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Superparticle Sum Rules in the presence of Hidden Sector Dynamics

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ABSTRACT: We derive sum rules among scalar masses for various boundary conditions of the hidden-visible couplings in the presence of hidden sector dynamics and show that they still can be useful probes of the MSSM and beyond.

KEYWORDS: Supersymmery, Hidden sector, Grand unified theory.

Contents

1. Introduction

The supersymmetric (SUSY) extension of the standard model (SM) has been attractive as physics beyond the weak scale [1, 2]. The gauge coupling unification can be realized within the framework of the minimal supersymmetric standard model (MSSM), under the assumption of 'desert' between the TeV scale and the unification scale [3, 4, 5, 6]. It is natural to expect that a similar unification occurs for soft SUSY breaking parameters at some high-energy scale, reflecting a physics beyond the MSSM [7, 8, 9, 10].

It is, however, pointed out that hidden sector interactions can give rise to sizable effects on renormalization group (RG) evolutions of soft SUSY breaking parameters and some modifications of ordinary analysis are necessary [11].[1] Cohen et al. have derived sfermion mass relations at the TeV scale in the presence of hidden sector dynamics, under the assumption that a coupling between the MSSM chiral fields and hidden vector superfield operators are universal at a unification scale and the hidden sector is not within the conformal regime [17]. It is important to examine whether sfermion masses can be useful probes for a high-energy physics, in the case that the coupling universality is relaxed with SUSY grand unified theories (GUTs) in mind.

[1] Conformal sequestering and its phenomenological implications were studied in Refs. [12, 13, 14, 15, 16].

In this paper, we derive sum rules among scalar masses for various boundary conditions of the hidden-visible couplings in the presence of hidden sector effects outside the conformal regime. We show that their sum rules still can be useful probes of the MSSM and beyond.

The contents of this paper are as follows. In section 2, we study a modification of RG evolution for scalar masses by the hidden sector interactions. In section 3, specific sum rules among scalar masses are derived for various boundary conditions of the hidden-visible couplings. In section 4, sum rules among sfermion masses are also studied for orbifold family unification models. Section 5 is devoted to conclusions and discussions.

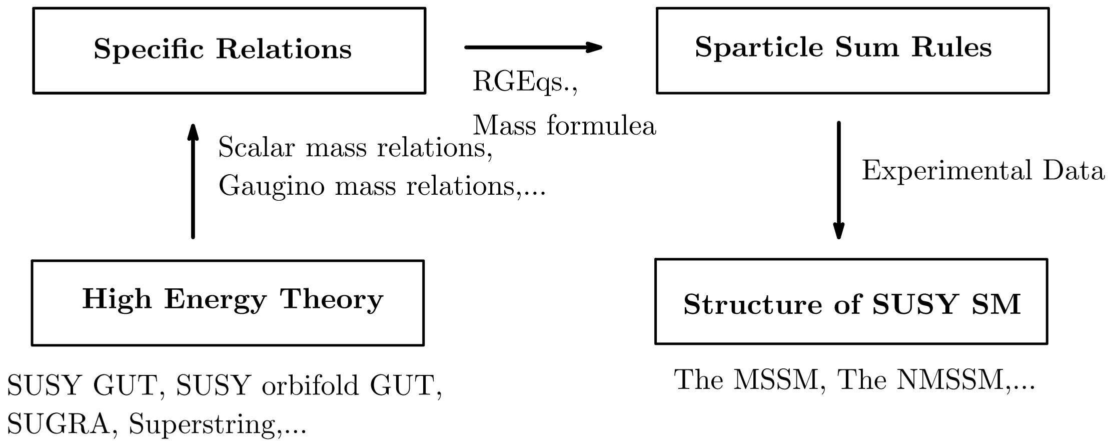
2. Renormalization group evolution of scalar masses

2.1 Basic assumptions

First we list assumptions adopted in our analysis.

1. The theory beyond the SM is the MSSM. Here the MSSM means the SUSY extension of the SM with the minimal particle contents, without specifying the structure of soft SUSY breaking terms. The superpartners and Higgs bosons have a mass whose magnitude is, at most, of order TeV scale. We neglect the threshold correction at the TeV scale due to the mass difference among the MSSM particles. Further the TeV scale is often identified with the weak scale (M_{EW}) for simplicity.
2. The MSSM holds from TeV scale to a high energy scale (M). Above M, there is a new physics. Possible candidates are supergravity (SUGRA), SUSY GUT and/or SUSY orbifold GUT. There is a big desert between M_{EW} and M in our visible sector.
3. The SUSY is broken in a hidden sector at the intermediate scale (M_I) and the effect is mediated to the visible sector as the appearance of soft SUSY breaking terms. The hidden sector fields are dynamical from M to M_I. The pattern of soft SUSY breaking parameters reflects on symmetries, the mechanism of SUSY breaking and the way of its mediation. We do not specify the mechanism of SUSY breaking. In most cases, we assume that the gravity mediation is dominant and soft SUSY breaking terms respect the gauge invariance. After the breakdown of gauge symmetry, there appear extra contributions to soft SUSY breaking parameters, which do not respect the gauge symmetry any more, e.g., D-term contributions [18, 19, 9, 10]. In most case, we consider only D-contribution for the electroweak symmetry breaking for simplicity.
4. The pattern of Yukawa couplings reflects flavor structure in a high-energy theory. We assume that a suitable pattern of Yukawa couplings is obtained in the low-energy effective theory. We neglect effects of Yukawa couplings concerning to the first two generations and those of the off-diagonal ones because they are small compared with the third generation ones.
5. The sufficient suppression of flavor-changing neutral currents (FCNC) processes requires the mass degeneracy for each squark and slepton species in the first two generations unless those masses are rather heavy or fermion and its superpartner mass matrices are aligned. We assume that the generation-changing entries in the sfermion mass matrices are sufficiently small in the basis where fermion mass matrices are diagonal. At first, we derive sum rules without the requirement of mass degeneracy and after that we give a brief comment

Figure 1: Outline of strategy



on the case with the degenerate masses.
6. After some parameters are made real by the rephasing of fields, CP violation occurs if the rest are complex. We assume that Yukawa couplings are dominant as a source of CP violation and other parameters are real.

2.2 Our strategy

We expect that Higgs bosons and superpartners are discovered and these masses and coupling constants are measured precisely in the large hadron collider (LHC) or e^+e^- linear collider. The process of Higgs bosons and superpartner hunting depends on the pattern of SUSY spectrum.[20, 21] The resultant particle contents and spectrum can answer the question whether the MSSM or its extension describes physics beyond the SM. If the answer is affirmative, values of various parameters are obtained from experimental data.

We explain how the sum rules can be tested and how a high energy physics can be revealed through future experimental data. Our strategy to explore the structure of SUSY SM and beyond is outlined in Figure 1. Let us construct a high energy theory with particular particle contents and symmetries. There, in general, exist specific relations among parameters at M reflecting the structure of high energy theory. Each parameter receives RG effects, and the value at M_{EW} is calculated by using RG equations. Hence sum rules among sparticle masses at M_{EW} are obtained from relevant specific relations at M using RG equations and mass formulae in the SUSY SM. By checking whether such sum rules hold or not using experimental data, we can find what kind of high energy theory is hopeful and see the particle assignment and symmetries at M indirectly. In this way, we expect that specific relations and sum rules can be useful to probe a physics beyond the MSSM in the near future and the structure of SUSY SM and a high energy theory is determined simultaneously. Our subject is now to derive peculiar sum rules for each high energy theory.

2.3 Renormalization group evolutiuon

We study RG evolution of scalar mass parameters in the presence of hidden sector dynamics [11, 17]. The general hidden sector fields are given by chiral superfield operators X_x and vector superfield operators V_v whose auxiliary components are F_x and D_v, respectively. Those fields are treated as dynamical down to the intermediate scale M_I. Visible sector fields consist of chiral superfields $\Phi_{\tilde{F}}$ and spinor superfields W_i whose lowest components are scalar fields $\tilde{F}$ and the MSSM gauginos λ_i $(i = 1, 2, 3)$, respectively. The $\tilde{F}$ represents a multiplet of $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$, which contains the scalar partner of the SM fermions and two Higgs doublets h_1 and h_2, and are written by,

$$\tilde{F} = \begin{cases} \tilde{q}_1,\ \tilde{u}^*_R,\ \tilde{d}^*_R,\ \tilde{l}_1,\ \tilde{e}^*_R, \\ \tilde{q}_2,\ \tilde{c}^*_R,\ \tilde{s}^*_R,\ \tilde{l}_2,\ \tilde{\mu}^*_R, \\ \tilde{q}_3,\ \tilde{t}^*_R,\ \tilde{b}^*_R,\ \tilde{l}_3,\ \tilde{\tau}^*_R, \\ h_1,\ h_2, \end{cases} \tag{2.1}$$

where $\tilde{q}_1$ means the first generation scalar quark (squark) doublet, $\tilde{u}^*_R$ up squark singlet, $\tilde{d}^*_R$ down squark singlet, $\tilde{l}_1$ the first generation scalar lepton (slepton) doublet, $\tilde{e}^*_R$ selectron singlet and so on. The astrisk means its complex conjugate.

