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Limitation on Magnitude of D-Components

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We study the magnitude of D-components in a generic supersymmetric field theory. There exists F-component whose vacuum expectation value is comparable to or higher than that of D-component, in the absence of the Fayet-Iliopoulos term, the large hierarchy in the charge spectrum and strongly interacting higher-dimensional couplings in the Kähler potential, if contributions from terms other than the F- and D-terms are negligible.

Subject Index: 111, 112

§1. Introduction

Much effort has been devoted to constructing a realistic model beyond the standard model (SM) based on supersymmetry (SUSY), which is broken softly in our visible world. The SUSY is broken by nonvanishing vacuum expectation values (VEVs) of some auxiliary fields (F and/or D) in a SUSY-breaking sector. The breakdown of SUSY is mediated to our visible world by some messengers. Then, soft SUSY-breaking parameters depend on the VEVs of F and D, reflecting on how to break SUSY and how to mediate the breakdown of SUSY.

Recently, the role of D-terms in the breakdown of SUSY has been attracting attention for general gauge mediation.[1),2)] The D-terms have also played an important role through the D-term contribution to scalar masses,[3),4)] in various models, e.g., SUSY grand unified theories,[5),6)] effective theories from string models,[7)–12)] effects due to the kinetic mixing,[13)] the gauge mediation,[14),15)] the anomaly mediation,[16),17)] the mirage mediation[18)] and models with Dirac gauginos.[19),20)] Hence, it would be useful to set a course of model building if we obtain constraints on the VEVs of F and D model-independently.

There is the theorem that if the VEVs of all F-components vanish, i.e., $\langle F_I \rangle = \langle \partial W/\partial \phi^I \rangle = 0$, where W is the superpotential and ϕ^I are scalar fields, there exists a SUSY-preserving solution satisfying the D-flat conditions, $\langle D^\alpha \rangle = \langle \phi_I^\dagger (T^\alpha \phi)^I \rangle = 0$.[21),22)] It is known that the VEV of the dominant F-component is comparable to or higher than that of any D-components in most SUSY breaking solutions through the analysis of explicit models. There are models that the VEV of the dominant D-component can be bigger than that of any F-components in the presence of the Fayet-Iliopoulos (FI) term[23)] or the large hierarchy in the charge spectrum.[24)] It is interesting to know whether these features hold in a more generic framework of the SUSY field theory. This is the motivation of our work.

In this paper, we study the magnitude of D-components model-independently,

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that is, without specifying the form of the Kähler potential (matter kinetic function), superpotential and gauge kinetic function. In the next section, we consider a generic global SUSY field theory in the absence of the FI term. In §3, we extend our discussion to the case with the FI term, soft SUSY-breaking terms and the local SUSY in order. In §4, we present conclusions and a discussion.

§2. Magnitude of D-component in global SUSY field theory

Let us consider the global SUSY Lagrangian density,

$$\mathcal{L}_{\mathrm{SUSY}} = \int d^2\theta d^2\overline{\theta} K(\Phi^I, \Phi^\dagger_J, V) + \left[\int d^2\theta W(\Phi^I) + \mathrm{h.c.}\right] + \left[\frac{1}{4}\int d^2\theta f_{\alpha\beta}(\Phi^I) W^\alpha W^\beta + \mathrm{h.c.}\right] , \tag{2·1}$$

where Φ^I, $\Phi^\dagger_J$ and $V = V^\alpha T^\alpha$ are chiral scalar superfields, anti-chiral scalar superfields and vector superfields, respectively, T^α are gauge transformation generators, h.c. stands for the hermitian conjugate and W^α are chiral spinor superfields constructed from V^α. $K(\Phi^I, \Phi^\dagger_J, V)$, $W(\Phi^I)$ and $f_{\alpha\beta}(\Phi^I)$ are Kähler potential (matter kinetic function), superpotential and gauge kinetic function, respectively. Both $K(\Phi^I, \Phi^\dagger_J, V)$ and $W(\Phi^I)$ are gauge invariant. The last terms on the right-hand side of (2·1) come from the following terms,

$$\left[\frac{1}{2}\int d^2\theta \mathrm{tr}\left(f(\Phi^I)(W^\alpha T^\alpha)(W^\beta T^\beta)\right) + \mathrm{h.c.}\right] , \tag{2·2}$$

where tr represents the trace over the gauge generators.

The scalar potential is given by

$$\begin{aligned} V_{\mathrm{SUSY}} &= -F^I K_I^J F_J - F^I \frac{\partial W}{\partial \phi^I} - F_J \frac{\partial \overline{W}}{\partial \phi^\dagger_J} - \frac{1}{2}\mathrm{Re} f_{\alpha\beta} D^\alpha D^\beta - D^\alpha (K_I (T^\alpha \phi)^I) \\ &= \frac{\partial \overline{W}}{\partial \phi^\dagger_J} \left(K^{-1}\right)^I_J \frac{\partial W}{\partial \phi^I} + \frac{1}{2}\left(\mathrm{Re} f^{-1}\right)_{\alpha\beta} (K_I (T^\alpha \phi)^I)(K_J (T^\beta \phi)^J) , \end{aligned} \tag{2·3}$$

where F^I, F_J and D^α are auxiliary components in Φ^I, $\Phi^\dagger_J$ and V^α. Here, $K = K(\phi^I, \phi^\dagger_J)$, $W = W(\phi^I)$, $\overline{W} = \overline{W}(\phi^\dagger_J)$, $f_{\alpha\beta} = f_{\alpha\beta}(\phi^I)$, $K_I = \partial K/\partial \phi^I$, $K_I^J = \partial^2 K/\partial \phi^I \partial \phi^\dagger_J$ etc. The ϕ^I and $\phi^\dagger_J$ are scalar components in Φ^I and $\Phi^\dagger_J$, respectively. $(\mathrm{Re} f^{-1})_{\alpha\beta}$ and $(K^{-1})^I_J$ are the inverse matrices of $\mathrm{Re} f_{\alpha\beta}$ and K_I^J, respectively. The last equality in (2·3) is derived using the equations of motion,

$$F^I K_I^J + \frac{\partial \overline{W}}{\partial \phi^\dagger_J} = 0 , \quad K_I^J F_J + \frac{\partial W}{\partial \phi^I} = 0 , \tag{2·4}$$

$$\mathrm{Re} f_{\alpha\beta} D^\beta + K_I (T^\alpha \phi)^I = 0 . \tag{2·5}$$

The scalar potential is rewritten as

$$V_{\mathrm{SUSY}} = F^I K_I^J F_J + \frac{1}{2}\mathrm{Re} f_{\alpha\beta} D^\alpha D^\beta , \tag{2·6}$$

where $F^I = -(K^{-1})^I_J \partial\overline{W}/\partial\phi^\dagger_J$, $F_J = -(K^{-1})^I_J \partial W/\partial\phi^I$ and $D^\alpha = -(\mathrm{Re}f^{-1})_{\alpha\beta}$ $(K_I(T^\beta\phi)^I)$.

