Fermion Mass Hierarchy in Lifshitz Type Gauge Theory

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Abstract

We study the origin of fermion mass hierarchy and flavor mixing in a Lifshitz type extension of the standard model including an extra scalar field. We show that the hierarchical structure can originate from renormalizable interactions. In contrast to the ordinary Froggatt-Nielsen mechanism, the higher the dimension of associated operators, the heavier the fermion masses. Tiny masses for left-handed neutrinos are obtained without introducing right-handed neutrinos.

The origin of fermion mass hierarchy and flavor mixing is one of the biggest problem in particle physics. In the standard model (SM), the hierarchical structure originates from the texture of Yukawa couplings. Because the Yukawa couplings are free parameters, their values should be determined by a theory beyond the SM. Hence the structure of Yukawa couplings can give us valuable clues for exploring an underlying theory.

Recently, an exotic theory beyond the SM and/or the minimal supersymmetric SM (MSSM) has been proposed.[1] The candidate theory is a Lifshitz type extension of the SM and/or the MSSM.[1] This type of theory is assumed

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[1] A Lifshitz type extension of gravity theory was proposed by Hořava.[2, 3, 4] Properties of Lifshitz type field theory have been investigated in Ref. [5, 6, 7, 8, 9, 10].

to have a fixed point with anisotropic scaling characterized by a dynamical critical exponent $z(>1)$ above a high-energy scale M_ℓ. The system does not possess the relativistic invariance for $z \neq 1$. The Lorentz invariance is expected to emerge after the transition from $z \neq 1$ to $z = 1$ around M_ℓ.[2]

In this letter, we study the origin of fermion mass hierarchy and flavor mixing in a Lifshitz type extension of the SM including an extra scalar field. We show that the hierarchical structure can originate from renormalizable interactions.

The basic idea is as follows. The Lifshitz type theory can be renormalizable by power counting, even though it contains higher-dimensional operators which make the theory with $z = 1$ non-renormalizable. Operators of dimensionality $4 + r$ $(r > 0)$, $O^{(4+r)}$, become irrelevant ones $O^{(4+r)}/M_\ell^r$ after the transition from $z \neq 1$ to $z = 1$ (and the dimensional reduction if extra dimensions exist) around M_ℓ. The contributions from these operators are, in general, negligibly small if M_ℓ is sufficiently large. For example, M_ℓ should be larger than $O(10^{15\sim16})$ GeV in order to suppress proton decay processes. Suppose that a symmetry is broken down spontaneously at a high-energy scale M_{SB} larger than M_ℓ and $O^{(4+r)}$ change into $M_{SB}^{4+r-q}O^{(q)}$. Then these operators can be relevant ones such as $(M_{SB}/M_\ell)^{4+r-q}M_\ell^{4-q}O^{(q)}$ for $q \leq 4$ below M_ℓ. Here, we assume that renormalizable terms including parameters with positive mass dimensions originate from a specific dynamics characterized by a scale M_ℓ and every parameter with mass dimension is given by a power of M_ℓ. In this case, the hierarchy among couplings related to $O^{(q)}$ can originate from the difference of exponents in $(M_{SB}/M_\ell)^{4+r-q}$. If we apply it to the Yukawa couplings, we find an interesting feature that *the higher the dimension of associated original operators, the heavier the fermion masses*.[3]

For the sake of completeness, we explain the ordinary Froggatt-Nielsen mechanism[16] for our framework. When we suppose that the Lifshitz type theory is an effective one derived from an underlying theory, non-renormalizable terms can appear after integrating out superheavy fields. That is, operators of dimensionality $D+z+p$, $O^{(D+z+p)}$, can be derived with the suppression factor Λ^p in the Lifshitz type theory on $D+1$-dimensional space-time, where Λ is a cut-off scale or a mass scale related to superheavy fields. After a symme-

