Prog. Theor. Exp. Phys. **2013**, 113B04 (13 pages)
DOI: 10.1093/ptep/ptt098

Naturalness, conformal symmetry, and duality

Yoshiharu Kawamura[1,*]

[1]*Department of Physics, Shinshu University, Matsumoto 390-8621, Japan*
E-mail: haru@azusa.shinshu-u.ac.jp

Received August 29, 2013; Accepted October 15, 2013; Published November 26, 2013

We reconsider naturalness from the viewpoint of effective field theories, motivated by the alternative scenario that the standard model holds up to a high-energy scale such as the Planck scale. We propose a calculation scheme of radiative corrections utilizing a hidden duality, in the expectation that the unnaturalness for scalar masses might be an artifact in the effective theory and it could be improved if features of an ultimate theory are taken in.

Subject Index B40, B53

1. Introduction

It might be a good time to reconsider various concepts concerning the Higgs boson mass $m_{\rm h}$ and the physics beyond the standard model (SM), on the basis of recent experimental results at the Large Hadron Collider (LHC).

The discovery of the Higgs boson at LHC with the observed value $m_{\rm h} \doteqdot 126\,{\rm GeV}$ [1,2] is quite suggestive. It rekindles the question whether $m_{\rm h}$ is a natural parameter or not [3–5]. Furthermore, evidence from new physics such as supersymmetry (SUSY), compositeness, and extra dimensions have not yet been discovered, and this fact could be the turning point of particle physics, because the gauge hierarchy problem [6,7] would be revisited.

Therefore, it is interesting to reexamine the validity of concepts relating $m_{\rm h}$ and the physics beyond the SM, from various aspects. In this paper, we reconsider the naturalness of $m_{\rm h}$ from the viewpoint of effective field theories, including the SM. Our study is motivated by the alternative scenario that the SM (modified with massive neutrinos) holds up to a high-energy scale such as the Planck scale $M_{\rm Pl}$ [8,9]. We expect that the unnaturalness for scalar masses might be an artifact in the effective theory, and it could be improved if features of an ultimate theory are brought in and the ingredients of the effective theory are enriched. We reanalyze radiative corrections on scalar masses using the ϕ^4 theory, and propose a calculation scheme utilizing a hidden duality.

The outline of this paper is as follows. In the next section, we review naturalness and its relevant symmetries. We give a suggestion for the subtraction of quadratic divergences by presenting a calculation scheme, considering a duality relating integration variables, in Sect. 3. In the last section, we give conclusions and discussions.

2. Naturalness and conformal symmetry

Let us first recall the concept of naturalness. According to ’t Hooft [4], naturalness is based on the dogma that “at any energy scale μ, a physical parameter or set of physical parameters $a_i(\mu)$ is allowed to be very small only if the replacement $a_i(\mu) = 0$ would increase the symmetry of the system." We refer to this type of parameter as a natural parameter.

We discuss the naturalness of fermion masses and scalar masses from the viewpoint of low-energy effective theories such as quantum electrodynamics (QED), the ϕ^4 theory, and the SM.

2.1. *Naturalness of fermion masses*

The electron mass $m_{\rm e}$ is listed as a natural parameter in QED. When we set $m_{\rm e} = 0$, the classical global chiral symmetry appears. Here, the chiral symmetry is the invariance of the action integral under the different phase changes for Weyl fermions $\psi_{\rm L}$ and $\psi_{\rm R}$ with the different chiralities, i.e., $\psi_{\rm L} \to e^{i\theta_{\rm L}}\psi_{\rm L}$ and $\psi_{\rm R} \to e^{i\theta_{\rm R}}\psi_{\rm R}$, $(\theta_{\rm L} \neq \theta_{\rm R})$. Note that the chiral symmetry is broken down, even in the massless case $m_{\rm e} = 0$, such that

$$\langle \partial_\mu(\psi_{\rm L}^\dagger \overline{\sigma}^\mu \psi_{\rm L})\rangle = \frac{e^2}{32\pi^2}\varepsilon_{\mu\nu\alpha\beta}F^{\mu\nu}F^{\alpha\beta}, \quad \langle \partial_\mu(\psi_{\rm R}^\dagger \sigma^\mu \psi_{\rm R})\rangle = -\frac{e^2}{32\pi^2}\varepsilon_{\mu\nu\alpha\beta}F^{\mu\nu}F^{\alpha\beta}, \tag{1}$$

in the presence of the axial $U(1)$ anomaly. In the massive case, the $U(1)$ vector current defined as $j_{\rm V}^\mu = \overline{\psi}\gamma^\mu\psi = \psi_{\rm L}^\dagger\overline{\sigma}^\mu\psi_{\rm L} + \psi_{\rm R}^\dagger\sigma^\mu\psi_{\rm R}$ is conserved, and the $U(1)$ axial vector current defined as $j_{\rm A}^\mu = \overline{\psi}\gamma_5\gamma^\mu\psi = \psi_{\rm L}^\dagger\overline{\sigma}^\mu\psi_{\rm L} - \psi_{\rm R}^\dagger\sigma^\mu\psi_{\rm R}$ is anomalous such that

$$\langle \partial_\mu j_{\rm A}^\mu\rangle = 2i(m_{\rm e} + \delta m_{\rm e})(\psi_{\rm L}^\dagger\psi_{\rm R} - \psi_{\rm R}^\dagger\psi_{\rm L}) + \frac{e^2}{16\pi^2}\varepsilon_{\mu\nu\alpha\beta}F^{\mu\nu}F^{\alpha\beta}, \tag{2}$$

where $\delta m_{\rm e}$ represents the radiative corrections on the tree-level mass $m_{\rm e}$.

$\delta m_{\rm e}$ at the one-loop level is given by

$$\delta m_{\rm e} = \frac{3\alpha}{4\pi}m_{\rm e}\left(\ln\frac{\Lambda^2}{m_{\rm e}^2} + \frac{1}{2}\right), \tag{3}$$

where $\alpha \equiv e^2/(4\pi)$ and Λ is a cutoff scale. In the limit of $m_{\rm e} \to 0$, $\delta m_{\rm e}$ also vanishes. This feature holds for higher-order corrections, and the chiral symmetry is not broken down perturbatively (although it is broken down anomalously without threatening the consistency of the theory). Hence, chiral symmetry is regarded as a powerful concept for controlling quantum corrections.

A classical conformal symmetry also appears in the limit of $m_{\rm e} \to 0$, and is broken down in the presence of anomaly. For instance, the scale invariance is broken down as

$$\langle T^\mu{}_\mu\rangle = (m_{\rm e} + \delta m_{\rm e})(\psi_{\rm L}^\dagger\psi_{\rm R} + \psi_{\rm R}^\dagger\psi_{\rm L}) + \frac{\beta_\alpha}{\alpha}F^{\mu\nu}F_{\mu\nu}, \tag{4}$$

where β_α is the β function for α. In QED, the conformal symmetry seems to play the same role as chiral symmetry does.

