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Analyses of $dN_{ch}/d\eta$ and dN_{ch}/dy distributions of BRAHMS Collaboration by means of the Ornstein-Uhlenbeck process

M. IDE[1], M. BIYAJIMA[1,2] and T. MIZOGUCHI[3]

[1]Department of Physics, Faculty of Science, Shinshu University, Matsumoto 390-8621, Japan
[2]The Niels Bohr Institute, DK-2100, Copenhagen, Denmark
[3]Toba National College of Maritime Technology, Toba 517-8501, Japan
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Abstract

Interesting data on $dN_{\rm ch}/d\eta$ in Au-Au collisions ($\eta=-\ln\tan(\theta/2)$) with the centrality cuts have been reported by BRAHMS Collaboration. Using the total multiplicity $N_{\rm ch}=\int(dN_{\rm ch}/d\eta)d\eta$, we find that there are scaling phenomena among $(N_{\rm ch})^{-1}dN_{\rm ch}/d\eta=dn/d\eta$ with different centrality cuts at $\sqrt{s_{NN}}=130$ GeV and 200 GeV, respectively. To explain these scaling behaviors of $dn/d\eta$, we consider the stochastic approach named the Ornstein-Uhlenbeck process with two sources. The following Fokker-Planck equation is adopted for the present analyses,

$$\frac{\partial P(x,\,t)}{\partial t}=\gamma\left[\frac{\partial}{\partial x}x+\frac{1}{2}\,\frac{\sigma^2}{\gamma}\frac{\partial^2}{\partial x^2}\right]P(x,\,t)$$

where x means the rapidity (y) or pseudo-rapidity (η). t, γ and σ^2 and the evolution parameter, the frictional coefficient and the variance, respectively. Introducing a variable of $z_r=\eta/\eta_{\rm rms}$ ($\eta_{\rm rms}=\sqrt{\langle\eta^2\rangle}$) we explain the dn/dz_r distributions in the present approach. Moreover, to explain the rapidity (y) distributions from η distributions at 200 GeV, we have derived the formula as

$$\frac{dn}{dy}=J^{-1}\frac{dn}{d\eta},$$

where $J^{-1}=\sqrt{M(1+\sinh^2 y)}/\sqrt{1+M\sinh^2 y}$ with $M=1+(m/p_t)^2$. Their data of pion and all hadrons are fairly well explained by the O-U process. To compare our approach with another one, a phenomenological formula by Eskola et al. is also used in calculations of $dn/d\eta$.

1 Introduction

Recently interesting data on $dN_{\rm ch}/d\eta(\eta=-\ln\tan(\theta/2))$ and $(0.5\langle N_{\rm part}\rangle)^{-1}dN_{\rm ch}/d\eta|_{\eta=0}$ in Au+Au collision at $\sqrt{s_{NN}}=130$ GeV and 200 GeV have been reported by

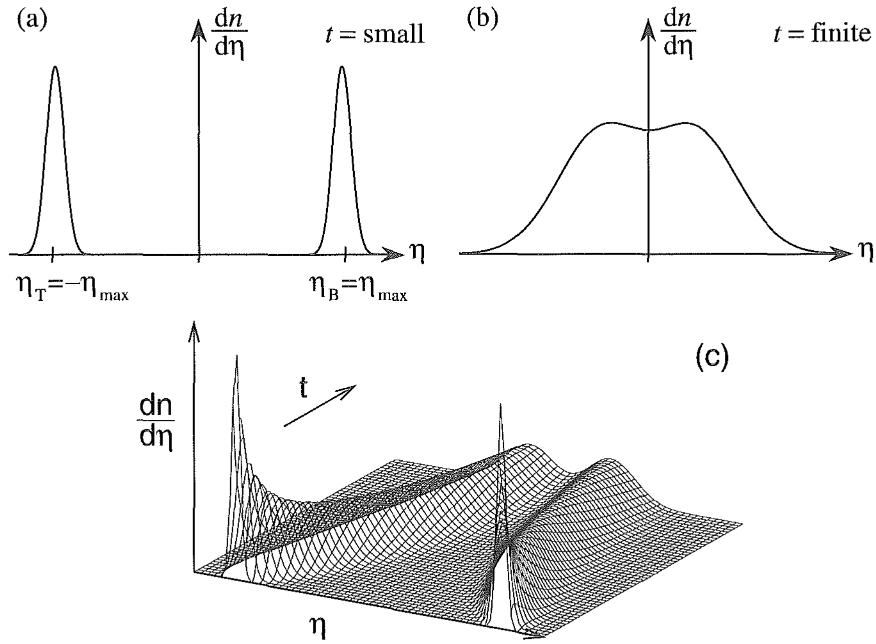

Figure 1 : (a) Initial distribution of Eq. (2). (b) Final distribution at t=finite. (c) Evolution of Eq. (2).

