

Scalar Mass Spectrum as a Probe of E_6 Gauge Symmetry Breaking

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We study the scalar mass spectrum within supersymmetric E_6 grand unified theory. It is demonstrated how the scalar mass spectrum gives useful information on the E_6 breaking patterns.

§ 1. Introduction

The Grand Unified Theory (GUT)¹⁾ has been attractive as the physics beyond standard model. However it has several problems. The notorious one is 'gauge hierarchy problem'.²⁾ This problem is partially solved by the introduction of supersymmetry (SUSY), that is, SUSY can stabilize the hierarchy of different energy scales against radiative corrections.³⁾

Interestingly, the recent precision measurements at LEP⁴⁾ have given strong support to the supersymmetric $SU(5)$ GUT,⁵⁾ that is, the three gauge coupling constants, g_3 , g_2 and g_1 of 'standard gauge group' $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$ meet at about 10^{16} GeV within the framework of the minimal supersymmetric standard model (MSSM).⁶⁾ Furthermore the present non-observation of the nucleon decay is shown to be still consistent with the minimal SUSY $SU(5)$ version.⁷⁾

There exist, however, many scenarios based on the various models of SUSY GUT consistent with the LEP data if the concept of 'simplicity' is put aside. For example, the direct breaking of the larger group down to G_{SM} and the models with extra heavy generations. Non-trivial examples are the models of SUSY $SO(10)$ GUT with the chain breaking.^{8),9)} It is difficult to distinguish these scenarios by the use of the precise measurements of gauge couplings alone. Hence it is an important task to study how we can discriminate among them by other experiments.

It is expected that the soft SUSY-breaking mass parameters can be novel probes of physics at higher energy scales. In fact, their utility has been examined in Ref. 9). Let us recall the results. The gaugino mass spectrum satisfies the GUT-relation¹⁰⁾ as far as 'standard gauge group' is embedded into a simple group, irrespective of the symmetry breaking pattern, while the squark and slepton mass spectrum carries the information on the breaking pattern of the gauge symmetry. Therefore, the gaugino and the scalar mass spectrum play a complementary role to select among the models of SUSY-GUT experimentally. As an explicit example, it is demonstrated how the scalar mass spectrum distinguishes various $SO(10)$ breaking patterns from each other.

There have been many attempts to extend the original $SU(5)$ GUT. Among them, the GUTs based on E_6 gauge group¹¹⁾ possess some interesting features:

1. The quark and lepton in each generation can be placed in the multiplet of a single representation **27** (or **$\bar{27}$**) of E_6 .
2. E_6 GUTs have no chiral anomaly.¹²⁾

3. There exist candidates for the composite Higgs states constructed from pairs of fermions, which could break E_6 down to G_{SM} dynamically.¹³⁾

Moreover the SUSY E_6 GUTs¹⁴⁾ have been studied with great interest because they are derived from superstring theory (SST) as the effective theories.¹⁵⁾ Thus the study of SUSY E_6 GUT models is an interesting subject in the search of realistic SUSY GUT.

In this paper, we apply the analysis in Ref. 9) to SUSY E_6 GUTs and obtain the specific relations among scalar masses which are useful to distinguish various E_6 breaking patterns from each other.

§ 2. Scalar masses

First we enumerate the chief assumptions adopted in our analysis.

1. The particle content is MSSM one below the nearest symmetry breaking scale M_{SB} above the SUSY breaking scale $m_{\text{SUSY}} = O(1)$ TeV. Here M_{SB} is much higher than m_{SUSY} .
2. The particle content above M_{SB} is not specified and the origin of gauge symmetry breakings above M_{SB} is not mentioned.
3. The matter multiplets of the three generations belong to three 27 (or $\overline{27}$) of E_6 respectively. There exist extra particles, which are called 'exotics', besides MSSM one and right-handed neutrino multiplets ν . Our particle assignments under E_6 maximal subgroups are shown in Table I.
4. As for the assignment of Higgs doublets H_1 and H_2 with hypercharge $-1/2$ and $+1/2$ respectively in MSSM, they belong to a multiplet of another 27 (or $\overline{27}$) of E_6 , and H_1 and H_2 correspond to L^c and L respectively.*)
5. The particles in the first two generations have negligible Yukawa coupling constants.