The derivative of V_{SUSY} by $\phi^{I'}$ is given by

$$\frac{\partial V_{\mathrm{SUSY}}}{\partial\phi^{I'}} = -F^I K^J_{II'} F_J - F^I \frac{\partial^2 W}{\partial\phi^I\partial\phi^{I'}} - \frac{1}{2}(\mathrm{Re}f_{\alpha\beta})_{I'} D^\alpha D^\beta - (\phi^\dagger T^\alpha)_I K^I_{I'} D^\alpha \ , \qquad (2{\cdot}7)$$

using the identity derived from the gauge invariance of Kähler potential,

$$K_I(T^\alpha\phi)^I = (\phi^\dagger T^\alpha)_I K^I \ . \qquad (2{\cdot}8)$$

From the stationary condition $\langle\partial V_{\mathrm{SUSY}}/\partial\phi^{I'}\rangle = 0$, we derive the formula:

$$\langle F^I\rangle\langle K^J_{II'}\rangle\langle F_J\rangle + \mu_{II'}\langle F^I\rangle + \frac{1}{2}\langle(\mathrm{Re}f_{\alpha\beta})_{I'}\rangle\langle D^\alpha\rangle\langle D^\beta\rangle + \langle(\phi^\dagger T^\alpha)_I\rangle\langle K^I_{I'}\rangle\langle D^\alpha\rangle = 0 \ , \qquad (2{\cdot}9)$$

where $\mu_{II'} \equiv \langle\partial^2 W/\partial\phi^I\partial\phi^{I'}\rangle$ is the SUSY mass coming from the superpotential.

By multiplying $(T^{\alpha'}\phi)^{I'}$ with (2·7), we obtain

$$\frac{\partial V_{\mathrm{SUSY}}}{\partial\phi^{I'}}(T^{\alpha'}\phi)^{I'} = -F^I (K_{I'}(T^{\alpha'}\phi)^{I'})^J_I F_J - \frac{1}{2}(\mathrm{Re}f_{\alpha\beta})_{I'}(T^{\alpha'}\phi)^{I'} D^\alpha D^\beta - (\phi^\dagger T^\alpha)_I K^I_{I'}(T^{\alpha'}\phi)^{I'} D^\alpha \ , \qquad (2{\cdot}10)$$

where we use (2·8) and the identities derived from the gauge invariance of the superpotential,

$$\frac{\partial W}{\partial\phi^{I'}}(T^{\alpha'}\phi)^{I'} = 0 \ , \quad \frac{\partial W}{\partial\phi^I\partial\phi^{I'}}(T^{\alpha'}\phi)^{I'} + \frac{\partial W}{\partial\phi^{I'}}(T^{\alpha'})^{I'}_I = 0 \ . \qquad (2{\cdot}11)$$

Taking its VEV and using the stationary condition, we derive the formula:

$$\langle F^I\rangle \left\langle (K_{I'}(T^{\alpha'}\phi)^{I'})^J_I \right\rangle \langle F_J\rangle + \frac{1}{2}\langle(\mathrm{Re}f_{\alpha\beta})_{I'}\rangle\langle(T^{\alpha'}\phi)^{I'}\rangle\langle D^\alpha\rangle\langle D^\beta\rangle + (\hat{M}^2_V)^{\alpha\alpha'}\langle D^\alpha\rangle = 0 \ , \qquad (2{\cdot}12)$$

where $(\hat{M}^2_V)^{\alpha\alpha'} = \langle(\phi^\dagger T^\alpha)_I K^I_{I'}(T^{\alpha'}\phi)^{I'}\rangle$ is the mass matrix of the gauge bosons up to the normalization due to the gauge coupling constants. The formula (2·12) is a counterpart of (B.13) in Ref. 25).

By multiplying $(K^{-1})^{I'}_{I''}K^{I''}$ with (2·7), we obtain

$$\frac{\partial V_{\mathrm{SUSY}}}{\partial\phi^{I'}}(K^{-1})^{I'}_{I''}K^{I''} = -F^I K^J_{II'} F_J (K^{-1})^{I'}_{I''}K^{I''} - F^I \frac{\partial^2 W}{\partial\phi^I\partial\phi^{I'}}(K^{-1})^{I'}_{I''}K^{I''} - \frac{1}{2}(\mathrm{Re}f_{\alpha\beta})_{I'}(K^{-1})^{I'}_{I''}K^{I''} D^\alpha D^\beta + \mathrm{Re}f_{\alpha\beta}D^\alpha D^\beta \ . \qquad (2{\cdot}13)$$

The relation (2·13) is a counterpart of the identity (4.5) in Ref. 2). Taking its VEV and using the stationary condition, we derive the formula:

$$\langle F^I\rangle\langle K^J_{II'}\rangle\langle F_J\rangle\langle (K^{-1})^{I'}_{I''}\rangle\langle K^{I''}\rangle + \mu_{II'}\langle F^I\rangle\langle (K^{-1})^{I'}_{I''}\rangle\langle K^{I''}\rangle$$
$$+ \frac{1}{2}\langle (\mathrm{Re}f_{\alpha\beta})_{I'}\rangle\langle (K^{-1})^{I'}_{I''}\rangle\langle K^{I''}\rangle\langle D^\alpha\rangle\langle D^\beta\rangle = \langle \mathrm{Re}f_{\alpha\beta}\rangle\langle D^\alpha\rangle\langle D^\beta\rangle \ . \quad (2\cdot14)$$