The hidden-visible couplings are given by

$$\begin{aligned} &\sum_{\tilde{F}} \int d^4\theta \sum_v k^{(v)}_{\tilde{F}} \frac{V_v}{M^2} \Phi^\dagger_{\tilde{F}} \Phi_{\tilde{F}} \\ &\quad + \sum_i \int d^2\theta \sum_x w^{(x)}_i \frac{X_x}{M} W_i W_i + \text{h.c.} \\ &\quad + \sum_r \int d^2\theta \sum_x a^{(x)}_r f_r \frac{X_x}{M} \Phi_{\tilde{F}} \Phi_{\tilde{F}'} \Phi_{\tilde{F}''} + \text{h.c.} \\ &\quad + \int d^2\theta \sum_x b^{(x)} \mu \frac{X_x}{M} H_1 H_2 + \text{h.c.}, \end{aligned} \tag{2.2}$$

where h.c. means the hermitian conjugate of the former term and r represents indices regarding trilinear couplings (and Yukawa couplings) among visible sector fields, e.g.,

$$r = \begin{cases} t \ \text{ for } \ (\tilde{q}_3, \tilde{t}^*_R, h_2), \\ b \ \text{ for } \ (\tilde{q}_3, \tilde{b}^*_R, h_1), \\ \tau \ \text{ for } \ (\tilde{l}_3, \tilde{\tau}^*_R, h_1). \end{cases} \tag{2.3}$$

In (2.2), we assume that there is no flavor mixing in the first term and trilinear couplings among visible sector fields exist only in the third generation. Scalar mass-squareds $m^2_{\tilde{F}}$, gaugino masses M_i, A-parameters and B-parameter are given by

$$m^2_{\tilde{F}}(t_I) = \sum_v k^{(v)}_{\tilde{F}}(t_I) \frac{\langle D_v \rangle}{M^2}, \tag{2.4}$$

$$M_i(t_I) = \sum_x w^{(x)}_i(t_I) \frac{\langle F_x \rangle}{M} g^2_i(t_I), \tag{2.5}$$

$$A_r(t_I) = \sum_x a_r^{(x)}(t_I)\frac{\langle F_x\rangle}{M}, \tag{2.6}$$

$$B(t_I) = \sum_x b^{(x)}(t_I)\frac{\langle F_x\rangle}{M}, \tag{2.7}$$

where $t_I \equiv \frac{1}{2\pi}\ln(M/M_I)$ and g_is are gauge couplings of $G_{\rm SM}$. The RG equation regarding $k_{\tilde{F}}^{(v)}$ is given by

$$\begin{aligned}\frac{\rm d}{{\rm d}t}k_{\tilde{F}}^{(v)} &= -\sum_{v'}\gamma_{vv'}k_{\tilde{F}}^{(v')} + \frac{1}{8\pi}\sum_i 8C_2^{(i)}(\tilde{F})g_i^6 G_i^{(v)} \\ &\quad - \frac{1}{4\pi}\frac{3}{5}Y(\tilde{F})g_1^2 k_S^{(v)} - \frac{1}{4\pi}\sum_r n_{\tilde{F}}^{(r)} f_r^2\left(k_r^{(v)} + h_r^{(v)}\right),\end{aligned} \tag{2.8}$$

where $t \equiv \frac{1}{2\pi}\ln(M/\mu)$ and μ is the renormalization scale. The $\gamma_{vv'}$ is the anomalous dimension matrix of V_v. The $C_2^{(i)}(\tilde{F})$ and $Y(\tilde{F})$ represent the eigenvalues of second Casimir operator (e.g., $C_2^{(3)}(\tilde{q}_1) = 4/3$, $C_2^{(2)}(\tilde{q}_1) = 3/4$ and $C_2^{(1)}(\tilde{q}_1) = 1/60$) and hypercharge for $\tilde{F}$, respectively. The $n_{\tilde{F}}^{(r)}$ are given by

$$\begin{aligned}&n_{\tilde{t}_L}^{(t)} = n_{\tilde{t}_L}^{(b)} = n_{\tilde{b}_L}^{(t)} = n_{\tilde{b}_L}^{(b)} = n_{\tilde{\nu}_{\tau L}}^{(\tau)} = n_{\tilde{\tau}_L}^{(\tau)} = n_{h_1}^{(\tau)} = 1, \\ &n_{\tilde{t}_R^*}^{(t)} = n_{\tilde{b}_R^*}^{(b)} = n_{\tilde{\tau}_R^*}^{(\tau)} = 2, \quad n_{h_1}^{(b)} = n_{h_2}^{(t)} = 3.\end{aligned} \tag{2.9}$$

The $G_i^{(v)}$, $k_S^{(v)}$, $k_r^{(v)}$ and $h_r^{(v)}$ are defined by

$$G_i^{(v)} \equiv \sum_{x,x'} w_i^{*(x)} J_{xx'}^{(v)} w_i^{(x')}, \quad k_S^{(v)} \equiv \sum_{\tilde{F}} Y(\tilde{F}) n_{\tilde{F}} k_{\tilde{F}}^{(v)}, \tag{2.10}$$

$$k_t^{(v)} \equiv k_{\tilde{q}_3}^{(v)} + k_{\tilde{t}_R^*}^{(v)} + k_{h_2}^{(v)}, \quad k_b^{(v)} \equiv k_{\tilde{q}_3}^{(v)} + k_{\tilde{b}_R^*}^{(v)} + k_{h_1}^{(v)}, \quad k_\tau^{(v)} \equiv k_{\tilde{l}_3}^{(v)} + k_{\tilde{\tau}_R^*}^{(v)} + k_{h_1}^{(v)}, \tag{2.11}$$

$$h_r^{(v)} \equiv \sum_{x,x'} a_r^{*(x)} J_{xx'}^{(v)} a_r^{(x')}, \tag{2.12}$$

where $J_{xx'}^{(v)}$ stands for a factor from the interaction among X_x, $X_{x'}$ and V_v, and $n_{\tilde{F}}$ represents degrees of freedom for $\tilde{F}$. The $k_S^{(v)}$ yields the following RG equation,

$$\frac{\rm d}{{\rm d}t}k_S^{(v)} = -\sum_{v'}\left(\gamma_{vv'} + b_1\alpha_1\delta_{vv'}\right)k_S^{(v')}. \tag{2.13}$$

By integrating (2.8) and (2.13), we obtain the following expressions for $k_{\tilde{F}}(t)$ and $k_S(t)$:

$$\begin{aligned}k_{\tilde{F}}(t) &= {\rm P}\exp\left(-\int_0^t dt'\gamma(t')\right)k_{\tilde{F}}(0) \\ &+ \frac{1}{8\pi}\sum_i 8C_2^{(i)}(\tilde{F})\int_0^t ds{\rm P}\exp\left(-\int_s^t dt'\gamma(t')\right)g_i^6(s)G_i(s) \\ &- \frac{1}{4\pi}\frac{3}{5}Y(\tilde{F})\int_0^t ds{\rm P}\exp\left(-\int_s^t dt'\gamma(t')\right)g_1^2(s)k_S(s)\end{aligned}$$

$$- \frac{1}{4\pi} \sum_r n_{\tilde{F}}^{(r)} \int_0^t ds \mathrm{P} \exp \left(- \int_s^t dt' \gamma(t') \right) f_r^2(s) \left(k_r(s) + h_r(s) \right), \tag{2.14}$$

$$k_S(t) = \mathrm{P} \exp \left(- \int_0^t dt' \left(\gamma(t') + b_1 \alpha_1(t') \right) \right) k_S(0), \tag{2.15}$$

where the index v is suppressed and P represents the path-ordered exponentials. Scalar mass squareds $m_{\tilde{F}}^2(t_I)$ are obtained by inserting (2.14) into the formula (2.4). Further the $m_{\tilde{F}}^2$ at the TeV scale (M_{EW}) are written by

$$\begin{aligned} m_{\tilde{F}}^2(t_{EW}) &= m_{\tilde{F}}^2(t_I) + \sum_{i=1}^3 \frac{2C_2^{(i)}(\tilde{F})}{b_i} \left(M_i^2(t_I) - M_i^2(t_{EW}) \right) \\ &\quad + \frac{3}{5b_1} Y(\tilde{F}) \left(S(t_{EW}) - S(t_I) \right) + \sum_r n_{\tilde{F}}^{(r)} \left(F_r(t_{EW}) - F_r(t_I) \right) \\ &= N_{\tilde{F}} + \sum_{i=1}^3 C_2^{(i)}(\tilde{F}) N_i + Y(\tilde{F}) N_S + \sum_r n_{\tilde{F}}^{(r)} N_r, \end{aligned} \tag{2.16}$$

where $t_{EW} \equiv \dfrac{1}{2\pi} \ln(M_I/M_{EW})$. In the final expression, $N_{\tilde{F}}$, N_i, N_S and N_r are defined by

$$N_{\tilde{F}} \equiv \sum_v \frac{\langle D \rangle}{M^2} \mathrm{P} \exp \left(- \int_0^{t_I} dt' \gamma(t') \right) k_{\tilde{F}}(0), \tag{2.17}$$