In the SM, chiral symmetry has a superior quality to conformal symmetry because chiral symmetry such as $SU(2)_{\rm L} \times U(1)_{\rm Y}$ becomes local and is not broken down either perturbatively or anomalously, whereas the conformal symmetry is broken down not only with the negative mass squared of the Higgs doublet explicitly, but also in the presence of anomalous terms. The chiral gauge symmetry is broken down spontaneously with the vacuum expectation value of the Higgs boson $v = 246$ GeV, and fermions ψ_f acquire masses $m_f = y_f v/\sqrt{2}$ via the Higgs mechanism, where y_f are Yukawa coupling constants. On the other hand, the global chiral symmetry enhances in the limit of $y_f \to 0$. In this way, the smallness of fermion masses, compared with a high-energy scale $M_{\rm U}$ such as the gravitational scale $M \equiv M_{\rm Pl}/\sqrt{8\pi} = 2.4 \times 10^{18}$ GeV, stems from the smallness of v compared with $M_{\rm U}$. Furthermore, the smallness of fermion masses, except for the top quark mass, compared with the weak gauge boson mass $M_W = gv/2$, originates from the smallness of y_f compared with the $SU(2)_L$ gauge coupling constant g. Hence, it is considered that chiral symmetry is responsible for the smallness of SM fermion masses.

2.2. Naturalness of scalar masses

Next, we study the relation between the relevant symmetry of scalar mass m_ϕ and radiative corrections on m_ϕ, in order to gain information on the naturalness of m_h. Unless a theory has dimensional parameters other than m_ϕ, the classical conformal symmetry appears in the limit of $m_\phi \to 0$. In the same way as for QED, the scale invariance is broken down as

$$\langle T^\mu{}_\mu \rangle = (m_\phi^2 + \delta m_\phi^2)\phi^2 + \sum_k \beta_k \mathcal{O}_k, \tag{5}$$

where δm_ϕ^2 represents the radiative corrections on m_ϕ^2, β_k are β functions for coupling constants a_k, and $\mathcal{O}_k$ are operators with mass dimension 4.

In the ϕ^4 theory, δm_ϕ^2 at the one-loop level is most commonly written by

$$\delta m_\phi^2 = \frac{\lambda_\phi}{32\pi^2}\left(\Lambda^2 - m_\phi^2 \ln \frac{\Lambda^2}{m_\phi^2}\right) + \cdots, \tag{6}$$

where λ_ϕ is the quartic self-coupling constant of ϕ, and the ellipsis stands for Λ independent terms.

For example, the unregularized one is given by

$$\delta m_\phi^2 = \frac{\lambda_\phi}{2}\int_{-\infty}^{\infty} \frac{d^4 p}{(2\pi)^4}\frac{1}{p^2 + m_\phi^2} = \frac{\lambda_\phi}{32\pi^2}\left(\int_0^\infty dp^2 + \int_0^\infty \frac{-m_\phi^2}{p^2 + m_\phi^2} dp^2\right), \tag{7}$$

where we rotate to Euclidean space and carry out the integration for the angles of momentum space. The δm_ϕ^2 of (6) is obtained by replacing ∞ by $\Lambda^2 - m_\phi^2$ in the final expression in (7).

As seen from (6), it is widely thought that m_ϕ is not a natural parameter, because δm_ϕ^2 does not vanish in the limit of $m_\phi^2 \to 0$ due to the appearance of the quadratic term of Λ.

However, if the quadratic term is subtracted or absent for some reason, m_ϕ can be a natural parameter. Bardeen reexamined naturalness in the SM and pointed out that the classical scale invariance implies the naturalness of the Higgs boson mass m_h [10]. The reasoning is illustrated as follows. In the SM, scale invariance is broken as

$$\langle T^\mu{}_\mu \rangle = (m_h^2 + \delta m_h^2)|H|^2 + \sum_k \beta_k \mathcal{O}_k, \tag{8}$$

where δm_h^2 represents the radiative corrections on m_h^2. The anomalous terms are quantum corrections induced from loop contributions due to particles with masses smaller than the reference energy scale. It is quite unlikely that the radiative corrections on masses affect them. Hence, the anomalous divergence of the scale current should remain in the limit of $m_h \to 0$, and δm_h^2 should be proportional not to Λ^2 but to m_h^2. In other words, the classical symmetries should be restored in the limits of $m_h \to 0$ and $\beta_k \to 0$.

In effective field theories, ambiguities can exist in the regularization procedure, and such ambiguities, in most cases, are resolved by considering symmetries realized manifestly at the low-energy scale [11]. If we had a theory with a high calculability and predictability, regularization-dependent quantities would be absent. In this regard, quantities depending on the regularization method should be subtracted or eliminated, unless the subtraction induces any physical effects.

The dimensional regularization is known as a regularization procedure that does not induce quadratic divergences for scalar masses. Using it, δm_ϕ^2 at the one-loop level is given by

$$\delta m_\phi^2 = \frac{\lambda_\phi}{32\pi^2} m_\phi^2 \left(-\frac{2}{\epsilon} + \gamma - 1\right) + \cdots, \tag{9}$$

where $\epsilon = 4 - D$ (D is the dimension of space time) and $\gamma = 0.577\cdots$ is the Euler constant. The δm_ϕ^2 becomes infinite in the limit of $\epsilon \to 0$, i.e., $D \to 4$. The $2/\epsilon$ corresponds to $\ln(\Lambda^2/m_\phi^2)$, and then the quadratic divergence is absent.

Fujikawa gave a scheme on the subtractive renormalization of the quadratic divergences of scalar mass [12]. In the case where the subtraction of quadratic divergences induces no physical effects on the low-energy theory, such a scheme is useful for treating physical quantities, including scalar masses.

Aoki and Iso studied the quadratic divergences of scalar mass from the viewpoint of the Wilsonian renormalization group, and found that they can be absorbed into a position of the critical surface, which means their subtraction [13].