Next let us discuss the selection of E_6 gauge symmetry breaking pattern,

$$E_6 \xrightarrow{M_U} \dots \longrightarrow G_n \xrightarrow{M_{\text{SB}}} G_{\text{SM}} \quad (1)$$

by using soft masses. The gaugino masses would give no information of gauge symmetry breaking pattern because they are common at the unification scale M_U and hence they satisfy GUT-relation irrespective of chain breakings as discussed in Ref. 9). Therefore we consider only the scalar masses hereafter. When MSSM is established as an effective theory at $O(1)$ TeV and the values of parameters (gauge couplings, gaugino masses the masses of squark, slepton and Higgs at m_{SUSY} and $\beta \equiv \tan^{-1} \langle H_2^0 \rangle / \langle H_1^0 \rangle$) are measured precisely in near future, we can easily know scalar mass spectrum at M_{SB} by using the solutions^{**)} of renormalization group equations (RGEs),¹⁰⁾

*) It is straightforward to generalize our analysis to other assignments.

***) We have neglected the Yukawa coupling contribution. This approximation should be valid for the first- and the second-generation fields. It is straightforward to generalize our results to the third generation and Higgs scalars by considering the effects of Yukawa couplings.

Table I. The particle assignments under maximal subgroups of E_6 . We refer to the chiral multiplets as q for left-handed quark, l left-handed lepton, u right-handed up type quark, d right-handed down type quark, e right-handed charged lepton and ν right-handed neutrino. The 'exotics' are denoted as D, D^c, L, L^c and N^c whose quantum numbers under G_{SM} can be read through Table III. The superscript c represents their charge conjugated states.

Maximal Subgroup	Particle Assignment
$SO(10) \times U(1)_1$	$\mathbf{16} = (d^c, l, q, u^c, e^c, \nu^c), \quad (\text{a})$ $\mathbf{10} = (D, L^c, D^c, L), \quad \mathbf{1} = N^c$
	$\mathbf{16} = (D^c, L, q, u^c, e^c, N^c), \quad (\text{b})$ $\mathbf{10} = (D, L^c, d^c, l), \quad \mathbf{1} = \nu^c$
$SU(6) \times SU(2)_R$	$(\mathbf{15}, \mathbf{1}) = (D, l, q, D^c, N^c),$ $(\bar{\mathbf{6}}, \mathbf{2}) = \begin{pmatrix} d^c & L^c & e^c \\ u^c & L & \nu^c \end{pmatrix}$
$SU(6) \times SU(2)_I$	$(\mathbf{15}, \mathbf{1}) = (D, L, q, d^c, \nu^c),$ $(\bar{\mathbf{6}}, \mathbf{2}) = \begin{pmatrix} D^c & L^c & e^c \\ u^c & l & N^c \end{pmatrix}$
$SU(6) \times SU(2)_J$	$(\mathbf{15}, \mathbf{1}) = (D, L^c, q, u^c, e^c),$ $(\bar{\mathbf{6}}, \mathbf{2}) = \begin{pmatrix} d^c & l & \nu^c \\ D^c & L & N^c \end{pmatrix}$
$SU(6) \times SU(2)_L$	$(\bar{\mathbf{15}}, \mathbf{1}) = (D^c, e^c, \nu^c, d^c, u^c, D, N^c),$ $(\mathbf{6}, \mathbf{2}) = (q, L^c, L, l)$
$SU(3)_C \times SU(3)_L \times SU(3)_R$	$(\mathbf{3}, \mathbf{3}, \mathbf{1}) = (q, D),$ $(\bar{\mathbf{3}}, \mathbf{1}, \bar{\mathbf{3}}) = (d^c, u^c, D^c),$ $(\mathbf{1}, \bar{\mathbf{3}}, \mathbf{3}) = \begin{pmatrix} L^c & L & l \\ e^c & \nu^c & N^c \end{pmatrix}$
$SU(3)_C \times SU(3)_L \times SU(3)_I$	$(\mathbf{3}, \mathbf{3}, \mathbf{1}) = (q, D),$ $(\bar{\mathbf{3}}, \mathbf{1}, \bar{\mathbf{3}}) = (D^c, u^c, d^c),$ $(\mathbf{1}, \bar{\mathbf{3}}, \mathbf{3}) = \begin{pmatrix} L^c & l & L \\ e^c & N^c & \nu^c \end{pmatrix}$
$SU(3)_C \times SU(3)_L \times SU(3)_J$	$(\mathbf{3}, \mathbf{3}, \mathbf{1}) = (q, D),$ $(\bar{\mathbf{3}}, \mathbf{1}, \bar{\mathbf{3}}) = (d^c, D^c, u^c),$ $(\mathbf{1}, \bar{\mathbf{3}}, \mathbf{3}) = \begin{pmatrix} l & L & L^c \\ \nu^c & N^c & e^c \end{pmatrix}$