BRAHMS Collaboration [1, 2]. ($\langle N_{\rm part}\rangle$ and $N_{\rm ch}$ mean the numbers of participants (nuclei) and charged particles produced in collisions, respectively.) Very recently the BRAHMS Collaboration has reported preliminary data on rapidity (y) distribution at 200 GeV in Ref. [3]. We are interested in theoretical analyses of these data.

On the other hand, in Refs. [4, 5] we have investigated the property of η scaling of $(N_{\rm ch})^{-1}dN_{\rm ch}/d\eta = dn/d\eta$ by PHOBOS Collaboration and found that the η scaling holds. As a possible theoretical approach, we have adopted the stochastic theory named the Ornstein-Uhlenbeck (O-U) process with two sources at $\pm y_{\rm max} = \ln(\sqrt{s_{NN}}/m_N)$. In this paper, we would like to analyses data [1, 2, 3] by the stochastic approach in terms of the pseudo-rapidity and/or rapidity variables.

The approach named the O-U process is described by the following Fokker-Planck equation,

$$\frac{\partial P(y,t)}{\partial t} = \gamma\left[\frac{\partial}{\partial y}y + \frac{1}{2}\frac{\sigma^2}{\gamma}\frac{\partial^2}{\partial y^2}\right]P(y,t), \tag{1}$$

where t, γ and σ^2 are the evolution parameter, the frictional coefficient and the variance, respectively[1]. Assuming two sources at $\pm y_{\rm max} = \ln(\sqrt{s_{NN}}/m_N)$ at $t=0$ and

[1]The equivalent Langevin stochastic equation with the white noise $f_{\rm W}(t)$ is given as

$$\frac{dy}{dt} = -\gamma y + f_{\rm W}(t).$$

$P(y, 0)=0.5[\delta(y+y_{\max})+\delta(y-y_{\max})]$, we obtain the following distribution function for $dn/d\eta$ (assuming $y\approx\eta$) using the probability density $P(y, t)$[6, 7, 8, 9]

$$P(y, y_{\max}, t)=\frac{1}{\sqrt{8\pi V^2(t)}}\left\{\exp\left[-\frac{(y+y_{\max}e^{-\gamma t})^2}{2V^2(t)}\right]+\exp\left[-\frac{(y-y_{\max}e^{-\gamma t})^2}{2V^2(t)}\right]\right\}, \tag{2}$$

where $V^2(t)=(\sigma^2/2\gamma)p$ with $p=1-e^{-2\gamma t}$. The physical picture of Eq. (2) with the assumption of $y\approx\eta$ are shown in Fig. 1. In our approach, it is assumed that $N_{\rm ch}/2$ particles are created at $\pm y_{\max}$ at $t=0$. Then these $N_{\rm ch}=(N_{\rm ch}/2+N_{\rm ch}/2)$ particles are evolved according to Eq. (2). It is worthwhile to mention that a similar approach for the proton spectra has been given in Ref. [11].

The contents of the present paper are organized as follows. In Sec. 2 η scaling of BRAHMS Collaboration is investigated. In Sec. 3 analyses of η distribution by means of Eq. (2) are performed. The physical meaning of evolution parameter γt with the frictional coefficient is also considered. In Sec. 4 $z_r=\eta/\eta_{\rm rms}(\eta_{\rm rms}=\sqrt{\langle\eta^2\rangle})$ scaling is considered. In Sec. 5 analysis of y distribution derived $dn/d\eta$ distribution is presented. In the final section concluding remarks are given.