We rearrange the fields into those forming irreducible representations such as $(T^\alpha)^J_I = T^\alpha_{(I)}\delta^J_I$ under gauge groups where $T^\alpha_{(I)}$ is the representation matrix for Φ^I and the same notation for Φ^I is used. In $K = K(\phi^I, \phi^\dagger_J)$, the fields with the same representation can be mixed such that

$$K = a^J_I \phi^\dagger_J \phi^I + \frac{a^J_{II'}}{\Lambda}\phi^\dagger_J\phi^I\phi^{I'} + \cdots \ , \quad (2\cdot15)$$

where a^J_I and $a^J_{II'}$ are coefficients and Λ is a high-energy scale. The VEV of K^J_I is estimated as

$$\langle K^J_I\rangle = a^J_I + \frac{a^J_{II'}}{\Lambda}\langle\phi^{I'}\rangle + \frac{a^J_{I'I}}{\Lambda}\langle\phi^{I'}\rangle + \cdots$$
$$= a^J_I + O(\langle\phi^{I'}\rangle/\Lambda) \ , \quad (2\cdot16)$$

where we assume that the magnitudes of $a^J_{II'}$ and other higher coefficients are at most $O(1)$ and the magnitude of $\langle\phi^{I'}\rangle$ is comparable to or less than Λ.

The nonvanishing VEV of D-component implies the breakdown of gauge symmetry by the VEV of some gauge nonsinglet scalar fields, in the absence of the FI term. The nonvanishing components in $\langle D\rangle \equiv \langle D^\alpha\rangle T^\alpha$ are those for diagonal generators T^a because $\langle D\rangle$ is transformed into $\langle D^a\rangle T^a$ by some unitary matrix U. Because the fields forming the same representation change in the same manner under the unitary transformation, the form of K is invariant after the redefinition of fields by U and we use the same notation for fields to avoid confusion. The VEV of D^a is written as

$$\langle D^a\rangle = -\langle(\mathrm{Re}f^{-1})_{aa}\rangle\langle K_I(T^a\phi)^I\rangle = -g_a^2 q^a_{(\phi^I)}\langle K_I\phi^I\rangle$$
$$= -g_a^2 q^a_{(\phi^I)}\left(a^J_I\langle\phi^\dagger_J\rangle\langle\phi^I\rangle + \frac{a^J_{II'}}{\Lambda}\langle\phi^\dagger_J\rangle\langle\phi^I\rangle\langle\phi^{I'}\rangle + \cdots\right)$$
$$= -g_a^2 q^a_{(\phi^I)}\left(\Big(\langle K^J_I\rangle + O(\langle\phi^{I'}\rangle/\Lambda)\Big)\langle\phi^\dagger_J\rangle\langle\phi^I\rangle + \frac{a^J_{II'}}{\Lambda}\langle\phi^\dagger_J\rangle\langle\phi^I\rangle\langle\phi^{I'}\rangle + \cdots\right), \quad (2\cdot17)$$

where $q^a_{(\phi^I)}$ is the value of $T^a_{(I)}$ for the nonvanishing component of ϕ^I and the gauge coupling constant g_a is given by $g_a^2 = \langle(\mathrm{Re}f^{-1})_{aa}\rangle$. We assume that the magnitude of $\langle K^J_I\rangle$ is $O(1)$. After the diagonalization of $\langle K^J_I\rangle$, $\langle D^a\rangle$ is written as

$$\langle D^a\rangle = -g_a^2 q^a_{(\phi^I)}\left|\langle\phi^I\rangle\right|^2\Big(1 + O(\langle\phi^{I'}\rangle/\Lambda)\Big) \ , \quad (2\cdot18)$$

where we also use the same notation for fields after their redefinition. Then the mass matrix of gauge bosons is diagonalized and the mass of gauge boson for T^a is given by

$$(M_V^2)^a = g_a^2(\hat{M}_V^2)^a = g_a^2(q^a_{(\phi^I)})^2\left|\langle\phi^I\rangle\right|^2 \ . \quad (2\cdot19)$$

The first term on the left-hand side of (2·12) for the diagonal generator T^a is written as

$$
\begin{aligned}
&\langle F^I\rangle \left\langle (K_{I'}(T^a\phi)^{I'})_I^J \right\rangle \langle F_J\rangle \\
&= \langle F^I\rangle\langle K^J_{I'}\rangle(T^a)^J_{I'}\langle F_J\rangle + \langle F^I\rangle\langle K^J_{I'I}\rangle\langle (T^a\phi)^{I'}\rangle\langle F_J\rangle \\
&= q^a_{(F^I)}\left|\langle F^I\rangle\right|^2 + \langle F^I\rangle\langle K^J_{I'I}\rangle q^a_{(\phi^{I'})}\langle\phi^{I'}\rangle\langle F_J\rangle \\
&= q^a_{(F^I)}\left|\langle F^I\rangle\right|^2\left(1 + O\left(\frac{q^a_{(\phi^{I'})}}{q^a_{(F^I)}}\frac{\langle\phi^{I'}\rangle}{\Lambda}\right)\right) \ ,
\end{aligned}
\tag{2·20}
$$

where $q^a_{(F^I)}$ is the value of $T^a_{(I)}$ for the nonvanishing component of F^I.

The second term on the left-hand side of (2·12) vanishes for T^a because the relation $\langle(\mathrm{Re}f_{bc})_I\rangle\langle(T^a\phi)^I\rangle = 0$ holds from the gauge invariance of $f_{bc}(\Phi^I)$. Here, a, b and c are indices for the Cartan subalgebra. Notice that $f_{bc}(\Phi^I)$, D^b and D^c are neutral under the $U(1)$ charges relating the Cartan subalgebra.

