Table I.

(1) The observable fields $\phi^\kappa$'s are classified into two types $\phi^s$ ($G_0$ singlets) and $\phi^P$ ($G_0$ nonsinglets) where $G_0$ is an unbroken gauge group at $M_I$. Among $\phi^s$'s, we denote scalar fields with large VEVs of $O(M_I)$ as $\phi^V$ and others as $\phi^X$. The $\phi^V$'s contribute to an extra gauge symmetry breaking. The indices $s, t, \dots$ run $G_0$ singlet observable fields and $P, Q, \dots$ run $G_0$ nonsinglet observable fields.

(2) We denote the solutions of the stationary conditions $\partial V/\partial\phi^I=0$ as $\langle\phi^I\rangle$. The difference between $\phi_0^\kappa$ and $\langle\phi^\kappa\rangle$ is estimated as follows. Both $\phi_0^P$ and $\langle\phi^P\rangle$ should not be large $[O(M_I)]$ since $G_0$ is unbroken at $M_I$. The fact that $\langle V^{(D)}\rangle$ should vanish up to $O(m_{3/2}^4)$ leads to the relation $\langle\phi^V\rangle=\phi_0^V+O(m_{3/2}^2/M_I)$. We can show that $\langle\phi^X\rangle=O(m_{3/2}^2/M_X)$ for $M_X>m_{3/2}$ and $\langle\phi^X\rangle=O(m_{3/2})$ for $m_{3/2}\geqslant M_X$ where $M_X$ is a SUSY fermionic mass of $\phi^X$ if there exists no large mixing between $\phi^X$ and $\phi^V$ in Kähler potential.[6] Hereafter, we use the angular brackets as the VEV of the quantity in most cases under the above assumptions.

(3) The general form of Kähler potential is given as

$$\begin{aligned}K=&\tilde{K}(\phi^i,\phi_j^*)+K_s^t(\phi^i,\phi_j^*)\phi^s\phi_t^*+H_{st}(\phi^i,\phi_j^*)\phi^s\phi^t+\text{H.c.}\\&+K_P^Q(\phi^i,\phi_j^*)\phi^P\phi_Q^*+H_{PQ}(\phi^i,\phi_j^*)\phi^P\phi^Q+\text{H.c.}\\&+K_{sP}^Q(\phi^i,\phi_j^*)\phi^s\phi^P\phi_Q^*+\cdots\end{aligned}\tag{20}$$

and the VEVs of $K_I^J$ are estimated as

$$\langle K_I^J\rangle=\begin{pmatrix}O(1) & O(M_I/M) & 0\\ O(M_I/M) & O(1) & 0\\ 0 & 0 & O(1)\end{pmatrix}\tag{21}$$

for $I=i,s,P$ and $J=j,t,Q$, and the magnitude of $\langle H_{IJ}\rangle$ is the same order as $\langle K_I^J\rangle$. The Kähler metric $\langle K_P^Q\rangle$ is diagonalized by a combination of some scale and unitary transformations (we denote it by $S_P^Q$ totally) such as

$$\phi^{(D)P}=(S^{-1})_Q^P\phi^Q,\quad S_P^{P'}\langle K_{P'}^{Q'}\rangle S_{Q'}^{\dagger Q}=\delta_P^Q,\tag{22}$$

where the $\phi^{(D)P}$'s are the scalar fields diagonalizing $\langle K_P^Q\rangle$.

(4) The SUSY fermionic mass $\mu_{PQ}$ is given as

$$\mu_{PQ}=\left.\frac{\partial^2 W_{\text{pert}}}{\partial\phi^{(D)P}\partial\phi^{(D)Q}}\right|_0\tag{23}$$

(in the SUSY limit). We take a basis of scalar fields $\hat{\phi}^P$ to diagonalize the SUSY fermion mass matrix $\mu_{PQ}$. Then, the scalars $\hat{\phi}^P$ are given as linear combinations of $\phi^{(D)P}$'s such as

$$\hat{\phi}^P=R_Q^P\phi^{(D)Q}.\tag{24}$$

Note that the Kähler metric $\langle K_P^Q\rangle$ is still diagonalized in terms of $\hat{\phi}^P$ because $R_P^Q$ is a unitary matrix. The scalar fields $\phi^P$ are classified either as ''heavy'' complex fields $\hat{\phi}^K,\hat{\phi}^L,\dots,$ ''light'' fields $\hat{\phi}^k,\hat{\phi}^l,\dots,$ such as $\mu_{KL}=O(M_I)$, $\mu_{kl}=O(m_{3/2})$, or Nambu-Goldstone fields $\hat{\phi}^{\hat{\alpha}^{(N)}},\hat{\phi}^{\hat{\beta}^{(N)}},\dots$ (which will be discussed just below).

(5) The gauge generators $T^\alpha$ are classified into ''heavy'' (those broken at $M_I$) $T^{\hat{\alpha}},T^{\hat{\beta}},\dots$ and ''light'' (which remain unbroken above $m_{3/2}$) $T^a,T^b,\dots$. Further, the broken generators are classified into those with off-diagonal elements $T^{\hat{\alpha}^{(N)}},T^{\hat{\beta}^{(N)}},\dots$ and diagonal ones $T^{\hat{\alpha}^{(D)}},T^{\hat{\beta}^{(D)}},\dots$. There exist the so-called Nambu-Goldstone multiplets $\hat{\phi}^{\hat{\alpha}}$'s corresponding to broken generators. $G_0$ singlet fields $\hat{\phi}^{\hat{\alpha}^{(D)}}$'s are given as some linear combinations of $\phi^s$'s. The $D$ components for $\hat{\alpha}^{(N)}$ are rewritten as

$$D^{\hat{\alpha}^{(N)}}=\hat{\phi}_P^\dagger[RS^{-1}(T^{\hat{\alpha}^{(N)}})\phi]^P+\cdots,\tag{25}$$

where we use the diagonalization of Kähler metric. We can take a basis which satisfies the relation

$$[RS^{-1}(T^{\hat{\alpha}^{(N)}})\phi]^P|_0=\mu^{\hat{\alpha}^{(N)}}\delta^{\hat{\alpha}^{(N)}P},\quad \mu^{\hat{\alpha}^{(N)}}=O(M_I).\tag{26}$$

The $\hat{\phi}^{\hat{\alpha}^{(N)}}$'s have no large VEVs due to $D$-vanishing condition (18). The equality from gauge invariance

$$\frac{\partial^2 W}{\partial\phi^I\partial\phi^J}(T^\alpha\phi)^J+\frac{\partial W}{\partial\phi^J}(T^\alpha)_I^J=0\tag{27}$$

leads to $\mu_{\hat{\alpha}I}=0$. The imaginary parts of $\hat{\phi}^{\hat{\alpha}}$'s are the would-be Nambu-Goldstone bosons which are absorbed into the gauge bosons, and the real parts acquire the same mass of order $M_I$ as that of the gauge bosons from $V^{(D)}$ in the SUSY limit. Hence, the Nambu-Goldstone multiplets belong to the heavy sector.

In the effective SUGRA derived from $Z_N$ orbifold models, the Kähler potential of matter parts $K^{(M)}$ is given as

$$\begin{aligned}K^{(M)}=&\sum_s(T+T^*)^{n_s}|\phi^s|^2+|\hat{\phi}^P|^2\\&+\sum_{P,Q}\hat{n}_P^Q\frac{\delta(T+T^*)}{\langle T+T^*\rangle}\hat{\phi}^P\hat{\phi}_Q^*\\&+\frac{1}{2}\sum_{P,Q,R}(\hat{n}_P^R\hat{n}_R^Q-\hat{n}_P^Q)\frac{[\delta(T+T^*)]^2}{\langle T+T^*\rangle}\hat{\phi}^P\hat{\phi}_Q^*+\cdots,\end{aligned}\tag{28}$$

where $\hat{n}_P^Q=(R^{-1})_P^R n_R R_R^Q$ and we expand the moduli $T$ in powers of $m_{3/2}$ around its VEV such as

$$T=\langle T\rangle+\delta T+\delta^2T+\cdots\tag{29}$$

[6] This condition is satisfied in SUGRAs from orbifold models if the Kähler potential has no mixing terms of order 1 between $\phi^V$ and $\phi^X$ in a holomorphic part of $\phi^\kappa$.

and we neglect the contribution of moduli fields $U_i$. $D$ terms for U(1) symmetries are given as

$$D^A=\frac{\delta^A_{\rm GS}}{S+S^*}+\sum_s (T+T^*)^{n_s}q^A_s|\phi^s|^2+\sum_{P,Q}(\hat{q}^A)^Q_P\hat{\phi}^P\hat{\phi}^*_Q + \sum_{P,Q,R}\hat{n}^R_P(\hat{q}^A)^Q_R\frac{\delta(T+T^*)}{\langle T+T^*\rangle}\hat{\phi}^P\hat{\phi}^*_Q+\cdots, \quad (30)$$

$$D^B=\sum_s (T+T^*)^{n_s}q^B_s|\phi^s|^2+\sum_{P,Q}(\hat{q}^B)^Q_P\hat{\phi}^P\hat{\phi}^*_Q + \sum_{P,Q,R}\hat{n}^R_P(\hat{q}^B)^Q_R\frac{\delta(T+T^*)}{\langle T+T^*\rangle}\hat{\phi}^P\hat{\phi}^*_Q+\cdots, \quad (31)$$

where $(\hat{q}^\alpha)^Q_P=(R^{-1})^R_P q^\alpha_R R^Q_R$. The parameters $\mu_{PQ}$ and $R^P_Q$, in general, depend on the VEV of the moduli field $T$ since Yukawa couplings have a moduli dependence.