2 Analysis of η scaling of $dn/d\eta$ by BRAHMS Collaboration

First of all, we consider the problem on η scaling in Fig. 2, plotting the data of $dn/d\eta$ at 130 GeV and 200 GeV. The η scaling seems to be held. These distributions show

$$\left.\frac{dn}{d\eta}\right|_{\eta=0}\approx c \text{ (constant)}. \tag{3}$$

Moreover, we examine the intercept at $\eta=0$. Authors of Ref. [11], WA98 Collaboration, noticed that the intercepts divided by $(0.5\langle N_{\rm part}\rangle)$ should be described by

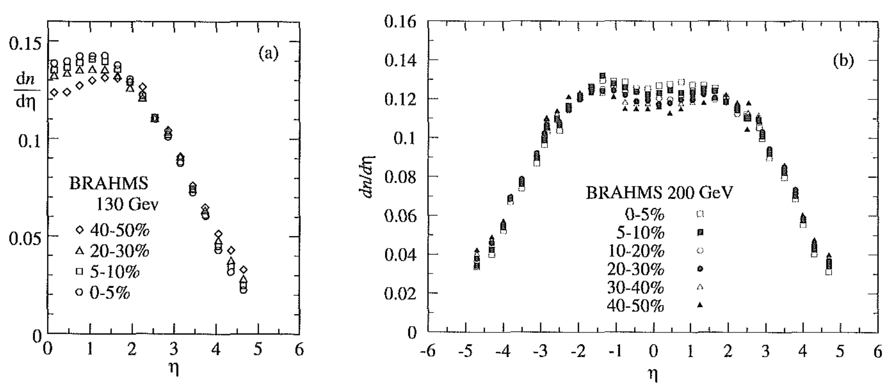

Figure 2 : (a) A set of $dn/d\eta$'s with different centrality cuts at $\sqrt{s_{NN}}=130$ Gev. Each symbols have error-bars of about 8~10% of the magnitude. (b) $dn/d\eta$ with different centrality cuts at $\sqrt{s_{NN}}=200$ Gev. About the error bars, the situation is the same as (a).

the power-like law, as

$$(0.5\langle N_{\rm part}\rangle)^{-1}\frac{dN_{\rm ch}}{d\eta}\bigg|_{\eta=0}=A\langle N_{\rm part}\rangle^{\alpha}, \tag{4}$$

provided that the participants (nuclei) have lost memory and every participant contribute a similar amount of energy to particle production in collisions. Actually it can be said that the power-like law holds, as seen in Fig. 3. See Tables 1 and 2. This physical picture with Eq. (4) indirectly supports the availability of the stochastic approach. Combining Eqs. (3) and (4), we have the following relations

$$c^{\rm Ex}=\frac{1}{N_{\rm ch}}\frac{dN_{\rm ch}}{d\eta}\bigg|_{\eta=0}, \tag{5}$$

$$c^{\rm Sp}=\frac{0.5\langle N_{\rm part}\rangle}{N_{\rm ch}}A\langle N_{\rm part}\rangle^{\alpha}, \tag{6}$$

where the suffix "Sp" means the semi-phenomenological formula. Comparisons between Eqs. (5) and (6) with A and α in Fig. 3 are shown in Tables 1 and 2.

As seen in Tables 1 and 2, the intercept at $\eta=0$ is fairly well explained by the semi -phenomenological expression, Eq. (6). This implies that the stochastic approach may be available, because the participants lost their memory in collision.

3 Analyses of data by Eq. (2)

Using the O-U process with two sources, Eq. (2), we have analyzed the data. The results at $\sqrt{s_{NN}}=130$ GeV and 200 GeV are shown in Figs. 4 and 5, and Tables 3 and 4. In our analyses we use Eq. (2) the pseudo-rapidity (η) instead of the rapidity (y). As seen in Tables 3 and 4, $R=N_{\rm ch}^{\rm (Th)}/N_{\rm ch}$ is always larger than 1. In the measurements of BRAHMS Collaboration, as the observable region is restricted with $|\eta|\leq 4.7$, we can conjecture the number of $N_{\rm ch}^{\rm (Th)}$ is always 3%~7% larger than $N_{\rm ch}$.

