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Consideration of the Hopf maps from viewpoint of groups

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Abstract

It is known that the real, complex and quaternionic Hopf maps are constructed by the canonical homomorphisms from $SO(2) \to SO(2)$, $SU(2) \to SO(3)$ and $Sp(2) \to SO(5)$, respectively. We write their proofs explicitly. We show that the Cayley Hopf map is also constructed by the canonical homomorphism $Spin(9) \to SO(9)$.

The Hopf maps are important and concrete materials in Algebra,Geometry and Topology. In the present note, we investigate the algebraic, exactly, Lie group theoretical properties of the Hopf maps applicable to Topology.

This note is based on [2] and gives a simpler proof.

Let $\boldsymbol{F} = \boldsymbol{R}, \boldsymbol{C}, \boldsymbol{H}, \mathfrak{C}$ denote the field or algebra of the real, complex, quaternionic, Cayley numbers, respectively. We set $d = \dim_{\boldsymbol{R}} \boldsymbol{F}$. The $2d-1$ and d dimensional spheres are defined by

$$S^{2d-1} = \{(a, b) \in \boldsymbol{F}^2 \mid |a|^2 + |b|^2 = 1\}, \ S^d = \{(\xi, x) \in \boldsymbol{R} \times \boldsymbol{F} \mid \xi^2 + |x|^2 = 1\},$$

respectively. The Hopf map $h_{\boldsymbol{F}} : S^{2d-1} \to S^d$ is defined by the equation

$$h_{\boldsymbol{F}}(a, b) = (|a|^2 - |b|^2, 2b\bar{a}) \quad ((a, b) \in S^{2d-1}).$$

Now, in this paper, we will show that the maps

$$\overline{\varphi}_{\boldsymbol{R}} : SO(2) \to SO(2), \qquad \overline{\varphi}_{\boldsymbol{C}} : SU(2) \to SO(3)/SO(2),$$
$$\overline{\varphi}_{\boldsymbol{H}} : Sp(2)/Sp(1) \to SO(5)/SO(4), \quad \overline{\varphi}_{\mathfrak{C}} : Spin(9)/Spin'(7) \to SO(9)/SO(8)$$

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induced by the natural homomorphisms

$$\varphi_{\boldsymbol{R}} : SO(2) \to SO(2), \qquad \varphi_{\boldsymbol{C}} : SU(2) \to SO(3),$$
$$\varphi_{\boldsymbol{H}} : Sp(2) \to SO(5), \qquad \varphi_{\mathfrak{C}} : Spin(9) \to SO(9)$$

coincide with the Hopf maps

$$h_{\boldsymbol{R}} : S^1 \to S^1, \quad h_{\boldsymbol{C}} : S^3 \to S^2, \quad h_{\boldsymbol{H}} : S^7 \to S^4, \quad h_{\mathfrak{C}} : S^{15} \to S^8,$$

respectively. Here $Spin'(7)$ will be defined later.

1. Case $\boldsymbol{F} = \boldsymbol{R}$.

Let $SO(2) = \{A \in M(2, \boldsymbol{R}) \mid A\,{}^tA = E, \det A = 1\}$ be the special orthogonal group. Then, $SO(2)$ is homeomorphic to S^1 by the correspondence

$$SO(2) \ni \begin{pmatrix} a & -b \\ b & a \end{pmatrix} \quad \to \quad (a, b) \in S^1$$

and we identify $SO(2)$ with S^1 under this correspondence. Now, we define a homomorphism $\varphi_{\boldsymbol{R}} : SO(2) \to SO(2)$ by

$$\varphi_{\boldsymbol{R}} \begin{pmatrix} a & -b \\ b & a \end{pmatrix} = \begin{pmatrix} 2a^2 - 1 & -2ab \\ 2ba & 2a^2 - 1 \end{pmatrix}.$$

Then $\varphi_{\boldsymbol{R}}$ induces the map $\overline{\varphi}_{\boldsymbol{R}} : S^1 = SO(2) \to SO(2) = S^1, \overline{\varphi}_{\boldsymbol{R}}(a, b) = (2a^2 - 1, 2ba)$, which is nothing but $h_{\boldsymbol{R}} : S^1 \to S^1$.