### C. Heavy-light mixing terms

In this subsection, we estimate magnitudes of heavy-light mixing mass terms $\langle V^k_K\rangle\equiv\langle\partial^2V/\partial\hat{\phi}^K\partial\hat{\phi}^*_k\rangle$ and $\langle V_{Kk}\rangle\equiv\langle\partial^2V/\partial\hat{\phi}^K\partial\hat{\phi}^k\rangle$.[7] After some calculations, $\langle V^k_K\rangle$ is expressed as

$$\langle V^k_K\rangle=-\hat{\mu}_{KL}\langle K^{Lk}_i\rangle\langle F^i\rangle+O(m^2_{3/2}), \quad (32)$$

where $\hat{\mu}_{PQ}\equiv\langle\exp(K/2M^2)\rangle\mu_{PQ}$. The mixing mass, Eq. (32), can be of $O(m_{3/2}M_I)$ if the Kähler potential has heavy-light mixing terms of order 1 in a holomorphic part of $\hat{\phi}^P$.

The chirality-flipped scalar mass $\langle V_{Kk}\rangle$ is written as

$$\langle V_{Kk}\rangle=m_{3/2}\langle G_{Kki}\rangle\langle F^i\rangle-\hat{\mu}_{KL}\langle K^L_{ki}\rangle\langle F^i\rangle+O(m^2_{3/2}). \quad (33)$$

If there are Yukawa couplings of order 1 among heavy, light, and moduli fields in the superpotential, the order of the first term in the right-hand side (RHS) of Eq. (33) can be $O(m_{3/2}M)$. The second term in the RHS of Eq. (33) can be of $O(m_{3/2}M_I)$ if the Kähler potential has mixing terms of order 1 among heavy, light, and moduli fields. If there are Yukawa couplings of order 1 among light and moduli fields in the superpotential, that is, $m_{3/2}\langle G_{kli}\rangle=O(1)$, the order of the scalar masses for light fields can be $O(m_{3/2}M)$ and the weak scale can be destabilized in the presence of weak scale Higgs doublets with such intermediate masses. This is the so-called ''gauge hierarchy problem.'' Only when $m_{3/2}\langle G_{IJJ'}\rangle\langle F^{J'}\rangle$'s meet some requirements, does the hierarchy survive. In many cases, we require the condition [34,35]

$$m_{3/2}\langle G_{kli}\rangle\langle F^i\rangle\le O(m^2_{3/2}). \quad (34)$$

If we impose the same condition to the case with $I=K$ and $J=l$, we neglect the effect of the first term in the RHS of Eq. (33). (In the following discussions, we require the above conditions.) In addition, unless there exist mixing terms among heavy, light, and moduli in the Kähler potential, there appear no heavy-light mass mixing terms of $O(m_{3/2}M_I)$. In string models, whether there exist such mixing terms or not is model dependent.

[7]The mixing mass between $\hat{\phi}^P$ and $\phi^{i(s)}$ is forbidden from gauge invariance. The order of the mixing mass between $\hat{\phi}^{K(k)}$ and $\hat{\phi}^{\hat{\alpha}(N)}$ is estimated as $O(m^2_{3/2})$ and hence its effect is negligible.

### D. Soft scalar masses

#### 1. Soft scalar mass terms

For convenience, we introduce new notation related to the classification of scalar fields:

$$\hat{\phi}^{\mathcal{H}}\equiv\begin{pmatrix}\hat{\phi}^H\\ \hat{\phi}^*_H\end{pmatrix},\quad \hat{\phi}^{\mathcal{L}}\equiv\begin{pmatrix}\hat{\phi}^l\\ \hat{\phi}^*_l\end{pmatrix}, \quad (35)$$

where heavy fields $\hat{\phi}^H$'s stand for $\hat{\phi}^K$ and $\hat{\phi}^{\hat{\alpha}(N)}$. The light fields can get VEVs of $O(m_{3/2})$ and induce an extra gauge symmetry breaking, but we treat them as a sum of the VEVs and fluctuations since both have the same order and our goal is to derive soft scalar mass formula at $M_I$. We can take effects of symmetry breaking at $O(m_{3/2})$ in a similar way.

Scalar mass terms are written as

$$V^{\rm mass}=\tfrac{1}{2}\{\hat{\phi}^*_{\mathcal{H}}H^{\mathcal{H}}_{\mathcal{H}'}\hat{\phi}^{\mathcal{H}'}+\hat{\phi}^*_{\mathcal{H}}M^{\mathcal{H}}_{\mathcal{L}'}\hat{\phi}^{\mathcal{L}'} + \hat{\phi}^*_{\mathcal{L}}M^{\dagger\mathcal{L}}_{\mathcal{H}'}\hat{\phi}^{\mathcal{H}'}+\hat{\phi}^*_{\mathcal{L}}L^{\mathcal{L}}_{\mathcal{L}'}\hat{\phi}^{\mathcal{L}'}\}, \quad (36)$$

where

$$H\equiv H^{\mathcal{H}}_{\mathcal{H}'}=\begin{pmatrix}\langle V^H_{H'}\rangle & \langle V^{HH'}\rangle\\ \langle V_{HH'}\rangle & \langle V^{H'}_H\rangle\end{pmatrix}, \quad (37)$$

$$M\equiv M^{\mathcal{H}}_{\mathcal{L}'}=\begin{pmatrix}\langle V^H_{l'}\rangle & \langle V^{Hl'}\rangle\\ \langle V_{Hl'}\rangle & \langle V^{l'}_H\rangle\end{pmatrix}, \quad (38)$$

$$L\equiv L^{\mathcal{L}}_{\mathcal{L}'}=\begin{pmatrix}\langle V^l_{l'}\rangle & \langle V^{ll'}\rangle\\ \langle V_{ll'}\rangle & \langle V^{l'}_l\rangle\end{pmatrix}. \quad (39)$$

The order of the above mass matrices is estimated as $H=O(M^2_I)$, $M=O(m_{3/2}M_I)$, and $L=O(m^2_{3/2})$. Scalar mass terms are rewritten as

$$V^{\rm mass}=\tfrac{1}{2}(\hat{\phi}^*_{\mathcal{H}}+\hat{\phi}^*_{\mathcal{L}}M^\dagger H^{-1})H(\hat{\phi}^{\mathcal{H}'}+H^{-1}M\hat{\phi}^{\mathcal{L}'}) -\tfrac{1}{2}\hat{\phi}^*_{\mathcal{L}}M^\dagger H^{-1}M\hat{\phi}^{\mathcal{L}'}+\tfrac{1}{2}\hat{\phi}^*_{\mathcal{L}}L\hat{\phi}^{\mathcal{L}'}. \quad (40)$$

We discuss the implication of each term in Eq. (40). The first term is the mass term among heavy fields. After the integration of the heavy fields $\hat{\phi}^{\mathcal{H}}+H^{-1}M\hat{\phi}^{\mathcal{L}}$, there appear $D$-term [36,28,35] and extra $F$-term contributions [28,35] to scalar masses which will be discussed later. The second term is new contributions which appear after the diagonalization of scalar mass terms. This contribution can be sizable, i.e., $O(m^2_{3/2})$, if the heavy-light mass mixing is $O(m_{3/2}M_I)$. The last term is a mass term among light fields. Note that the heavy fields $\hat{\phi}^{\mathcal{H}}+H^{-1}M\hat{\phi}^{\mathcal{L}}$ and the light fields $\hat{\phi}^{\mathcal{L}}$ used here are different from properly diagonalized fields up to

$O(m_{3/2}/M_I)$ terms. The final expressions of scalar masses of $O(m_{3/2}^2)$ for light fields are the same whichever we use as a definition of the scalar fields. Hence, we use the same notation $\hat{\phi}^k$ and $\hat{\phi}_l^*$ for light scalar fields which are properly diagonalized. The extra contribution due to the existence of heavy-light mass mixing terms is expressed as

$$V_{\rm soft\ mass}^{\rm mix} \equiv -\tfrac{1}{2}\hat{\phi}_{\mathcal{L}}^* M^\dagger H^{-1} M \hat{\phi}^{\mathcal{L}'} = (V_{\rm soft\ mass}^{\rm mix})^k_l \hat{\phi}^l \hat{\phi}_k^* + \tfrac{1}{2}(V_{\rm soft\ mass}^{\rm mix})_{kl}\hat{\phi}^k\hat{\phi}^l + \text{H.c.}, \tag{41}$$

where

$$(V_{\rm soft\ mass}^{\rm mix})^k_l = -\langle V_{lK}(V^{-1})^K_L V^{Lk}\rangle - \langle V_{lK}(V^{-1})^{KL}V^k_L\rangle - \langle V^K_l (V^{-1})_{KL}V^{Lk}\rangle - \langle V^K_l (V^{-1})^L_K V^k_L\rangle, \tag{42}$$

$$(V_{\rm soft\ mass}^{\rm mix})_{kl} = -\langle V_{kK}(V^{-1})^K_L V^L_l\rangle - \langle V_{kK}(V^{-1})^{KL}V_{Ll}\rangle - \langle V^K_k (V^{-1})_{KL}V^L_l\rangle - \langle V^K_k (V^{-1})^L_K V_{Ll}\rangle. \tag{43}$$

After the diagonalization of $V^{\rm mass}$, there exists an extra term such as $-\mu_{KL}\hat{\phi}^K (H^{-1}M)^L_{\mathcal{L}}\hat{\phi}^{\mathcal{L}}$ in $W$ where we denote properly diagonalized fields as $\hat{\phi}^K$ and $\hat{\phi}^{\mathcal{L}}$ again. This term gives a sizable contribution to soft terms through $\partial W/\partial \hat{\phi}^K$.