$$m_a(\mu)^2 = m_a(\mu_0)^2 - \sum_i \frac{2}{b_i} C_2(R_i^a) (M_i(\mu)^2 - M_i(\mu_0)^2) + \frac{3}{5b_1} Y_a (S(\mu) - S(\mu_0)), \quad (2)$$

$$S(\mu) = \frac{\alpha_i(\mu)}{\alpha_i(\mu_0)} S(\mu_0), \quad (3)$$

$$S \equiv \sum_a Y_a n_a m_a^2. \quad (4)$$

Here a represents the species of the scalar, i the gauge group, b_i the coefficient of the β -function, $C_2(R_i^a)$ the second order Casimir invariant of the gauge group i for the species a , Y_a the hypercharge and n_a the multiplicity of the species a .

Now the question which we should answer is how the scalar mass spectrum at M_{SB} reflects the pattern of gauge symmetry breaking. The mass formulae at M_{SB} are given by^{*)}

$$m_a(M_{\text{SB}})^2 = m_{R(a)}^2 + \sum_I g_I^2 Q_I(\phi_a) D_I. \quad (5)$$

Here the $m_{R(a)}$'s represent the soft mass parameters of the scalar fields ϕ_a included in $R(a)$ representation of G_n and show a kind of 'unification' in the unified theory based on the gauge group G_n . (Note that the assumption that the soft mass parameters have a universal structure is not imposed on. It is only assumed that the $m_{R(a)}$'s respect the gauge symmetries.) The second term on the right-hand side of Eq. (5) represents the D -term contributions to scalar masses on the symmetry breaking which violate the 'unification'.^{16),17)} The g_I 's and $Q_I(\phi_a)$'s are the gauge coupling constants and the diagonal charges related to the broken gauge symmetry respectively, and the D_I 's are the quantities which depend on the heavy field condensations. One can show that the sizable D -term contributions generally exist once the soft SUSY breaking terms in the scalar potential are non-universal.^{**,18)} (Although there also exist, in general, F -term contributions to scalar masses, we assume that they are negligible. This assumption is justified for the unified theory with a certain type of non-universal soft SUSY breaking terms when Yukawa couplings with heavy fields are negligible and there exist no heavy fields with the same quantum number as usual matter fields.) The D -term contributions arise when the rank of the group is reduced due to the gauge symmetry breakings. When E_6 breaks down to G_{SM} , the rank is reduced by two and the D -term contributions are expressed by two parameters.

§ 3. Scalar mass relations

In this section, we use the particle assignments given in Table I and get specific relations among scalar masses in various E_6 breaking patterns by eliminating unknown parameters $m_{R(a)}$ and D_I .

In the "direct" breaking, the scalar masses satisfy

$$m_a^2 = m_{R(a)}^2 + g^2 Q_1(\phi_a) D_1 + g^2 Q_2(\phi_a) D_2 \quad (6)$$

at the E_6 unification scale M_U . Here g is E_6 gauge coupling constant and $Q_{1(2)}$ are broken $U(1)$ charges. For a simplicity, we used the particle assignment and quantum numbers given in Table II. Since M_U can be determined from the RGEs of gauge coupling constants as well as the gaugino masses, the only free parameters are $m_{R(a)}$, D_1 and D_2 . Having seven kinds of measurable scalar masses,^{***)} we can eliminate

*) We assume that the threshold effects and "gravitational" corrections are negligible.

***) It is known that the non-universal soft SUSY breaking parameters emerge in the effective theory derived from superstring theory.¹⁹⁾ Even if they are universal at Planck scale M_{Pl} as in minimal supergravity or SUSY breaking by dilaton F -term, the radiative corrections between M_{Pl} and M_{SB} generally induce non-universality.

****) It is probably impossible to measure the SUSY-breaking part of masses for 'exotics' and ν as far as they have SUSY-invariant masses of the intermediate scale.