Extensions of the SM have been proposed by adopting the classical conformal invariance as a guiding principle [14–18].[1]

2.3. *Naturalness of Higgs boson mass*

Before we study the subtraction of quadratic divergences from the viewpoint of hidden symmetry, we discuss the naturalness of the Higgs boson mass. In the SM, the radiative corrections on the Higgs mass squared m_h^2 at the one-loop level are given by

$$\delta m_\mathrm{h}^2 = c_\mathrm{h}\Lambda^2 + c'_\mathrm{h} m_\mathrm{h}^2 \ln\frac{\Lambda^2}{m_\mathrm{h}^2} + \cdots, \tag{10}$$

where c_h and c'_h are functions of the SM parameters such that

$$c_\mathrm{h} = \frac{1}{16\pi^2}\left(6\lambda + \frac{9}{4}g^2 + \frac{3}{4}g'^2 - 6y_\mathrm{t}^2\right), \quad c'_\mathrm{h} = \frac{1}{16\pi^2}\left(6\lambda - \frac{9}{4}g^2 - \frac{3}{4}g'^2 + 3y_\mathrm{t}^2\right). \tag{11}$$

Here, λ is the quartic self-coupling constant of the Higgs boson, g' is the $U(1)_\mathrm{Y}$ gauge-coupling constant, y_t is the top Yukawa coupling constant, and contributions from other fermions are omitted.

If we face the quadratic divergences squarely, fine tuning among parameters is necessary to explain the observed value $m_\mathrm{h} \doteqdot 126\,\mathrm{GeV}$, unless $\Lambda \le O(1)$ TeV or $c_\mathrm{h} = 0$ is realized. Here, the condition $\Lambda \le O(1)$ TeV means that a new physics beyond the SM must exist around the terascale, unless nature requires fine tuning. The condition $c_\mathrm{h} = 0$ is equivalent to the Veltman condition $m_\mathrm{h}^2 = 4m_\mathrm{t}^2 - 2M_\mathrm{W}^2 - M_Z^2$ [5],[2] which leads to a value $m_\mathrm{h} \doteqdot 320\,\mathrm{GeV}$ at the weak scale.[3]

If all quadratic divergences are subtracted, δm_h^2 also vanishes in the limit of $m_\mathrm{h} \to 0$. Then, the classical conformal symmetry seems to control quantum corrections as the chiral symmetry does.

[1] A model where both the Planck scale and the weak scale emerge as quantum effects has been proposed [15]. As an extension including dark matter candidates, a model with a strongly interacting hidden sector, to trigger the breakdown of electroweak symmetry, has been constructed [16]. Recently, various models to generate the weak scale and provide dark matter candidates have been proposed [19–22].

[2] The same type of condition was derived through a tadpole diagram concerning the Higgs boson [23].

[3] Recently, it was pointed out that $c_\mathrm{h} = 0$ holds around M_Pl and there is a possibility that the bare Higgs mass vanishes there [24].

If this feature holds for higher-order corrections, the classical conformal symmetry is not broken down perturbatively (although it is broken down anomalously without threatening the consistency of the theory). In this way, the conformal symmetry might be responsible for the smallness of the Higgs boson mass, compared with a high-energy scale M_{U}. Or it might be said that the regularization ambiguities can be resolved by the conformal symmetry.

Hence, the problem of *whether the weak scale relating the Higgs boson mass is stabilized against radiative corrections in the framework of the SM* (a narrow definition of the naturalness problem)[4] can be solved by the subtraction of quadratic divergences. Then, the naturalness can become a powerful guiding principle for constructing an effective theory.[5] In other words, symmetries such as the chiral symmetry, the gauge symmetry, and the conformal symmetry become powerful tools for realistic model-building, from the viewpoint of the effective field theory. There is a possibility that all fields, in our low-energy world, are massless at M_{U}.

At this stage, the following questions (other parts of the naturalness problem) arise.

One is, *what induces the negative mass squared of the Higgs boson around the weak scale, starting the massless state at M_{U}, i.e., what is the origin of the weak scale?* A possible solution has been proposed based on the extension of the SM with the $U(1)_{B-L}$ gauge symmetry and new particles around the terascale [14,17]. In particular, the TeV scale $B-L$ model proposed in [18] has several excellent features such as classical conformality, the flatness of the Higgs potential at M_{U}, and predictability in relating $m_{\mathrm{h}} \doteqdot 126\,\mathrm{GeV}$.

The other is the problem of *whether the weak scale is stabilized against large radiative corrections due to heavy particles in the framework of field theory including a high-energy physics, e.g., a grand unified theory.* This is (the technical side of) the gauge hierarchy problem [6,7]. For instance, in the presence of heavy particles with masses M_I and some SM gauge quantum numbers, m_{h}^2 generally receives large radiative corrections of $O(M_I^2)$ in addition to the quadratic term of Λ such that

$$\delta m_{\mathrm{h}}^2 = \tilde{c}_{\mathrm{h}}\Lambda^2 + c'_{\mathrm{h}} m_{\mathrm{h}}^2 \ln\frac{\Lambda^2}{m_{\mathrm{h}}^2} + \sum_I c''_{\mathrm{h}I} M_I^2 \ln\frac{\Lambda^2}{M_I^2} + \cdots, \tag{12}$$

and the stability of the weak scale is threatened. Here, $\tilde{c}_{\mathrm{h}}$ and $c''_{\mathrm{h}I}$ are also functions of parameters. Then, the fine tuning is indispensable for $M_I^2 \gg m_{\mathrm{h}}^2$ in the appearance of the quadratic term of M_I (part of the logarithmic divergences), even if the quadratic divergences of $O(\Lambda^2)$ are removed and unless some miraculous cancellation mechanism works among corrections due to heavy particles. We will come back to this problem in Sect. 3.3.

3. Naturalness and duality

Let us explore the possibility that the quadratic divergences are removed, in the expectation that *the quadratic divergences might be artifacts of the regularization procedure and the calculation scheme can be selected by the physics*.

[4] We use the terminology "the naturalness problem" in a wider sense; that is, it should be regarded as a collective term for a fine-tuning problem concerning mass parameters of scalar fields such as the Higgs mass, which contains the gauge hierarchy problem.

[5] Wells presented an interesting observation toward the SM from QED using naturalness as the guiding principle [25].

Concretely, we pursue other reasoning to suggest the subtraction of quadratic divergences, based on the conjecture that *an ultimate theory does not induce any large radiative corrections for low-energy fields owing to a symmetry, and such a symmetry is hidden in the SM.*[6] We present a (tricky) calculation scheme that rules out quadratic divergences thanks to a hidden duality.

In the following, it is shown that the logarithmic corrections on a scalar mass can be picked out by specifying the duality in the effective field theory.

3.1. Basic idea

Our method is based on the following assumptions relating features of an underlying theory:[7]

(a) There is an ultimate theory, which has a fundamental energy scale. We denote the scale as Λ, for simplicity.

(b) The ultimate theory has a duality between the physics at a higher-energy scale ($E \gtrsim \Lambda$) and that at a lower-energy scale ($E \lesssim \Lambda$). It consists of the following two features:
  (b1) The physics is invariant under a duality transformation, e.g., $E \to E' = \Lambda^2/E$.
  (b2) The physics is only described by one of the two energy regions, relating with each other by the transformation.