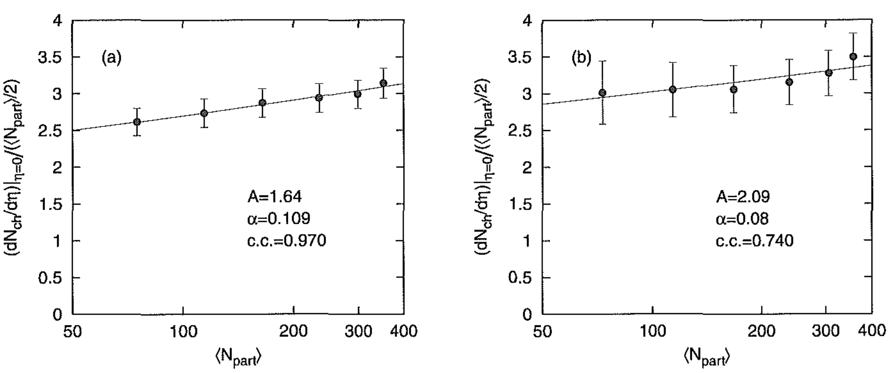

Figure 3: (a) Estimation of parameters A and α at $\sqrt{s_{NN}}=130$ Gev. The method of linear-regression is used. A=1.64, $\alpha=0.109$, and the correlation coefficient (c. c.) is 0.970. A power-like law is seen. (b) $\sqrt{s_{NN}}=200$ Gev. A=2.09, $\alpha=0.08$, and (c. c)=0.740.

Table 1: Empirical examination of Eqs. (5) and (6) at $\sqrt{s_{NN}}=130$ GeV. $\delta c_e=0.013\sim0.015$ and $\delta c_s=0.010\sim0.016$.

centrality (%)	40-50	30-40	20-30	10-20	5-10	0-5
$\langle N_{\rm part}\rangle$	75	114	165	235	299	352
$N_{\rm ch}$	750±60	1160±90	1720±130	2470±190	3180±250	3860±430
$c^{\rm Ex}$	$0.131\pm\delta c_e$	$0.134\pm\delta c_e$	$0.138\pm\delta c_e$	$0.141\pm\delta c_e$	$0.143\pm\delta c_e$	$0.137\pm\delta c_e$
$c^{\rm Sp}$	$0.131\pm\delta c_s$	$0.135\pm\delta c_s$	$0.137\pm\delta c_s$	$0.141\pm\delta c_s$	$0.144\pm\delta c_s$	$0.141\pm\delta c_s$

Table 2: The same as Table 1 but $\sqrt{s_{NN}}=200$ GeV, $\delta c_e=0.014\sim0.016$ and $\delta c_s=0.011\sim0.016$.

centrality (%)	40-50	30-40	20-30	10-20	5-10	0-5
$\langle N_{\rm part}\rangle$	73±8	114±9	168±9	239±10	306±11	357±8
$N_{\rm ch}$	890±70	1380±110	2020±160	2920±230	3810±300	4630±370
$c^{\rm Ex}$	$0.124\pm\delta c_e$	$0.126\pm\delta c_e$	$0.127\pm\delta c_e$	$0.131\pm\delta c_e$	$0.135\pm\delta c_e$	$0.129\pm\delta c_e$
$c^{\rm Sp}$	$0.121\pm\delta c_s$	$0.126\pm\delta c_s$	$0.131\pm\delta c_s$	$0.133\pm\delta c_s$	$0.133\pm\delta c_s$	$0.129\pm\delta c_s$

The different values of χ^2 in Tables 3 and 4 are attributed to the magnitude of the error bars at 130 GeV and 200 GeV.

The intercepts of $dn/d\eta$ at $\eta=0$ is explained by the following expression in the O-U process,

$$c^{(\rm Th)}=\frac{1}{\sqrt{2\pi V^2(t)}}\left\{\exp\left[-\frac{(\pm\eta_{\rm max}\sqrt{1-p})^2}{2V^2(t)}\right]\right\} \qquad (7)$$

Since our theory is based on the O-U process, the intercept $c^{(\rm Th)}$ is relating to $y_{\rm max}$, the width of $dn/d\eta$ and the evolution parameter.

Next we consider physical meaning of the evolution parameter γt. When we assign the dimension of time [sec] to t, γ is in units of [sec^{-1}]. For the magnitude of the

Table 3: Estimated parameters at $\sqrt{s_{NN}}=130$ GeV in our analyses by Eq. (2) with two sources. Evolution of Eq. (2) is stopped at minimum χ^2s. $\eta_{\rm max}=4.8$. $R=N_{\rm ch}^{(\rm Th)}/N_{\rm ch}$. $\eta_{\rm rms}=\sqrt{\langle\eta^2\rangle}$.