Using (2·18) and (2·20), the magnitudes of $\langle D^a\rangle$ and $\langle F^I\rangle \left\langle (K_{I'}(T^a\phi)^{I'})_I^J \right\rangle \langle F_J\rangle$ are bounded as

$$
|\langle D^a\rangle| \le g_a^2|q^a_{(\phi^I)}|\left|\langle\phi^I\rangle\right|^2\left|1 + O(\langle\phi^{I'}\rangle/\Lambda)\right|
\tag{2·21}
$$

and

$$
\langle F^I\rangle \left\langle (K_{I'}(T^a\phi)^{I'})_I^J \right\rangle \langle F_J\rangle \le |q^a_{(F^I)}|\left|\langle F^I\rangle\right|^2\left|1 + O\left(\frac{q^a_{(\phi^{I'})}}{q^a_{(F^I)}}\frac{\langle\phi^{I'}\rangle}{\Lambda}\right)\right| \ ,
\tag{2·22}
$$

respectively. Using (2·12), (2·19), (2·21) and (2·22), the magnitude of $\langle D^a\rangle^2$ is bounded as

$$
\begin{aligned}
\overline{q^a_{(\phi)}}\langle D^a\rangle^2 &\le (M_V^2)^a|\langle D^a\rangle|\left|1 + O(\langle\phi^{I'}\rangle/\Lambda)\right| \\
&\le g_a^2|q^a_{(F^I)}|\left|\langle F^I\rangle\right|^2\left|1 + O(\langle\phi^{I'}\rangle/\Lambda) + O\left(\frac{q^a_{(\phi^{I'})}}{q^a_{(F^I)}}\frac{\langle\phi^{I'}\rangle}{\Lambda}\right)\right| \ ,
\end{aligned}
\tag{2·23}
$$

where $\overline{q^a_{(\phi)}}$ is defined by

$$
\overline{q^a_{(\phi)}} \equiv \frac{(\hat{M}_V^2)^a}{|q^a_{(I)}(\phi)|\,|\langle\phi^I\rangle|^2} = \frac{(q^a_{(\phi^I)})^2\left|\langle\phi^I\rangle\right|^2}{|q^a_{(I)}(\phi)|\,|\langle\phi^I\rangle|^2} \ .
\tag{2·24}
$$

Equation (2·23) is our master formula and, from (2·23), we find that *the magnitude of $\langle D^a\rangle$ is comparable to**) *or smaller than that of dominant $\langle F^I\rangle$***) *if the condition $\overline{q^a_{(\phi)}} \ge O(g_a^2|q^a_{(F^I)}|)$ is fulfilled.* Here, we restate our basic assumptions:

$$
\left|\langle\phi^I\rangle\right| \le \Lambda \ , \quad \left|\langle K^{J_1J_2\cdots J_m}_{I_1I_2\cdots I_n}\rangle\right| = O\left(\frac{1}{\Lambda^{n+m-2}}\right) \ . \ \ (n+m \ge 2)
\tag{2·25}
$$

*) There are several models that generate comparable $\langle D^a\rangle$ and $\langle F^I\rangle$.[2),26)–28)]

**) We assume that the number of $\langle F^I\rangle$ contributing SUSY breaking dominantly is not so large.

These mean that the breakdown of gauge symmetry occurs below the scale Λ and there are no strongly interacting higher-dimensional couplings in K, respectively.

In the case of $g_a^2|q^a_{(F^I)}| \gg \overline{q^a_{(\phi)}}$, the magnitude of $\langle D^a\rangle$ can be much bigger than that of $\langle F^I\rangle$ if the equalities in (2·23) hold approximately. Actually, an explicit model has been constructed with the large hierarchy in the charge spectrum.[24] We explain it briefly. Let us take the O' Raifertaigh model with the following superpotential W,

$$W = \lambda_1 \Phi_0 (\Phi_1 \Phi_{-1/N}^N - 1) + \lambda_2 \Phi_1 \Phi_{-1} + \lambda_3 \Phi_0' \Phi_{1/N} \Phi_{-1/N} \ , \tag{2·26}$$

where Φ_0, Φ_0', Φ_1, Φ_{-1} $\Phi_{1/N}$ and $\Phi_{-1/N}$ are chiral superfields with $U(1)$ charges 0, 0, 1, -1, $1/N$ and $-1/N$. The relation $|\langle D\rangle|^2 \sim N|\langle F^I\rangle|^2$ is derived, and it leads to $|\langle D\rangle| \gg |\langle F^I\rangle|$ if $\sqrt{N} \gg 1$. Here, D is the D-component of $U(1)$. As the relation suggests, $\langle F_1\rangle$ dominates in $\langle F^I\rangle$ and $\langle \phi_{-1/N}\rangle$ dominates in $\langle D\rangle$.

In the case of $|\langle K^J_{I'I}\rangle\langle\phi^{I'}\rangle| \gg O(1)$, the term $\langle F^I\rangle\langle K^J_{I'I}\rangle\langle (T^a\phi)^{I'}\rangle\langle F_J\rangle$ dominates in $\langle F^I\rangle \left\langle (K_{I'}(T^a\phi)^{I'})^J_I \right\rangle \langle F_J\rangle$ and $\langle D^a\rangle^2$ is bounded as

$$\overline{q^a_{(\phi)}}\langle D^a\rangle^2 \le g_a^2 \left| \langle K^J_{I'I}\rangle q^a_{(\phi^{I'})}\langle\phi^{I'}\rangle\langle F^I\rangle\langle F_J\rangle \right| \ . \tag{2·27}$$

Then the magnitude of $\langle D^a\rangle$ can be much bigger than that of $\langle F^I\rangle$ if the equality in (2·27) holds approximately and $g_a^2|\langle K^J_{I'I}\rangle q^a_{(\phi^{I'})}\langle\phi^{I'}\rangle| \gg \overline{q^a_{(\phi)}}$.

In the case that all F-components vanish, we obtain the relation

$$\frac{1}{2}\langle(\mathrm{Re}f_{\alpha\beta})_{I'}\rangle\langle D^\alpha\rangle\langle D^\beta\rangle + \langle(\phi^\dagger T^\alpha)_I\rangle\langle K^I_{I'}\rangle\langle D^\alpha\rangle = 0 \ , \tag{2·28}$$

from (2·9) or

$$\frac{1}{2}\langle(\mathrm{Re}f_{\alpha\beta})_{I'}\rangle\langle(K^{-1})^{I'}_{I''}\rangle\langle K^{I''}\rangle\langle D^\alpha\rangle\langle D^\beta\rangle = \langle \mathrm{Re}f_{\alpha\beta}\rangle\langle D^\alpha\rangle\langle D^\beta\rangle \ , \tag{2·29}$$

from (2·14). Unless $\langle\mathrm{Re}f_{\alpha\beta}\rangle$ equals to $\frac{1}{2}\langle(\mathrm{Re}f_{\alpha\beta})_{I'}\rangle\langle(K^{-1})^{I'}_{I''}\rangle\langle K^{I''}\rangle$,[*)] the D-flat conditions, $\langle D^\alpha\rangle = 0$, are derived and then the SUSY is unbroken.