Table II. The particle assignment (case (a)) and quantum numbers under the series of E_6 subgroups $E_6 \supset SO(10) \times U(1)_1 \supset SU(5) \times U(1)_2 \times U(1)_1$. We take the following normalization for the $U(1)$ charges Q_1 and Q_2 , $\sum_{27} Q_1^2 = \sum_{27} Q_2^2 = 3$.

E_6	$SO(10)$	$SU(5)$	$2\sqrt{10}Q_2$	$2\sqrt{6}Q_1$	Species	
27	16	$\bar{5}$	3	1	d^c	
					l	
					q	
		10	-1		u^c	
					e^c	
		1	-5		ν^c	
	10		5	2	-2	D
						L^c
		$\bar{5}$	-2			D^c
						L
	1	1	0	4	N^c	

free parameters and have the following relations among scalar masses,

$$m_{\tilde{d}}^2 = m_{\tilde{l}}^2, \tag{7}$$

$$m_{\tilde{u}}^2 = m_{\tilde{q}}^2 = m_{\tilde{e}}^2, \tag{8}$$

$$m_{\tilde{d}}^2 - m_{\tilde{u}}^2 = m_2^2 - m_1^2 \tag{9}$$

at the E_6 unification scale M_U . Here the tilde represents the scalar component of supermultiplet. m_1^2 and m_2^2 stand for the soft SUSY breaking masses of the Higgs bosons H_1 and H_2 respectively. The same result is obtained by the use of other particle assignments in Table I.

Next we will study chain breakings. There are many breaking patterns of E_6 down to G_{SM} , which cannot be selected unless the dynamics of symmetry breakings is clarified. We pay attention to the final stage of symmetry breaking,

$$G_n \xrightarrow{M_{SB}} G_{SM}, \tag{10}$$

where G_n is some subgroup of E_6 . Table I shows our particle assignments under the maximal subgroups of E_6 , and the quantum numbers under arbitrary subgroups are easily known. The ‘exotics’ D^c , L and N^c have the same quantum number of d^c , l and ν^c respectively under G_{SM} . So there is an arbitrariness in the particle assignment. For example, d^c and l might be some mixtures with the states in **16** and **10** of $SO(10)$ subgroup. In this paper we consider only a particular case where both d^c and l lie in the same $\bar{5}$ of $SU(5)$ subgroup. (The assignment that d^c and l lie in **16** of $SO(10)$ is case (a) and the assignment that they lie in **10** of $SO(10)$ is case (b).) The pairs (d^c, u^c) , (L^c, L) and (e^c, ν^c) make up the doublets of $SU(2)_R$, and (d^c, u^c, D^c) , (L^c, L, l) and (e^c, ν^c, N^c) make up the triplets of $SU(3)_R$. We define the gauge group $SU(2)_I$, $SU(3)_I$ and $SU(2)_J$ as follows. The multiplets of $SU(2)_I$ and $SU(3)_I$ are obtained

Table III. The particle assignment and quantum numbers under the series of E_6 subgroups $E_6 \supset SU(3)_c \times SU(3)_L \times SU(3)_f \supset SU(3)_c \times SU(2)_L \times U(1)_L \times SU(2)_f \times U(1)_f$. We take the following normalization for the $U(1)$ charges Q_L and Q_f , $\sum_{27} Q_L^2 = \sum_{27} Q_f^2 = 3$. The hypercharge is defined as $Y \equiv (1/\sqrt{3})Q_L - (2/\sqrt{3})Q_f$.

E_6	$SU(3)_c$	$SU(3)_L$	$SU(3)_f$	$SU(2)_L$	$2\sqrt{3}Q_L$	$SU(2)_f$	$2\sqrt{3}Q_f$	$6Y$	Species
27	3	3	1	2	1	1	0	1	q
				1	-2			-2	D
	$\bar{3}$	1	$\bar{3}$	1	0	2	-1	2	(d^c, D^c)
						1	2	-4	u^c
	1	$\bar{3}$	3	2	-1	2	1	-3	(l, L)
						1	-2	3	L^c
				1	2	2	1	0	(ν^c, N^c)
						1	-2	6	e^c

from the corresponding ones of $SU(2)_R$ and $SU(3)_R$ by the following exchange of particle assignment,

$$\begin{aligned}
 d^c &\leftrightarrow D^c, \\
 l &\leftrightarrow L, \\
 \nu^c &\leftrightarrow N^c.
 \end{aligned}
 \tag{11}$$

The above exchange symmetry is generated by $SU(2)_f$, that is, the pairs (d^c, D^c) , (l, L) and (ν^c, N^c) make up the doublets of $SU(2)_f$.