centrality (%)	40-50	20-30	5-10	0-5
$N_{\rm ch}^{(\rm Th)}$	789±17	1775±37	3273±68	3952±83
R	1.05	1.03	1.03	1.04
$\eta_{\rm rms}$	2.32±0.12	2.27±0.12	2.24±0.12	2.21±0.12
p	0.841±0.007	0.858±0.007	0.865±0.007	0.871±0.007
$V^2(t)$	2.79±0.23	2.80±0.23	2.64±0.21	2.56±0.20
$c^{(\rm Th)}$	0.124±0.007	0.133±0.007	0.136±0.008	0.139±0.008
χ^2/n.d.f.	0.877/13	0.434/13	0.507/13	0.758/13

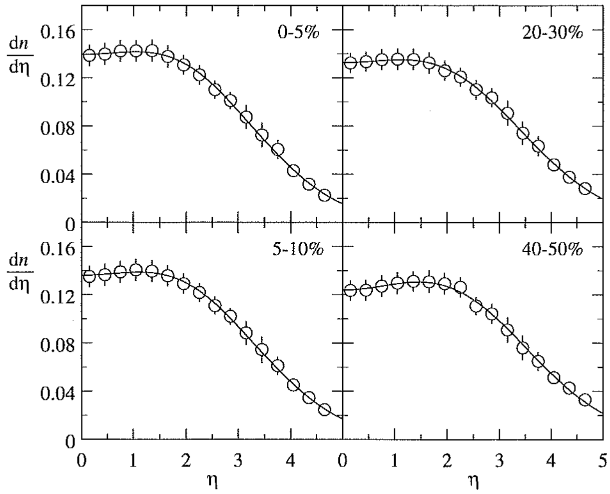

Figure 4 : Analysis of $dn/d\eta$ at $\sqrt{s_{NN}}$=130 GeV by Eq. (2) See Table 3.

Table 4 : Estimated parameters at $\sqrt{s_{NN}}$=200 GeV in our analyses by Eq. (2) with two sources. Evolution of Eq. (2) is stopped at minimum χ^2's. $\eta_{\rm max}=5.4$. $R=N_{\rm ch}^{\rm (Th)}/N_{\rm ch}$, $\eta_{\rm rms}=\sqrt{\langle\eta^2\rangle}$, $\delta p\approx 0.005$ and $\delta c_t=0.004\sim 0.005$.

centrality (%)	40–50	30–40	20–30	10–20	5–10	0–5
$N_{\rm ch}^{\rm (Th)}$	955±15	1477±24	2158±34	3101±49	4034±63	4881±76
R	1.07	1.07	1.07	1.06	1.06	1.05
$\eta_{\rm rms}$	2.41±0.08	2.40±0.06	2.39±0.09	2.37±0.08	2.35±0.08	2.32±0.08
p	0.854±δp	0.859±δp	0.862±δp	0.866±δp	0.871±δp	0.878±δp
$V^2(t)$	3.169±0.20	3.17±0.14	3.15±0.19	3.16±0.19	3.10±0.19	3.08±0.19
$c^{\rm (Th)}$	0.115±δc_t	0.117±δc_t	0.118±δc_t	0.121±δc_t	0.123±δc_t	0.128±δc_t
χ^2/n.d.f.	7.2/33	5.2/33	4.3/33	5.4/33	4.9/33	5.1/33

interaction region of Au+Au collisions, we assume to be 10 fm. See discussions in Ref. [12]. See also Tables 5 and 6. The averaged γ [fm^{-1}] are almost the same as estimated values from PHOBOS Collaboration [13, 14] and ones estimated from the proton spectra at SPS energies in Ref. [10].

4 The $z_r=\eta/\eta_{\rm rms}$ scaling

To investigate the $z_r=\eta/\eta_{\rm rms}$ scaling which has been proposed in Ref. [4], we use $\eta_{\rm rms}=\sqrt{\langle\eta^2\rangle}=\sqrt{\sum\eta^2 dn/d\eta}$ at $\sqrt{s_{NN}}$ =130 GeV and 200 GeV. We can consider the following formula with z_r :