[2] In Ref. [11, 12, 13], properties and renormalizability for quantum field theories with Lorentz symmetry breaking terms have been studied intensively on the basis of "weighted power counting". Furthermore, extensions of the SM have been proposed for this framework.[14, 15]

[3] This feature changes into an opposite one if M_{SB} is smaller than M_ℓ.

try breaking at M_{SB} smaller than Λ, $O^{(D+z+p)}$ change into $(M_{SB}/\Lambda)^p O^{(D+z)}$ and the hierarchy among couplings related to $O^{(D+z)}$ can originate from the difference of exponents in $(M_{SB}/\Lambda)^p$.[4] In this case, it is known that *the higher the dimension of associated original operators, the lighter the fermion masses.*

First, let us explain the fermion mass hierarchy and the flavor mixing. In the SM, the Yukawa interactions for quarks and charged leptons are given by

$$\mathcal{L}_Y = f_{ij}^{(u)} \bar{q}_{Li} h_u u_{Rj} + f_{ij}^{(d)} \bar{q}_{Li} h d_{Rj} + f_{ij}^{(e)} \bar{l}_{Li} h e_{Rj} + \text{h.c.} \ , \tag{1}$$

where $f_{ij}^{(X)}$ ($X = u, d, e$) are the Yukawa couplings, i, j are family indices ($i, j = 1, 2, 3$), $\bar{q}_{Li}$ are Hermitian conjugates of left-handed quark doublets, u_{Ri} are right-handed up type quark singlets, d_{Ri} are right-handed down type quark singlets, $\bar{l}_{Li}$ are Hermitian conjugates of left-handed lepton doublets, e_{Ri} are right-handed electron type lepton singlets, h (or $h_u \equiv i\tau_2 h^*$) is a weak Higgs doublet and h.c. represents Hermitian conjugates of former terms. Quark masses and charged lepton masses are the eigenvalues of fermion mass matrices M_X given by

$$(M_u)_{ij} = f_{ij}^{(u)} \frac{v}{\sqrt{2}} \ , \quad (M_d)_{ij} = f_{ij}^{(d)} \frac{v}{\sqrt{2}} \ , \quad (M_e)_{ij} = f_{ij}^{(e)} \frac{v}{\sqrt{2}} \ , \tag{2}$$

where $v(= 246\text{GeV})$ is the vacuum expectation value (VEV) of neutral component (h^0) of h.[5] Using unitary matrices S_X and T_X, M_X are diagonalized as

$$\begin{aligned} &S_u^\dagger M_u T_u = \text{diag}(m_u, m_c, m_t) \ , \quad S_d^\dagger M_d T_d = \text{diag}(m_d, m_s, m_b) \ , \\ &S_e^\dagger M_e T_e = \text{diag}(m_e, m_\mu, m_\tau) \ . \end{aligned} \tag{3}$$

The quark flavor mixing is given by the Kobayashi-Maskawa matrix $V_{\text{KM}} = S_u^\dagger S_d$.[17] Experimental data[18] show the existence of fermion mass hierarchy and flavor mixing. Because the flavor structure originates from the texture of Yukawa couplings and the Yukawa couplings are free parameters in the

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[4]Strictly speaking, an extra factor such as $(M_\ell/\Lambda)^{\gamma p}$ appears after the dimensional reduction and the redefinition of fields where γ is a constant related to z and the dimensionality of extra space. See (19).

[5]In the MSSM, $v/\sqrt{2}$ is replaced by the corresponding one, i.e., either VEV for neutral components (h_u^0, h_d^0) of two Higgs doublets.