### 2. Parametrization

For the analysis of soft SUSY-breaking parameters, it is convenient to introduce the following parametrization:

$$\langle e^{G/2}(K^S_S)^{-1/2}G^S\rangle = \sqrt{3}Cm_{3/2}e^{i\alpha_S}\sin\theta, \tag{44}$$

$$\langle e^{G/2}(K^T_T)^{-1/2}[G^T + (K^T_T)(K^{-1})^T_\kappa G^\kappa]\rangle = \sqrt{3}Cm_{3/2}e^{i\alpha_T}\cos\theta. \tag{45}$$

Then the vacuum energy $V_0$ is written as

$$V_0 = \langle e^G[G^I(G^{-1})^J_I G_J - 3]\rangle = 3(C^2-1)m_{3/2}^2 + V_0^{(M)}, \tag{46}$$

$$V_0^{(M)} = m_{3/2}^2\langle f(1)\rangle(3C^2-2) \tag{47}$$

up to $\langle f(a_\kappa)\rangle^2$. Relation (2) implies the relation $\langle G^\kappa\rangle \ll \langle G^S\rangle, \langle G^T\rangle$. In this case, the parametrization (45) becomes simpler such as

$$\langle e^{G/2}(K^T_T)^{-1/2}G^T\rangle = \sqrt{3}Cm_{3/2}e^{i\alpha_T}\cos\theta \tag{48}$$

and $V_0$ is dominated by the first part, i.e.,

$$V_0 = 3(C^2-1)m_{3/2}^2. \tag{49}$$

Soft SUSY-breaking scalar mass terms are given as

$$V_{\rm soft\ mass}^{(0)} = (m_{3/2}^2 + V_0)\sum_\kappa \langle (T+T^*)^{n_\kappa}\rangle |\phi^\kappa|^2 + \sum_\kappa \langle F^I\rangle\langle K^{I'}_{I\kappa}(K^{-1})^{J'}_{I'}K^{J\kappa}_{J'} - K^{J\kappa}_{I\kappa}\rangle\langle F^*_J\rangle \times \langle (T+T^*)^{n_\kappa}\rangle |\phi^\kappa|^2 \tag{50}$$

before heavy fields are integrated out. By the use of the parametrization, $V_{\rm soft\ mass}^{(0)}$ for $\hat{\phi}^P$ is rewritten as

$$V_{\rm soft\ mass}^{(0)} = \sum_P (m_{3/2}^2 + V_0)|\hat{\phi}^P|^2 + \sum_{P,Q} m_{3/2}^2 C^2 \cos^2\theta \hat{n}^Q_P \hat{\phi}^P \hat{\phi}^*_Q. \tag{51}$$

After heavy fields are integrated out,[8] we have the following mass terms for light fields at the energy scale $M_I$:

$$V_{\rm soft\ mass} = \sum_k (m_{3/2}^2 + V_0)|\hat{\phi}^k|^2 + \sum_{k,l} m_{3/2}^2 C^2\cos^2\theta \hat{n}^l_k \hat{\phi}^k\hat{\phi}^*_l + V^D_{\rm soft\ mass} + V^{{\rm extra}\ F}_{\rm soft\ mass} + V^{\rm mix}_{\rm soft\ mass} + V^{\rm ren}_{\rm soft\ mass}, \tag{52}$$

where $V^D_{\rm soft\ mass}$, $V^{{\rm extra}\ F}_{\rm soft\ mass}$, $V^{\rm mix}_{\rm soft\ mass}$, and $V^{\rm ren}_{\rm soft\ mass}$ are $D$-term contributions at the tree level, extra $F$-term contributions which will be discussed in the following subsections, the contributions due to the existence of heavy-light mass mixing discussed in the previous section and contributions of renormalization effects from $M$ to $M_I$, respectively.

### 3. D-term contributions

The $D$-term contributions are given as [36,28,35],

$$V^D_{\rm soft\ mass} = \sum_{k,l}\sum_{\hat{\alpha}^{(D)}} g^2_{\hat{\alpha}^{(D)}}\langle D^{\hat{\alpha}^{(D)}}\rangle (\hat{q}^{\hat{\alpha}^{(D)}})^l_k \hat{\phi}^k\hat{\phi}^*_l, \tag{53}$$

where $g_{\hat{\alpha}^{(D)}}$'s are gauge coupling constants related to diagonal generators broken at $M_I$ [and $M$ for $U(1)_A$] and we use the relation $\langle {\rm Re}S\rangle = 1/g^2_{\hat{\alpha}^{(D)}}$. We omit the terms whose magnitudes are less than $O(m_{3/2}^4)$. Note that the VEVs of $D$ terms vanish for unbroken generators and broken ones with off-diagonal elements. Hereafter, we omit the index $(D)$ which means the diagonal generators.

Next, we rewrite $V^D_{\rm soft\ mass}$ using the parametrization introduced before. For this purpose, it is useful to adapt to the following formula of $D$-term condensations [35]:

$$g_{\hat{\alpha}}\langle D^{\hat{\alpha}}\rangle = 2(M_V^{-2})^{\hat{\alpha}\hat{\beta}} g_{\hat{\beta}}\langle F^I\rangle\langle F^*_J\rangle\langle (D^{\hat{\beta}})^J_I\rangle, \tag{54}$$

where $(M_V^{-2})^{\hat{\alpha}\hat{\beta}}$ is the inverse matrix of gauge boson mass matrix $(M_V^2)^{\hat{\alpha}\hat{\beta}}$ given as

$$(M_V^2)^{\hat{\alpha}\hat{\beta}} = 2g_{\hat{\alpha}}g_{\hat{\beta}}\langle (T^{\hat{\beta}}(\phi^\dagger))_I K^I_J (T^{\hat{\alpha}}(\phi))^J\rangle. \tag{55}$$

Here, the gauge transformation of $\phi^I$ is given as $\delta_\varepsilon \phi^I = i g_\alpha (T^\alpha(\phi))^I$ up to space-time-dependent infinitesimal parameters. The above relation (54) is true at the tree level.

[8]The procedure is the same as that in Ref. [38].

After some straightforward calculations, $D$-term condensations are written as

$$g_{\hat\alpha}^2\langle D^{\hat\alpha}\rangle=2g_{\hat\alpha}m_{3/2}^2\Bigg\{(M_V^{-2})^{\hat\alpha A}g_A(1-6C^2\sin^2\theta)\times\Bigg\langle\sum_\kappa q_\kappa^A(T+T^*)^{n_\kappa}|\phi^\kappa|^2\Bigg\rangle-\sum_{\hat\beta}(M_V^{-2})^{\hat\alpha\hat\beta}g_{\hat\beta}C^2\cos^2\theta\times\Bigg\langle\sum_\kappa q_\kappa^{\hat\beta}n_\kappa(T+T^*)^{n_\kappa}|\phi^\kappa|^2\Bigg\rangle\Bigg\}. \quad (56)$$

We give some comments. We need to introduce three kinds of model-dependent quantities such as

$$\Bigg\langle\sum_\kappa q_\kappa^A(T+T^*)^{n_\kappa}|\phi^\kappa|^2\Bigg\rangle, \quad (57)$$

$$\Bigg\langle\sum_\kappa q_\kappa^{\hat\alpha}q_\kappa^{\hat\beta}(T+T^*)^{n_\kappa}|\phi^\kappa|^2\Bigg\rangle, \quad (58)$$

$$\Bigg\langle\sum_\kappa q_\kappa^{\hat\alpha}n_\kappa(T+T^*)^{n_\kappa}|\phi^\kappa|^2\Bigg\rangle. \quad (59)$$

Their magnitudes are of order $O(M_I^2)$. Note that $\langle\Sigma_\kappa q_\kappa^{\hat\alpha'}(T+T^*)^{n_\kappa}|\phi^\kappa|^2\rangle=O(m_{3/2}^2)$ due to the $D$-vanishing condition in the presence of SUSY breaking. Here, $\hat\alpha'$ runs over only *nonanomalous* diagonal-broken generators. The $\langle D^{\hat\alpha}\rangle$'s are estimated as $O(m_{3/2}^2)$ from Eq. (54) or Eq. (56). Note that they do not necessarily vanish.

The above formula (56) is practical. The same result is obtained up to $O(m_{3/2}^4/M_I^2)$ even if $\phi_0^\kappa$'s are used in place of $\langle\phi^\kappa\rangle$ on the calculation of $g_{\hat\alpha}^2\langle D^{\hat\alpha}\rangle$. [On the other hand, the situation is different if we use the formula $D^{\hat\alpha'}=\Sigma_\kappa q_\kappa^{\hat\alpha'}(T+T^*)^{n_\kappa}|\phi^\kappa|^2$ directly, that is, $D^{\hat\alpha'}$'s always vanish in the SUSY limit.] Actually, we will calculate $D$-term condensations using the formula (54) and $\phi_0^\kappa$'s in the next section.