In this way, we obtain the following results.

(1) The magnitude of $\langle D^\alpha\rangle$ is comparable to or smaller than that of dominant $\langle F^I\rangle$ under the assumptions (2·25), unless the magnitude of the broken charge of F-components that contribute SUSY breaking is much bigger than that of the broken charge of scalar components that contribute gauge symmetry breaking.

(2) There always exists a SUSY vacuum in the case that all F-components vanish and $\langle\mathrm{Re}f_{\alpha\beta}\rangle$ is different from $\frac{1}{2}\langle(\mathrm{Re}f_{\alpha\beta})_{I'}\rangle\langle(K^{-1})^{I'}_{I''}\rangle\langle K^{I''}\rangle$.

§3. Several extensions

We extend our discussion to several cases.

*) As an example, the relation $\langle\mathrm{Re}f_{\alpha\beta}\rangle = \frac{1}{2}\langle(\mathrm{Re}f_{\alpha\beta})_{I'}\rangle\langle(K^{-1})^{I'}_{I''}\rangle\langle K^{I''}\rangle$ holds for the canonical Kähler potential $K = |\phi^I|^2$ and the nonminimal gauge kinetic function $f_{\alpha\beta} = c_{\alpha\beta}(\phi^I)^2$.

3.1. *Case with FI term*

For $U(1)$ gauge symmetries, the following term called Fayet-Iliopoulos term can be added to $\mathcal{L}_{\rm SUSY}$,

$$\mathcal{L}_{\rm FI} = \int d^2\theta d^2\overline{\theta}\xi_r V^r = \xi_r D^r \ , \tag{3·1}$$

where ξ_r are constants, V^r are $U(1)$ vector superfields and D^r are the auxiliary components in V^r. The equations of motions for D-components are modified as

$$\mathrm{Re} f_{\alpha\beta} D^\beta + K_I (T^\alpha \phi)^I + \xi_r \delta^{\alpha r} = 0 \ . \tag{3·2}$$

Then the scalar potential is modified as

$$\begin{aligned} V_{\rm SUSY} &= -F^I K_I^J F_J - F^I \frac{\partial W}{\partial \phi^I} - F_J \frac{\partial \overline{W}}{\partial \phi_J^\dagger} \\ &\qquad - \frac{1}{2}\mathrm{Re} f_{\alpha\beta} D^\alpha D^\beta - D^\alpha \left(K_I (T^\alpha\phi)^I + \xi_r \delta^{\alpha r}\right) \\ &= \frac{\partial \overline{W}}{\partial \phi_J^\dagger} \left(K^{-1}\right)_J^I \frac{\partial W}{\partial \phi^I} \\ &\qquad + \frac{1}{2}\left(\mathrm{Re} f^{-1}\right)_{\alpha\beta} (K_I (T^\alpha\phi)^I + \xi_r\delta^{\alpha r})(K_I (T^\beta \phi)^I + \xi_r \delta^{\beta r}) \ . \end{aligned} \tag{3·3}$$

Although the same types of formulae (2·9) and (2·12) are derived, the inequalities on $\langle D^r\rangle^2$ are different from (2·23) such that

$$\begin{aligned} \overline{q^r_{(\phi)}}\langle D^r\rangle^2 &\le \eta_r (M_V^2)^r |\langle D^r\rangle| \left|1 + O(\langle\phi^{I'}\rangle/\Lambda)\right| \\ &\le \eta_r g_r^2 |q^r_{(F^I)}| \left|\langle F^I\rangle\right|^2 \left|1 + O(\langle\phi^{I'}\rangle/\Lambda) + O\left(\frac{q^r_{(\phi^{I'})}}{q^r_{(F^I)}}\frac{\langle\phi^{I'}\rangle}{\Lambda}\right)\right| \ , \end{aligned} \tag{3·4}$$

where $g_r^2 = \langle(\mathrm{Re} f^{-1})_{rr}\rangle$, and $\overline{q^r_{(\phi)}}$ and η_r are defined by

$$\overline{q^r_{(\phi)}} \equiv \frac{(\hat{M}_V^2)^r}{|q^r_{(I)}(\phi)|\left|\langle\phi^I\rangle\right|^2} = \frac{(q^r_{(\phi^I)})^2 \left|\langle\phi^I\rangle\right|^2}{|q^r_{(\phi^I)}|\left|\langle\phi^I\rangle\right|^2} \tag{3·5}$$

and

$$\eta_r \equiv \frac{|q^r_{(\phi^I)}|\left|\langle\phi^I\rangle\right|^2 + |\xi_r|}{|q^r_{(\phi^I)}|\left|\langle\phi^I\rangle\right|^2} \ , \tag{3·6}$$

respectively. Here, $q^r_{(\phi^I)}$ and $q^r_{(F^I)}$ are values of $T^r_{(I)}$ for the nonvanishing components of ϕ^I and F^I, respectively. In the case of $\eta_r = O(1)$, the same result (1) is obtained. If $\eta_r \gg 1$*) and the equalities in (3·4) hold approximately, the magnitude of $\langle D^r\rangle$

*) In an extreme case, there is a possibility that the VEV of D^r is ξ_r itself and nonvanishing but the $U(1)$ gauge symmetry is not broken with $\langle(T^r\phi)^I\rangle = 0$ and $\langle F^I\rangle = 0$, where T^r is the $U(1)$ charge operator.

can be much bigger than that of $\langle F^I \rangle$ such that

$$|\langle D^r \rangle| = O(|\xi_r|) \gg |\langle F^I \rangle| \ . \tag{3·7}$$

In the case that all F-components vanish, we obtain the relation (2·28) or

$$\frac{1}{2}\langle(\mathrm{Re}f_{\alpha\beta})_{I'}\rangle\langle(K^{-1})^{I'}_{I''}\rangle\langle K^{I''}\rangle\langle D^\alpha\rangle\langle D^\beta\rangle + \langle(\phi^\dagger T^\alpha)_I K^I\rangle\langle D^\alpha\rangle = 0 \ . \tag{3·8}$$

There can appear a non-SUSY vacuum with $\langle D^r \rangle \neq 0$, in which the gauge symmetry is unbroken with $\langle(\phi^\dagger T^r)_I\rangle = 0$, in the case that $\langle(\mathrm{Re}f_{rr'})_{I'}\rangle = 0$ and all F-components vanish with $\langle(\phi^\dagger T^r)_I\rangle = 0$.