SM, we need a theory beyond the SM to strip the structure of its aura of mystery. Suppose that an underlying theory holds above $O(10^{15\sim 16})$ GeV. Considering renormalization effects in the SM or the MSSM, the magnitude of each fermion mass and each entry in V_{KM} at $O(10^{16})$ GeV can be roughly represented as

$$(m_u, m_c, m_t) \sim (\lambda^7, \lambda^4, 1)\frac{v}{\sqrt{2}} , \quad (m_d, m_s, m_b) \sim (\lambda^6, \lambda^4, \lambda^2)\frac{v}{\sqrt{2}} ,$$
$$(m_e, m_\mu, m_\tau) \sim (\lambda^7, \lambda^4, \lambda^2)\frac{v}{\sqrt{2}} \tag{4}$$

and

$$(V_{\mathrm{KM}})_{ij} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} , \tag{5}$$

where we use the Cabibbo angle $\lambda \equiv \sin\theta_C \sim 0.23$.[19] For V_{KM}, recall the Wolfenstein parameterization.[20] Our present goal is to derive the structure (4) and (5) using a specific theory beyond the SM. An exotic candidate beyond the SM is a Lifshitz type extension of the SM or the MSSM.

Next let us explain a framework of Lifshitz type field theory [1, 2, 3, 4, 5, 6, 7, 8, 9, 10] briefly. Space-time is assumed to be factorized into a product of 3-dimensional Euclidean space $\boldsymbol{R}^3$, extra n-dimensional compact space and time $\boldsymbol{R}$, whose coordinates are denoted by x^i $(i = 1, 2, 3)$, y^k $(k = 1, \cdots, n)$ and t. The notation x^I $(I = 1, \cdots, n+3)$ is also used for the $(n+3)$-dimensional space coordinates. The dimensions of x^i, y^k and t are defined as

$$[x^i] = [y^k] = -1 , \quad [t] = -z , \tag{6}$$

where z is the dynamical critical exponent, which characterizes anisotropic scaling $x^i \to bx^i$, $y^k \to by^k$ and $t \to b^z t$ at the fixed point. The system does not possess the relativistic invariance for $z \neq 1$ but it possesses spatial rotational invariance and translational invariance. The kinetic terms for a complex scalar field Φ and a spinor field Ψ are given by

$$\int dt d^3x d^n y \left[\left| \frac{\partial \Phi}{\partial t} \right|^2 + \bar{\Psi} i \Gamma^0 \frac{\partial}{\partial t} \Psi + \cdots \right] , \tag{7}$$

where Γ^0 corresponds to the time component of the gamma matrices and the ellipsis stands for terms including spatial derivatives. Ψ is a spinor defined on $\boldsymbol{R}^{n+3}$ and it transforms as

$$\Psi(x) \to \Psi' \to \Psi'(x') = S(O)\Psi(x) \ , \tag{8}$$

$$S(O) \equiv e^{-\frac{i}{4}\omega_{IJ}\Sigma^{IJ}} \ , \quad \Sigma^{IJ} \equiv \frac{i}{2}\left(\Gamma^I\Gamma^J - \Gamma^J\Gamma^I\right) \ , \tag{9}$$

under the spatial rotation $x^I \to x'^I = O^I_J x^J$. Here, the Γ^I are gamma matrices, the ω_{IJ} are parameters related to the rotation angles θ^I with $\omega_{IJ} = -\varepsilon_{IJK}\theta^K$ and O^I_J is the orthogonal matrix given by $O^I_J = (e^\omega)^I_J$. $S(O)$ satisfies the following relations:

$$S^\dagger(O)\Gamma^I S(O) = O^I_J \Gamma^J \ , \quad S^\dagger(O)S(O) = \mathcal{I} \ , \tag{10}$$

where $\mathcal{I}$ is the unit matrix. Chiral fermions on $\boldsymbol{R}^3$ are assumed to appear after the dimensional reduction, e.g., through the orbifold breaking mechanism. The engineering dimensions of Φ and Ψ are given by

$$[\Phi] = \frac{3+n-z}{2} \ , \quad [\Psi] = \frac{3+n}{2} \ , \tag{11}$$

respectively. Then the dimension of operator $\Psi^\dagger\Phi^N\Psi$ is given by