(c) A remnant of the duality is hidden in quantities of the low-energy physics involved with Λ, e.g., radiative corrections on parameters.

To illustrate our idea, let us consider quantum corrections on a parameter a at the one-loop level given by

$$\delta a = \int_0^\infty f(p^2) \mathrm{d}p^2, \tag{13}$$

where p^2 is a Euclidean momentum squared for a massless virtual particle running in the loop, and $f(p^2)$ is a function of p^2.

In case that δa diverges, the infinities come from $p^2 = \infty$ and/or $p^2 = 0$, and hence it is ordinarily regularized as

$$\delta a = \int_{\mu_0^2}^{\Lambda^2} f(p^2) \mathrm{d}p^2, \tag{14}$$

where μ_0 is a fictitious mass parameter.

Here, let us show that expression (14) is necessarily obtained and the form of δa is restricted, based on the above assumptions, by specifying the duality transformation.

First, we rewrite (13) as

$$\delta a = \int_{\mu_0^2}^{\Lambda^2} f(p^2) \mathrm{d}p^2 + \int_{\Lambda^2}^{\Lambda^4/\mu_0^2} f(p^2) \mathrm{d}p^2. \tag{15}$$

Note that (15) is reduced to (13) in the limit of $\mu_0^2 \to 0$. Using assumption (b), (14) is obtained if the domain of integration $[\mu_0^2, \Lambda^2]$ is transformed into $[\Lambda^2, \Lambda^4/\mu_0^2]$ under a remnant of duality and

[6] This conjecture corresponds to one of the guiding principles in solving the gauge hierarchy problem and the cosmological constant problem, without SUSY and extra dimensions, proposed by Dienes [26].

[7] Our idea is inspired by the world-sheet modular invariance in string theory. We will comment on world-sheet modular invariance in Sect. 3.3.

the following relation holds,

$$\int_{\mu_0^2}^{\Lambda^2} f(p^2)\mathrm{d}p^2 = \int_{\Lambda^2}^{\Lambda^4/\mu_0^2} f(p^2)\mathrm{d}p^2. \tag{16}$$

Next, we take $p^2 \to p'^2 = \Lambda^4/p^2$ as the remnant of duality transformation. Hereafter, we refer to the remnant of duality transformation as the duality transformation or the duality, in most cases. Then, using assumptions (b1) and (c), the the following relation is derived:

$$\int_{\mu_0^2}^{\Lambda^2} f(p^2)\mathrm{d}p^2 = \int_{\Lambda^4/\mu_0^2}^{\Lambda^2} f(p'^2)\mathrm{d}p'^2 = \int_{\Lambda^2}^{\Lambda^4/\mu_0^2} f(\Lambda^4/p^2)\frac{\Lambda^4}{p^4}\mathrm{d}p^2. \tag{17}$$

From (16) and (17), the form of $f(p^2)$ is restricted as $f(p^2) = c(p^2)/p^2$, where $c(p^2)$ is a function invariant under the change $p^2 \to \Lambda^4/p^2$, e.g., $c(p^2) = p^2 + \Lambda^4/p^2$. Unless we consider effects of heavy particles with masses of $O(\Lambda)$ such as threshold corrections, $f(p^2)$ does not contain Λ and then δa is determined as

$$\delta a = c_{-1} \ln \frac{\Lambda^2}{\mu_0^2}, \tag{18}$$

where c_{-1} is a p^2-independent quantity.

Our procedure can be regarded not as a mere regularization but as a recipe to obtain finite physical values, because Λ is (large but) finite and infinities are taken away by the symmetry relating integration variables, like string theory. It is also regarded as the operation to pick out parts that satisfy assumptions. In the case that $f(p^2)$ does not contain Λ, it is simply denoted by

$$\delta a = \mathrm{Du}\left[\int_0^\infty f(p^2)\mathrm{d}p^2\right] = \mathrm{Du}\left[\int_0^\infty \sum_n c_n \left(p^2\right)^n \mathrm{d}p^2\right] = c_{-1} \ln \frac{\Lambda^2}{\mu_0^2}, \tag{19}$$

where Du[∗] represents the operation, and $f(p^2)$ is expanded in a series.

3.2. *Radiative corrections on scalar mass*

We apply our method to radiative corrections on m_ϕ^2.

In case that the bare mass is zero, the unregularized one is given by

$$\delta m_\phi^2 = \frac{\lambda_\phi}{2}\int_{-\infty}^{\infty} \frac{\mathrm{d}^4 p}{(2\pi)^4}\frac{1}{p^2} = \frac{\lambda_\phi}{32\pi^2}\int_0^\infty \mathrm{d}p^2. \tag{20}$$

If we demand that the duality $p^2 \to \Lambda^4/p^2$ is hidden in δm_ϕ^2 and the physics can be described by the region below Λ, δm_ϕ^2 turns out to be zero, such that

$$\delta m_\phi^2 = \mathrm{Du}\left[\frac{\lambda_\phi}{32\pi^2}\int_0^\infty \mathrm{d}p^2\right] = 0. \tag{21}$$

Next, we study the case with a non-zero bare mass, based on the momentum cutoff method and the proper time method.

(i) The momentum cutoff method. First, we separate the original into quadratic and logarithmic divergent parts such that

$$\delta m_\phi^2 = \frac{\lambda_\phi}{32\pi^2}\int_0^{\Lambda_\phi^2} \mathrm{d}p^2 - \frac{\lambda_\phi m_\phi^2}{32\pi^2}\int_0^{\Lambda_\phi^2} \frac{\mathrm{d}p^2}{p^2 + m_\phi^2}, \tag{22}$$

where Λ_ϕ is a provisional cutoff parameter ($\Lambda_\phi^2 \gg m_\phi^2$) which goes to infinity in the limit of $m_\phi^2 \to 0$. In the case with $\Lambda_\phi^2 = (\Lambda^4/m_\phi^2) - m_\phi^2$, we find that the second term on the right-hand side in (22) is invariant under the change $p^2 + m_\phi^2 \to \Lambda^4/(p^2 + m_\phi^2)$, but the first term is not. Furthermore, in this case, the integration from $p^2 = 0$ to $p^2 = \Lambda_\phi^2$ is divided into that from $p^2 = 0$ to $p^2 = \Lambda^2 - m_\phi^2 (\ll \Lambda_\phi^2)$ and that from $p^2 = \Lambda^2 - m_\phi^2$ to $p^2 = \Lambda_\phi^2$, and these integrals for the second term take the same value. Note that the duality transformation reduces to that in the massless case, in the limit of $m_\phi^2 \to 0$.