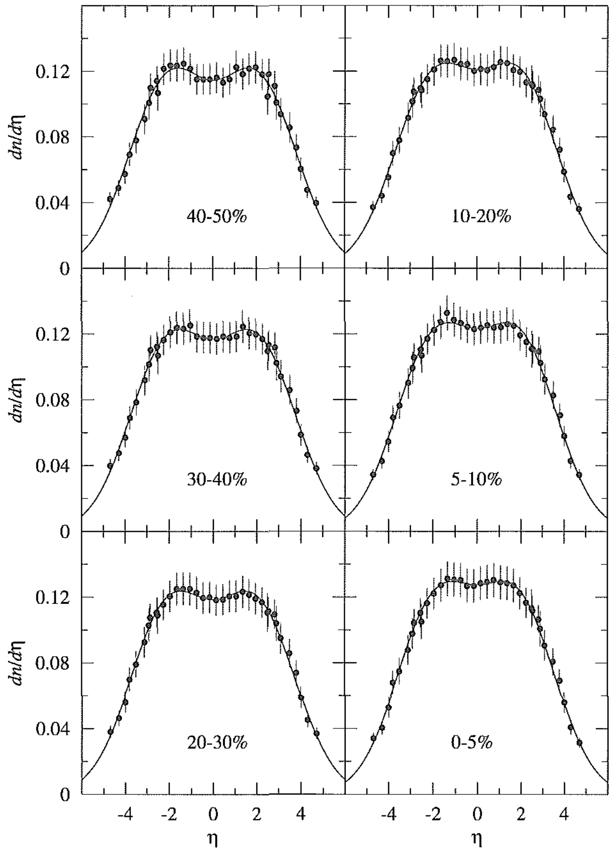

Figure 5 : The same as Fig. 4, but 200 GeV. See Table 4.

Table 5 : Values of γ and σ^2 at $\sqrt{s_{NN}}$=130 GeV provided that $t \approx 3.3 \times 10^{-23}$ sec.

centrality (%)	40-50	20-30	5-10	0-5	average
γ[fm^{-1}]	0.092	0.098	0.100	0.102	0.098
σ^2[fm^{-1}]	0.560	0.601	0.648	0.656	0.616
σ^2/γ	6.09	6.15	6.47	6.41	6.28

Table 6 : Values of γ and σ^2 at $\sqrt{s_{NN}}$=200 GeV provided that $t \approx 3.3 \times 10^{-23}$ sec.

centrality (%)	40-50	30-40	20-30	10-20	5-10	0-5	average
γ[fm^{-1}]	0.096	0.098	0.100	0.101	0.102	0.105	0.100
σ^2[fm^{-1}]	0.714	0.723	0.724	0.733	0.729	0.738	0.727
σ^2/γ	7.42	7.38	7.81	7.30	7.11	7.02	7.26

$$\eta_{\rm rms}\frac{dn}{d\eta}=\frac{dn}{dz_r}=f(z_r=\eta/\eta_{\rm rms}). \tag{8}$$

The right hand side with multiplying $\eta_{\rm rms}$ is obtained from Eq (2), as

$$\frac{dn}{dz_r}=\frac{1}{\sqrt{8\pi V_\eta^2(t)}}\left\{\exp\left[-\frac{(z_r+z_{\rm max}e^{-\gamma t})^2}{2V_\eta^2(t)}\right]+\exp\left[-\frac{(z_r-z_{\rm max}e^{-\gamma t})^2}{2V_\eta^2(t)}\right]\right\}, \tag{9}$$

where $z_{\rm max}=\eta_{\rm max}/\langle\eta_{\rm rms}\rangle$ and $V_\eta^2(t)=V^2(t)/\eta_{\rm rms}^2$. $\langle\eta_{\rm rms}\rangle$ is the averaged quantity in the set of data. In concrete analyses of data, $V_\eta^2(t)$ and p are treated as the free parameters. The z_r scaling at 130 GeV are compared with that of the hemisphere ($0\leq\eta\leq6$) at 200 GeV in Fig. 6 (b). It is difficult to distinguish them without the labels of incident energies. The behavior of full space is given in Fig. 6 (c). This situation is also observed in analyses of data at 130 GeV and 200 GeV by PHOBOS Collaboration [13, 14].