Here we pick up one example where $G_n = SU(3)_c \times SU(3)_L \times SU(3)_f$. The particle assignment and quantum numbers are shown in Table III. The scalar masses satisfy

$$m_{\bar{q}}^2 = m_{(3,3,1)}^2 + 2g_{3L}^2 D, \tag{12}$$

$$m_{\bar{d}}^2 = m_{(\bar{3},1,\bar{3})}^2 - g_{3f}^2 D + \frac{1}{2}g_{3f}^2 D_f, \tag{13}$$

$$m_{\bar{u}}^2 = m_{(\bar{3},1,\bar{3})}^2 + 2g_{3f}^2 D, \tag{14}$$

$$m_{\bar{l}}^2 = m_{(1,\bar{3},3)}^2 + (-2g_{3L}^2 + g_{3f}^2)D + \frac{1}{2}g_{3f}^2 D_f, \tag{15}$$

$$m_{\bar{e}}^2 = m_{(1,\bar{3},3)}^2 + (4g_{3L}^2 - 2g_{3f}^2)D, \tag{16}$$

$$m_{1^c}^2 = m_{(1,\bar{3},3)(H)}^2 + (-2g_{3L}^2 + g_{3f}^2)D - \frac{1}{2}g_{3f}^2 D_f, \tag{17}$$

$$m_{2^c}^2 = m_{(1,\bar{3},3)(H)}^2 + (-2g_{3L}^2 - 2g_{3f}^2)D \tag{18}$$

at M_{SB} . Recall the gauge couplings g_{3L} and g_{3f} of $SU(3)_L$ and $SU(3)_f$ can be computed from the weak-scale coupling constants as a function of M_{SB} alone. After eliminating free parameters, we obtain two relations,

Table IV. The list of scalar mass relations. Here g_{NX} is the gauge coupling constant of $SU(N)_X$. The gauge group $SU(5)_F \times U(1)_2$ corresponds to that of flipped $SU(5)$ model.²⁰⁾ $SU(5)_{F'} \times U(1)_3 \times SU(2)_R$ and $SU(5) \times U(1)_{3''} \times SU(2)_I$ are one of subgroups of $SU(6) \times SU(2)_R$ and $SU(6) \times SU(2)_I$, respectively. $SU(5)_{F''} \times U(1)_{3'''} \times SU(2)_L$ is one of subgroups of $SU(6) \times SU(2)_L$ and the two kinds of particle assignments exist corresponding to the choice of $SU(2)_R$ or $SU(2)_I$ as the subgroup of $SU(5)_{F''}$. The asterisk (*) represents the scalar mass relation derived under the assumption of ‘flavor’ universality at M_{SB} .