$$\begin{aligned} N_i &\equiv \frac{1}{\pi} \sum_v \frac{\langle D \rangle}{M^2} \int_0^{t_I} ds \mathrm{P} \exp \left(- \int_s^{t_I} dt' \gamma(t') \right) g_i^6(s) G_i(s) \\ &\quad + \frac{2}{b_i} \left(M_i^2(t_I) - M_i^2(t_{EW}) \right), \end{aligned} \tag{2.18}$$

$$\begin{aligned} N_S &\equiv -\frac{1}{4\pi} \frac{3}{5} \sum_v \frac{\langle D \rangle}{M^2} \int_0^{t_I} ds \mathrm{P} \exp \left(- \int_s^{t_I} dt' \gamma(t') \right) g_1^2(s) k_S(s) \\ &\quad + \frac{3}{5b_1} \left(S(t_{EW}) - S(t_I) \right), \end{aligned} \tag{2.19}$$

$$\begin{aligned} N_r &\equiv -\frac{1}{4\pi} \sum_v \frac{\langle D \rangle}{M^2} \int_0^{t_I} ds \mathrm{P} \exp \left(- \int_s^{t_I} dt' \gamma(t') \right) f_r^2(s) \left(k_r(s) + h_r(s) \right) \\ &\quad + F_r(t_{EW}) - F_r(t_I), \end{aligned} \tag{2.20}$$

where we use the conventional RG equations in the MSSM from t_I to t_{EW} such that[22, 23, 24, 25]

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t} m_{\tilde{F}}^2 &= 4 \sum_{i=1}^3 C_2^{(i)}(\tilde{F}) \alpha_i M_i^2 - \frac{3}{5} Y(\tilde{F}) \alpha_1 S \\ &\quad - \sum_r n_{\tilde{F}}^{(r)} \frac{f_r^2}{4\pi} \left({\sum}'_{\tilde{F}} m_{\tilde{F}}^2 + A_r^2 \right), \end{aligned} \tag{2.21}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} S = -b_1 \alpha_1 S, \quad S \equiv \sum_{\tilde{F}} Y(\tilde{F}) n_{\tilde{F}} m_{\tilde{F}}^2. \tag{2.22}$$

Here $\sum'_{\tilde{F}}$ means a sum among scalar masses relating to Yukawa interactions. The F_rs in (2.16) and (2.20) stand for contributions from Yukawa interactions and satisfy the following equations

$$\frac{\mathrm{d}}{\mathrm{d}t} F_t = \frac{f_t^2}{4\pi} \left(m_{\tilde{q}_3}^2 + m_{\tilde{t}_R}^2 + m_{h_2}^2 + A_t^2 \right), \tag{2.23}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} F_b = \frac{f_b^2}{4\pi} \left(m_{\tilde{q}_3}^2 + m_{\tilde{b}_R}^2 + m_{h_1}^2 + A_b^2 \right), \tag{2.24}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} F_\tau = \frac{f_\tau^2}{4\pi} \left(m_{\tilde{l}_3}^2 + m_{\tilde{\tau}_R}^2 + m_{h_1}^2 + A_\tau^2 \right). \tag{2.25}$$

Complete analytic solutions for F_t, F_b and F_τ are not known and those values are determined numerically by solving RG equations of sparticle masses and coupling constants. We treat $N_{\tilde{F}}$, N_i, N_S and N_r as free parameters because $\gamma(t')$ is an unknown function, which reflects on the hidden sector dynamics.

After the breakdown of electroweak symmetry, two kinds of contributions are added to sfermion masses, i.e., fermion masses (m_f) and the D-term contribution ($D_W(\tilde{f})$) relating to the generator of the broken symmetry $(SU(2)_L \times U(1)_Y)/U(1)_{EM}$. The diagonal elements ($M_{\tilde{f}}^2$) of sfermion mass-squared matrices at M_{EW} are written as

$$\begin{aligned} M_{\tilde{f}}^2 &= m_{\tilde{F}}^2 + m_f^2 + D_W(\tilde{f}), \\ &= N_{\tilde{F}} + \sum_{i=1}^{3} C_2^{(i)}(\tilde{F}) N_i + Y(\tilde{F}) N_S + \sum_r n_{\tilde{F}}^{(r)} N_r + m_f^2 + D_W(\tilde{f}). \end{aligned} \tag{2.26}$$

where $\tilde{f}$ means the scalar partner of fermion species f. The fs are given by

$$f = \begin{cases} u_L,\ d_L,\ u_R,\ d_R,\ \nu_{eL},\ e_L,\ e_R, \\ c_L,\ s_L,\ c_R,\ s_R,\ \nu_{\mu L},\ \mu_L,\ \mu_R, \\ t_L,\ b_L,\ t_R,\ b_R,\ \nu_{\tau L},\ \tau_L,\ \tau_R. \end{cases} \tag{2.27}$$

The $D_W(\tilde{f})$ are given by

$$\begin{aligned} D_W(\tilde{f}) &= \left(T_L^3(\tilde{f}) - Q(\tilde{f}) \sin^2\theta_W \right) M_Z^2 \cos 2\beta \\ &= \left(\left(T_L^3 - Q(\tilde{f}) \right) M_Z^2 + Q(\tilde{f}) M_W^2 \right) \cos 2\beta \quad (f = u_L, \cdots \tau_L), \end{aligned} \tag{2.28}$$

$$\begin{aligned} D_W(\tilde{f}) &= Q(\tilde{f}) \sin^2\theta_W M_Z^2 \cos 2\beta \\ &= Q(\tilde{f}) \left(M_Z^2 - M_W^2 \right) \cos 2\beta \qquad\qquad (f = u_R, \cdots \tau_R). \end{aligned} \tag{2.29}$$

The off-diagonal elements of sfermion mass-squared matrices are proportional to the corresponding fermion mass. For the first two generations, the diagonal ones $M_{\tilde{f}}^2$ are regarded as 'physical masses' which are eigenvalues of mass-squared matrices because the off-diagonal ones are negligibly small. Using the mass formula (2.26), values of $m_{\tilde{F}}^2$ can be determined for the first two generations. For the third generation, mass-squared matrices are given by

$$\begin{pmatrix} m_{\tilde{t}_L}^2 + m_t^2 + D_W(\tilde{t}_L) & -m_t(A_t + \mu \cot\beta) \\ -m_t(A_t + \mu\cot\beta) & m_{\tilde{t}_R}^2 + m_t^2 + D_W(\tilde{t}_R) \end{pmatrix} \quad \text{(for top squarks)}, \tag{2.30}$$

$$\begin{pmatrix} m^2_{\tilde{b}_L} + m_b^2 + D_W(\tilde{b}_L) & -m_b(A_b + \mu\tan\beta) \\ -m_b(A_b + \mu\tan\beta) & m^2_{\tilde{b}_R} + m_b^2 + D_W(\tilde{b}_R) \end{pmatrix} \text{(for bottom squarks)}, \tag{2.31}$$

$$\begin{pmatrix} m^2_{\tilde{\tau}_L} + m_\tau^2 + D_W(\tilde{\tau}_L) & -m_\tau(A_\tau + \mu\tan\beta) \\ -m_\tau(A_\tau + \mu\tan\beta) & m^2_{\tilde{\tau}_R} + m_\tau^2 + D_W(\tilde{\tau}_R) \end{pmatrix} \quad \text{(for tau sleptons)}. \tag{2.32}$$

By diagonalized the above mass-squared matrices, we obtain mass eigenstates whose masses are physical ones, $(M_{\tilde{t}_1},\ M_{\tilde{t}_2})$ for top squarks, $(M_{\tilde{b}_1},\ M_{\tilde{b}_2})$ for bottom squarks and $(M_{\tilde{\tau}_1},\ M_{\tilde{\tau}_2})$ for tau sleptons. By using the feature of trace, we have the relations,

$$\begin{aligned} &M^2_{\tilde{t}_1} + M^2_{\tilde{t}_2} = M^2_{\tilde{t}_L} + M^2_{\tilde{t}_R}, \quad M^2_{\tilde{b}_1} + M^2_{\tilde{b}_2} = M^2_{\tilde{b}_L} + M^2_{\tilde{b}_R}, \\ &M^2_{\tilde{\tau}_1} + M^2_{\tilde{\tau}_2} = M^2_{\tilde{\tau}_L} + M^2_{\tilde{\tau}_R}. \end{aligned} \tag{2.33}$$

By diagonalizing the mass-squared matrices, we have the relations,

$$\left(M^2_{\tilde{t}_1} - M^2_{\tilde{t}_2}\right)^2 = \left(M^2_{\tilde{t}_L} - M^2_{\tilde{t}_R}\right)^2 + 4m_t^2\left(A_t + \mu\cot\beta\right)^2, \tag{2.34}$$

$$\left(M^2_{\tilde{b}_1} - M^2_{\tilde{b}_2}\right)^2 = \left(M^2_{\tilde{b}_L} - M^2_{\tilde{b}_R}\right)^2 + 4m_b^2\left(A_b + \mu\tan\beta\right)^2, \tag{2.35}$$

$$\left(M^2_{\tilde{\tau}_1} - M^2_{\tilde{\tau}_2}\right)^2 = \left(M^2_{\tilde{\tau}_L} - M^2_{\tilde{\tau}_R}\right)^2 + 4m_\tau^2\left(A_\tau + \mu\tan\beta\right)^2, \tag{2.36}$$