In the case that there are no mixing elements between $U(1)_A$ and other symmetries in $(M_V^2)^{\hat\alpha\hat\beta}$, the mass of $U(1)_A$ gauge boson is given as

$$(M_V^2)^A=2g_A^2\Bigg\{\Bigg\langle\sum_\kappa q_\kappa^A(T+T^*)^{n_\kappa}|\phi^\kappa|^2\Bigg\rangle^2+\Bigg\langle\sum_\kappa (q_\kappa^A)^2(T+T^*)^{n_\kappa}|\phi^\kappa|^2\Bigg\rangle\Bigg\} \quad (60)$$

by the use of the relation $Q^AS=\delta_{GS}$ and $(Q^A\phi)^\lambda=q_\lambda^A\phi^\lambda$. Under the assumption that $\langle\phi^\kappa\rangle\ll M$, $(M_V^2)^A$ is simplified as

$$(M_V^2)^A=2g_A^2\Bigg\langle\sum_\kappa (q_\kappa^A)^2(T+T^*)^{n_\kappa}|\phi^\kappa|^2\Bigg\rangle \quad (61)$$

and $D$-term condensation of $U(1)_A$ is written as

$$g_A^2\langle D^A\rangle=\frac{m_{3/2}^2}{\Big\langle\sum_\kappa (q_\kappa^A)^2(T+T^*)^{n_\kappa}|\phi^\kappa|^2\Big\rangle}\Bigg\{(1-6C^2\sin^2\theta)\times\Bigg\langle\sum_\kappa q_\kappa^A(T+T^*)^{n_\kappa}|\phi^\kappa|^2\Bigg\rangle-C^2\cos^2\theta\Bigg\langle\sum_\kappa q_\kappa^An_\kappa(T+T^*)^{n_\kappa}|\phi^\kappa|^2\Bigg\rangle\Bigg\}. \quad (62)$$

Furthermore, at that time, $D$-term condensations of nonanomalous symmetries are given as

$$g_{\hat\alpha'}^2\langle D^{\hat\alpha'}\rangle=-m_{3/2}^2C^2\cos^2\theta\frac{\Big\langle\sum_\kappa q_\kappa^{\hat\alpha'}n_\kappa(T+T^*)^{n_\kappa}|\phi^\kappa|^2\Big\rangle}{\Big\langle\sum_\kappa (q_\kappa^{\hat\alpha'})^2(T+T^*)^{n_\kappa}|\phi^\kappa|^2\Big\rangle}, \quad (63)$$

where broken charges are redefined by the use of diagonalization of $(M_V^2)^{\hat\alpha'\hat\beta'}$. Using the expression (63), we can show that there appears no sizable $D$-term contribution to scalar masses at the tree level if a broken symmetry is nonanomalous and SUSY is broken by the dilaton $F$ term, i.e., $\cos^2\theta=0$. In the case that a large mass splitting arises among soft scalar masses due to radiative corrections, sizable $D$-term condensations can appear and survive even in the limit of dilaton-dominant SUSY breaking as will be shown in formula (78).

In a simple case that only one field $X$, which has no charges except the $U(1)_A$ charge, gets VEV to cancel the contribution of $S$ in $D^A$, the above result is reduced to the previous one obtained in Ref. [24]. Note that our result is not reduced to that obtained from the theory with the Fayet-Iliopoulos $D$ term, which is derived from the effective SUGRA by taking the flat limit first [23], even in the limit that $|\delta_{GS}^A/q_X^A|\ll1$ unless one treats $S$ as a dynamical field, because we use the condition $\langle\partial V/\partial\phi^I(Q^A\phi)^I\rangle=0$ with the contribution from $S$ to calculate $D$-term condensations.

The dominant $D$-term contributions to mixing mass terms are obtained as

$$\sum_{\hat\alpha} g_{\hat\alpha}^2\langle D^{\hat\alpha}\rangle\langle K_{kl}^\lambda\rangle\langle\phi_\lambda^*\rangle\phi^k(T^{\hat\alpha}\phi)^l+\text{H.c.} \quad (64)$$

The magnitudes are estimated as $O(m_{3/2}^4(M_I/M)^2)$ and so they are negligible in the case that $M_I\ll M$. Note that the contribution of $O(m_{3/2}^4)$ such as $g_\alpha^2\langle D^\alpha\rangle\langle H_{\kappa\lambda}\rangle\phi^\kappa(T^\alpha\phi)^\lambda$ vanishes from gauge invariance of the holomorphic part of $\phi^\kappa$ in the Kähler potential.

### 4. Extra *F*-term contributions

After the integration of complex heavy fields $\hat{\phi}^K$ and Nambu-Goldstone multiplets $\hat{\phi}^{\hat{\alpha}}$, the following $F$-term contributions appear in the low-energy effective scalar potential[9] [28,35]:

$$V^{\text{extra } F} = -|\delta^2(\hat{D}_K\hat{W})|^2 + \delta^2(\hat{D}^K\hat{W}^*)[\tfrac{1}{2}\hat{h}_{Kkl}\hat{\phi}^k\hat{\phi}^l + m_{3/2}\delta K_K - \langle F_i\rangle \delta K_K^i - \hat{\mu}_{KL}(H^{-1}M)^L_l\hat{\phi}^l - \hat{\mu}_{KL}(H^{-1}M)^{Ll}\hat{\phi}_l^*] + \text{H.c.} + |\delta^2(\hat{D}_{\hat{\alpha}(N)}\hat{W})|^2 + \langle F^s\rangle \frac{\partial \hat{W}}{\partial \hat{\phi}^s} + \text{H.c.}, \tag{65}$$

where $\delta^2(\hat{D}_K\hat{W})$, $\delta^2(\hat{D}_{\alpha(N)}\hat{W})$, and $\langle F^s\rangle$ are given as

$$\delta^2(\hat{D}_K\hat{W}) = -(\hat{\mu}^{-1})_{KL}\delta(D_s\hat{W})\hat{h}^{sLl}\hat{\phi}_l^*, \tag{66}$$

$$\delta^2(\hat{D}_{\hat{\alpha}(N)}\hat{W}) = m_{3/2}\langle H_{\hat{\alpha}(N)k}\rangle\hat{\phi}^k - \langle F_i\rangle\langle H^i_{\hat{\alpha}(N)k}\rangle\hat{\phi}^k - \langle F_i\rangle\langle K^{il}_{\hat{\alpha}(N)}\rangle\hat{\phi}_l^* \tag{67}$$

and

$$\langle F^s\rangle = \langle (K^{-1})^s_t\rangle(m_{3/2}^*\langle K^t\rangle - \langle F^i\rangle\langle K^t_i\rangle), \tag{68}$$

respectively. Here, $\delta(D_s\hat{W})$ is defined as

$$\delta(D_s\hat{W}) \equiv \left\langle \left(\partial_s + \frac{K_s}{M^2}\right)\hat{W}\right\rangle. \tag{69}$$

Further, $\hat{W} \equiv \langle \exp(K/2M^2)\rangle W$ and $\hat{W}$ is, in general, written as

$$\hat{W} = \tfrac{1}{2}\hat{\mu}_{PQ}\hat{\phi}^P\hat{\phi}^Q + \frac{1}{3!}\hat{h}_{PQR}\hat{\phi}^P\hat{\phi}^Q\hat{\phi}^R + \cdots. \tag{70}$$

Scalar masses are given as

$$V^{\text{extra } F}_{\text{soft mass}} = (V^{\text{extra } F}_{\text{soft mass}})^l_k\hat{\phi}^k\hat{\phi}_l^*, \tag{71}$$

$$(V^{\text{extra } F}_{\text{soft mass}})^l_k \equiv -(\hat{\mu}^{-1})^{KL}\delta(D^s\hat{W}^*)\hat{h}_{sLk}(\hat{\mu}^{-1})_{KM}\delta(D_t\hat{W})\hat{h}^{tMl} - \{(\hat{\mu}^{-1})^{KL}\delta(D^s\hat{W}^*)\hat{h}_{sLk}[\hat{\mu}_{KM}(H^{-1}M)^{Ml} + \langle F_i\rangle\langle K^{il}_K\rangle] + \text{H.c.}\} + (m_{3/2}\langle H_{\hat{\alpha}(N)k}\rangle - \langle F_i\rangle\langle H^i_{\hat{\alpha}(N)k}\rangle)(m^*_{3/2}\langle H^{\dagger\hat{\alpha}(N)l}\rangle - \langle F^j\rangle\langle H_j^{\dagger\hat{\alpha}(N)l}\rangle) + \langle F^i\rangle\langle K^{\hat{\alpha}(N)}_{ik}\rangle\langle F_j\rangle\langle K^{jl}_{\hat{\alpha}(N)}\rangle. \tag{72}$$

We discuss conditions that $(V^{\text{extra } F}_{\text{soft mass}})^l_k$ is neglected. If Yukawa couplings among heavy, light, and $G_0$ singlet fields are small enough $[O(m_{3/2}/M_I)]$, the first and second terms are neglected. If we impose $R$-parity conservation, the third and last terms are forbidden since bilinear couplings between Nambu-Goldstone and light fields are $R$-parity odd. Here, we define the $R$ parity of $\hat{\phi}^P$ as

$$R(\hat{\phi}^{\hat{\alpha}(N)}) = +1, \quad R(\hat{\phi}^K) = R(\hat{\phi}^k) = -1.$$