G_n	Scalar Masses
E_6	$m_{\bar{d}}^2 = m_{\bar{l}}^2,$ $m_{\bar{u}}^2 = m_{\bar{q}}^2 = m_{\bar{e}}^2,$ $m_2^2 - m_1^2 = m_{\bar{d}}^2 - m_{\bar{u}}^2$
$SO(10) \times U(1)_1$	$m_{\bar{d}}^2 = m_{\bar{l}}^2,$ $m_{\bar{u}}^2 = m_{\bar{q}}^2 = m_{\bar{e}}^2,$ $m_2^2 - m_1^2 = m_{\bar{d}}^2 - m_{\bar{u}}^2$ (a)
	$m_{\bar{d}}^2 = m_{\bar{l}}^2,$ $m_{\bar{u}}^2 = m_{\bar{q}}^2 = m_{\bar{e}}^2,$ $m_2^2 - m_1^2 = m_{\bar{d}}^2 - m_{\bar{u}}^2$ (*) (b)
$SU(5) \times U(1)_2 \times U(1)_1$	$m_{\bar{d}}^2 = m_{\bar{l}}^2,$ $m_{\bar{u}}^2 = m_{\bar{q}}^2 = m_{\bar{e}}^2$
$SU(5)_F \times U(1)_2 \times U(1)_1$	$m_{\bar{q}}^2 - m_{\bar{d}}^2 = m_{\bar{u}}^2 - m_{\bar{l}}^2$
$SU(4) \times SU(2)_L \times SU(2)_R \times U(1)_1$	$m_{\bar{l}}^2 - m_{\bar{q}}^2 = m_{\bar{d}}^2 - m_{\bar{e}}^2,$ $g_{2R}^2(m_{\bar{l}}^2 - m_{\bar{q}}^2) = g_4^2(m_{\bar{d}}^2 - m_{\bar{u}}^2),$ $m_2^2 - m_1^2 = m_{\bar{d}}^2 - m_{\bar{u}}^2$
$SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times U(1)_1$	$m_2^2 - m_1^2 = m_{\bar{d}}^2 - m_{\bar{u}}^2$
$SU(4) \times SU(2)_L \times SU(2)_I \times U(1)_1$	$g_{2I}^2(m_{\bar{q}}^2 - m_1^2) = g_4^2(m_{\bar{l}}^2 - m_2^2),$ (*) $g_{2I}^2(m_{\bar{e}}^2 - m_{\bar{u}}^2) = (g_4^2 - g_{2I}^2)(m_{\bar{l}}^2 - m_1^2)$ (*)
$SU(6) \times SU(2)_R$	$m_{\bar{l}}^2 - m_{\bar{q}}^2 = m_{\bar{d}}^2 - m_{\bar{e}}^2,$ $g_{2R}^2(m_{\bar{l}}^2 - m_{\bar{q}}^2) = g_6^2(m_{\bar{d}}^2 - m_{\bar{u}}^2),$ $m_2^2 - m_1^2 = m_{\bar{d}}^2 - m_{\bar{u}}^2$
$SU(5)_{F'} \times U(1)_3 \times SU(2)_R$	$m_2^2 - m_1^2 = m_{\bar{d}}^2 - m_{\bar{u}}^2$
$SU(6) \times SU(2)_I$	$m_{\bar{q}}^2 - m_{\bar{d}}^2 = m_{\bar{u}}^2 - m_{\bar{l}}^2$
$SU(5)_F \times U(1)_{3'} \times SU(2)_I$	$m_{\bar{q}}^2 - m_{\bar{d}}^2 = m_{\bar{u}}^2 - m_{\bar{l}}^2$
$SU(6) \times SU(2)_I$	$m_{\bar{d}}^2 = m_{\bar{l}}^2,$ $m_{\bar{u}}^2 = m_{\bar{q}}^2 = m_{\bar{e}}^2$
$SU(5) \times U(1)_{3''} \times SU(2)_I$	$m_{\bar{d}}^2 = m_{\bar{l}}^2,$ $m_{\bar{u}}^2 = m_{\bar{q}}^2 = m_{\bar{e}}^2$
$SU(6) \times SU(2)_L$	$m_2^2 - m_1^2 = m_{\bar{d}}^2 - m_{\bar{u}}^2$ $= m_{\bar{l}}^2 - m_{\bar{q}}^2,$ $m_{\bar{u}}^2 = m_{\bar{e}}^2$
$SU(5)_{F''} \times U(1)_{3'''} \times SU(2)_L$	$m_2^2 - m_1^2 = m_{\bar{d}}^2 - m_{\bar{u}}^2,$ ($SU(2)_R$) or $m_{\bar{d}}^2 - m_{\bar{e}}^2 = m_{\bar{l}}^2 - m_{\bar{q}}^2,$ ($SU(2)_I$)
$SU(3)_C \times SU(3)_L \times SU(3)_R$	$m_2^2 - m_1^2 = m_{\bar{d}}^2 - m_{\bar{u}}^2,$ $g_{3R}^2(m_2^2 - m_{\bar{e}}^2) = g_{3L}^2(m_{\bar{d}}^2 - m_{\bar{u}}^2 + m_2^2 - m_{\bar{l}}^2)$ (*)
$SU(3)_C \times SU(3)_L \times SU(2)_R \times U(1)_R$	$m_2^2 - m_1^2 = m_{\bar{d}}^2 - m_{\bar{u}}^2$
$SU(3)_C \times SU(2)_L \times U(1)_L \times SU(3)_R$	$m_2^2 - m_1^2 = m_{\bar{d}}^2 - m_{\bar{u}}^2$