If A parameters are measured precisely, $m^2_{\tilde{F}}$s (and $M^2_{\tilde{f}}$s) in the third generation can be fixed by using the mass-squared matrices (2.30) - (2.32). From the fact that left-handed fermions (and its superpartners) form $SU(2)_L$ doublets, e.g., $q_1 = (u_L, d_L)$ (and $\tilde{q}_1 = (\tilde{u}_L, \tilde{d}_L)$), we obtain following sum rules among $SU(2)_L$ doublet sfermions [7, 8]:

$$M^2_{\tilde{u}_L} - M^2_{\tilde{d}_L} = m_u^2 - m_d^2 + M_W^2\cos 2\beta \simeq M_W^2\cos 2\beta, \tag{2.37}$$

$$M^2_{\tilde{\nu}_{eL}} - M^2_{\tilde{e}_L} = m^2_{\nu_{eL}} - m_e^2 + M_W^2\cos 2\beta \simeq M_W^2\cos 2\beta, \tag{2.38}$$

$$M^2_{\tilde{c}_L} - M^2_{\tilde{s}_L} = m_c^2 - m_s^2 + M_W^2\cos 2\beta \simeq M_W^2\cos 2\beta, \tag{2.39}$$

$$M^2_{\tilde{\nu}_{\mu L}} - M^2_{\tilde{\mu}_L} = m^2_{\nu_{\mu L}} - m_\mu^2 + M_W^2\cos 2\beta \simeq M_W^2\cos 2\beta, \tag{2.40}$$

$$M^2_{\tilde{t}_L} - M^2_{\tilde{b}_L} = m_t^2 - m_b^2 + M_W^2\cos 2\beta \simeq m_t^2 + M_W^2\cos 2\beta, \tag{2.41}$$

$$M^2_{\tilde{\nu}_{\tau L}} - M^2_{\tilde{\tau}_L} = m^2_{\nu_{\tau L}} - m_\tau^2 + M_W^2\cos 2\beta \simeq M_W^2\cos 2\beta, \tag{2.42}$$

where we neglect fermion masses except for the top quark mass in the final expressions. The above sum rules (2.37) - (2.42) are irrelevant to the structure of models beyond the MSSM, and hence the sfermion sector (and the breakdown of electroweak symmetry) in the MSSM can be tested by using them. We refer to these sum rules (2.37) - (2.42) as the electroweak symmetry (EWS) sum rules.

3. Sparticle sum rules

First of all, we write down the formula for each scalar mass at M_{EW} using the mass formula (2.26),

$$M^2_{\tilde{u}_L} = N_{\tilde{q}_1} + \frac{4}{3}N_3 + \frac{3}{4}N_2 + \frac{1}{60}N_1 + \frac{1}{6}N_S + \left(\frac{2}{3}M_W^2 - \frac{1}{6}M_Z^2\right)\cos 2\beta, \tag{3.1}$$

$$M^2_{\tilde{d}_L} = N_{\tilde{q}_1} + \frac{4}{3}N_3 + \frac{3}{4}N_2 + \frac{1}{60}N_1 + \frac{1}{6}N_S + \left(-\frac{1}{3}M_W^2 - \frac{1}{6}M_Z^2\right)\cos 2\beta, \tag{3.2}$$

$$M^2_{\tilde{u}_R} = N_{\tilde{u}^*_R} + \frac{4}{3}N_3 + \frac{4}{15}N_1 - \frac{2}{3}N_S + \left(-\frac{2}{3}M_W^2 + \frac{2}{3}M_Z^2\right)\cos 2\beta, \tag{3.3}$$

$$M^2_{\tilde{d}_R} = N_{\tilde{d}^*_R} + \frac{4}{3}N_3 + \frac{1}{15}N_1 + \frac{1}{3}N_S + \left(\frac{1}{3}M_W^2 - \frac{1}{3}M_Z^2\right)\cos 2\beta, \tag{3.4}$$

$$M^2_{\tilde{\nu}_{eL}} = N_{\tilde{l}_1} + \frac{3}{4}N_2 + \frac{3}{20}N_1 - \frac{1}{2}N_S + \frac{1}{2}M_Z^2\cos 2\beta, \tag{3.5}$$

$$M^2_{\tilde{e}_L} = N_{\tilde{l}_1} + \frac{3}{4}N_2 + \frac{3}{20}N_1 - \frac{1}{2}N_S + \left(-M_W^2 + \frac{1}{2}M_Z^2\right)\cos 2\beta, \tag{3.6}$$

$$M^2_{\tilde{e}_R} = N_{\tilde{e}^*_R} + \frac{3}{5}N_1 + N_S + \left(M_W^2 - M_Z^2\right)\cos 2\beta, \tag{3.7}$$

$$M^2_{\tilde{c}_L} = N_{\tilde{q}_2} + \frac{4}{3}N_3 + \frac{3}{4}N_2 + \frac{1}{60}N_1 + \frac{1}{6}N_S + \left(\frac{2}{3}M_W^2 - \frac{1}{6}M_Z^2\right)\cos 2\beta, \tag{3.8}$$

$$M^2_{\tilde{s}_L} = N_{\tilde{q}_2} + \frac{4}{3}N_3 + \frac{3}{4}N_2 + \frac{1}{60}N_1 + \frac{1}{6}N_S + \left(-\frac{1}{3}M_W^2 - \frac{1}{6}M_Z^2\right)\cos 2\beta, \tag{3.9}$$

$$M^2_{\tilde{c}_R} = N_{\tilde{c}^*_R} + \frac{4}{3}N_3 + \frac{4}{15}N_1 - \frac{2}{3}N_S + \left(-\frac{2}{3}M_W^2 + \frac{2}{3}M_Z^2\right)\cos 2\beta, \tag{3.10}$$

$$M^2_{\tilde{s}_R} = N_{\tilde{s}^*_R} + \frac{4}{3}N_3 + \frac{1}{15}N_1 + \frac{1}{3}N_S + \left(\frac{1}{3}M_W^2 - \frac{1}{3}M_Z^2\right)\cos 2\beta, \tag{3.11}$$

$$M^2_{\tilde{\nu}_{\mu L}} = N_{\tilde{l}_2} + \frac{3}{4}N_2 + \frac{3}{20}N_1 - \frac{1}{2}N_S + \frac{1}{2}M_Z^2\cos 2\beta, \tag{3.12}$$

$$M^2_{\tilde{\mu}_L} = N_{\tilde{l}_2} + \frac{3}{4}N_2 + \frac{3}{20}N_1 - \frac{1}{2}N_S + \left(-M_W^2 + \frac{1}{2}M_Z^2\right)\cos 2\beta, \tag{3.13}$$

$$M^2_{\tilde{\mu}_R} = N_{\tilde{\mu}^*_R} + \frac{3}{5}N_1 + N_S + \left(M_W^2 - M_Z^2\right)\cos 2\beta, \tag{3.14}$$

$$\begin{aligned} M^2_{\tilde{t}_L} &= N_{\tilde{q}_3} + \frac{4}{3}N_3 + \frac{3}{4}N_2 + \frac{1}{60}N_1 + \frac{1}{6}N_S + \left(\frac{2}{3}M_W^2 - \frac{1}{6}M_Z^2\right)\cos 2\beta \\ &\quad + N_t + N_b + m_t^2, \end{aligned} \tag{3.15}$$

$$\begin{aligned} M^2_{\tilde{b}_L} &= N_{\tilde{q}_3} + \frac{4}{3}N_3 + \frac{3}{4}N_2 + \frac{1}{60}N_1 + \frac{1}{6}N_S + \left(-\frac{1}{3}M_W^2 - \frac{1}{6}M_Z^2\right)\cos 2\beta \\ &\quad + N_t + N_b + m_b^2, \end{aligned} \tag{3.16}$$

$$\begin{aligned} M^2_{\tilde{t}_R} &= N_{\tilde{t}^*_R} + \frac{4}{3}N_3 + \frac{4}{15}N_1 - \frac{2}{3}N_S + \left(-\frac{2}{3}M_W^2 + \frac{2}{3}M_Z^2\right)\cos 2\beta \\ &\quad + 2N_t + m_t^2, \end{aligned} \tag{3.17}$$

$$\begin{aligned} M^2_{\tilde{b}_R} &= N_{\tilde{b}^*_R} + \frac{4}{3}N_3 + \frac{1}{15}N_1 + \frac{1}{3}N_S + \left(\frac{1}{3}M_W^2 - \frac{1}{3}M_Z^2\right)\cos 2\beta \\ &\quad + 2N_b + m_b^2, \end{aligned} \tag{3.18}$$