### 5. Formula of soft scalar masses

Using scalar mass terms (52), we have the following mass formula for light scalar fields at the energy scale $M_I$:

$$(m^2)^l_k|_{M_I} = (m^2_{3/2} + V_0)\delta^l_k + m^2_{3/2}C^2\cos^2\theta \hat{n}^l_k + \sum_{\hat{\alpha}} g^2_{\hat{\alpha}}\langle D^{\hat{\alpha}}\rangle(\hat{q}^{\hat{\alpha}})^l_k + (V^{\text{extra } F}_{\text{soft mass}})^l_k + (V^{\text{mix}}_{\text{soft mass}})^l_k + (V^{\text{ren}}_{\text{soft mass}})^l_k, \tag{73}$$

where $(V^{\text{ren}}_{\text{soft mass}})^l_k$ is a sum of contributions related to renormalization effects from $M$ to $M_I$ and consists of the following two parts. One is a radiative correction between $M$ and $M_I$. This contribution $(\Delta m^2)^\lambda_\kappa|_{M\to M_I}$ is given as [39]

$$(\Delta m^2)^\lambda_\kappa|_{M\to M_I} = -\sum_\alpha \frac{2}{b_\alpha}C_2(R^\alpha_\kappa)[M^2_\alpha(M_I) - M^2_\alpha(M)]\delta^\lambda_\kappa + \sum_B \frac{1}{b_B}Q^{(B)}_{R_\kappa}[S_B(M_I) - S_B(M)]\delta^\lambda_\kappa, \tag{74}$$

$$S_B(M_I) = \frac{\alpha_B(M_I)}{\alpha_B(M)}S_B(M), \tag{75}$$

$$S_B(\mu) \equiv \sum_{R_\kappa} Q^{(B)}_{R_\kappa}n_{R_\kappa}(m^2)^\kappa_\kappa(\mu), \tag{76}$$

where $\alpha$ runs all the gauge groups but $B$ runs only nonanomalous U(1) gauge groups [if $G'_{SM}$ includes U(1)'s, their contributions should be added] whose charge operators are $Q^{(\alpha)}_{R_\kappa}$, $C_2(R^\alpha_\kappa)$'s are the second order Casimir invariants, $M_\alpha$'s are gaugino masses, and $n_{R_\kappa}$ is the multiplicity. Here, we neglect effects of Yukawa couplings. It is straightforward to generalize our results to the case with large Yukawa couplings. Here, we use the anomaly cancellation condition $\Sigma_{R_\kappa}C_2(R^\alpha_\kappa)Q^{(B)}_{R_\kappa}n_{R_\kappa} = 0$ and the relation of orthgonality

[9] Here, we neglect the terms of $O(m^4_{3/2}(M_I/M)^2)$ as $M_I \ll M$.

$\Sigma_{R_\kappa} Q^{(B)}_{R_\kappa} Q^{(B')}_{R_\kappa} n_{R_\kappa} = b_B \delta_{BB'}$. Note that there is no contribution related to $U(1)_A$ symmetry since it is broken at $M$.

The other is $D$-term contribution due to mass splitting which is induced by mass renormalization. We denote it by $(\Delta m_D^2)^l_k|_{M\to M_I}$. This contribution is given as

$$(\Delta m_D^2)^l_k|_{M\to M_I} = \sum_{\hat{\alpha}} g^2_{\hat{\alpha}} \langle \Delta D^{\hat{\alpha}} \rangle (\hat{q}^{\hat{\alpha}})^l_k, \tag{77}$$

where

$$g_{\hat{\alpha}} \langle \Delta D^{\hat{\alpha}} \rangle \equiv -2 (M_V^{-2})^{\hat{\alpha}\hat{\beta}} g_{\hat{\beta}} \times \left\langle \sum_{\kappa,\lambda,\mu} \hat{\phi}^*_\lambda (\Delta m^2)^\lambda_\mu |_{M\to M_I} (\hat{q}^{\hat{\beta}})^\mu_\kappa \hat{\phi}^\kappa \right\rangle. \tag{78}$$

In the same way, we must consider the effects of radiative corrections $(\Delta m^2)^l_k|_{M_I\to M_{I'}}$ and $(\Delta m_D^2)^l_k|_{M_I\to M_{I'}}$ to obtain scalar mass formula in the case with a lower symmetry-breaking scale $M_{I'}$.

### E. Degeneracy and positivity

Here, we discuss phenomenological implications of our soft scalar mass formula, especially $D$-term contributions at the tree level, considering simple cases. In general, $D$-term contributions are comparable with $F$-term contributions [the first and second terms in the RHS of Eq. (73)]. Our formula could lead to a strong nonuniversality of soft scalar masses. Recently, much work is devoted to phenomenological implications of the nonuniversality [27,28,40]. In addition, various researches of soft scalar masses have been done in the presence of anomalous U(1) symmetry [23–26,41].

Experiments for the process of flavor-changing neutral current (FCNC) require that $\Delta m^2_{\tilde{q}}/m^2_{3/2} \sim 10^{-2}$ for the squarks $(\tilde{q}_1, \tilde{q}_2)$ of the first and the second families with equal quantum numbers under $G_{\rm SM}$ in the case with $m^2_{\tilde{q}} \sim 1$ TeV [42]. Hence, we should derive $\Delta m^2_{\tilde{q}}/m^2_{3/2} \approx 0$ within the level of $\sim 10^{-2}$. Here and hereafter, $\Delta X_{\tilde{q}} \equiv |X_{\tilde{q}_1} - X_{\tilde{q}_2}|$ and $a \approx 0$ denote such meaning. We examine the degeneracy and the positivity of squared soft scalar masses, which is an initial condition at $M_I$, in the case that there are neither particle mixing in the Kähler potential, nor heavy-light mass mixing effects, nor extra $F$-term contributions. Further, we take $V_0 = 0$, i.e., $C^2 = 1$, and consider the case that there are no mixing elements between $U(1)_A$ and other symmetries in $(M_V^2)^{\hat{\alpha}\hat{\beta}}$. There exist many other sources which threaten the suppression of FCNC process, e.g., radiative corrections and $D$-term contributions due to horizontal symmetries broken at the scale below $M_I$. They are not discussed here, but should be considered in the search of a realistic model.

#### *1. Anomaly-free symmetry case*

Here, we consider models with an anomaly-free symmetry. In this case, soft scalar masses are obtained at the tree level as

$$m_k^2 = m_{3/2}^2 \left[ 1 + \cos^2\theta \times \left( n_k - q_k \frac{\left\langle \sum_\lambda q_\lambda^{\hat{\alpha}'} n_\lambda (T+T^*)^{n_\lambda} |\phi^\lambda|^2 \right\rangle}{\left\langle \sum_\lambda (q_\lambda^{\hat{\alpha}'})^2 (T+T^*)^{n_\lambda} |\phi^\lambda|^2 \right\rangle} \right) \right]. \tag{79}$$

If $q_k \langle \Sigma_\lambda q_\lambda^{\hat{\alpha}'} n_\lambda (T+T^*)^{n_\lambda} |\phi^\lambda|^2 \rangle > 0$, squared soft masses $m_k^2$ can easily become negative, especially for larger value of $\cos\theta$.

In the limit that $\cos^2\theta \to 0$, i.e., the dilaton-dominant SUSY breaking, we obtain universal soft squark masses with equal quantum numbers under $G_{\rm SM}$, $m_k^2 = m_{3/2}^2$ [43,21]. In order to realize degenerate soft scalar masses in the other values of $\cos\theta$, one needs a ''fine-tuning'' condition as

$$\Delta n_{\tilde{q}} = \Delta q_{\tilde{q}} \frac{\left\langle \sum_\lambda q_\lambda^{\hat{\alpha}'} n_\lambda (T+T^*)^{n_\lambda} |\phi^\lambda|^2 \right\rangle}{\left\langle \sum_\lambda (q_\lambda^{\hat{\alpha}'})^2 (T+T^*)^{n_\lambda} |\phi^\lambda|^2 \right\rangle}. \tag{80}$$

If fields with nonvanishing VEVs have the same modular weight, i.e., the same Kähler metric, the $D$-term contributions on soft scalar masses vanish due to the $D$-flatness condition. This fact is important. This situation can happen in some cases. In these cases, degeneracy of soft masses is realized for fields with the same values of modular weights. One example for the vanishing $D$-term contribution is shown in the next section.

Another interesting example is the case where enhanced gauge symmetries break by VEVs of moduli fields in orbifold models. Gauge symmetries are enhanced at specific points of moduli spaces, where some massless states $\eta_i$ also appear in the untwisted sector. For example, $Z_3$ orbifold models have enhanced $U(1)^6$ symmetries. Here, we expand moduli fields $\tau_i$ around these points so that vanishing or nonvanishing VEVs $\langle \tau_i \rangle$ correspond to unbroken or broken enhanced symmetries. Neither $\tau_i$ nor $\eta_i$ has well-defined charges under the U(1)'s and we take linear combinations $s_i$, which have definite U(1) charges [44]. These fields $s_i$ have the same Kähler metric. If only these fields $s_i$ develop VEVs and no symmetry other than enhanced symmetries breaks, $D$-term contributions on soft scalar masses vanish, because enhanced symmetries are anomaly-free and fields developing VEVs have the same Kähler metric.

#### *2. Anomalous U(1) case*

Here, we study models with an anomalous U(1) symmetry. In this case, soft scalar masses are obtained as[10]

[10]Throughout this subsection, we omit the superscript $A$ in the $U(1)_A$ charge $q_k^A$ and the subscript $\tilde{q}$ in $\Delta n_{\tilde{q}}$ and $\Delta q_{\tilde{q}}$.

$$m_k^2 = m_{3/2}^2 \left[ 1 + n_k \cos^2\theta + \frac{q_k}{\langle f(q_\lambda^2)\rangle} \{ \langle f(q_\lambda)\rangle (1 - 6\sin^2\theta) - \langle f(q_\lambda n_\lambda)\rangle \cos^2\theta \} \right], \tag{81}$$

where $f(a_\lambda)$ denotes Eq. (14). Here, $\langle f(q_\lambda)\rangle$ does not vanish for a finite value of $\langle S\rangle$ because of the $D$-vanishing condition of $U(1)_A$. It is remarkable that even if $\cos\theta = 0$, the $D$-term contribution does not vanish. That is different from $D$-term contributions due to the breakdown of anomaly-free U(1) symmetries. In general, nonuniversal soft scalar masses are obtained even if $\cos\theta = 0$.