Here, we impose the duality relating $p^2 + m_\phi^2 \to \Lambda^4/(p^2 + m_\phi^2)$ on quantities relevant to Λ. If the physics from $p^2 = 0$ to $p^2 = \Lambda^2 - m_\phi^2$ is same as that from $p^2 = \Lambda^2 - m_\phi^2$ to $p^2 = \Lambda_\phi^2$, and the physics is only described by one of the two regions, Λ is naturally introduced and the desired expression is obtained as

$$\delta m_\phi^2 = -\frac{\lambda_\phi m_\phi^2}{32\pi^2} \int_0^{\Lambda^2 - m_\phi^2} \frac{\mathrm{d}p^2}{p^2 + m_\phi^2} = -\frac{\lambda_\phi}{32\pi^2} m_\phi^2 \ln \frac{\Lambda^2}{m_\phi^2}. \tag{23}$$

Note that δm_ϕ^2 vanishes in the limit of $m_\phi^2 \to 0$.

(ii) The proper time method. Using the proper time method, δm_ϕ^2 is given as

$$\delta m_\phi^2 = \frac{\lambda_\phi}{2} \int_{-\infty}^{\infty} \frac{\mathrm{d}^4 p}{(2\pi)^4} \int_0^\infty \mathrm{e}^{-(p^2 + m_\phi^2)t} \mathrm{d}t = \frac{\lambda_\phi}{32\pi^2} \int_0^\infty \frac{\mathrm{e}^{-m_\phi^2 t}}{t^2} \mathrm{d}t, \tag{24}$$

where t is a parameter called a proper time.

First, we separate δm_ϕ^2 into quadratic and logarithmic divergent parts by expanding the exponential factor such that

$$\delta m_\phi^2 = \frac{\lambda_\phi}{32\pi^2} \int_{1/\tilde{\Lambda}_\phi^2}^{1/m_\phi^2} \frac{\mathrm{d}t}{t^2} - \frac{\lambda_\phi m_\phi^2}{32\pi^2} \int_{1/\tilde{\Lambda}_\phi^2}^{1/m_\phi^2} \frac{\mathrm{d}t}{t} + \frac{\lambda_\phi m_\phi^4}{64\pi^2} \int_{1/\tilde{\Lambda}_\phi^2}^{1/m_\phi^2} \mathrm{d}t + \cdots, \tag{25}$$

where $\tilde{\Lambda}_\phi$ is a provisional cutoff parameter ($\tilde{\Lambda}_\phi^2 \gg m_\phi^2$) which goes to infinity in the limit of $m_\phi^2 \to 0$. In the case with $\tilde{\Lambda}_\phi^2 = \Lambda^4/m_\phi^2$, we find that the second term on the right-hand side in (25) is invariant under the change $t \to 1/(\Lambda^4 t)$, but the others are not. Furthermore, in this case, the integration from $t = 1/\tilde{\Lambda}_\phi^2$ to $t = 1/m_\phi^2$ is divided into that from $t = 1/\tilde{\Lambda}_\phi^2$ to $t = 1/\Lambda^2$ and that from $t = 1/\Lambda^2$ to $t = 1/m_\phi^2$, and these integrals for the second term take a same value.

Here, we impose the duality relating $t \to 1/(\Lambda^4 t)$ on quantities relevant to Λ. If the physics from $t = 1/\tilde{\Lambda}_\phi^2$ to $t = 1/\Lambda^2$ is same as that from $t = 1/\Lambda^2$ to $t = 1/m_\phi^2$ and the physics is only described by one of the two regions, Λ is naturally introduced and the desired expression is obtained as

$$\delta m_\phi^2 = \mathrm{Du}\left[\frac{\lambda_\phi}{32\pi^2} \int_0^\infty \frac{\mathrm{e}^{-m_\phi^2 t}}{t^2} \mathrm{d}t\right] = -\frac{\lambda_\phi m_\phi^2}{32\pi^2} \int_{1/\Lambda^2}^{1/m_\phi^2} \frac{\mathrm{d}t}{t} = -\frac{\lambda_\phi}{32\pi^2} m_\phi^2 \ln \frac{\Lambda^2}{m_\phi^2}. \tag{26}$$

In a similar way to the momentum cutoff method, δm_ϕ^2 vanishes in the massless limit.

The region around $p^2 = \Lambda_\phi^2$ or $t = 1/\tilde{\Lambda}_\phi^2$ corresponds to the ultraviolet (UV) region, and that around $p^2 = 0$ or $t = 1/m_\phi^2$ corresponds to the infrared (IR). Hence, the symmetry relating $p^2 + m_\phi^2 \to \Lambda^4/(p^2 + m_\phi^2)$ or $t \to 1/(\Lambda^4 t)$ might suggest that Λ has a physical meaning as a fundamental scale and that, in an ultimate theory, there is an equivalence between the physics in the UV region and that in the IR one.

3.3. *Different choice*

It is important to examine the applicable scope of our scheme. Here, we point out that the result depends on the choice of duality transformation, by giving an example.

Based on the proper time method, δm_ϕ^2 is rewritten as

$$\delta m_\phi^2 = \frac{\lambda_\phi}{32\pi^2}\int_0^\infty \frac{\mathrm{e}^{-m_\phi^2 t}}{t^2}\mathrm{d}t = \frac{\lambda_\phi}{32\pi^2}\int_0^\infty \mathrm{d}\tau_2 \int_{-1/2}^{1/2}\mathrm{d}\tau_1 \frac{\Lambda^2}{\tau_2^2}\exp\left[-\frac{m_\phi^2}{\Lambda^2}\tau_2\right], \tag{27}$$

where $\tau_2 = \Lambda^2 t$. Let us make the complex parameter $\tau = \tau_1 + \mathrm{i}\tau_2$ play the role of the modular parameter in string theory. The world-sheet modular transformation is given by

$$\tau \to \frac{a\tau + b}{c\tau + d}, \quad (ad - bc = 1) \tag{28}$$

where a, b, c, and d are integers, and the transformation is generated by the compositions of two types of transformations $\tau \to \tau + 1$ and $\tau \to -1/\tau$. If we require invariance under the transformation (28) and assume that the physics is only described by an independent region, which is not connected with by the transformation, the following expression is obtained,

$$\delta m_\phi^2 = \mathrm{Du}\left[\frac{\lambda_\phi}{32\pi^2}\int_0^\infty \mathrm{d}\tau_2 \int_{-1/2}^{1/2}\mathrm{d}\tau_1 \frac{\Lambda^2}{\tau_2^2}\exp\left(-\frac{m_\phi^2}{\Lambda^2}\tau_2\right)\right] = \frac{\lambda_\phi}{32\pi^2}\Lambda^2\int_{\mathcal{F}}\frac{\mathrm{d}^2\tau}{\tau_2^2} = \frac{\lambda_\phi}{32\pi^2}\frac{\pi}{2}\Lambda^2, \tag{29}$$