$$[\bar{\Psi}\Phi^N\Psi] = \frac{3+n-z}{2}N + 3 + n \ . \tag{12}$$

The operator $\bar{\Psi}\Phi^N\Psi$ becomes relevant or a renormalizable term if its dimension is less than or equals to $3+n+z$. The operator including higher spatial derivatives such as $\Phi^\dagger\nabla^{2z}\Phi$ can also become relevant. The theory can be renormalizable by power counting, even though it contains higher-dimensional operators which make the theory with $z=1$ non-renormalizable. The Lorentz invariance is expected to emerge as an accidental symmetry after the transition from a high-energy theory with $z \neq 1$ to that with $z = 1$ around M_ℓ.[6] The magnitude of Lorentz symmetry breaking terms is estimated and it gives constraints on parameters.[23, 24, 25, 26],[1]

Now let us explore the origin of texture of Yukawa couplings in a Lifshitz type extension of the SM including an extra scalar field Φ.[7] We introduce an

[6]There has been a proposal that the Lorentz invariance appears at an attractive infrared fixed point.[21, 22]

[7]The study based on the Lifshitz type extension of the MSSM can be carried out by the introduction of two Higgs doublets and similar results can be obtained.

extra $U(1)$ symmetry denoted by $U(1)_A$ and assume that Φ is a singlet under the SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ but has a non-zero $U(1)_A$ charge and $U(1)_A$ is spontaneously broken down by the non-vanishing VEV of Φ above M_ℓ. The interactions such as $\Phi^{m_{ij}^{(X)}} \bar{\Psi}_i H_{(u)} \Psi_j$ are determined by $U(1)_A$. Here, $m_{ij}^{(X)}$ are zero or positive integers, and Ψ_i and $H_{(u)}$ are fermions and a boson which contain SM fermions ψ_i and a weak Higgs boson $h_{(u)}$ as zero modes, respectively. The origin of extra $U(1)$ symmetry is not specified in our analysis for simplicity.[8] The action is given by

$$\int dtd^3xd^ny \Big[|D_t\Phi|^2 - \frac{1}{\kappa^2}\Phi^\dagger(D_I^\dagger D_I)^z\Phi - C_\Phi |D_I\Phi|^2 + |D_tH|^2 - \frac{1}{\kappa_h^2}H^\dagger(D_I^\dagger D_I)^z H - C_H |D_IH|^2 + \bar{\Psi}_i i\Gamma^0 D_t\Psi_i - \frac{1}{\xi_i^2}\bar{\Psi}_i(i\Gamma^I D_I)^z\Psi_i - C_{\Psi_i}\bar{\Psi}_i i\Gamma^I D_I\Psi_i + \gamma_{ij}^{(X)}\Phi^{m_{ij}^{(X)}}\bar{\Psi}_i H_{(u)}\Psi_j + \cdots \Big] , \quad (13)$$

where κ^2, κ_h^2 and ξ_i^2 are dimensionless parameters concerning Lorentz symmetry violating terms, D_t and D_I are covariant derivatives and the ellipsis stands for other terms. The engineering dimensions of fields $(\Phi, H_{(u)}, \Psi_i)$ and parameters $(C_\Phi, C_H, C_{\Psi_i}, \gamma_{ij}^{(X)})$ are given by

$$[\Phi] = [H_{(u)}] = \frac{3+n-z}{2} \ , \quad [\Psi_i] = \frac{3+n}{2} \quad (14)$$

and

$$[C_\Phi] = [C_H] = 2(z-1) \ , \ [C_{\Psi_i}] = z-1 \ , \quad [\gamma_{ij}^{(X)}] = z - \frac{(3+n-z)(m_{ij}^{(X)}+1)}{2} \ , \quad (15)$$

respectively. We assume that renormalizable terms including parameters with positive mass dimensions originate from a specific dynamics characterized by a scale M_ℓ and the parameters are given by a power of M_ℓ, which is similar to the soft supersymmetry (SUSY) breaking parameters in SUSY models.[9] The magnitude of parameters is not necessarily $O(1)$ in the unit

[8] If $U(1)_A$ is an anomalous gauge symmetry, we need to introduce extra fields in order for the theory to be harmless.