3.2. *Case with soft SUSY-breaking terms*

In the case that SUSY is broken in another sector at some high-energy scale, soft SUSY-breaking terms can appear after mediation by some messengers. We consider the following type of soft SUSY breaking terms for the scalar potential,[*)]

$$V_{\mathrm{soft}} = (m^2)^J_I \phi^\dagger_J \phi^I + \left[U(\phi^I) + \mathrm{h.c.}\right] \ . \tag{3·9}$$

In the presence of V_{soft}, (2·9) and (2·12) are modified as

$$\begin{aligned}\langle F^I\rangle\langle K^J_{II'}\rangle\langle F_J\rangle + \mu_{II'}\langle F^I\rangle + \frac{1}{2}\langle(\mathrm{Re}f_{\alpha\beta})_{I'}\rangle\langle D^\alpha\rangle\langle D^\beta\rangle \\ + \langle(\phi^\dagger T^\alpha)_I\rangle\langle K^I_{I'}\rangle\langle D^\alpha\rangle = (m^2)^J_{I'}\langle\phi^\dagger_J\rangle + \langle U_{I'}\rangle\end{aligned} \tag{3·10}$$

and

$$\begin{aligned}\langle F^I\rangle \left\langle (K_{I'}(T^{\alpha'}\phi)^{I'})^J_I \right\rangle \langle F_J\rangle + \frac{1}{2}\langle(\mathrm{Re}f_{\alpha\beta})_{I'}\rangle\langle(T^{\alpha'}\phi)^{I'}\rangle\langle D^\alpha\rangle\langle D^\beta\rangle \\ + (\hat{M}^2_V)^{\alpha\alpha'}\langle D^\alpha\rangle = (m^2)^J_{I'}\langle\phi^\dagger_J\rangle\langle(T^{\alpha'}\phi)^{I'}\rangle \ ,\end{aligned} \tag{3·11}$$

respectively. The formula (3·11) is a counterpart of (3.54) in Ref. 6).

If the soft SUSY-breaking terms are related to the SUSY extension of SM directly, the magnitude of $(m^2)^J_{I'}$ should be the same size as or less than $O(1)$ TeV^2. In this case with $(\hat{M}^2_V)^a \gg (m^2)^J_{I'}$, the soft SUSY-breaking terms are treated as a perturbation. Then the same argument as that in the previous section is applied, and the same result (1) is obtained if $\langle F^I\rangle \left\langle (K_{I'}(T^a\phi)^{I'})^J_I \right\rangle \langle F_J\rangle$ is bigger than $(m^2)^J_{I'}\langle\phi^\dagger_J\rangle\langle(T^a\phi)^{I'}\rangle$.

3.3. *Case with local SUSY*

In the Einstein supergravity, the scalar potential is given by[29),30)]

$$V_{\mathrm{SG}} = M^2 e^{G/M^2}(G^I(G^{-1})^J_I G_J - 3M^2) + \frac{1}{2}\mathrm{Re}f_{\alpha\beta}D^\alpha D^\beta \ , \tag{3·12}$$

[*)] The form of $U(\phi^I)$ is constrained by requiring that the gauge hierarchy achieved by a fine-tuning in the superpotential should not be violated by soft SUSY breaking terms.[6)] where M is a gravitational scale defined by $M \equiv M_{\rm Pl}/\sqrt{8\pi}$ using the Planck scale $M_{\rm Pl}$, $G(\phi^I, \phi^\dagger_J)$ is the total Kähler potential defined by

$$G(\phi^I, \phi^\dagger_J) \equiv K(\phi^I, \phi^\dagger_J) + M^2 \ln \frac{|W(\phi^I)|^2}{M^6} \tag{3·13}$$

and D-auxiliary fields are defined by

$$D^\alpha \equiv -({\rm Re}f^{-1})_{\alpha\beta} G_I (T^\beta \phi)^I = -({\rm Re}f^{-1})_{\alpha\beta} (\phi^\dagger T^\beta)_J G^J \ . \tag{3·14}$$

The F-auxiliary fields are given by

$$F_J = -M e^{G/2M^2} (G^{-1})^I_J G_I \ . \tag{3·15}$$

The scalar potential is rewritten as

$$V_{\rm SG} = F^I K^J_I F_J - 3M^4 e^{G/M^2} + \frac{1}{2}{\rm Re}f_{\alpha\beta} D^\alpha D^\beta \ , \tag{3·16}$$

where D^α and F^I are given by (3·14) and (3·15), respectively.

The derivative of V by $\phi^{I'}$ is given by

$$\begin{aligned}\frac{\partial V_{\rm SG}}{\partial \phi^{I'}} &= G_{I'} \left(\frac{V_F}{M^2} + M^2 e^{G/M^2}\right) - F^I K^J_{II'} F_J - M e^{G/2M^2} G_{II'} F^I \\ &\quad - \frac{1}{2}({\rm Re}f_{\alpha\beta})_{I'} D^\alpha D^\beta - (\phi^\dagger T^\alpha)_I G^I_{I'} D^\alpha \ ,\end{aligned} \tag{3·17}$$

where $V_F \equiv F^I K^J_I F_J - 3M^4 e^{G/M^2}$. Taking its VEV and using the stationary condition, we derive the formula:

$$\begin{aligned}&\langle F^I\rangle\langle K^J_{II'}\rangle\langle F_J\rangle + m_{3/2}\langle G_{II'}\rangle\langle F^I\rangle + \frac{1}{2}\langle({\rm Re}f_{\alpha\beta})_{I'}\rangle\langle D^\alpha\rangle\langle D^\beta\rangle \\ &\quad + \langle(\phi^\dagger T^\alpha)_I\rangle\langle G^I_{I'}\rangle\langle D^\alpha\rangle = \langle G_{I'}\rangle\left(\frac{\langle V_F\rangle}{M^2} + m^2_{3/2}\right) \ ,\end{aligned} \tag{3·18}$$

where $m_{3/2}$ is the gravitino mass given by

$$m_{3/2} = \langle M e^{G/2M^2}\rangle = |\langle e^{K/2M^2} W/M^2\rangle| \ . \tag{3·19}$$