where $\mathcal{F}$ stands for the fundamental region defined by

$$\mathcal{F} = \{\tau : |\mathrm{Re}\,\tau| \le 1/2, 1 \le |\tau|\}. \tag{30}$$

The value of (29) is different from that of (26). The difference of values comes from that of the invariant measures, i.e., the invariant measure for $\tau \to -1/\tau$ is $\mathrm{d}^2\tau/\tau_2^2$, but that for $t \to 1/(\Lambda^4 t)$ is $\mathrm{d}t/t$ up to a sign factor. Note that both $\tau \to -1/\tau$ and $t \to 1/(\Lambda^4 t)$ connect the UV region to the IR one, and $\tau \to -1/\tau$ reduces to $\tau_2 \to 1/\tau_2$, which corresponds to $t \to 1/(\Lambda^4 t)$, in the case with $\tau_1 = 0$.

In this way, we find that the value of δm_ϕ^2 depends on the form of duality transformation, and we need to specify it in order to obtain a physically meaningful value. We expect that the form of duality transformation is determined by matching the counterpart in the ultimate theory.

We add a comment on radiative corrections in string theory. From the world-sheet modular invariance for the closed string, δm_ϕ^2 (radiative corrections of the scalar mass including contributions from innumerable string states) should be given by

$$\delta m_\phi^2 = \int_{\mathcal{F}}\frac{\mathrm{d}^2\tau}{\tau_2^2}G(\tau), \tag{31}$$

where $G(\tau)$ is a world-sheet modular invariant function, i.e., $G(\tau) = G(\tau + 1)$ and $G(\tau) = G(-1/\tau)$. In cases where SUSY holds exactly, $G(\tau)$ vanishes, and then $\delta m_\phi^2 = 0$. Even if SUSY is broken down, there is a possibility that $G(\tau)$ vanishes in conspiracy with infinite towers of massive particles, as suggested in Ref. [26].

In string theory, the world-sheet modular invariance is deeply connected to the consistency of the theory, and radiative corrections should be given in the world-sheet modular invariant form for the closed string. On the other hand, in the effective field theory, a corresponding symmetry stays in the background if it exists at all, and the consistency of the theory would not necessarily be threatened if it is overlooked. Hence, radiative corrections in the effective theory are not generally given in the duality invariant form, and the projection to pick out the invariant parts would be required.

Finally, we give a conjecture on a solution for one side of the naturalness problem, *whether the weak scale is stabilized against radiative corrections*, taking string theory as a candidate for the ultimate theory. In string theory, the world-sheet conformal invariance induces the massless string states, and the world-sheet modular invariance guarantees the finiteness of physical quantities. We conjecture that, owing to some powerful symmetry (such as SUSY) in addition to the world-sheet modular invariance, the masslessness of scalar particles would be protected against quantum corrections, and the abovementioned problem would not be caused. This could be understood in the framework of low-energy effective field theory, as follows. We assume that the theory is described by only massless string states, effects of massive string states are introduced as non-renormalizable interactions among massless particles, and they do not cause (the technical side of) the gauge hierarchy problem. In the field theory limit, if τ reduces to τ_2 with $\tau_1 = 0$, the duality transformation $\tau \to -1/\tau$ reduces to $\tau_2 \to 1/\tau_2$, which corresponds to $t \to 1/(\Lambda^4 t)$. Then, massless scalar fields do not receive any radiative corrections on their masses, as seen from (26). This matches the conjecture based on string theory. For the case with massive light scalar fields, more careful consideration is needed which is beyond the scope of this paper, because it is deeply related to the other side of the naturalness problem, *what is the origin of the weak scale or the Higgs mass?*

4. Conclusions

We have reconsidered naturalness and its relevant symmetries from the viewpoint of effective field theories including the SM, in the expectation that unnaturalness for scalar masses might be an artifact in the effective theory and they could be improved if features of an ultimate theory are introduced and the ingredients of the effective theory enriched. We have given a suggestion for the subtraction of quadratic divergences, based on assumptions relating features of the ultimate theory. The assumptions are summarized as follows. Beyond the SM, there is an ultimate theory with a fundamental scale Λ and a duality between the physics at the UV region beyond Λ and that at the IR region, and a remnant of the duality is hidden in the lower-energy theory. We have shown that the logarithmic corrections can be picked out by specifying the duality transformation. Because the logarithmic corrections are compatible with a specific duality, it is expected that the subtraction of quadratic divergences could be justified in the ultimate theory.

If the quadratic divergences of scalar fields are artifacts of the regularization procedure, the problem *whether the weak scale is stabilized against radiative corrections in the framework of SM* can be solved by the subtraction of quadratic divergences. Note that, even if the quadratic divergences are eliminated, the physics beyond the SM can induce the gauge hierarchy problem, i.e., *the effective field theory becomes unnatural, because fine tuning is required to obtain the weak scale and/or to stabilize the weak scale, if there is a high-energy physics relevant to the SM.* The sources of large radiative corrections, which can ruin the stability of the weak scale, are logarithmic divergences due to heavy particles. There is a possibility that the SM (or the extension of the SM with new particles around the terascale and without new concepts such as SUSY, compositeness, and extra dimensions) holds until Λ and an ultimate theory protects masses of low-energy fields against large quantum corrections by some mechanism and/or symmetry. This is the background of our previous work [27].

Finally, we discuss the applicable scope of our method. The issue is whether our calculation scheme is applicable to other systems and ordinary results are obtained or not. We anticipate that it is applicable to calculate logarithmic corrections including Λ on quantities.

We have applied our method to the radiative corrections on vacuum energy density, and shown that the logarithmic corrections can be picked out, in Appendix A. Under the assumption that QED holds until Λ in the broken phase of electroweak symmetry, we have also applied it to the self-energy of the electron, and obtained the well-known result, in Appendix B.

Because our procedure contains a provisional cutoff parameter depending on a mass, it looks like an artifact or a temporary expedient to pick out specific corrections. It is important to examine whether it is applicable to radiative corrections with higher loops by introducing several proper times and more complex models including several fields.

Even if our scheme has a limit of application or the hidden duality is a product of fantasy, our expectation would survive that the calculation scheme can be selected by the physics, and radiative corrections can be constrained by a remnant of symmetries in an ultimate theory.

Acknowledgements

This work was supported in part by scientific grants from the Ministry of Education, Culture, Sports, Science, and Technology under Grant Nos. 22540272 and 21244036.