[9] For example, the soft supersymmetry breaking terms are given by a power of the gravitino mass $m_{3/2}$ in the gravity mediation.

of M_ℓ but can be much smaller like most A terms in SUSY models. On the other hand, we assume that parameters in non-renormalizable terms are suppressed by a power of cutoff scale Λ as usual. To become relativistic below M_ℓ, finetuning among parameters is required such as $C_\Phi = C_H = C_{\Psi_i}^2$ for all species. In this setup, parameters are expressed as

$$C_\Phi = C_H = M_\ell^{2(z-1)} \ , \quad C_{\Psi_i} = M_\ell^{z-1} \ ,$$
$$\gamma_{ij}^{(X)} = \begin{cases} \gamma_{ij}^{0(X)} M_\ell^{z-\frac{(3+n-z)(m_{ij}^{(X)}+1)}{2}} & \left(z \geq \frac{(3+n-z)(m_{ij}^{(X)}+1)}{2}\right) \ , \\ \gamma_{ij}^{0(X)} \Lambda^{z-\frac{(3+n-z)(m_{ij}^{(X)}+1)}{2}} & \left(z < \frac{(3+n-z)(m_{ij}^{(X)}+1)}{2}\right) \ , \end{cases} \tag{16}$$

after a suitable rescaling of fields. Here, $\gamma_{ij}^{0(X)}$ is a dimensionless parameter. We assume that the volume of extra n-dimensional space is $1/M_\ell^n$. After the redefinition of time variable and fields as

$$x_0 \equiv M_\ell^{z-1} t \ , \quad \tilde{\Phi} \equiv M_\ell^{\frac{z-n-1}{2}} \Phi = \phi + \cdots \ ,$$
$$\tilde{H}_{(u)} \equiv M_\ell^{\frac{z-n-1}{2}} H_{(u)} = h_{(u)} + \cdots \ , \quad \tilde{\Psi}_i \equiv M_\ell^{-\frac{n}{2}} \Psi_i = \psi_i + \cdots \tag{17}$$

and the dimensional reduction of extra dimensions, the following action for zero modes is derived from (13),

$$\int d^4x \left[|D_\mu h|^2 + \bar{\psi}_i i\gamma^\mu D_\mu \psi_i + \gamma_{ij}^{0(X)} \left(\frac{\langle\phi\rangle}{M_\ell}\right)^{m_{ij}^{(X)}} \bar{\psi}_i h_{(u)} \psi_j + \cdots \right] \ , \tag{18}$$

where the ellipsis stands for the Yukawa interactions from non-renormalizable terms and so on. The dimensions of ϕ, $h_{(u)}$ and ψ_i are $[\phi] = [h_{(u)}] = 1$ and $[\psi_i] = 3/2$. The Yukawa couplings are given by[10]

$$f_{ij}^{(X)} = \begin{cases} \gamma_{ij}^{0(X)} \left(\frac{\langle\phi\rangle}{M_\ell}\right)^{m_{ij}^{(X)}} & \left(z \geq \frac{(3+n-z)(m_{ij}^{(X)}+1)}{2}\right) \ , \\ \gamma_{ij}^{0(X)} \left(\frac{M_\ell}{\Lambda}\right)^{\frac{3+n-3z}{2}} \left(\left(\frac{M_\ell}{\Lambda}\right)^{\frac{1+n-z}{2}} \frac{\langle\phi\rangle}{\Lambda}\right)^{m_{ij}^{(X)}} & \left(z < \frac{(3+n-z)(m_{ij}^{(X)}+1)}{2}\right) \ . \end{cases} \tag{19}$$

[10]The contributions from volume suppression factor can also appear after the dimensional reduction.[27] In this case, the difference of field configurations related to interactions can be important to study the origin of mass hierarchy. Here, we do not consider them for simplicity.