2. Case $\boldsymbol{F} = \boldsymbol{C}$.

We define a 3 dimensional $\boldsymbol{R}$-vector space V^3 by

$$\begin{aligned} V^3 = \mathfrak{J}(2, \boldsymbol{C})_0 &= \{X \in M(2, \boldsymbol{C}) \mid X^* = X, \mathrm{tr}(X) = 0\} \\ &= \left\{X = \begin{pmatrix} \xi & \overline{x} \\ x & -\xi \end{pmatrix} \,\middle|\, \xi \in \boldsymbol{R}, x \in \boldsymbol{C}\right\} \end{aligned}$$

with the norm (X, X) given by

$$(X, X) = \frac{1}{2}\mathrm{tr}(XX) = \xi^2 + x\overline{x}.$$

In V^3, we define a 2 dimensional sphere S_2 by $\{X \in V^3 \mid (X, X) = 1\}$, and we identify S^2 with S_2 under the correspondence

$$S^2 \ni (\xi, x) \quad \to \quad \begin{pmatrix} \xi & \overline{x} \\ x & -\xi \end{pmatrix} \in S_2.$$

In $M(2,\boldsymbol{C})$, we adopt the notation $E_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, $E_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$. We define special orthogonal groups $SO(3)$ and $SO(2)$ by $SO(3) = SO(V^3) = \{\alpha \in \mathrm{Iso}_{\boldsymbol{R}}(V^3) \,|\, (\alpha X, \alpha X) = (X, X), \det\alpha = 1\}$ and $SO(2) = \{A \in SO(3) \,|\, \alpha(E_1 - E_2) = E_1 - E_2\}$, respectively. Then the correspondence $\alpha \in SO(3)$ to $\alpha(E_1 - E_2) \in S_2$ induces the homeomorphism $SO(3)/SO(2) = S_2$. The special unitary group $SU(2)$ is usually defined by $\{A \in M(2,\boldsymbol{C}) \,|\, A^*A = E, \det A = 1\}$. Then, $SU(2)$ is homeomorphic to S^3 by the correspondence

$$SU(2) \ni \begin{pmatrix} a & -\overline{b} \\ b & \overline{a} \end{pmatrix} \quad \to \quad (a, b) \in S^3$$

and we identify $SU(2)$ with S^3 under this correspondence. We define a homomorphism $\varphi_{\boldsymbol{C}} : SU(2) \to SO(3)$ by

$$\varphi_{\boldsymbol{C}}(A)X = AXA^*, \quad X \in V^3.$$

Now, for $(a, b) \in S^3$, construct a matrix $A = \begin{pmatrix} a & -\overline{b} \\ b & \overline{a} \end{pmatrix} \in SU(2)$. Then

$$\begin{aligned} &\varphi_{\boldsymbol{C}}(A)(E_1 - E_2) = A(E_1 - E_2)A^* \\ &\quad = \begin{pmatrix} a & -\overline{b} \\ b & \overline{a} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \overline{a} & \overline{b} \\ -b & a \end{pmatrix} = \begin{pmatrix} 2|a|^2 - 1 & 2a\overline{b} \\ 2b\overline{a} & -2|a|^2 + 1 \end{pmatrix}. \end{aligned}$$

Thus we have the map $\overline{\varphi}_{\boldsymbol{C}} : S^3 = SU(2) \to SO(3)/SO(2) = S^2, \overline{\varphi}_{\boldsymbol{C}}(a, b) = (2|a|^2 - 1, 2b\overline{a}) \in S^2$, which is nothing but $h_{\boldsymbol{C}} : S^3 \to S^2$.