Appendix A. Radiative corrections on vacuum energy density

We apply our procedure to radiative corrections on the vacuum energy density $\delta\Lambda_{\rm V}$. For the scalar field ϕ, $\delta\Lambda_{\rm V}$ at the one-loop level is commonly written as

$$\delta\Lambda_{\rm V} = -\frac{1}{2}\int_{-\infty}^{\infty}\frac{{\rm d}^4 p}{(2\pi)^4}\ln(p^2+m_\phi^2) = \frac{1}{2}\int_{-\infty}^{\infty}\frac{{\rm d}^4 p}{(2\pi)^4}\int_0^\infty \frac{{\rm e}^{-(p^2+m_\phi^2)t}}{t}{\rm d}t$$
$$= \frac{1}{32\pi^2}\int_0^\infty \frac{{\rm e}^{-m_\phi^2 t}}{t^3}{\rm d}t, \qquad \text{(A1)}$$

and it contains infinities. We carry out the same procedure as for scalar masses.

First, we separate $\delta\Lambda_{\rm V}$ into the quartic, quadratic, and logarithmic divergent parts by expanding the exponential factor such that

$$\delta\Lambda_{\rm V} = \frac{1}{32\pi^2}\int_{1/\tilde{\Lambda}_\phi^2}^{1/m_\phi^2}\frac{{\rm d}t}{t^3} - \frac{1}{32\pi^2}m_\phi^2\int_{1/\tilde{\Lambda}_\phi^2}^{1/m_\phi^2}\frac{{\rm d}t}{t^2}$$
$$+\frac{1}{64\pi^2}m_\phi^4\int_{1/\tilde{\Lambda}_\phi^2}^{1/m_\phi^2}\frac{{\rm d}t}{t} - \frac{1}{128\pi^2}m_\phi^6\int_{1/\tilde{\Lambda}_\phi^2}^{1/m_\phi^2}{\rm d}t + \cdots, \qquad \text{(A2)}$$

where $\tilde{\Lambda}_\phi$ is a provisional cutoff parameter ($\tilde{\Lambda}_\phi^2 \gg m_\phi^2$), which goes to infinity in the limit of $m_\phi^2 \to 0$. In case with $\tilde{\Lambda}_\phi^2 = \Lambda^4/m_\phi^2$, we find that the third term on the right-hand side of (A2) is invariant under the change $t \to 1/(\Lambda^4 t)$, but the others are not.

Here, we impose the duality relating $t \to 1/(\Lambda^4 t)$ on quantities relevant to Λ. If the physics from $t = 1/\tilde{\Lambda}_\phi^2$ to $t = 1/\Lambda^2$ is same as that from $t = 1/\Lambda^2$ to $t = 1/m_\phi^2$ and the physics is only described by one of the two regions, we obtain the relation

$$\delta\Lambda_{\rm V} = {\rm Du}\left[\frac{1}{32\pi^2}\int_0^\infty\frac{{\rm e}^{-m_\phi^2 t}}{t^3}{\rm d}t\right] = \frac{1}{64\pi^2}m_\phi^4\int_{1/\Lambda^2}^{1/m_\phi^2}\frac{{\rm d}t}{t} = \frac{1}{64\pi^2}m_\phi^4\ln\frac{\Lambda^2}{m_\phi^2}. \qquad \text{(A3)}$$

In this way, the quartic and quadratic divergences in $\delta\Lambda_{\rm V}$ are eliminated by requiring that the effective theory should have a hidden symmetry on the proper time. Note that $\delta\Lambda_{\rm V}$ also vanishes in the massless limit $m_\phi = 0$. Because the subtraction of the quartic and quadratic divergences in $\delta\Lambda_{\rm V}$

induces the effect that the cosmological constant shifts, a more careful consideration is required to justify our procedure.[8]

Appendix B. Self-energy of electron

The self-energy of an electron with the momentum p at the one-loop level is given by [29]

$$\Sigma(p) = -e^2 \int_{-\infty}^{\infty} \frac{\mathrm{d}^4 q}{(2\pi)^4} \left(\frac{\mathrm{i}}{q^2 + \mu_\gamma^2} \gamma_\nu \frac{\mathrm{i}}{\not{p} - \not{q} - m_\mathrm{e}} \gamma^\nu \right), \tag{B1}$$

where we rotate to the Euclidean space, q is a momentum of virtual photon, $p - q$ is the momentum of a virtual electron, and μ_γ is a fictitious photon mass for a regularization of IR divergences occurring at $q^2 = 0$. Using the proper time method, $\Sigma(p)$ is written by

$$\Sigma(p) = e^2 \int_0^\infty \mathrm{d}z_1 \int_0^\infty \mathrm{d}z_2 \int_{-\infty}^{\infty} \frac{\mathrm{d}^4 q}{(2\pi)^4} \left[\gamma_\nu (\not{p} - \not{q} + m_\mathrm{e}) \gamma^\nu \right] \times \exp\left[-z_1 \left(q^2 + \mu_\gamma^2 \right) - z_2 \left((p - q)^2 + m_\mathrm{e}^2 \right) \right]. \tag{B2}$$

By changing the integration variable q into the following one,

$$\tilde{q} \equiv q - \frac{z_2}{z_1 + z_2} p = q - p + \frac{z_1}{z_1 + z_2} p, \tag{B3}$$

and integrating out $\tilde{q}$, we obtain the expression

$$\Sigma(p) = \frac{e^2}{16\pi^2} \int_0^\infty \int_0^\infty \frac{\mathrm{d}z_1 \mathrm{d}z_2}{(z_1 + z_2)^2} \left(-2 \frac{z_1}{z_1 + z_2} \not{p} + 4 m_\mathrm{e} \right) \times \exp\left[-\left(\frac{z_1 z_2}{z_1 + z_2} p^2 + z_1 \mu_\gamma^2 + z_2 m_\mathrm{e}^2 \right) \right]. \tag{B4}$$

Furthermore, we insert the following identity relating the delta function into the integrand,

$$\int_0^\infty \mathrm{d}\xi \, \delta(\xi - z_1 - z_2) = \int_0^\infty \frac{\mathrm{d}\xi}{\xi} \delta\left(1 - \frac{z_1 + z_2}{\xi} \right) = 1, \tag{B5}$$

and integrate out z_2 after changing the scale of the proper time parameters z_i as ξz_i, we obtain the expression

$$\Sigma(p) = \frac{e^2}{8\pi^2} \int_0^1 \mathrm{d}z_1 \left(-z_1 \not{p} + 2 m_\mathrm{e} \right) \int_0^\infty \frac{\mathrm{d}\xi}{\xi} \mathrm{e}^{-\xi \tilde{m}^2}, \tag{B6}$$

where $\tilde{m}^2$ is a function of p^2 and z_1, defined by

$$\tilde{m}^2 \equiv z_1(1 - z_1) p^2 + z_1 \mu_\gamma^2 + (1 - z_1) m_\mathrm{e}^2. \tag{B7}$$