The exponents $m_{ij}^{(X)}$ are determined from the $U(1)_A$ charge conservation:

$$m_{ij}^{(u)} Q_A(\phi) + Q_A(\bar{q}_{Li}) + Q_A(u_{Rj}) + Q_A(h_u) = 0 \ , \tag{20}$$

$$m_{ij}^{(d)} Q_A(\phi) + Q_A(\bar{q}_{Li}) + Q_A(d_{Rj}) + Q_A(h) = 0 \ , \tag{21}$$

$$m_{ij}^{(e)} Q_A(\phi) + Q_A(\bar{l}_{Li}) + Q_A(e_{Rj}) + Q_A(h) = 0 \ , \tag{22}$$

where Q_A represents the charge of $U(1)_A$. The first one in (19) comes from renormalizable terms and the ratio $M_\ell/\langle\phi\rangle$ can play a role of λ, in the case that there is no hierarchy among each entry in $\gamma_{ij}^{0(X)}$ and $\langle\phi\rangle$ is larger than M_ℓ. In other words, there is a build-in mechanism to generate the hierarchy using the ratio $M_\ell/\langle\phi\rangle$ on the basis of relevant operators. On the other hand, the second one in (19) comes from non-renormalizable terms which might originate from some renormalizable interactions after integrating out superheavy fields. The mechanism to generate the hierarchy using the ratio $(M_\ell/\Lambda)^{\frac{1+n-z}{2}} \langle\phi\rangle/\Lambda$ is regarded as the Lifshitz type extended version of the Froggatt-Nielsen mechanism.[11] There is a possibility that the hierarchy of Yukawa couplings stems from the mixture of first and second ones.

Next we consider the case with $z = 4$ and $n = 2$ as an example. The Yukawa couplings are given by

$$f_{ij}^{(X)} = \begin{cases} \gamma_{ij}^{0(X)} \left(\dfrac{\langle\phi\rangle}{M_\ell}\right)^{m_{ij}^{(X)}} & (0 \le m_{ij}^{(X)} \le 7) \ , \\ \gamma_{ij}^{0(X)} \left(\dfrac{\Lambda}{M_\ell}\right)^{\frac{7}{2}} \left(\dfrac{\langle\phi\rangle}{\sqrt{M_\ell \Lambda}}\right)^{m_{ij}^{(X)}} & (m_{ij}^{(X)} > 7) \ . \end{cases} \tag{23}$$

If $M_\ell/\langle\phi\rangle \sim \lambda$, the fermion mass hierarchy (4) can be derived from the $U(1)_A$ charge assignment:[12]

$$\begin{aligned} & Q_A(\bar{q}_{Li}) = (0,1,3) \ , \quad Q_A(u_{Ri}) = (0,2,4) \ , \quad Q_A(d_{Ri}) = (1,2,2) \ , \\ & Q_A(\bar{l}_{Li}) = (0,1,1) \ , \quad Q_A(e_{Ri}) = (0,2,4) \ , \quad Q_A(h_{(u)}) = 0 \ , \\ & Q_A(\phi) = -1 \ , \end{aligned} \tag{24}$$

[11]We need some selection rule related to interactions in order for the Froggatt-Nielsen mechanism to work. In most cases, one uses an anomalous $U(1)$ gauge symmetry,[**?**] whose anomalies are canceled via the Green-Schwarz mechanism,[30] motivated by superstring theories. A discrete horizontal symmetry is also used.[31]

[12]The $U(1)_A$ charge assignment is not unique.