$$M^2_{\tilde{\nu}_{\tau L}} = N_{\tilde{l}_3} + \frac{3}{4}N_2 + \frac{3}{20}N_1 - \frac{1}{2}N_S + \frac{1}{2}M_Z^2\cos 2\beta + N_\tau, \tag{3.19}$$

$$M^2_{\tilde{\tau}_L} = N_{\tilde{l}_3} + \frac{3}{4}N_2 + \frac{3}{20}N_1 - \frac{1}{2}N_S + \left(-M_W^2 + \frac{1}{2}M_Z^2\right)\cos 2\beta + N_\tau + m_\tau^2, \tag{3.20}$$

$$M^2_{\tilde{\tau}_R} = N_{\tilde{\tau}^*_R} + \frac{3}{5}N_1 + N_S + \left(M_W^2 - M_Z^2\right)\cos 2\beta + 2N_\tau + m_\tau^2, \tag{3.21}$$

$$m_{h_1}^2 = N_{h_1} + \frac{3}{4}N_2 + \frac{3}{20}N_1 + \frac{1}{2}N_S + N_\tau + 3N_b, \tag{3.22}$$

$$m_{h_2}^2 = N_{h_2} + \frac{3}{4}N_2 + \frac{3}{20}N_1 - \frac{1}{2}N_S + 3N_t, \tag{3.23}$$

where we neglect effects of Yukawa couplings in the first two generations. Extra D-term contributions are not written because they depend on a large gauge group beyond the SM one. Hereafter we neglect m_b and m_τ for simplicity.

In the next section, we derive specific sum rules (except for the EWS sum rules) reflecting the structure of hidden-visible couplings for various ultra-violet (UV) boundary conditions

3.1 Universal type

Let us discuss the case with a universal hidden-visible coupling at M, i.e., $k_{\tilde{F}}^{(v)}(0) = k_0$. In this case, $N_{\tilde{F}}$ takes a common value and $N_S = 0$. There exists a specific sum rule among the first generation sfermion masses such as[8, 17]

$$2M_{\tilde{u}_R}^2 - M_{\tilde{d}_R}^2 - M_{\tilde{d}_L}^2 + M_{\tilde{e}_L}^2 - M_{\tilde{e}_R}^2 = \frac{10}{3}\left(M_Z^2 - M_W^2\right)\cos 2\beta. \tag{3.24}$$

There exist five kinds of sum rules among first and second generations sfermion masses such that

$$\begin{aligned} M_{\tilde{u}_L}^2 - M_{\tilde{c}_L}^2 = M_{\tilde{u}_R}^2 - M_{\tilde{c}_R}^2 = M_{\tilde{d}_R}^2 - M_{\tilde{s}_R}^2 \\ = M_{\tilde{e}_L}^2 - M_{\tilde{\mu}_L}^2 = M_{\tilde{e}_R}^2 - M_{\tilde{\mu}_R}^2 = 0. \end{aligned} \tag{3.25}$$

Further we obtain four kinds of sum rules including third generation sfermion masses and/or Higgs masses such that

$$2\left(M_{\tilde{u}_L}^2 - M_{\tilde{t}_L}^2 + m_t^2\right) = M_{\tilde{u}_R}^2 + M_{\tilde{d}_R}^2 - M_{\tilde{t}_R}^2 - M_{\tilde{b}_R}^2 + m_t^2, \tag{3.26}$$

$$2\left(M_{\tilde{e}_L}^2 - M_{\tilde{\tau}_L}^2\right) = M_{\tilde{e}_R}^2 - M_{\tilde{\tau}_R}^2, \tag{3.27}$$

$$2\left(m_{h_1}^2 - m_{h_2}^2\right) = 2\left(M_{\tilde{\tau}_L}^2 - M_{\tilde{e}_L}^2\right) + 3\left(M_{\tilde{b}_R}^2 - M_{\tilde{d}_R}^2 + M_{\tilde{u}_R}^2 - M_{\tilde{t}_R}^2 + m_t^2\right), \tag{3.28}$$

$$2\left(m_{h_2}^2 - M_{\tilde{e}_L}^2\right) = 3\left(M_{\tilde{t}_R}^2 - M_{\tilde{u}_R}^2 - m_t^2\right) + \left(2M_W^2 - M_Z^2\right)\cos 2\beta. \tag{3.29}$$

If all paremeters were measured precisely enough, these sum rules can be powerful tools to test the universality of $k_{\tilde{F}}^{(v)}$ at M.

Here we give comments for a later convenience. In the case with a non-vanishing N_S, the sum rules (3.25), (3.26), (3.27) and (3.29) hold on.[2] In the case that D-term contributions are independent of the generation, the sum rules (3.25), (3.26) and (3.27) still hold in their presence. In the case that all Yukawa couplings except for the top Yukawa is negligibly small, the following two extra sum rules are derived,

$$M_{\tilde{e}_R}^2 - M_{\tilde{\tau}_R}^2 = 0, \quad M_{\tilde{d}_R}^2 - M_{\tilde{b}_R}^2 = 0. \tag{3.30}$$

[2]If there were extra heavy scalar particles with hypercharge that couple to the hidden sector fields non-universally, N_S would not vanish.

The six predictions (3.26) – (3.30) have been derived in [17]. The most common sum rules are derived in the case with the universal hidden-visible coupling at M.[7]

In the following subsections, we will find that some of sum rules (3.24) – (3.29) survive after the coupling universality is relaxed. The less universality among couplings the hidden and visible fields yield, the less sum rules hold. Sum rules survived depend on the boundary condition for hidden-visible couplings as shown for $SU(5)$ type, $SO(10)$ type, $SU(5)\times U(1)_F$ type, $SU(4)\times SU(2)_L\times SU(2)_R$ type and $SU(3)_C\times SU(3)_L\times SU(3)_R$ type. Hence they still can be useful probes of the MSSM and beyond.

3.2 $SU(5)$ type

We consider the case with $SU(5)$ symmetry in the hidden-visible couplings. In this case, the following relations hold,

$$\begin{aligned}&N_{\tilde{q}_1}=N_{\tilde{u}_R^*}=N_{\tilde{e}_R^*},\quad N_{\tilde{d}_R^*}=N_{\tilde{l}_1},\quad N_{\tilde{q}_2}=N_{\tilde{c}_R^*}=N_{\tilde{\mu}_R^*},\quad N_{\tilde{s}_R^*}=N_{\tilde{l}_2},\\&N_{\tilde{q}_3}=N_{\tilde{t}_R^*}=N_{\tilde{\tau}_R^*},\quad N_{\tilde{b}_R^*}=N_{\tilde{l}_3}.\end{aligned}\tag{3.31}$$

Using these relations (3.31), we derive the following three kinds of sum rules

$$M^2_{\tilde{u}_L}-M^2_{\tilde{c}_L}=M^2_{\tilde{u}_R}-M^2_{\tilde{c}_R}=M^2_{\tilde{e}_R}-M^2_{\tilde{\mu}_R},\tag{3.32}$$

$$M^2_{\tilde{d}_R}-M^2_{\tilde{s}_R}=M^2_{\tilde{e}_L}-M^2_{\tilde{\mu}_L}.\tag{3.33}$$

If $N_S=0$, (3.24) holds.

3.3 $SO(10)$ type

We consider the case with $SO(10)$ symmetry in the hidden-visible couplings. In this case, the following relations hold,

$$\begin{aligned}&N_{\tilde{q}_1}=N_{\tilde{u}_R^*}=N_{\tilde{e}_R^*}=N_{\tilde{d}_R^*}=N_{\tilde{l}_1},\quad N_{\tilde{q}_2}=N_{\tilde{c}_R^*}=N_{\tilde{\mu}_R^*}=N_{\tilde{s}_R^*}=N_{\tilde{l}_2},\\&N_{\tilde{q}_3}=N_{\tilde{t}_R^*}=N_{\tilde{\tau}_R^*}=N_{\tilde{b}_R^*}=N_{\tilde{l}_3},\quad N_{h_1}=N_{h_2}.\end{aligned}\tag{3.34}$$

Using these relations (3.34), we derive (3.24) and (3.26) and the following four kinds of sum rules

$$M^2_{\tilde{u}_L}-M^2_{\tilde{c}_L}=M^2_{\tilde{u}_R}-M^2_{\tilde{c}_R}=M^2_{\tilde{e}_R}-M^2_{\tilde{\mu}_R}=M^2_{\tilde{d}_R}-M^2_{\tilde{s}_R}=M^2_{\tilde{e}_L}-M^2_{\tilde{\mu}_L}.\tag{3.35}$$

In the presence of D-term contribution related to $SO(10)/SU(5)$ generator, the above sum rules (3.26) and (3.35) still hold on. A similar feature holds on for the following partially unified types.