We expand the exponential factor such that

$$\Sigma(p) = \frac{e^2}{8\pi^2} \int_0^1 \mathrm{d}z_1 \left(-z_1 \not{p} + 2 m_\mathrm{e} \right) \int_{1/\tilde{\Lambda}_p^2}^{1/\tilde{m}^2} \frac{\mathrm{d}\xi}{\xi} - \frac{e^2}{8\pi^2} \int_0^1 \mathrm{d}z_1 \left(-z_1 \not{p} + 2 m_\mathrm{e} \right) \tilde{m}^2 \int_{1/\tilde{\Lambda}_p^2}^{1/\tilde{m}^2} \mathrm{d}\xi + \cdots, \tag{B8}$$

where $\tilde{\Lambda}_p$ is a provisional cutoff parameter ($\tilde{\Lambda}_p^2 \gg \tilde{m}^2$). In the case with $\tilde{\Lambda}_p^2 = z_1^2 \Lambda^4 / \tilde{m}^2$, we find that the first term in the right-hand side of (B8) is invariant under the change $\xi \to 1/(z_1^2 \Lambda^4 \xi)$, but

[8] As another work to show the importance of the trans-Planckian physics, Volovik gave the observation that the sub-Planckian and trans-Planckian contributions to the vacuum energy are canceled by the thermodynamical argument [28].

others are not. Furthermore, in this case, the integration from $\xi = \tilde{m}^2/z_1^2\Lambda^4$ to $\xi = 1/\tilde{m}^2$ is divided into that from $\xi = \tilde{m}^2/z_1^2\Lambda^4$ to $\xi = 1/(z_1\Lambda^2)$ and that from $\xi = 1/(z_1\Lambda^2)$ to $\xi = 1/\tilde{m}^2$, and these integrals for the first term take the same value. If we identify $z_1\xi$ as a proper time t, the transformation is the same form $t \to 1/(\Lambda^4 t)$ as that in case of the scalar mass m_ϕ.

Here, we impose the duality relating $\xi \to 1/(z_1^2\Lambda^4\xi)$ on quantities relevant to Λ. If the physics from $\xi = \tilde{m}^2/z_1^2\Lambda^4$ to $\xi = 1/(z_1\Lambda^2)$ is same as that from $\xi = 1/(z_1\Lambda^2)$ to $\xi = 1/\tilde{m}^2$ and the physics is only described by one of the two regions, the desired expression is obtained as

$$\Sigma(p) = \frac{e^2}{8\pi^2}\int_0^1 dz_1\,(-z_1\not{p} + 2m_e)\int_{1/(z_1\Lambda^2)}^{1/\tilde{m}^2}\frac{d\xi}{\xi}$$
$$= \frac{e^2}{8\pi^2}\int_0^1 (-z_1\not{p} + 2m_e)\ln\frac{z_1\Lambda^2}{(1-z_1)m_e^2 + z_1\mu_\gamma^2 + (1-z_1)z_1p^2}dz_1. \qquad \text{(B9)}$$

Using (B9), we obtain the ordinary expression for radiative corrections on the electron mass such that

$$\delta m_e = \Sigma(p)|_{\not{p}=m_e} = \frac{e^2 m_e}{8\pi^2}\int_0^1 (2-z_1)\ln\frac{z_1\Lambda^2}{(1-z_1)^2m_e^2}dz_1 = \frac{3\alpha}{4\pi}m_e\left(\ln\frac{\Lambda^2}{m_e^2} + \frac{1}{2}\right), \qquad \text{(B10)}$$

where we take the limit of $\mu_\gamma^2 \to 0$.

References

[1] ATLAS Collaboration, Phys. Lett. B **716**, 1 (2012).
[2] CMS Collaboration, Phys. Lett. B **716**, 30 (2012).
[3] L. Susskind, Phys. Rev. D **19**, 2619 (1979).
[4] G. ’t Hooft, Proc. NATO Advanced Study Institute on Recent Developments in Gauge Theories, 135 (1979).
[5] M. Veltman, ACTA Phys. Pol. B **12**, 437 (1981).
[6] E. Gildener, Phys. Rev. D **14**, 1667 (1976).
[7] S. Weinberg, Phys. Lett. B **82**, 387 (1979).
[8] M. Shaposhnikov, arXiv:0708.3550 [hep-th].
[9] H. B. Nielsen, arXiv:1212.5716 [hep-ph].
[10] W. A. Bardeen, FERMILAB-CONF-95-391-T.
[11] R. Jackiw, Int. J. Mod. Rev. B **14**, 2011 (2000).
[12] K. Fujikawa, Phys. Rev. D **83**, 105012 (2011).
[13] H. Aoki and S. Iso, Phys. Rev. D **86**, 013001 (2012).
[14] K. A. Meissner and H. Nicolai, Phys. Lett. B **648**, 312 (2007).
[15] R. Foot, A. Kobakhidze, K. L. McDonald, and R. R. Volkas, Phys. Rev. D **77**, 035006 (2008).
[16] T. Hur and P. Ko, Phys. Rev. Lett. **106**, 141802 (2011).
[17] S. Iso, N. Okada, and Y. Orikasa, Phys. Lett. B **676**, 81 (2009).
[18] S. Iso and Y. Orikasa, Prog. Theor. Exp. Phys. **2**, 023B08 (2013).
[19] M. Heikinheimo, A. Racioppi, M. Raidal, C. Spethmann, and K. Tuominen, arXiv:1304.7006 [hep-ph].
[20] T. Hambye and A. Strumia, Phys. Rev. D **88**, 055022 (2013).
[21] C. D. Carone and R. Ramos, Phys. Rev. D **88**, 055020 (2013).
[22] A. Farzinnia, H.-J. He, and J. Ren, arXiv:1308.0295 [hep-ph].
[23] R. Decker and J. Pestieau, UCL-IPT-79-19, reprinted in hep-ph/0512126.
[24] Y. Hamada, H. Kawai, and K. Oda, Phys. Rev. D **87**, 053009 (2013).
[25] J. D. Wells, arXiv:1305.3434 [hep-ph].
[26] K. Dienes, Nucl. Phys. B **611**, 146 (2001).
[27] Y. Kawamura, Prog. Theor. Exp. Phys. **8**, 081B01 (2013).
[28] G. E. Volovik, arXiv:1111.1155 [gr-qc].
[29] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (MacGraw-Hill, New York, 1964).