The $D$-term contribution vanishes if the following fine-tuning condition is satisfied:

$$[6\langle f(q_\lambda)\rangle - \langle f(q_\lambda n_\lambda)\rangle] \sin^2\theta = \langle f(q_\lambda)\rangle - \langle f(q_\lambda n_\lambda)\rangle. \tag{82}$$

We have $\langle f(q_\lambda n_\lambda)\rangle = n_\lambda \langle f(q_\lambda)\rangle$ in the case where the fields in the summation $\langle f(q_\lambda n_\lambda)\rangle$ have the same modular weight $n_\lambda$. In this case Eq. (82) reduces as

$$(6 - n_\lambda)\sin^2\theta = 1 - n_\lambda. \tag{83}$$

For $n_\lambda = 1$, the moduli-dominant SUSY breaking, i.e., $\sin\theta = 0$, satisfies this condition although this modular weight $n_\lambda = 1$ is not naturally obtained [18,45]. The modular weight satisfying $n_\lambda \leqslant 0$ leads to $0 < \sin^2\theta < 1$ for Eq. (83).

Degenerate soft squark masses are obtained for differences of modular weights and $U(1)_A$ charges $\Delta n$ and $\Delta q$ in the case where the following fine-tuning condition is satisfied:

$$[\langle f(q_\lambda^2)\rangle \Delta n + 6\langle f(q_\lambda)\rangle \Delta q - \langle f(q_\lambda n_\lambda)\rangle] \cos^2\theta = 5\langle f(q_\lambda)\rangle \Delta q. \tag{84}$$

Soft scalar masses are written for two extreme cases of the SUSY breaking, i.e., $\cos\theta = 0$ and 1 as

$$\frac{m_k^2}{m_{3/2}^2} = 1 - 5 q_k \frac{\langle f(q_\lambda)\rangle}{\langle f(q_\lambda^2)\rangle} \quad \text{for} \quad \cos\theta = 0, \tag{85}$$

$$\frac{m_k^2}{m_{3/2}^2} = 1 + n_k + q_k \frac{\langle f(q_\lambda)\rangle - \langle f(q_\lambda n_\lambda)\rangle}{\langle f(q_\lambda^2)\rangle} \quad \text{for} \quad \cos\theta = 1. \tag{86}$$

Squared soft scalar masses become easily negative for $q_k \langle f(q_\lambda)\rangle > 0$ in the former case. On the other hand, we obtain likely negative $m_k^2$ for $q_k\{\langle f(q_\lambda)\rangle - \langle f(q_\lambda n_\lambda)\rangle\} < 0$ in the latter case.

We have discussed a simple case where only one field $X$ develops its VEV in Ref. [24].

## IV. ANALYSIS ON EXPLICIT MODEL

### A. Flat direction

In this section, we study $U(1)_A$-breaking effects and derive specific scalar mass relations by using an explicit model [46]. The model we study is the $Z_3$ orbifold model with a shift vector $V$ and Wilson lines $a_1$ and $a_3$ such as

$$V = \tfrac{1}{3}(1,1,1,1,2,0,0,0)(2,0,0,0,0,0,0,0)',$$

$$a_1 = \tfrac{1}{3}(0,0,0,0,0,0,0,2)(0,0,1,1,0,0,0,0)',$$

$$a_3 = \tfrac{1}{3}(1,1,1,2,1,1,1,0)(1,1,0,0,0,0,0,0)'.$$

This model has a gauge group as

$$G = SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)^7 \times SO(8)' \times SU(2)'.$$

One of $U(1)^7$ is anomalous. This model has matter multiplets as

$$\text{U-sec.:} \quad 3[(3,2,1)_0 + (\overline{3},1,2)_0 + (1,2,2)_0] + 3[(8,2)'_6 + (1,1)'_{-12}],$$

$$\text{T-sec.}(N_{OSC}=0): \quad 9[(3,1,1)_4 + (\overline{3},1,1)_4] + 15[(1,2,1)_4 + (1,1,2)_4] + 3(1,2,2)_4 + 3[(1,2,1)(1,2)'_{-2} + (1,1,2)(1,2)'_{-2}] + 24(1,2)'_{-2} + 60(1,1,1)_4 + 3(1,1,1)_{-8},$$

$$\text{T-sec.}(N_{OSC}=-1/3): \quad 9(1,1,1)_4,$$

where the number of suffix denotes the anomalous U(1) charge defined as $Q^A \equiv Q_5 - Q_6$ and $N_{OSC}$ is the oscillator number. This model has $\mathrm{Tr}Q^A = 864$. The U(1) charge generators of $U(1)^7$ are defined in Table II.

This model has many $SU(3)_C \times SU(2)_L \times SU(2)_R$ singlets as shown above. These fields are important for flat directions leading to realistic vacua. For example, this model includes the following $SU(3)_C \times SU(2)_L \times SU(2)_R$ singlets:

$$u: \quad Q_a = (0,0,0,0,-6,6,0),$$

$$Y: \quad Q_a = (0,-4,-4,0,2,-2,0),$$

$$S_1: \quad Q_a = (0,-4,-4,0,-4,4,0),$$

$$D_3': \quad Q_a = (6,4,0,-2,-2,0,-2),$$

TABLE II. U(1) charge generators in terms of $E_8 \times E_8'$ lattice vectors.

| |
|---|
| $Q_1 = 6(1,1,1,0,0,0,0,0)(0,0,0,0,0,0,0,0)'$ |
| $Q_2 = 6(0,0,0,1,-1,0,0,0)(0,0,0,0,0,0,0,0)'$ |
| $Q_3 = 6(0,0,0,0,0,1,1,0)(0,0,0,0,0,0,0,0)'$ |
| $Q_4 = 6(0,0,0,0,0,0,0,1)(0,0,0,0,0,0,0,0)'$ |
| $Q_5 = 6(0,0,0,0,0,0,0,0)(1,0,0,0,0,0,0,0)'$ |
| $Q_6 = 6(0,0,0,0,0,0,0,0)(0,1,0,0,0,0,0,0)'$ |
| $Q_7 = 6(0,0,0,0,0,0,0,0)(0,0,1,1,0,0,0,0)'$ |

$$D_4': \quad Q_a = (6,4,0,2,-2,0,2),$$

$$D_5': \quad Q_a = (-6,0,4,-2,0,2,-2),$$

$$D_6': \quad Q_a = (-6,0,4,2,0,2,2),$$

where $U(1)^7$ charges $Q_a$ ($a=1,2,\ldots,7$) are represented in the basis of Table II. Here, we follow the notation of the fields in Ref. [46] except the $D_i'$ fields. These $D_i'$ fields are $SU(2)'$ doublets in the nonoscillated twisted sector with $n_\kappa = -2$, corresponding to $T_i$ fields in Ref. [46]. The others are singlets under any non-Abelian group. The $u$ field corresponds to the untwisted sector with $n_u = -1$. In addition, $S_1$ corresponds to the nonoscillated twisted sector with $n_\kappa = -2$ and $Y$ corresponds to the twisted sector with a nonvanishing oscillator number. Thus, the field $Y$ has the modular weight $n_Y = -3$. There exist vacuum solutions up to $O(m_{3/2}^2)$ [46],

$$\langle (T+T^*)^{-1}|u|^2\rangle = v_1, \tag{87}$$

$$\langle (T+T^*)^{-3}|Y|^2\rangle = \langle (T+T^*)^{-2}|D_3'|^2\rangle = \langle (T+T^*)^{-2}|D_6'|^2\rangle = v_2,$$

$$\langle (T+T^*)^{-2}|S_1|^2\rangle = \langle (T+T^*)^{-2}|D_4'|^2\rangle = \langle (T+T^*)^{-2}|D_5'|^2\rangle = v_3,$$

where $v_i \geqslant 0$. Along this flat direction, the gauge symmetries break as $U(1)^7 \times SU(2)' \to U(1)^3$. One of the unbroken $U(1)^3$ charges corresponds to $Q_{B-L}$. Here, we define the broken charges as

$$Q_1' \equiv \tfrac{1}{3}(2Q_1 + Q_2 - Q_3 - Q_5 - Q_6),$$

$$Q_2' \equiv \frac{1}{\sqrt{3}}(2Q_4 + Q_7),$$

$$Q_3' \equiv \frac{1}{\sqrt{2}}(Q_2 + Q_3),$$

$$Q^A \equiv Q_5 - Q_6, T'^3,$$

where $T'^3$ is a third component of generators of $SU(2)'$. Note that the gauge boson mass matrix is not diagonalized in this definition of charges. The modular weights and broken charges of the light scalar fields and the fields with VEVs are given in Table III. For the light fields, we follow the notation of fields in Ref. [46]. Chiral multiplets are denoted as $Q_L$ for left-handed quarks, $Q_R$ right-handed quarks, $L$ for left-handed leptons, and $R$ for right-handed leptons. In addition, $H$ are Higgs doublets.