3.4 $SU(5)\times U(1)_{\mathrm{F}}$ type

We consider the case with a flipped $SU(5)$ symmetry in the hidden-visible couplings. In this case, the following relations hold,

$$\begin{aligned}&N_{\tilde{q}_1}=N_{\tilde{d}_R^*},\quad N_{\tilde{u}_R^*}=N_{\tilde{l}_1},\quad N_{\tilde{q}_2}=N_{\tilde{s}_R^*},\quad N_{\tilde{c}_R^*}=N_{\tilde{l}_2},\\&N_{\tilde{q}_3}=N_{\tilde{b}_R^*},\quad N_{\tilde{t}_R^*}=N_{\tilde{l}_3}.\end{aligned}\tag{3.36}$$

Using these relations (3.36), we derive the following two kinds of sum rules

$$M^2_{\tilde{u}_L}-M^2_{\tilde{c}_L}=M^2_{\tilde{d}_R}-M^2_{\tilde{s}_R},\quad M^2_{\tilde{u}_R}-M^2_{\tilde{c}_R}=M^2_{\tilde{e}_L}-M^2_{\tilde{\mu}_L}.\tag{3.37}$$

3.5 $SU(4) \times SU(2)_L \times SU(2)_R$ type

We consider the case with $SU(4) \times SU(2)_L \times SU(2)_R$ symmetry in the hidden-visible couplings. In this case, the following relations hold,

$$
\begin{aligned}
& N_{\tilde{q}_1} = N_{\tilde{l}_1}, \quad N_{\tilde{u}_R^*} = N_{\tilde{d}_R^*} = N_{\tilde{e}_R^*}, \quad N_{\tilde{q}_2} = N_{\tilde{l}_2}, \quad N_{\tilde{c}_R^*} = N_{\tilde{s}_R^*} = N_{\tilde{\mu}_R^*}, \\
& N_{\tilde{q}_3} = N_{\tilde{l}_3}, \quad N_{\tilde{t}_R^*} = N_{\tilde{b}_R^*} = N_{\tilde{\tau}_R^*}, \quad N_{h_1} = N_{h_2}.
\end{aligned} \tag{3.38}
$$

Using these relations (3.38), we derive the following three kinds of sum rules

$$
M_{\tilde{u}_L}^2 - M_{\tilde{c}_L}^2 = M_{\tilde{e}_L}^2 - M_{\tilde{\mu}_L}^2, \quad M_{\tilde{u}_R}^2 - M_{\tilde{c}_R}^2 = M_{\tilde{d}_R}^2 - M_{\tilde{s}_R}^2 = M_{\tilde{e}_R}^2 - M_{\tilde{\mu}_R}^2. \tag{3.39}
$$

If $N_S = 0$, (3.24) holds.

3.6 $SU(3)_C \times SU(3)_L \times SU(3)_R$ type

We consider the case with $SU(3)_C \times SU(3)_L \times SU(3)_R$ symmetry in the hidden-visible couplings. For sfermions in the first generation, $\tilde{q}_1$ belongs to $(\mathbf{3}, \mathbf{3}, \mathbf{1})$, $\tilde{u}_R^*$ and $\tilde{d}_R^*$ belong to $(\overline{\mathbf{3}}, \mathbf{1}, \overline{\mathbf{3}})$ and $\tilde{l}_L$ and $\tilde{e}_R^*$ belong to $(\mathbf{1}, \overline{\mathbf{3}}, \mathbf{3})$ of $SU(3)_C \times SU(3)_L \times SU(3)_R$. The same assignment holds on for other generations. In this case, the following relations hold,

$$
\begin{aligned}
& N_{\tilde{u}_R^*} = N_{\tilde{d}_R^*}, \quad N_{\tilde{e}_R^*} = N_{\tilde{l}_1}, \quad N_{\tilde{c}_R^*} = N_{\tilde{s}_R^*}, \quad N_{\tilde{\mu}_R^*} = N_{\tilde{l}_2}, \\
& N_{\tilde{t}_R^*} = N_{\tilde{b}_R^*}, \quad N_{\tilde{\tau}_R^*} = N_{\tilde{l}_3}.
\end{aligned} \tag{3.40}
$$

Using these relations (3.40), we derive the following two kinds of sum rules

$$
M_{\tilde{u}_R}^2 - M_{\tilde{c}_R}^2 = M_{\tilde{d}_R}^2 - M_{\tilde{s}_R}^2, \quad M_{\tilde{e}_L}^2 - M_{\tilde{\mu}_L}^2 = M_{\tilde{e}_R}^2 - M_{\tilde{\mu}_R}^2. \tag{3.41}
$$

4. Sfermion sum rules in orbifold family unification

We study sfermion sum rules in orbifold family unification models. Here the orbifold family unification models are refered as those derived from $SU(N)$ gauge theory on $M^4 \times (S^1/Z_2)$, with the gauge symmetry breaking pattern $SU(N) \to SU(3) \times SU(2) \times SU(r) \times SU(s) \times U(1)^n$, which is realized with the Z_2 parity assignment

$$
P_0 = \mathrm{diag}(+1, +1, +1, +1, +1, -1, \ldots, -1, -1, \ldots, -1), \tag{4.1}
$$

$$
P_1 = \mathrm{diag}(+1, +1, +1, -1, -1, \underbrace{+1, \ldots, +1}_{r}, \underbrace{-1, \ldots, -1}_{s}), \tag{4.2}
$$

where $s = N - 5 - r$ and $N \geq 6$ [27].[3] The matrices P_0 and P_1 are the representation matrices (up to sign factors) of the fundamental representation of the Z_2 transformation ($y \to -y$) and the Z_2' transformation ($y \to 2\pi R - y$), respectively. Here, y is the coordinate of S^1/Z_2, and R is the radius of S^1. After the breakdown of $SU(N)$, the rank-k completely

[3] In the absence of hidden dynamics, sfermion mass relations and sum rules were studied in this framework [28, 29]. Sfermion masses have also been studied from the viewpoint of flavor symmetry and its violation in $SU(5)$ SUSY orbifold GUT [30].

antisymmetric tensor representation $[N,k]$, whose dimension is ${}_NC_k$, is decomposed into a sum of multiplets of the subgroup $SU(3)\times SU(2)\times SU(r)\times SU(s)$ as

$$[N,k]=\sum_{l_1=0}^{k}\sum_{l_2=0}^{k-l_1}\sum_{l_3=0}^{k-l_1-l_2}\left({}_3C_{l_1},{}_2C_{l_2},{}_rC_{l_3},{}_sC_{l_4}\right), \tag{4.3}$$

where l_1, l_2 and l_3 are integers, we have the relation $l_4=k-l_1-l_2-l_3$, and our notation is such that ${}_nC_l=0$ for $l>n$ and $l<0$. We define the Z_2 parity for the representation $({}_pC_{l_1},{}_qC_{l_2},\,{}_rC_{l_3},\,{}_sC_{l_4})$ as

$$\mathcal{P}_0=(-1)^{l_1+l_2}(-1)^k\eta_k,\quad \mathcal{P}_1=(-1)^{l_1+l_3}(-1)^k\eta'_k, \tag{4.4}$$

where η_k and η'_k are the intrinsic Z_2 parities and each takes the value $+1$ or -1 by definition. We find that all zero modes of mirror particles are eliminated when we take $(-1)^k\eta_k=+1$. Hereafter, we consider such a case.

We write the flavor numbers of $(d_R)^c$, l_L, $(u_R)^c$, $(e_R)^c$ and q_L as $n_{\bar{d}}$, n_l, $n_{\bar{u}}$, $n_{\bar{e}}$ and n_q. Both left-handed and right-handed Weyl fermions having even Z_2 parities, $\mathcal{P}_0=\mathcal{P}_1=+1$, compose chiral fermions in the SM. We list the flavor number of each chiral fermion derived from $[N,k]$ in Table 1 and 2.

We add the following assumptions in our analysis.

1. Three families in the MSSM come from zero modes of the bulk field with the representation $[N,k]$ and some brane fields. Higgs fields originate from other multiplets. Chiral anomalies may arise at the boundaries with the appearance of chiral fermions. Such anomalies must be canceled in the four-dimensional effective theory by the contribution of the brane chiral fermions and/or counterterms, such as the Chern-Simons term [31, 32, 33].
2. We do not specify the mechanism by which the $N=1$ SUSY is broken in four dimensions.[4] Soft SUSY breaking terms respect the gauge invariance.
3. Extra gauge symmetries are broken by the Higgs mechanism simultaneously with the orbifold breaking at the scale $M=O(1/R)$. Then there can appear extra contributions to soft SUSY breaking parameters. We need to specify the particle assignment and interactions in order to consider such contributions. We do not consider them for simplicity.
4. Chiral fermions are first and/or second generation ones in the case that the flavor number of each chiral fermion is less than three.

Under the above assumptions, specific sum rules among sfermion masses are derived and listed in 8-th column of Table 1 and 2. Some of them are model dependent and can be useful probes to select Z_2 orbifold family unification models.

5. Conclusions

We have derived sum rules among scalar masses for various boundary conditions for hidden-visible sector couplings in the presence of hidden sector dynamics. The most common sum

[4] The Scherk-Schwarz mechanism, in which SUSY is broken by the difference between the BCs of bosons and fermions, is typical [34, 35]. This mechanism on S^1/Z_2 leads to a restricted type of soft SUSY breaking parameters, such as $M_i=\beta/R$ for bulk gauginos and $m^2_{\tilde{F}}=(\beta/R)^2$ for bulk scalar particles, where β is a real parameter and R is the radius of S^1.