where we assume $\gamma_{ij}^{0(X)} = O((M_\ell/\langle\phi\rangle)^7)$. If $\langle\phi\rangle/\sqrt{M_\ell\Lambda} \sim \lambda$, the fermion mass hierarchy (4) can be derived from the $U(1)_A$ charge assignment:

$$\begin{aligned}
&Q_A(\bar{q}_{Li}) = (11, 10, 8)\ , \quad Q_A(u_{Ri}) = (4, 2, 0)\ , \quad Q_A(d_{Ri}) = (3, 2, 2)\ , \\
&Q_A(\bar{l}_{Li}) = (9, 8, 8)\ , \quad Q_A(e_{Ri}) = (6, 4, 2)\ , \quad Q_A(h_{(u)}) = 0\ , \\
&Q_A(\phi) = -1\ ,
\end{aligned} \tag{25}$$

where we assume $\gamma_{ij}^{0(X)} = O(\Lambda^{1/2}M_\ell^{15/2}/\langle\phi\rangle^8)$. In either case, the quark flavor mixing due to the Kobayashi-Maskawa matrix (5) can be obtained by

$$(V_{\mathrm{KM}})_{ij} \sim \lambda^{|Q_A(\bar{q}_{Li})-Q_A(\bar{q}_{Lj})|} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} . \tag{26}$$

Finally, we discuss the neutrino sector. The Majorana mass matrix M_ν of left-handed neutrinos is usually obtained throught the see-saw mechanism such that[32, 33, 34]

$$(M_\nu)_{ij} = f_{ia}^{(\nu)} \frac{v}{\sqrt{2}} \left(M_R^{-1}\right)_{ab} f_{bj}^{(\nu)} \frac{v}{\sqrt{2}}\ , \tag{27}$$

where $f_{ia}^{(\nu)}$ is the Yukawa coupling among l_{Li}, right-handed neutrinos ν_{Ra} and h_u and $(M_R)_{ab}$ is the superheavy Majorana mass matrix of right-handed neutrinos. In our Lifshitz type extension of the SM including Φ, M_ν can be obtained without introducing right-handed neutrinos from the following relevant interactions:

$$\gamma_{ij}^{(\nu)} \Phi^{m_{ij}^{(\nu)}} \bar{L}_i i\tau_2 \boldsymbol{\tau} L_j^c \cdot H_u^t i\tau_2 \boldsymbol{\tau} H_u\ , \tag{28}$$

where L_j are fermions whose zero modes are l_{Li}, and superscripts c and t stand for the charge conjugation and transpose of the relevant field, respectively. The exponent $m_{ij}^{(\nu)}$ is determined by

$$m_{ij}^{(\nu)} Q_A(\phi) + Q_A(\bar{l}_{Li}) + Q_A(\bar{l}_{Lj}) + 2Q_A(h_u) = 0\ . \tag{29}$$

In our model, M_ν is given by

$$(M_\nu)_{ij} = \begin{cases} \gamma_{ij}^{0(\nu)} \left(\dfrac{\langle\phi\rangle}{M_\ell}\right)^{m_{ij}^{(\nu)}} \dfrac{v^2}{2M_\ell} & \left(z \geq \frac{(3+n-z)(m_{ij}^{(\nu)}+2)}{2}\right)\ , \\ \gamma_{ij}^{0(\nu)} \left(\dfrac{M_\ell}{\Lambda}\right)^{2+n-2z} \left(\left(\dfrac{M_\ell}{\Lambda}\right)^{\frac{1+n-z}{2}} \dfrac{\langle\phi\rangle}{\Lambda}\right)^{m_{ij}^{(\nu)}} \dfrac{v^2}{2\Lambda} & \left(z < \frac{(3+n-z)(m_{ij}^{(\nu)}+2)}{2}\right)\ , \end{cases} \tag{30}$$

where $\gamma_{ij}^{0(\nu)}$ is a dimensionless parameter. Using a unitary matrix S_ν, M_ν is diagonarized as

$$S_\nu^\dagger M_\nu S_\nu = \mathrm{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) \ , \tag{31}$$

where $m_{\nu_1} < m_{\nu_2} < m_{\nu_3}$. The lepton flavor mixing is given by the Maki-Nakagawa-Sakata matrix $V_{\mathrm{MNS}} = S_e^\dagger S_\nu$.[35] Using the experimental data for neutrinos,[18] we find that there are two large mixings:

$$\sin^2 2\theta_{12} \sim 0.88 \ , \quad \sin^2 2\theta_{23} > 0.92 \tag{32}$$

and the hierarchy between mass-squared differences for solar neutrinos $\Delta m_\odot^2$ and for the atmospheric neutrinos $\Delta m_\oplus^2$:

$$\frac{\Delta m_\odot^2}{\Delta m_\oplus^2} = \frac{|m_{\nu_2}^2 - m_{\nu_1}^2|}{|m_{\nu_3}^2 - m_{\nu_2}^2|} \sim \lambda^2 \ . \tag{33}$$

If three neutrino masses do not degenerate, the hierarchy (33) suggests the relation:

$$(m_{\nu_2}, m_{\nu_3}) \sim (\lambda, 1) \times 0.05\mathrm{eV} \ . \tag{34}$$

Using our mechanism with the $U(1)$ charge assignment (24) or the ordinary Froggatt-Nielsen mechanism with the $U(1)$ charge assignment (25), M_ν and V_{MNS} are estimated as

$$(M_\nu)_{ij} \propto \lambda^{\mp(Q_A(\bar{l}_{Li}) + Q_A(\bar{l}_{Lj}))} \propto \begin{pmatrix} \lambda^2 & \lambda & \lambda \\ \lambda & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix} \ , \tag{35}$$

$$V_{\mathrm{MNS}} \sim \lambda^{|Q_A(\bar{l}_{Li}) - Q_A(\bar{l}_{Lj})|} S_\nu \sim \begin{pmatrix} 1 & \lambda & \lambda \\ \lambda & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix} S_\nu \ , \tag{36}$$

where the minus and the plus sign in (35) for our mechanism and the Froggatt-Nielsen mechanism, respectively. This type of neutrino mass matrix has been proposed and studied in Refs. [36, 37, 38]. The above matrices (35) and (36) have an interesting property that the bi-large mixing can be naturally derived if we obtain the mass relation (34) after the diagonalization of M_ν.

Finally, we discuss the lepton number and/or baryon number violating process through four fermi interactions. The four fermi interactions originate from the operator such as $\Phi^N\Psi^\dagger\Psi\Psi^\dagger\Psi$ after Φ acquires the VEV, and there appear the lepton number and/or baryon number violating terms such as $qqql$ and $udde$. The dimension of $\Phi^N\Psi^\dagger\Psi\Psi^\dagger\Psi$ is given by

$$[\Phi^N\Psi^\dagger\Psi\Psi^\dagger\Psi] = \left(\frac{N}{2}+1\right)(3+n-z)+3+n+z\ . \tag{37}$$

The operator $\Phi^N\Psi^\dagger\Psi\Psi^\dagger\Psi$ becomes a non-renormalizable term by a power-counting if z is less than $3+n$, and then the interaction is suppressed by a power of Λ.

The origin of fermion mass hierarchy and flavor mixing has been studied using the Froggatt-Nielsen mechanism in the framework of (supersymmetric) grand unified theory.[36, 39, 40, 41, 42] A similar analysis can be carried out in the framework of Lifshitz type extension of (supersymmetric) grand unified theory.

In conclusion, we have studied the origin of fermion mass hierarchy and flavor mixing in a Lifshitz type extension of the SM including an extra scalar field. We have found a mechanism to generate the hierarchical structure from renormalizable interactions. The mechanism is similar to the Froggatt-Nielsen mechanism in the sense that the hierarchy originates from operators of dimensionality u ($u > 4$). But, there is a difference in characters between them. In the ordinary Froggatt-Nielsen mechanism, the higher the dimension of associated operators, the lighter the fermion masses. In our mechanism, the higher the dimension of associated operators, the heavier the fermion masses. Furthermore we have found that tiny masses for left-handed neutrinos can be obtained without introducing right-handed neutrinos.

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