TABLE III. The modular weights and broken U(1) charges for the light scalar fields and the scalar fields with large VEVs. An anomalous U(1) charge $Q^A$ is defined as $Q^A \equiv Q_5 - Q_6$. We denote the third component of generators of $SU(2)'$ as $T^{3'}$ and the number in the seventh column represents the eigenvalue of $2T^{3'}$ for the field component with VEV.

| String states | $n_\kappa$ | $Q'^1$ | $\sqrt{3}Q'^2$ | $\sqrt{2}Q'^3$ | $Q^A$ | $2T^{3'}$ |
|---|---|---|---|---|---|---|
| $Q_L$ | $-1$ | 6 | 0 | $-6$ | 0 | 0 |
| $Q_R$ | $-1$ | $-6$ | 0 | $-6$ | 0 | 0 |
| $H$ | $-1$ | 0 | 0 | 12 | 0 | 0 |
| $L$ ($L_4$) | $-2$ | $-2$ | 0 | $-2$ | 4 | 0 |
| $R$ ($R_5$) | $-2$ | 2 | 0 | $-2$ | 4 | 0 |
| $L'$ ($L_3$) | $-2$ | $-2$ | 0 | $-2$ | 4 | 0 |
| $\overline{L'}$ ($L_5$) | $-2$ | $-2$ | 0 | $-2$ | 4 | 0 |
| $R'$ ($R_4$) | $-2$ | 2 | 0 | $-2$ | 4 | 0 |
| $\overline{R'}$ ($R_1$) | $-2$ | 2 | 0 | $-2$ | 4 | 0 |
| $u$ | $-1$ | 0 | 0 | 0 | $-12$ | 0 |
| $Y$ | $-3$ | 0 | 0 | $-8$ | 4 | 0 |
| $S_1$ | $-2$ | 0 | 0 | $-8$ | $-8$ | 0 |
| $D_3'$ | $-2$ | 6 | $-6$ | 4 | $-2$ | 1 |
| $D_4'$ | $-2$ | 6 | 6 | 4 | $-2$ | $-1$ |
| $D_5'$ | $-2$ | $-6$ | $-6$ | 4 | $-2$ | 1 |
| $D_6'$ | $-2$ | $-6$ | 6 | 4 | $-2$ | $-1$ |

The $D$-vanishing condition for $U(1)_A$ requires

$$\left\langle \frac{\delta_{GS}^A}{S+S^*} \right\rangle - 12v_1 - 12v_3 = 0. \tag{88}$$

Using the solution (87), we have

$$\langle f(n_\lambda)\rangle = -v_1 - 7v_2 - 6v_3, \tag{89}$$

$$\langle f(1)\rangle = v_1 + 3v_2 + 3v_3. \tag{90}$$

For simplicity, we study the solution,

$$v_1 = \frac{1}{12}\left\langle \frac{\delta_{GS}^A}{S+S^*} \right\rangle, \quad v_2 = O(m_{3/2}^2), \quad v_3 = O(m_{3/2}^2). \tag{91}$$

Using $\mathrm{Tr}Q^A = 864$ and $\langle \mathrm{Re}S\rangle \sim 2$, we estimate $v_1 \sim M^2/53$. In this case, a single symmetry $U(1)_A$ is broken at $M$. The other extra symmetries could break radiatively at some intermediate scales.

### B. Soft mass relations

We derive specific relations among soft scalar masses. The basic idea and the strategy are the same as those in Refs. [27–29]. The SUSY spectrum at the weak scale, which is expected to be measured in the near future, is translated into the soft SUSY-breaking parameters. The values of these parameters at higher energy scales are obtained by using the renormalization group equations (RGEs) [39]. In many cases, there exist some relations among these parameters. They reflect the structure of high-energy physics. Hence, we can specify the high-energy physics by checking these relations.

The generic formula of scalar mass is given as Eq. (73). We have the same number of observable scalar masses as that of species of scalar fields, e.g., 17 observables in the MSSM. There are several model-dependent parameters in the RHS of Eq. (73) such as $m_{3/2}^2+V_0$, $\cos\theta$, and so on. If the number of independent equations is more than that of unknown parameters, nontrivial relations exist among scalar masses.

We assume that Yukawa couplings among heavy and light fields are small enough and the $R$ parity is conserved. In such a case, we can neglect the effect of extra $F$-term contributions. Since the light fields $\hat{\phi}^k$ are equal to just string states in this model, there are no mixing terms among heavy and light fields in the Kähler potential. As discussed in Sec. III C, there appear no heavy-light mixing terms of $O(m_{3/2}M_I)$ if Yukawa couplings among heavy, light, and moduli fields are suppressed sufficiently, i.e., $\langle \hat{W}_{Hki}\rangle = O(m_{3/2}/M)$. At that time, the quantities $\hat{n}_k^l$ and $(\hat{q}^\alpha)_k^l$ are simplified as

$$\hat{n}_k^l = n_k \delta_k^l, \quad (\hat{q}^\alpha)_k^l = q_k^\alpha \delta_k^l. \tag{92}$$

Under the above assumptions and excellent features, our soft scalar mass formula is written in a simple form such as

$$\begin{aligned}(m^2)_k|_{M_I} &= m_{3/2}^2 + m_{3/2}^2 n_k \cos^2\theta + \sum_{\hat{\alpha}} g_{\hat{\alpha}}^2 \langle D^{\hat{\alpha}}\rangle q_k^{\hat{\alpha}} \\ &= m_{3/2}^2 \left\{ 1 + n_k\cos^2\theta + \frac{q_k^A}{12}(5-7\cos^2\theta)\right\}, \end{aligned} \tag{93}$$

where we take $V_0=0$, i.e., $C=1$. Here, we use the formula of $D$-term condensation (62) and the values

$$\langle f(q^A)\rangle = -12 v_1, \quad \langle f(q^A n_\kappa)\rangle = 12 v_1,$$

$$\langle f((q^A)^2)\rangle = 144 v_1.$$

In this model, the gauge boson mass matrix is diagonalized for the components of $\mathrm{U}(1)_A$ and $\mathrm{U}(1)_3'$ up to $m_{3/2}^2/M_I^2$.

In Table IV, we give a ratio $m_k^2/m_{3/2}^2$ at $M$ for all light species except $G_{\mathrm{SM}}'$ singlets in two extreme cases, $\cos^2\theta=0$ and $\cos^2\theta=1$. For $\cos^2\theta=1$, $L_i$ $(i=3,4,5)$ and $R_j$ $(j=1,4,5)$ fields acquire negative squared masses and they could trigger a ''larger'' symmetry breaking including the dangerous charge symmetry breaking. In addition, we have a strong nonuniversality of soft masses. However, in this model, soft masses are degenerate for squarks and sleptons with same quantum numbers under $G_{\mathrm{SM}}$ because they have same quantum numbers under the gauge group $G$ and same modular weights. Hence, it does not lead to a dangerous FCNC process.

We have the following relations at $M$ by eliminating model-dependent parameters:

TABLE IV. The particle contents and the ratios of $m_k^2/m_{3/2}^2$. We refer to the chiral multiplets as $Q_L$ for left-handed quarks, $Q_R$ for right-handed quarks, $H$ for Higgs doublets, $L$ for left-handed leptons, and $R$ for right-handed leptons. The fields $L'$, $\overline{L'}$ and $R'$, $\overline{R'}$ are extra $\mathrm{SU}(2)_L$ and $\mathrm{SU}(2)_R$ doublets, respectively.

| | | | $m_k^2/m_{3/2}^2\vert_M$ | |
|---|---|---|---|---|
| | Rep. | $q_k^A$ | $\cos^2\theta=0$ | $\cos^2\theta=1$ |
| U-sec. | $Q_L$ (3,2,1) | 0 | 1 | 0 |
| | $Q_R$ ($\overline{3}$,1,2) | 0 | 1 | 0 |
| | $H$ (1,2,2) | 0 | 1 | 0 |
| T-sec. | $L$ (1,2,1) | 4 | 8/3 | −5/3 |
| ($N_{OSC}=0$) | $R$ (1,1,2) | 4 | 8/3 | −5/3 |
| | $L'$ (1,2,1) | 4 | 8/3 | −5/3 |
| | $\overline{L'}$ (1,2,1) | 4 | 8/3 | −5/3 |
| | $R'$ (1,1,2) | 4 | 8/3 | −5/3 |
| | $\overline{R'}$ (1,1,2) | 4 | 8/3 | −5/3 |

$$m_{\tilde{Q}_L}^2 = m_{\tilde{Q}_R}^2 = m_H^2, \quad m_{\tilde{L}}^2 = m_{\tilde{R}}^2, \quad 13 m_{\tilde{Q}_L}^2 = 3 m_{\tilde{L}}^2 + 5 m_{3/2}^2, \tag{94}$$

where the tilde represents scalar components.

On the top of that, the gaugino mass $M_{1/2}$ is obtained as [21]

$$M_{1/2}^2 = 3 m_{3/2}^2 \sin^2\theta. \tag{95}$$

We can use this gaugino mass to obtain a relation not including $m_{3/2}$ as

$$3 m_{\tilde{Q}_L}^2 = M_{1/2}^2. \tag{96}$$

In the case that the SUSY breaking is induced by the dilaton $F$ term, there are no modular weight dependence. Hence, we have a more specific relation such that

$$8 m_{\tilde{Q}_L}^2 = 3 m_{\tilde{L}}^2. \tag{97}$$

Further, various contributions should be added at lower energy scales. For example, the $D$-term contribution can appear after the breakdown of extra gauge symmetries.

In general, original string states are different from the MSSM fields in string models including $G_{\mathrm{SM}}$ [46]. The coefficients $R_P^Q$ of linear combinations depend on the VEVs of moduli fields. A study of soft masses in such a situation has been carried out by using an explicit model [47].

## V. REMARKS ON EXTENSION OF KÄHLER POTENTIAL

Here we discuss extensions of our soft mass formula for different types of Kähler potentials. At the one-loop level, the dilaton field $S$ and the moduli field $T$ are mixed in the Kähler potential as

$$-\ln[S+S^*+\Delta(T+T^*)] - 3\ln(T+T^*). \tag{98}$$

In this case we can obtain the same parametrization of soft scalar masses as the case without the mixing, i.e., $\Delta(T+T^*)=0$, except replacing $\cos^2\theta$ as

$$\cos^2\theta \rightarrow \left[1-\frac{(T+T^*)^2\Delta''(T+T^*)}{3[S+S^*+\Delta(T+T^*)]}\right]\cos^2\theta, \tag{99}$$

where $\Delta''(T+T^*)$ is the second derivative of $\Delta(T+T^*)$ by $T$.