3. Case $\boldsymbol{F} = \boldsymbol{H}$. Although the result is obtained in [1] by another method and this case is similar to the complex case, we will write again.

We define a 5 dimensional $\boldsymbol{R}$-vector space V^5 by

$$\begin{aligned} V^5 = \mathfrak{J}(2, \boldsymbol{H})_0 &= \{X \in M(2, \boldsymbol{H}) \,|\, X^* = X, \mathrm{tr}(X) = 0\} \\ &= \left\{ X = \begin{pmatrix} \xi & \overline{x} \\ x & -\xi \end{pmatrix} \,|\, \xi \in \boldsymbol{R}, x \in \boldsymbol{H} \right\} \end{aligned}$$

with the norm (X, X) given by

$$(X, X) = \frac{1}{2}\mathrm{tr}(XX) = \xi^2 + x\overline{x}.$$

In V^5, we define a 4 dimensional sphere S_4 by $\{X \in V^5 \,|\, (X,X) = 1\}$, and we identify S^4 with S_4 under the correspondence

$$S^4 \ni (\xi, x) \quad \to \begin{pmatrix} \xi & \overline{x} \\ x & -\xi \end{pmatrix} \in S_4.$$

We define the special orthogonal groups $SO(5)$ and $SO(4)$ by $SO(5) = SO(V^5) = \{\alpha \in \mathrm{Iso}_{\boldsymbol{R}}(V^5) \,|\, (\alpha X,\, \alpha X) = (X, X), \det\alpha = 1\}$ and $SO(4) = \{\alpha \in SO(5) \,|\, \alpha(E_1 - E_2) = E_1 - E_2\}$, respectively. Then the correspondence $\alpha \in SO(5)$ to $\alpha(E_1 - E_2) \in S_4$ induces the homeomorphism $SO(5)/SO(4) = S_4$. The symplectic groups $Sp(2)$ and $Sp(1)$ are usually defined by $\{A \in M(2, \boldsymbol{H}) \,|\, A^*A = E\}$ and $Sp(1) = \left\{A \in Sp(2) \,\middle|\, A\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}\right\} = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & d \end{pmatrix} \,\middle|\, d \in \boldsymbol{H}, |d| = 1\right\}$, respectively. Then the correspondence $A \in Sp(2)$ to $A\begin{pmatrix} 1 \\ 0 \end{pmatrix} \in S_7$ induces the homeomorphism $Sp(2)/Sp(1) = S_7$. We define a homomorphism $\varphi_{\boldsymbol{H}} : Sp(2) \to SO(5)$ by

$$\varphi_{\boldsymbol{H}}(A)X = AXA^*, \quad X \in V^5.$$

Now, for $(a, b) \in S^7$, construct a matrix $A = \begin{pmatrix} a & c \\ b & d \end{pmatrix} \in Sp(2)$ (for example, $c = |b|, d = -b\overline{a}/|b|$ ($b/|b|$ means 1 if $b = 0$). Then we have

$$\begin{aligned} \varphi_{\boldsymbol{H}}(A)(E_1 - E_2) &= A(E_1 - E_2)A^* = \begin{pmatrix} a & c \\ b & d \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} \overline{a} & \overline{b} \\ \overline{c} & \overline{d} \end{pmatrix} \\ &= \begin{pmatrix} |a|^2 - |c|^2 & a\overline{b} - c\overline{d} \\ b\overline{a} - d\overline{c} & |b|^2 - |d|^2 \end{pmatrix} = \begin{pmatrix} 2|a|^2 - 1 & 2a\overline{b} \\ 2b\overline{a} & -2|a|^2 + 1 \end{pmatrix}, \end{aligned}$$

Thus we have the map $\overline{\varphi}_{\boldsymbol{H}} : S^7 = Sp(2)/Sp(1) \to SO(5)/SO(4) = S^4, \overline{\varphi}_{\boldsymbol{H}}(a, b) = (2|a|^2 - 1, 2b\overline{a}) \in S^2$, which is nothing but $h_{\boldsymbol{H}} : S^7 \to S^4$.