Table 1: The flavor number of each chiral fermion with $(-1)^k\eta_k = (-1)^k\eta'_k = +1$ and sum rules.

rep.	(r,s)	$n_{\bar{d}}$	n_l	$n_{\bar{u}}$	$n_{\bar{e}}$	n_q	Sum rules
$[6,3]$	(0,1)	0	0	2	2	0	$M^2_{\tilde{u}_R} - M^2_{\tilde{c}_R} = M^2_{\tilde{e}_R} - M^2_{\tilde{\mu}_R} = 0$
$[7,3]$	(2,0)	1	0	1	1	2	$M^2_{\tilde{u}_L} - M^2_{\tilde{c}_L} = 0$
	(1,1)	0	1	2	2	1	$M^2_{\tilde{u}_R} - M^2_{\tilde{c}_R} = M^2_{\tilde{e}_R} - M^2_{\tilde{\mu}_R} = 0$
	(0,2)	1	0	3	3	0	$M^2_{\tilde{u}_R} - M^2_{\tilde{c}_R} = M^2_{\tilde{e}_R} - M^2_{\tilde{\mu}_R} = 0$
$[8,3]$	(3,0)	3	0	1	1	3	$M^2_{\tilde{d}_R} - M^2_{\tilde{s}_R} = M^2_{\tilde{u}_L} - M^2_{\tilde{c}_L} = 0$
	(2,1)	1	2	2	2	2	$M^2_{\tilde{u}_R} - M^2_{\tilde{c}_R} = M^2_{\tilde{e}_R} - M^2_{\tilde{\mu}_R}$ $= M^2_{\tilde{e}_L} - M^2_{\tilde{\mu}_L} = M^2_{\tilde{u}_L} - M^2_{\tilde{c}_L} = 0$
	(1,2)	1	2	3	3	1	$M^2_{\tilde{u}_R} - M^2_{\tilde{c}_R} = M^2_{\tilde{e}_R} - M^2_{\tilde{\mu}_R}$ $= M^2_{\tilde{e}_L} - M^2_{\tilde{\mu}_L} = 0$
	(0,3)	3	0	4	4	0	$M^2_{\tilde{u}_R} - M^2_{\tilde{c}_R} = M^2_{\tilde{e}_R} - M^2_{\tilde{\mu}_R}$ $= M^2_{\tilde{d}_R} - M^2_{\tilde{s}_R} = 0$
$[8,4]$	(3,0)	1	1	3	3	3	$M^2_{\tilde{u}_R} - M^2_{\tilde{c}_R} = M^2_{\tilde{e}_R} - M^2_{\tilde{\mu}_R}$ $= M^2_{\tilde{u}_L} - M^2_{\tilde{c}_L} = 0$
	(2,1)	2	0	2	2	4	$M^2_{\tilde{u}_R} - M^2_{\tilde{c}_R} = M^2_{\tilde{e}_R} - M^2_{\tilde{\mu}_R}$ $= M^2_{\tilde{d}_R} - M^2_{\tilde{s}_R} = M^2_{\tilde{u}_L} - M^2_{\tilde{c}_L} = 0$
	(1,2)	1	1	3	3	3	$M^2_{\tilde{u}_R} - M^2_{\tilde{c}_R} = M^2_{\tilde{e}_R} - M^2_{\tilde{\mu}_R}$ $= M^2_{\tilde{u}_L} - M^2_{\tilde{c}_L} = 0$
	(0,3)	2	0	6	6	0	$M^2_{\tilde{u}_R} - M^2_{\tilde{c}_R} = M^2_{\tilde{e}_R} - M^2_{\tilde{\mu}_R}$ $= M^2_{\tilde{d}_R} - M^2_{\tilde{s}_R} = 0$
$[9,3]$	(4,0)	6	0	1	1	4	$M^2_{\tilde{u}_L} - M^2_{\tilde{c}_L} = M^2_{\tilde{d}_R} - M^2_{\tilde{s}_R} = 0$
	(3,1)	3	3	2	2	3	$M^2_{\tilde{u}_R} - M^2_{\tilde{c}_R} = M^2_{\tilde{e}_R} - M^2_{\tilde{\mu}_R}$ $= M^2_{\tilde{d}_R} - M^2_{\tilde{s}_R} = M^2_{\tilde{e}_L} - M^2_{\tilde{\mu}_L}$ $= M^2_{\tilde{u}_L} - M^2_{\tilde{c}_L} = 0$
	(2,2)	2	4	3	3	2	$M^2_{\tilde{u}_R} - M^2_{\tilde{c}_R} = M^2_{\tilde{e}_R} - M^2_{\tilde{\mu}_R}$ $= M^2_{\tilde{d}_R} - M^2_{\tilde{s}_R} = M^2_{\tilde{e}_L} - M^2_{\tilde{\mu}_L} = 0,$ $2\left(M^2_{\tilde{e}_L} - M^2_{\tilde{\tau}_L}\right) = M^2_{\tilde{e}_R} - M^2_{\tilde{\tau}_R}$
	(1,3)	3	3	4	4	1	$M^2_{\tilde{u}_R} - M^2_{\tilde{c}_R} = M^2_{\tilde{e}_R} - M^2_{\tilde{\mu}_R}$ $= M^2_{\tilde{d}_R} - M^2_{\tilde{s}_R} = M^2_{\tilde{e}_L} - M^2_{\tilde{\mu}_L} = 0,$ $2\left(M^2_{\tilde{e}_L} - M^2_{\tilde{\tau}_L}\right) = M^2_{\tilde{e}_R} - M^2_{\tilde{\tau}_R}$
	(0,4)	6	0	5	5	0	$M^2_{\tilde{u}_R} - M^2_{\tilde{c}_R} = M^2_{\tilde{e}_R} - M^2_{\tilde{\mu}_R}$ $= M^2_{\tilde{d}_R} - M^2_{\tilde{s}_R} = 0$

rules are derived in the case with the universal hidden-visible coupling at M.[7] The less universality among couplings the hidden and visible fields yield, the less sum rules hold. We find that sum rules survived depend on the boundary condition for hidden-visible couplings as shown for $SU(5)$ type, $SO(10)$ type, $SU(5) \times U(1)_F$ type, $SU(4) \times SU(2)_L \times SU(2)_R$ type and $SU(3)_C \times SU(3)_L \times SU(3)_R$ type and/or orbifold family unification models shown

Table 2: The flavor number of each chiral fermion with $(-1)^k\eta_k = +1, (-1)^k\eta'_k = -1$ and sum rules.