In general, string models have several moduli fields other than one overall moduli field $T$. In this case, their $F$ terms could contribute on the SUSY breaking and one needs more goldstino angles to parametrize these $F$ terms. For example, we discuss the models with three diagonal moduli fields $T_i$ $(i=1,2,3)$. These moduli fields have the following Kähler potential:

$$-\sum_i \ln(T_i+T_i^*) \tag{100}$$

instead of $-3\ln(T+T^*)$ in the case of the overall moduli field. Here, we parametrize their contributions on the SUSY breaking as [22]

$$\langle e^{G/2}(K_{T_i}^{T_i})^{-1/2}G^{T_i}\rangle=\sqrt{3}Cm_{3/2}e^{i\alpha_{T_i}}\cos\theta\Theta_i, \tag{101}$$

where $\Sigma_i\Theta_i^2=1$. Using these parameters, $F$-term contributions on soft scalar masses are written as

$$m_{3/2}^2+V_0+3m_{3/2}^2C^2\sum_i n_{i\kappa}\cos^2\theta\Theta_i^2, \tag{102}$$

where $n_{i\kappa}$ is a modular weight of $\phi^\kappa$ for the $i$th moduli field $T_i$. Similarly, $D$-term contributions can be written by the use of these parameters. For example, the $D$-term condensations (63) are extended as

$$g_{\hat{\alpha}'}^2\langle D^{\hat{\alpha}'}\rangle=-3m_{3/2}^2C^2\cos^2\theta \times\sum_i \Theta_i^2\frac{\left\langle\sum_\kappa q_\kappa^{\hat{\alpha}'}n_{i\kappa}(T+T^*)^{n_\kappa}|\phi^\kappa|^2\right\rangle}{\left\langle\sum_\kappa (q_\kappa^{\hat{\alpha}'})^2(T+T^*)^{n_\kappa}|\phi^\kappa|^2\right\rangle}, \tag{103}$$

where $(T+T^*)^{n_\kappa}$ means $\Pi_{i=1}^3(T_i+T_i^*)^{n_{i\kappa}}$.

Some orbifold models have complex structure moduli fields $U_i$. In such models, a Kähler potential includes holomorphic parts of $\phi^\kappa$ as [48]

$$\frac{1}{(T_i+T_i^*)(U_i+U_i^*)}\phi\phi'. \tag{104}$$

We can extend our formula into these models. These holomorphic parts are important for mixing of fields. Further, they could originate the $\mu$ term with a suitable order, naturally.

The Kähler potential can receive radiative corrections and be modified by nonperturbative effects. Our approach is generic and basically available to other types of Kähler potential although one might need more complicated parametrization than Eqs. (44) and (45).

## VI. CONCLUSIONS AND DISCUSSIONS

We have derived the formula of soft SUSY-breaking scalar masses from the effective SUGRA derived from 4D string models within a more generic framework. The gauge group contains extra gauge symmetries including the anomalous U(1), some of which are broken at a higher energy scale. Such breaking is related to the flat direction breaking in the SUSY limit. It is supposed that there are two types of matter multiplets classified by supersymmetric fermion mass and $D$ term, i.e., heavy fields and light ones. The physical scalar fields are, in general, linear combinations of original fields corresponding to massless states in string models.

The mass formula contains the effects of extra gauge symmetry breaking, i.e., $D$-term and extra $F$-term contributions, particle mixing effects, and heavy-light mass mixing effects. The $D$-term contributions to soft scalar masses are parametrized in terms of three types of new parameters in addition to the goldstino angle, gravitino mass, and vacuum energy. These contributions, in general, are sizable. In particular, $D$-term contribution of $U(1)_A$ survives even in the case of the dilaton-dominant SUSY breaking. The $D$-term contributions for anomaly-free U(1) symmetries vanish at the tree level if the fields developing VEVs have the same modular weight. Extra $F$-term contributions are neglected in the case where Yukawa couplings among heavy, light, and $G_0$ singlet fields are suppressed and the $R$ parity is conserved. In the case that there exist mixing terms among heavy, light, and moduli fields in the Kähler potential, the extra contributions can appear after the diagonalization of scalar mass terms in the presence of heavy-light mass mixings of $O(m_{3/2}M_I)$.

We have discussed the degeneracy and the positivity of squared scalar masses in special cases where there is neither particle mixing in the Kähler potential, nor heavy-light mass mixing effects, nor extra $F$-term contributions. We find that the $F$-term contribution from the difference among modular weights and the $D$-term contribution to scalar masses can destroy universality among scalar masses at $M$ and/or $M_I$. This nondegeneracy among squark masses of first and second families endangers the discussion of the suppression of FCNC process. On the other hand, the difference among U(1) charges is crucial for the generation of fermion mass hierarchy [31]. It seems to be difficult to make two discussions compatible. As a byway, we can take a model that the fermion mass hierarchy is generated due to nonanomalous U(1) symmetries and SUSY is broken by the dilaton $F$-term condensation. For example, it is supposed that anomalies from contributions of the MSSM matter fields are canceled out by those of extra matter fields in such a model. Further ''stringy'' symmetries are also useful for fermion mass generation leading to degenerate soft scalar masses [32], because these symmetries do not induce $D$ terms.

Many fields could acquire negative squared masses and they could trigger a ''larger'' symmetry breaking including the dangerous color and/or charge symmetry breaking. This type of symmetry breaking might be favorable in the case where $G'_{SM}$ is a large group like a grand unified group. These results might be useful for model building.

We have calculated $D$-term condensations and derived specific scalar mass relations by taking an explicit string model. It is expected that such relations can be novel probes to select a realistic string model since they are model dependent.

The moduli fields have a problem in string cosmology because their masses are estimated as of $O(m_{3/2})$ and they weakly couple with the observable matter fields, i.e., through the gravitational couplings [49]. They decay slowly to the observable matter fields. That makes the standard nucleosynthesis dangerous. In our model, some linear combinations of $S$, $T$, and other fields such as $X$ remain light whose $F$ terms are of $O(m_{3/2}M)$ and break the SUSY. It is supposed that the couplings between such fields and observable fields are strongly suppressed to guarantee the stability of the weak scale. Such a problem have to be considered for the light linear combinations, too.

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## APPENDIX: KÄHLER POTENTIAL AND ITS DERIVATIVES IN STRING MODELS

The Kähler potential $K$ in $Z_N$ orbifold models is given as [4–6]

$$K=-\ln(S+S^*)-3\ln(T+T^*)+\sum_{\kappa}(T+T^*)^{n_\kappa}|\phi^\kappa|^2 \tag{A1}$$

in the case of overall moduli. Here, we neglect higher order terms related to matter fields. The derivatives of $K$ are given as

$$K_S^S=\frac{1}{(S+S^*)^2},\quad K_S^T=0,\quad K_S^\kappa=0,$$

$$K_T^T=\frac{3}{(T+T^*)^2}+\sum_{\kappa} n_\kappa(n_\kappa-1)(T+T^*)^{n_\kappa-2}|\phi^\kappa|^2,$$

$$K_T^\lambda=n_\lambda(T+T^*)^{n_\lambda-1}\phi^\lambda,\quad K_\kappa^\lambda=(T+T^*)^{n_\kappa}\delta_\kappa^\lambda.$$

The determinant of $K_I^J$ is calculated as

$$\Delta\equiv\det K_I^J$$
$$=\frac{3\prod_\lambda(T+T^*)^{n_\lambda}}{(S+S^*)^2(T+T^*)^2}\left\{1-\sum_\kappa\frac{n_\kappa}{3}(T+T^*)^{n_\kappa}|\phi^\kappa|^2\right\}. \tag{A2}$$

The inverses $(K^{-1})_I^J$ are given as

$$(K^{-1})_S^S=(S+S^*)^2,\quad (K^{-1})_S^T=0,\quad (K^{-1})_S^\kappa=0,$$

$$(K^{-1})_T^T=\frac{\prod_\kappa(T+T^*)^{n_\kappa}}{(S+S^*)^2\Delta}$$
$$=\frac{(T+T^*)^2}{3}\left\{1+\sum_\kappa\frac{n_\kappa}{3}(T+T^*)^{n_\kappa}|\phi^\kappa|^2\right\}$$
$$+O(|\phi|^4),$$

$$(K^{-1})_\kappa^T=-\frac{n_\kappa(T+T^*)\phi_\kappa^*}{3-\sum_\lambda n_\lambda(T+T^*)^{n_\lambda}|\phi^\lambda|^2}$$
$$=-\frac{n_\kappa}{3}(T+T^*)\phi_\kappa^*\left\{1+\sum_\lambda\frac{n_\lambda}{3}(T+T^*)^{n_\lambda}|\phi^\lambda|^2\right\}$$
$$+O(\phi^*|\phi|^4),$$

$$(K^{-1})_\kappa^\lambda=\frac{3(T+T^*)^{-n_\kappa}\delta_\kappa^\lambda}{3-\sum_\lambda n_\lambda(T+T^*)^{n_\lambda}|\phi^\lambda|^2}$$
$$=(T+T^*)^{-n_\kappa}\delta_\kappa^\lambda+O(|\phi|^2),$$

where $\phi$ represents scalar field of matter multiplet.

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