4. Case $\boldsymbol{F} = \mathfrak{C}$.

Let $\mathfrak{J} = \{X \in M(3, \mathfrak{C}) \,|\, X^* = X\}$ be the exceptional Jordan algebra with the Jordan multiplication $X \circ Y$ and the inner product (X, Y) respectively defined by

$$X \circ Y = \frac{1}{2}(XY + YX), \quad (X, Y) = \frac{1}{2}\mathrm{tr}(X \circ Y).$$

In $\mathfrak{J}$, we adopt the following notation.

$$E_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad E_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad E_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$
$$F_1(x) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & x \\ 0 & \overline{x} & 0 \end{pmatrix}, \quad F_2(x) = \begin{pmatrix} 0 & 0 & \overline{x} \\ 0 & 0 & 0 \\ x & 0 & 0 \end{pmatrix}, \quad F_3(x) = \begin{pmatrix} 0 & x & 0 \\ \overline{x} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

We define $\boldsymbol{R}$-vector subspaces V^9 and V^{16} of $\mathfrak{J}$ respectively by

$$V^9 = \{X \in \mathfrak{J} \,|\, E_1 \circ X = 0, \mathrm{tr}(X) = 0\} = \left\{ \begin{pmatrix} 0 & 0 & 0 \\ 0 & \xi & x \\ 0 & \overline{x} & -\xi \end{pmatrix} \,\middle|\, \xi \in \boldsymbol{R}, x \in \mathfrak{C} \right\},$$
$$V^{16} = \{Y \in \mathfrak{J} \,|\, 2E_1 \circ Y = Y\} = \left\{ \begin{pmatrix} 0 & y_3 & \overline{y}_2 \\ \overline{y}_3 & 0 & 0 \\ y_2 & 0 & 0 \end{pmatrix} \,\middle|\, y_2, y_3 \in \mathfrak{C} \right\}.$$

In V^9 and V^{16}, we define 8 and 15 dimensional spheres S_8 and S_{15} respectively by

$$S_8 = \{X \in V^9 \,|\, (X, X) = 2\} = \{\xi(E_2 - E_3) + F_1(x) \,|\, \xi \in \boldsymbol{R}, x \in \mathfrak{C}, \xi^2 + |x|^2 = 1\},$$
$$S_{15} = \{Y \in V^{15} \,|\, (Y, Y) = 2\} = \{F_2(y_2) + F_3(y_3) \,|\, y_2, y_3 \in \mathfrak{C}, |y_2|^2 + |y_3|^2 = 1\}.$$

We identify S^8 with S_8 and S^{15} with S_{15} under the correspondences

$$\begin{aligned} S^8 \ni (\xi, x) \quad &\to \quad \xi(E_2 - E_3) + F_1(x) \in S_8, \\ S^{15} \ni (a, b) \quad &\to \quad F_2(a) - F_3(\overline{b}) \in S_{15} \end{aligned}$$

We define special orthogonal groups $SO(9)$ and $SO(8)$ by $SO(9) = SO(V^9) = \{\alpha \in \mathrm{Iso}_{\boldsymbol{R}}(V^9) \,|\, (\alpha X_1,\ \alpha X_2) = (X_1, X_2), \det\alpha = 1\}$ and $SO(8) = \{\alpha \in SO(9) \,|\, \alpha(E_2 - E_3) = E_2 - E_3\}$, respectively. Then the correspondence $\alpha \in SO(9)$ to $\alpha(E_2 - E_3)$ induces the homeomorphism $SO(9)/SO(8) = S_8$.