By multiplying $(T^{\alpha'}\phi)^{I'}$ with (3·17) and using the identities derived from the gauge invariance of the total Kähler potential,

$$G_{II'}(T^{\alpha'}\phi)^{I'} + G_{I'}(T^{\alpha'})^{I'}_I - K^J_I(\phi^\dagger T^{\alpha'})_J = 0 \ , \tag{3·20}$$

$$K^J_{II'}(T^{\alpha'}\phi)^{I'} + K^J_{I'}(T^{\alpha'})^{I'}_I - ((\phi^\dagger T^{\alpha'})_{J'} G^{J'})^J_I = 0 \ , \tag{3·21}$$

we obtain

$$\begin{aligned}\frac{\partial V}{\partial \phi^{I'}}(T^{\alpha'}\phi)^{I'} &= \left(\frac{V_F}{M^2} + 2M^2 e^{G/M^2}\right) G_{I'}(T^{\alpha'}\phi)^{I'} - F^I (G_{I'}(T^{\alpha'}\phi)^{I'})^J_I F_J \\ &\quad - \frac{1}{2}({\rm Re}f_{\alpha\beta})_{I'}(T^{\alpha'}\phi)^{I'} D^\alpha D^\beta - (\phi^\dagger T^\alpha)_I G^I_{I'}(T^{\alpha'}\phi)^{I'} D^\alpha \ .\end{aligned} \tag{3·22}$$

Taking its VEV and using the stationary condition, we derive the formula:[31)]

$$\langle F^I \rangle \left\langle (G_{I'}(T^{\alpha'}\phi)^{I'})^J_I \right\rangle \langle F_J \rangle + \frac{1}{2}\langle (\mathrm{Re}f_{\alpha\beta})_{I'} \rangle \langle (T^{\alpha'}\phi)^{I'} \rangle \langle D^\alpha \rangle \langle D^\beta \rangle$$
$$+ \left((\hat{M}_V^2)^{\alpha\alpha'} + \left(\frac{\langle V_F \rangle}{M^2} + 2m_{3/2}^2 \right) \langle \mathrm{Re}f_{\alpha\alpha'} \rangle \right) \langle D^\alpha \rangle = 0 \ . \tag{3·23}$$

The VEV of V_{SG} is given by

$$\langle V_{\mathrm{SG}} \rangle \equiv \langle F^I \rangle \langle K_I^J \rangle \langle F_J \rangle - 3m_{3/2}^2 M^2 + \frac{1}{2}\langle \mathrm{Re}f_{\alpha\beta} \rangle \langle D^\alpha \rangle \langle D^\beta \rangle \ . \tag{3·24}$$

By the requirement of $\langle V_{\mathrm{SG}} \rangle = 0$, the relations $\langle F^I \rangle = O(m_{3/2}M)$ and/or $\langle D^\alpha \rangle = O(m_{3/2}M)$ are derived for some components. If the soft SUSY-breaking terms are related to the SUSY extension of SM directly, the magnitude of $m_{3/2}$ should be the same size as or less than $O(1)$ TeV. In this case with $(\hat{M}_V^2)^a \gg m_{3/2}^2$, the soft SUSY breaking terms are treated as a perturbation and the same result (1) is obtained with the following upper bound for the magnitude of dominant SUSY breaking F component,

$$\langle D^a \rangle \leq O(\langle F^I \rangle) \leq O(m_{3/2}M) \ . \tag{3·25}$$

If the gauge symmetry breaking scale is $O(M)$, the following strong constraint is derived,[31)]

$$\langle D^a \rangle \leq O(m_{3/2}^2) \ . \tag{3·26}$$

In this case, the relation $m_{3/2}^2 = \frac{\langle F^I \rangle \langle K_I^J \rangle \langle F_J \rangle}{3M^2}$ holds.

By multiplying $(K^{-1})^{I'}_{I''} G^{I''}$ with (3·17), taking its VEV and using the stationary condition, we derive the formula:

$$\langle F^I \rangle \langle K^J_{II'} \rangle \langle F_J \rangle \langle (K^{-1})^{I'}_{I''} \rangle \langle G^{I''} \rangle - \langle G_{II'} \rangle \langle F^I \rangle \langle F^{I'} \rangle$$
$$+ \frac{1}{2m_{3/2}} \langle (\mathrm{Re}f_{\alpha\beta})_{I'} \rangle \langle F^{I'} \rangle \langle D^\alpha \rangle \langle D^\beta \rangle$$
$$= \frac{\langle F^I \rangle \langle K_I^J \rangle \langle F_J \rangle}{m_{3/2}^2} \left(\frac{\langle V_F \rangle}{M^2} + m_{3/2}^2 \right) + \langle \mathrm{Re}f_{\alpha\beta} \rangle \langle D^\alpha \rangle \langle D^\beta \rangle \ . \tag{3·27}$$

If the VEVs of all F-components vanish and $m_{3/2} \neq 0$, i.e., $\langle W \rangle \neq 0$, we obtain the relation

$$\langle \mathrm{Re}f_{\alpha\beta} \rangle \langle D^\alpha \rangle \langle D^\beta \rangle = 0 \ , \tag{3·28}$$

which means the D-flat conditions, $\langle D^\alpha \rangle = 0$. There exists a SUSY AdS vacuum if $\langle F^I \rangle = 0$ and $\langle W \rangle \neq 0$. This fact is directly understood from the definition (3·14) as follows. From (3·15), the conditions $\langle F^I \rangle = \langle F_J \rangle = 0$ for all species are equivalent to $\langle G^I \rangle = \langle G_J \rangle = 0$ for $\langle W \rangle \neq 0$. Then the D-flat conditions are derived from (3·14).