Table IV. continuation

$SU(3)_c \times SU(2)_L \times U(1)_L \times SU(2)_R \times U(1)_R$	$m_2^2 - m_1^2 = m_{\tilde{d}}^2 - m_{\tilde{u}}^2$
$SU(3)_c \times SU(3)_L \times SU(3)_I$	$m_2^2 - m_1^2 = m_{\tilde{d}}^2 - m_{\tilde{u}}^2$, $g_{3I}^2(m_2^2 - m_{\tilde{e}}^2) = g_{3L}^2(m_{\tilde{d}}^2 - m_{\tilde{u}}^2 + m_2^2 - m_{\tilde{t}}^2)$ (*)
$SU(3)_c \times SU(2)_L \times U(1)_L \times SU(3)_I$	$m_2^2 - m_1^2 = m_{\tilde{d}}^2 - m_{\tilde{u}}^2$
$SU(3)_c \times SU(3)_L \times SU(3)_I$	$m_2^2 - m_1^2 = m_{\tilde{d}}^2 - m_{\tilde{u}}^2$, $g_{3I}^2(m_2^2 - m_{\tilde{e}}^2) = g_{3L}^2(m_{\tilde{d}}^2 - m_{\tilde{u}}^2 + m_2^2 - m_{\tilde{t}}^2)$ (*)
$SU(3)_c \times SU(2)_L \times U(1)_L \times SU(3)_I$	$m_2^2 - m_1^2 = m_{\tilde{d}}^2 - m_{\tilde{u}}^2$

$$m_{\tilde{d}}^2 - m_{\tilde{u}}^2 = m_2^2 - m_1^2, \quad (19)$$

$$g_{3I}^2(m_2^2 - m_{\tilde{e}}^2) = g_{3L}^2(m_{\tilde{d}}^2 - m_{\tilde{u}}^2 + m_2^2 - m_{\tilde{t}}^2) \quad (20)$$

at M_{SB} . Here Eq. (20) is derived in the case that the $m_{R(\alpha)}$'s take the same value for the same type of representation, that is, $m_{(1, \bar{3}, 3)(H)} = m_{(1, \bar{3}, 3)}$. (We shall call this feature 'flavor' universality. It can give the sufficient suppression of flavor changing neutral currents.²¹⁾ When M_{SB} is unknown, one of them should be used to determine M_{SB} .

In the same way, we can obtain specific relations among scalar masses at M_{SB} in other breaking patterns. The results are summarized in Table IV. The same results hold for its $U(1)$ subgroup in place of $SU(2)_{R(I,J)}$. We notice that the common relations appear in the wide class of E_6 breakings. This fact originates from the G_n gauge symmetry and the matter assignment. Here we explain it by taking two examples.

(1) The relations $m_{\tilde{d}}^2 = m_{\tilde{t}}^2$ and $m_{\tilde{u}}^2 = m_{\tilde{q}}^2 = m_{\tilde{e}}^2$ are obtained for $G_n = E_6$, $SO(10) \times U(1)_1$, $SU(5) \times U(1)_2 \times U(1)_1$, $SU(6) \times SU(2)_I$ and $SU(5) \times U(1)_{3''} \times SU(2)_I$. This is due to the fact that the above groups include $SU(5)$ as a subgroup, and (\tilde{d}, \tilde{t}) and $(\tilde{u}, \tilde{q}, \tilde{e})$ belong to $\bar{5}$ and $\mathbf{10}$ of the $SU(5)$ respectively.

(2) The relation $m_{\tilde{d}}^2 - m_{\tilde{u}}^2 = m_2^2 - m_1^2$ appears for the models with G_n which includes $SU(2)_R$ or $SU(3)_{I(U)}$ as a subgroup. It results from the fact that both (\tilde{u}, \tilde{d}) and (H_1, H_2) belong to $\mathbf{2}$ of $SU(2)_R$ or they lie in $\mathbf{3}$ of $SU(3)_{I(U)}$.

§ 4. Summary

We have obtained specific scalar mass relations in various E_6 breaking patterns. It is important that the specific relations hold without specifying the particle content above the symmetry breaking scale. We can select the final stage of some chain breakings by checking scalar mass relations. But it is not easy to carry out the complete selection since the same relations hold in the wide class of SUSY E_6 GUTs. The other powerful information is needed to specify the pattern of symmetry breakings.

Our mass relations would not be applied when some of our assumptions are relaxed. For example, if extra large Yukawa couplings exist, the contributions from F -term become important and the 'flavor' universality may be destroyed by renormal-

ization effects. When the symmetry breaking scale is near the Planck scale, the additional mass splittings can be induced by the non-renormalizable interactions.