We consider the compact exceptional group F_4 :

$$F_4 = \{\alpha \in \mathrm{Iso}_{R}(\mathfrak{J}) \,|\, \alpha(X \circ Y) = \alpha X \circ \alpha Y\}.$$

This contains subgroups

$$\begin{aligned} (F_4)_{E_1} &= \{\alpha \in F_4 \,|\, \alpha E_1 = E_1\}, \\ (F_4)_{E_1, E_2} &= \{\alpha \in F_4 \,|\, \alpha E_1 = E_1, \alpha E_2 = E_2\}. \\ (F_4)_{E_1, F_2(1)} &= \{\alpha \in F_4 \,|\, \alpha E_1 = E_1, \alpha F_2(1) = F_2(1)\}. \end{aligned}$$

The group $(F_4)_{E_1}$ is isomorphic to the spinor group $Spin(9)$ as the covering group of $SO(9)$ and the group $(F_4)_{E_1,E_2}$ is isomorphic to the spinor group $Spin(8)$ as the covering group of $SO(8)$. Furthermore the group $(F_4)_{E_1,F_2(1)}$ (hereafter this group is denoted by $Spin'(7)$) is isomorphic to the group $(F_4)_{E_1,E_2,F_1(1)}$ which is isomorphic to the spinor group $Spin(7)$ as the covering group of $SO(7) = \{\alpha \in SO(8) \,|\, \alpha F_1(1) = F_1(1)\}$:

$$(F_4)_{E_1} = Spin(9), \quad (F_4)_{E_1,E_2} = Spin(8), \quad (F_4)_{E_1,F_2(1)} = Spin'(7)$$

(in detail, [3],[6]). Note that $(F_4)_{E_1,F_2(1)}$ is a subgroup of $(F_4)_{E_1,E_2}$: $Spin'(7) \subset Spin(8)$. Indeed, let $\alpha \in (F_4)_{E_1,F_2(1)}$. Applying α on $F_2(1) \circ F_2(1) = E_1 + E_3$ we have $F_2(1) \circ F_2(1) = E_1 + \alpha E_3$, so we get $\alpha E_3 = E_3$. Since α always satisfies $\alpha(E_2 + E_3) = E_2 + E_3$, we have $\alpha E_2 = E_2$, and hence $\alpha \in (F_4)_{E_1,E_2}$.

Now, we will show that $Spin(9)$ acts transitively on S_{15}. To this end, we need the following [4].

Lemma 1 *Suppose that an element $A \in S_8$ is given. Choose any element $X_0 \in S_8$ such that $(A, X_0) = 0$. Choose any element $Y_0 \in S_{15}$ such that $2A \circ Y_0 = -Y_0$. Let*

$$Z_0 = 2X_0 \circ Y_0.$$

Choose any element $X_1 \in S_8$ such that $(A, X_1) = (X_0, X_1) = 0$. Choose any element $X_2 \in S_8$ such that $(A, X_2) = (X_0, X_2) = (X_1, X_2) = 0$. Let

$$Y_1 = -2Z_0 \circ X_1, \quad Z_2 = -2X_2 \circ Y_0, \quad X_3 = -2Y_1 \circ Z_2.$$

Finally, choose any element $X_4 \in S_8$ such that $(A, X_4) = (X_0, X_4) = (X_1, X_4) = (X_2, X_4) = (X_3, X_4) = 0$. Let

$$\begin{aligned} Z_4 &= -2X_4 \circ Y_0, \quad Y_2 = -2Z_0 \circ X_2, \quad Y_3 = -2Z_0 \circ X_3, \\ X_5 &= -2Y_1 \circ Z_4, \quad X_6 = 2Y_2 \circ Z_4, \quad\; X_7 = -2Y_3 \circ Z_4 \end{aligned}$$

and moreover let

$$\begin{aligned} Y_i &= -2Z_0 \circ X_i, \quad i = 4, 5, 6, 7, \\ Z_i &= -2X_i \circ Y_0, \quad i = 1, 3, 5, 6, 7. \end{aligned}$$

Then, an $\boldsymbol{R}$-linear map $\alpha : \mathfrak{J} \to \mathfrak{J}$ satisfying

$$\begin{aligned} &\alpha E = E, \qquad\quad \alpha E_1 = E_1, \qquad \alpha(E_2 - E_3) = A, \\ &\alpha F_1(e_i) = X_i, \quad \alpha F_2(e_i) = Y_i, \quad \alpha F_3(e_i) = Z_i, \quad i = 0, 1, \cdots, 7 \end{aligned}$$

belongs to $(F_4)_{E_1} = Spin(9)$.