Finally, we comment on models with the Kachru-Kallosh-Linde-Trivedi (KKLT) moduli stabilization. In the KKLT compactification, the extra potential $V_{\rm lift}$ is introduced in order to uplift SUSY AdS vacua to dS vacua.[32] In this case, (3·23) is modified as

$$\begin{aligned}\langle F^I\rangle \left\langle (G_{I'}(T^{\alpha'}\phi)^{I'})^J_I\right\rangle \langle F_J\rangle + \frac{1}{2}\langle({\rm Re}f_{\alpha\beta})_{I'}\rangle\langle(T^{\alpha'}\phi)^{I'}\rangle\langle D^\alpha\rangle\langle D^\beta\rangle \\ + \left((\hat{M}_V^2)^{\alpha\alpha'} + \left(\frac{\langle V_F\rangle}{M^2} + 2m_{3/2}^2\right)\langle{\rm Re}f_{\alpha\alpha'}\rangle\right)\langle D^\alpha\rangle \\ = \langle\partial V_{\rm lift}/\partial\phi^{I'}\rangle\langle(T^{\alpha'}\phi)^{I'}\rangle\ . \qquad (3\cdot29)\end{aligned}$$

The formula (3·29) is a counterpart of (3.7) in Ref. 18). If the magnitude of the new term $\langle\partial V_{\rm lift}/\partial\phi^{I'}\rangle\langle(T^{\alpha'}\phi)^{I'}\rangle$ is negligibly small compared with those of other terms, the same result (1) holds.

§4. Conclusions and discussion

We have studied the magnitude of D-components in a generic framework of SUSY field theory. We have found that there exists F-component whose VEV is comparable to or higher than that of D-component in the absence of the FI term, the large hierarchy in the charge spectrum and strongly interacting higher-dimensional couplings in the Kähler potential, if contributions from terms other than F- and D-terms, such as soft SUSY-breaking terms or the uplifting potential, are negligible. If all F-components vanish, the SUSY is unbroken in most cases. Hence, F-components have the initiative in the breakdown of SUSY.

We have shown that the features of magnitude on $\langle D^\alpha\rangle$ and $\langle F^I\rangle$, which are obtained through explicit models, also hold in models with a generic Kähler potential and a generic gauge kinetic function if the Kähler potential contains no strongly interacting couplings and contributions from terms other than the F- and D-terms are negligibly small. Although we do not obtain completely new constraints, it would be meaningful to report our results and clarify our statement because it is applicable to a broad class of SUSY field theory, including effective theories derived from a fundamental theory.

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References

1) K. Intriligator and M. Sudano, J. High Energy Phys. **06** (2010), 047.
2) T. T. Dumitrescu, Z. Komargodski and M. Sudano, arXiv:1007.5352.
3) M. Drees, Phys. Lett. B **181** (1986), 279.
4) J. S. Hagelin and S. Kelley, Nucl. Phys. B **342** (1990), 95.
5) Y. Kawamura, H. Murayama and M. Yamaguchi, Phys. Lett. B **324** (1994), 52.
6) Y. Kawamura, H. Murayama and M. Yamaguchi, Phys. Rev. D **51** (1995), 1337.

7) Y. Kawamura and T. Kobayashi, Phys. Lett. B **375** (1996), 141; Phys. Rev. D **56** (1997), 3844.
8) E. Dudas, S. Pokorski and C. A. Savoy, Phys. Lett. B **369** (1996), 255.
9) E. Dudas, C. Grojean, S. Pokorski and C. A. Savoy, Nucl. Phys. B **481** (1996), 85.
10) Y. Kawamura, Phys. Lett. B **446** (1999), 228.
11) T. Higaki, Y. Kawamura, T. Kobayashi and H. Nakano, Phys. Rev. D **69** (2004), 086004.
12) E. Dudas and S. K. Vempati, Nucl. Phys. B **727** (2005), 139.
13) K. R. Dienes, C. Kolda and J. March-Russell, Nucl. Phys. B **492** (1997), 104.
14) I. Antoniadis and S. Hohenegger, Nucl. Phys. B (Proc. Suppl.) **171** (2007), 176.
15) Y. Nakayama, M. Taki, T. Watari and T. T. Yanagida, Phys. Lett. B **655** (2007), 58.
16) A. Pomarol and R. Rattazzi, J. High Energy Phys. **05** (1999), 013.
17) I. Jack and D. R. T. Jones, Phys. Lett. B **482** (2000), 167.
18) K. Choi and K.-S. Jeong, J. High Energy Phys. **08** (2006), 007.
19) P. J. Fox, A. E. Nelson and N. Weiner, J. High Energy Phys. **08** (2002), 035.
20) A. E. Nelson, N. Rius, V. Sanz and M. Unsal, J. High Energy Phys. **08** (2002), 039
21) F. Buccella, J. P. Derendinger, S. Ferrara and C. A. Savoy, Phys. Lett. B **115** (1982), 375.
22) J. Wess and J. Bagger, *Supersymmetry and Supergravity*, 2nd Edition (Princeton University Press, 1992).
23) P. Fayet and J. Iliopoulos, Phys. Lett. B **51** (1974), 461.
24) T. Gregoire, R. Rattazzi and C. A. Scrucca, Phys. Lett. B **624** (2005), 260.
25) J. Polchinski and L. Susskind, Phys. Rev. D **26** (1982), 3661.
26) M. Dine, A. E. Nelson, Y. Nir and Y. Shirman, Phys. Rev. D **53** (1996), 2658.
27) L. M. Carpenter, P. J. Fox and D. E. Kaplan, hep-ph/0503093.
28) L. Matos, arXiv:0910.0451.
29) E. Cremmer, B. Julia, J. Scherk, S. Ferrara, L. Girardello and P. van Nieuwenhuizen, Phys. Lett. B **79** (1978), 231; Nucl. Phys. B **147** (1979), 105.
30) E. Cremmer, S. Ferrara, L. Girardello and A. van Proeyen, Phys. Lett. B **116** (1982), 231; Nucl. Phys. B **212** (1983), 413.
31) I. Joichi, Y. Kawamura and M. Yamaguchi, hep-ph/9407385.
32) S. Kachru, R. Kallosh, A. Linde and S. P. Trivedi, Phys. Rev. D **68** (2003), 046005.