In conclusion, it is expected that the measurements of scalar masses give some useful information on SUSY GUTs.

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References

- 1) H. Georgi and S. L. Glashow, Phys. Rev. Lett. **32** (1974), 438.
- 2) E. Gildener, Phys. Rev. **D14** (1976), 1667.
- 3) M. J. G. Veltman, Acta Phys. Pol. **B12** (1981), 437.
L. Maiani, *Gif-sur-Yvette Summer School on Particle Physics, 11th, Gif-sur-Yvette, France, 1979* (Inst. Nat. Phys. Nucl. Phys. Particules, Paris, 1979).
S. Dimopoulos and S. Raby, Nucl. Phys. **B192** (1981), 353.
E. Witten, Nucl. Phys. **B188** (1981), 513.
M. Dine, W. Fischler and M. Srednicki, Nucl. Phys. **B189** (1981), 575.
- 4) LEP Collaborations, Phys. Lett. **B276** (1992), 247.
- 5) S. Dimopoulos and H. Georgi, Nucl. Phys. **B193** (1981), 150.
N. Sakai, Z. Phys. **C11** (1981), 153.
- 6) P. Langacker and M.-X. Luo, Phys. Rev. **D44** (1991), 817.
U. Amaldi, W. de Boer and H. Fürstenau, Phys. Lett. **B260** (1991), 447.
W. J. Marciano, Brookhaven Preprint, BNL-45999, April 1991.
- 7) J. Hisano, H. Murayama and T. Yanagida, Phys. Rev. Lett. **69** (1992), 1014; Nucl. Phys. **B402** (1993), 46.
- 8) N. G. Deshpande, E. Keith and T. G. Rizzo, Phys. Rev. Lett. **70** (1993), 3189.
- 9) Y. Kawamura, H. Murayama and M. Yamaguchi, Phys. Lett. **B324** (1994), 52.
- 10) K. Inoue, A. Kakuto, H. Komatsu and S. Takeshita, Prog. Theor. Phys. **68** (1982), 927; **71** (1984), 413.
- 11) F. Gürsey, P. Ramond and P. Sikivie, Phys. Lett. **B60** (1976), 177.
F. Gürsey and M. Serdaroglu, Nuovo Cim. Lett. **21** (1978), 28; Nuovo Cim. **A65** (1981), 337.
- 12) S. Okubo, Phys. Rev. **D16** (1977), 3528.
- 13) R. Barbieri and D. V. Nanopoulos, Phys. Lett. **B95** (1980), 43.
R. Barbieri, D. V. Nanopoulos and A. Masiero, Phys. Lett. **B104** (1981), 194.
- 14) J. L. Hewett and T. G. Rizzo, Phys. Rep. **183** (1989), 193 and references therein.
- 15) P. Candelas, G. Horowitz, A. Strominger and E. Witten, Nucl. Phys. **B258** (1985), 46.
- 16) M. Drees, Phys. Lett. **B181** (1986), 279.
J. S. Hagelin and S. Kelley, Nucl. Phys. **B342** (1990), 95.
- 17) A. E. Faraggi, J. S. Hagelin, S. Kelley and D. V. Nanopoulos, Phys. Rev. **D45** (1992), 3272.
- 18) Y. Kawamura, H. Murayama and M. Yamaguchi, in preparation.
- 19) L. E. Ibáñez and D. Lüst, Nucl. Phys. **B382** (1992), 305.
B. de Carlos, J. A. Casas and C. Muñoz, Phys. Lett. **B299** (1993), 234.
V. S. Kaplunovsky and J. Louis, Phys. Lett. **B306** (1993), 269.
- 20) S. M. Barr, Phys. Lett. **B112** (1982), 219.
J. P. Deredinger, J. E. Kim and D. V. Nanopoulos, Phys. Lett. **B139** (1984), 170.
I. Antoniadis, J. Ellis, J. Hagelin and D. V. Nanopoulos, Phys. Lett. **B194** (1987), 231.
- 21) J. Ellis and D. V. Nanopoulos, Phys. Lett. **B110** (1982), 44.
R. Barbieri and R. Gatto, Phys. Lett. **B110** (1982), 211.
T. Inami and C. S. Lim, Nucl. Phys. **B207** (1982), 533.