Next we shall show (Example 5.6 in [5])

Proposition 2 $Spin(9)/Spin'(7) = S_{15}$.

PROOF. Let $\alpha \in Spin(9) = (F_4)_{E_1}$. Since $(\alpha Y, \alpha Y) = (Y, Y) = 2$ for $Y \in S_{15}$, the group $(F_4)_{E_1}$ acts on S_{15}. We shall show that this action is transitive. Let $Y_0 \in S_{15}$. Choose any element $A \in S_8$ such that $2A \circ Y_0 = -Y_0$. Using these A and Y_0, construct X_i, Y_i, Z_i and α of Lemma 1, then $\alpha \in Spin(9)$ and satisfies $\alpha F_2(1) = Y_0$, which shows the transitivity. The isotropy subgroup $Spin(9)_{F_2(1)}$ of $Spin(9)$ at $F_2(1) \in S_{15}$ is $(F_4)_{E_1, F_2(1)} = Spin'(7)$. Thus we have the homeomorphism $Spin(9)/Spin'(7) = S_{15}$. □

From the fact that $\alpha E_1 = E_1$ and $\mathrm{tr}(\alpha X) = \alpha X$ for $\alpha \in Spin(9)$ and $X \in \mathfrak{J}$ [6, Lemma 2.2.(2)], the restriction $\alpha_{|V^9}$ belongs to $SO(9)$. Define a map $\varphi_{\mathfrak{C}} : Spin(9) \to SO(9)$ by $\varphi_{\mathfrak{C}}(\alpha) = \alpha_{|V^9}$. Then, $\varphi_{\mathfrak{C}}$ is a homomorphism and induces a map $\overline{\varphi}_{\mathfrak{C}} : S_{15} = Spin(9)/Spin'(7) \to Spin(9)/Spin(8) = SO(9)/SO(8) = S_8$. Now, for $(a, b) \in S^{15}$, we set $Y_0 = F_2(a) - F_3(\overline{b}) \in S_{15}$ and $A = (2|a|^2 - 1)(E_2 - E_3) + 2F_1(b\overline{a}) \in S_8$. Then we have (Table of the Jordan products on p.249 in [5])

$$\begin{aligned}
2A \circ Y_0 &= 2((2|a|^2 - 1)(E_2 - E_3) + 2F_1(b\overline{a})) \circ (F_2(a) - F_3(\overline{b})) \\
&= -(2|a|^2 - 1)F_2(a) - (2|a|^2 - 1)F_3(\overline{b}) + 2F_3(\overline{(b\overline{a})a}) - 2F_2(\overline{\overline{b}(b\overline{a})}) \\
&= (-2|a|^2 + 1)(F_2(a) + F_3(\overline{b})) + 2|a|^2 F_3(\overline{b}) - 2|b|^2 F_2(a) \\
&= -F_2(a) + F_3(\overline{b}) = -Y_0.
\end{aligned}$$

Using these A and Y_0 in Lemma 1, we construct $\alpha \in Spin(9)$. Then we have

$$\alpha(E_2 - E_3) = A = (2|a|^2 - 1)(E_2 - E_3) + 2F_1(b\overline{a}).$$

Thus, for $(a, b) \in S^{15}$, we have $\overline{\varphi}_{\mathfrak{C}}(a, b) = (2|a|^2 - 1, 2b\overline{a})$, which is nothing but $h_{\mathfrak{C}} : S^{15} \to S^8$.

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