rep.	(r,s)	$n_{\bar{d}}$	n_l	$n_{\bar{u}}$	$n_{\bar{e}}$	n_q	Sum rules
$[6,3]$	(0,1)	0	0	0	0	2	$M^2_{\tilde{u}_L} - M^2_{\tilde{c}_L} = 0$
$[7,3]$	(2,0)	0	1	2	2	1	$M^2_{\tilde{u}_R} - M^2_{\tilde{c}_R} = M^2_{\tilde{e}_R} - M^2_{\tilde{\mu}_R} = 0$
	(1,1)	1	0	1	1	2	$M^2_{\tilde{u}_L} - M^2_{\tilde{c}_L} = 0$
	(0,2)	0	1	0	0	3	$M^2_{\tilde{u}_L} - M^2_{\tilde{c}_L} = 0$
$[8,3]$	(3,0)	0	3	3	3	1	$M^2_{\tilde{u}_R} - M^2_{\tilde{c}_R} = M^2_{\tilde{e}_R} - M^2_{\tilde{\mu}_R}$ $= M^2_{\tilde{e}_L} - M^2_{\tilde{\mu}_L} = 0,$ $2\left(M^2_{\tilde{e}_L} - M^2_{\tilde{\tau}_L}\right) = M^2_{\tilde{e}_R} - M^2_{\tilde{\tau}_R}$
	(2,1)	2	1	2	2	2	$M^2_{\tilde{u}_R} - M^2_{\tilde{c}_R} = M^2_{\tilde{e}_R} - M^2_{\tilde{\mu}_R}$ $= M^2_{\tilde{d}_R} - M^2_{\tilde{s}_R} = M^2_{\tilde{u}_L} - M^2_{\tilde{c}_L} = 0$
	(1,2)	2	1	1	1	3	$M^2_{\tilde{d}_R} - M^2_{\tilde{s}_R} = M^2_{\tilde{u}_L} - M^2_{\tilde{c}_L} = 0$
	(0,3)	0	3	0	0	4	$M^2_{\tilde{e}_L} - M^2_{\tilde{\mu}_L} = M^2_{\tilde{u}_L} - M^2_{\tilde{c}_L} = 0$
$[8,4]$	(3,0)	1	1	3	3	3	$M^2_{\tilde{u}_R} - M^2_{\tilde{c}_R} = M^2_{\tilde{e}_R} - M^2_{\tilde{\mu}_R}$ $= M^2_{\tilde{u}_L} - M^2_{\tilde{c}_L} = 0$
	(2,1)	0	2	4	4	2	$M^2_{\tilde{u}_R} - M^2_{\tilde{c}_R} = M^2_{\tilde{e}_R} - M^2_{\tilde{\mu}_R}$ $= M^2_{\tilde{e}_L} - M^2_{\tilde{\mu}_L} = M^2_{\tilde{u}_L} - M^2_{\tilde{c}_L} = 0$
	(1,2)	1	1	3	3	3	$M^2_{\tilde{u}_R} - M^2_{\tilde{c}_R} = M^2_{\tilde{e}_R} - M^2_{\tilde{\mu}_R}$ $= M^2_{\tilde{u}_L} - M^2_{\tilde{c}_L} = 0$
	(0,3)	0	2	0	0	6	$M^2_{\tilde{e}_L} - M^2_{\tilde{\mu}_L} = M^2_{\tilde{u}_L} - M^2_{\tilde{c}_L} = 0$
$[9,3]$	(4,0)	0	6	4	4	1	$M^2_{\tilde{u}_R} - M^2_{\tilde{c}_R} = M^2_{\tilde{e}_R} - M^2_{\tilde{\mu}_R}$ $= M^2_{\tilde{e}_L} - M^2_{\tilde{\mu}_L} = 0,$ $2\left(M^2_{\tilde{e}_L} - M^2_{\tilde{\tau}_L}\right) = M^2_{\tilde{e}_R} - M^2_{\tilde{\tau}_R}$
	(3,1)	3	3	3	3	2	$M^2_{\tilde{u}_R} - M^2_{\tilde{c}_R} = M^2_{\tilde{e}_R} - M^2_{\tilde{\mu}_R}$ $= M^2_{\tilde{d}_R} - M^2_{\tilde{s}_R} = M^2_{\tilde{e}_L} - M^2_{\tilde{\mu}_L}$ $= M^2_{\tilde{u}_L} - M^2_{\tilde{c}_L} = 0,$ $2\left(M^2_{\tilde{e}_L} - M^2_{\tilde{\tau}_L}\right) = M^2_{\tilde{e}_R} - M^2_{\tilde{\tau}_R}$
	(2,2)	4	2	2	2	3	$M^2_{\tilde{u}_R} - M^2_{\tilde{c}_R} = M^2_{\tilde{e}_R} - M^2_{\tilde{\mu}_R}$ $= M^2_{\tilde{d}_R} - M^2_{\tilde{s}_R} = M^2_{\tilde{e}_L} - M^2_{\tilde{\mu}_L}$ $= M^2_{\tilde{u}_L} - M^2_{\tilde{c}_L} = 0$
	(1,3)	3	3	1	1	4	$M^2_{\tilde{d}_R} - M^2_{\tilde{s}_R} = M^2_{\tilde{e}_L} - M^2_{\tilde{\mu}_L}$ $= M^2_{\tilde{u}_L} - M^2_{\tilde{c}_L} = 0$
	(0,4)	0	6	0	0	5	$M^2_{\tilde{e}_L} - M^2_{\tilde{\mu}_L} = M^2_{\tilde{u}_L} - M^2_{\tilde{c}_L} = 0$

in Table 1. Hence they still can be useful probes of the MSSM and beyond.

The sum rules were derived for various gauge symmetry breaking $SU(5) \to G_{SM}$, $SO(10) \to G_{SM}$, $SU(5)\times U(1)_F \to G_{SM}$, $\cdots$, $SU(3)_C \times SU(3)_L \times SU(3)_R \to G_{SM}$ in four-dimensional GUTs and orbifold family unification models in the absence of hidden sector dynamics.[28, 29] The sum rules in the presence of hidden sector dynamics, in general, form a subset of those in the absence of hidden sector dynamics. Hence they can be useful to determine whether the hidden sector dynamics is present or not.

We classify scalar sum rules into following types to make easier to select models.

(Type A) The EWS sum rules [7, 8]:

$$\begin{aligned} M_{\tilde{u}_L}^2 - M_{\tilde{d}_L}^2 &= M_{\tilde{\nu}_{eL}}^2 - M_{\tilde{e}_L}^2 = M_{\tilde{c}_L}^2 - M_{\tilde{s}_L}^2 = M_{\tilde{\nu}_{\mu L}}^2 - M_{\tilde{\mu}_L}^2 \\ &= M_{\tilde{t}_L}^2 - M_{\tilde{b}_L}^2 - m_t^2 = M_{\tilde{\nu}_{\tau L}}^2 - M_{\tilde{\tau}_L}^2 = M_W^2 \cos 2\beta. \end{aligned} \tag{5.1}$$

These sum rules are derived from the fact that left-handed fermions (and its superpartner) form $SU(2)_L$ doublets, and they are irrelevant to the structure of models beyond the MSSM. The sfermion sector (and the breakdown of electroweak symmetry) in the MSSM can be tested by using them.

(Type B) Intrafamily sfermion sum rule [8, 17]:

$$2M_{\tilde{u}_R}^2 - M_{\tilde{d}_R}^2 - M_{\tilde{d}_L}^2 + M_{\tilde{e}_L}^2 - M_{\tilde{e}_R}^2 = \frac{10}{3}\left(M_Z^2 - M_W^2\right)\cos 2\beta. \tag{5.2}$$

In the case with $N_S = 0$, the universality in each family can be checked by using it.

(Type C) Outer-family sfermion sum rules [17]:

$$M_{\tilde{u}_L}^2 - M_{\tilde{c}_L}^2 = M_{\tilde{u}_R}^2 - M_{\tilde{c}_R}^2 = M_{\tilde{d}_R}^2 - M_{\tilde{s}_R}^2 = M_{\tilde{e}_L}^2 - M_{\tilde{\mu}_L}^2 = M_{\tilde{e}_R}^2 - M_{\tilde{\mu}_R}^2, \tag{5.3}$$

$$2\left(M_{\tilde{u}_L}^2 - M_{\tilde{t}_L}^2 + m_t^2\right) = M_{\tilde{u}_R}^2 + M_{\tilde{d}_R}^2 - M_{\tilde{t}_R}^2 - M_{\tilde{b}_R}^2 + m_t^2, \tag{5.4}$$

$$2\left(M_{\tilde{e}_L}^2 - M_{\tilde{\tau}_L}^2\right) = M_{\tilde{e}_R}^2 - M_{\tilde{\tau}_R}^2. \tag{5.5}$$

Some of these sum rules are derived from the case that some chiral multiplets form a member of multiple under some large gauge group. Hence the sfermion sector with the grand unification can be tested and the gauge group can be specified by using them.

(Type D) Z_2 orbifold sfermion sum rules:

$$M_{\tilde{u}_R}^2 - M_{\tilde{c}_R}^2 = M_{\tilde{e}_R}^2 - M_{\tilde{\mu}_R}^2. \tag{5.6}$$

This sum rule is a piece of type C and it is derived on the orbifold breaking of $SU(N)$ gauge symmetry for bulk fields with an antisymmetric representation if the bulk field contains $\mathbf{10}_L$ or $\overline{\mathbf{10}}_R$ under the subgroup $SU(5)$, and $SU(2)_L$ singlets have even Z_2 parities in the five-dimensional orbifold grand unification. This relation can be useful as a judgement condition for the Z_2 orbifold breaking of $SU(N)$ gauge symmetry.

It is known that the dangerous FCNC processes can be avoided if the sfermion masses in the first two families are degenerate or rather heavy or fermion and its superpartner mass matrices are aligned. We have derived sfermion sum rules without a requirement of the mass degeneracy for each squark and slepton species in the first two generations. If we require the mass degeneracy, we obtain the following relations, in most GUTs,

$$M^2_{\tilde{u}_L} = M^2_{\tilde{c}_L}, \quad M^2_{\tilde{u}_R} = M^2_{\tilde{c}_R}, \quad M^2_{\tilde{e}_R} = M^2_{\tilde{\mu}_R}, \quad M^2_{\tilde{d}_R} = M^2_{\tilde{s}_R}, \quad M^2_{\tilde{e}_L} = M^2_{\tilde{\mu}_L}. \tag{5.7}$$

In this case, sum rules including third generation sfermions could be useful to specify models.

In the case that the gauge mediation is dominant, the couplings $k^{(v)}_{\tilde{F}}$ parametrize as $k^{(v)}_{\tilde{F}}(0) = \sum_i C^{(i)}_2(\tilde{F})K_i$ using SUSY breaking and messenger dependent functions K_i. Hence the following extra sum rule is derived,[17]

$$3\left(M^2_{\tilde{d}_R} - M^2_{\tilde{u}_R}\right) + M^2_{\tilde{e}_R} = 4\left(M^2_Z - M^2_W\right)\cos 2\beta, \tag{5.8}$$

as intrafamily sfermion sum rule in addition (3.24) for universal type. For outer-family sfermion sum rules, the degeneracy occurs in the first and second generation sfermion masses and then some of (5.7) are derived.

If the hidden sector dynamics were strong and superconformal, conformal sequestering can occur and anomaly mediation can be dominant.[12, 13, 14, 15, 16][5]

The scalar mass relations and sum rules have been also derived in various models.[38, 39, 40, 41, 42, 44, 43, 45, 46, 47] We expect that these specific relations and sum rules can also be useful to probe a physics beyond the MSSM.

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