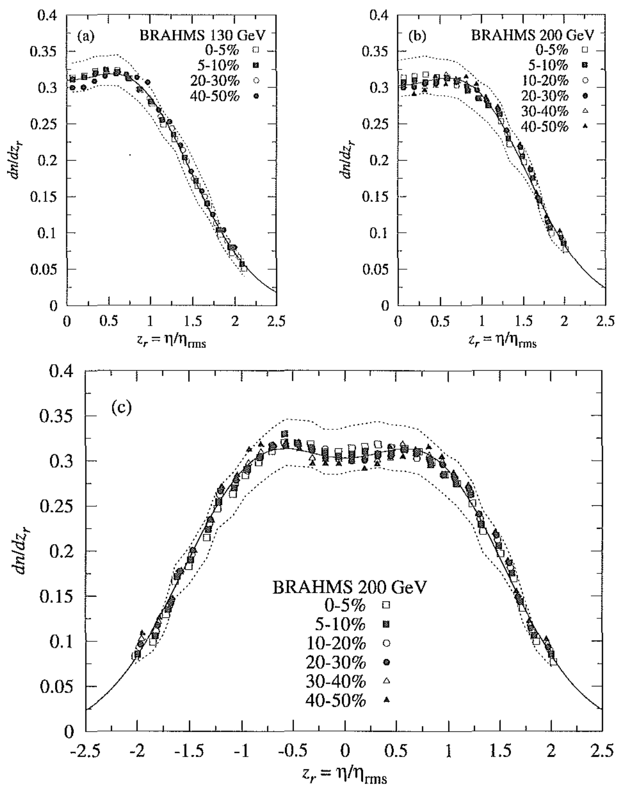

Figure 6 : Normalized distribution of dn/dz_r with $z_r=\eta/\eta_{\rm rms}$ scaling and estimated parameters using Eq. (9). (a) $\sqrt{s_{NN}}=130$ GeV, $p=1-e^{-2\gamma t}=0.889\pm0.003$, $V_\eta^2(t)=0.527\pm0.021$ and χ^2/n.d.f.=5.4/61. (b) and (c) $\sqrt{s_{NN}}=200$ GeV, $p=1-e^{-2\gamma t}=0.865\pm0.002$, $V_\eta^2(t)=0.559\pm0.015$ and χ^2/n.d.f.=32.1/189. (b) is taken from hemisphere data ($0\leq\eta\leq6$) of Fig. 6 (c). (c) The full space of dn/dz_r. The dotted lines represent the magnitude of error-bars in the centrality cut 0–5%.

5 Rapidity (y) distribution derived from η distribution

It is well known that one can usually calculate the η distribution from the y distribution. In this present study, on the contrary, we consider an inverse problem as follows. First we regard Eq. (2) as the correct description of the data, because of small χ^2 values. Using the following formula we can obtain the y distribution[2] as

$$\frac{dn}{dy}=\frac{\sqrt{M(1+\sinh^2 y)}}{\sqrt{1+M\sinh^2 y}}\frac{dn}{d\eta}, \tag{10}$$

where $M=1+m^2/p_t^2$. The right hand side, $dn/d\eta$, is given as

$$\frac{dn}{dy}=\frac{1}{\sqrt{8\pi V^2(t)}}\left\{\exp\left[-\frac{(\eta(y)+y_{\max}e^{-\gamma t})^2}{2V^2(t)}\right]+\exp\left[-\frac{(\eta(y)-y_{\max}e^{-\gamma t})^2}{2V^2(t)}\right]\right\}, \tag{11}$$

where $\eta(y)=\mathrm{arcsinh}(\sqrt{M}\sinh y)$. From Eq. (2) with the averaged parameters p and $V^2(t)$, we obtain y distributions at 200 GeV for π meson and all hadrons ($\pi^{\pm}$, $K^{\pm}$, p and $\bar{p}$). They are compared with the data in Ref. [3] in Fig. 7. The small peak is due to the inverse Jacobian factor. Indeed the data at 200 GeV show these behaviors at $y\approx 0$, even large error bars. To confirm these phenomena, measurements in wider region as well as $y\approx 0$ are necessary.

A phenomenological approach proposed in Ref. [15] (which is named as EKRT) is also shown in Fig 7.

$$\frac{dn}{dy}(\mathrm{EKRT})=\frac{1}{c_N}\frac{(1+e^{-y_0/d})^2}{(1+e^{(-y-y_0)/d})(1+e^{(y-y_0)/d})}, \tag{12}$$

where c_N is the normalization factor[3]. $y_0=3.3$ and $d=0.65$ are parameters[4] given in Ref. [15]. Eq. (12) also reproduces the both data in Fig. 7. From Eq. (12) we can calculate $dn/d\eta$ (centrality cut 20–30%) at 130 GeV and 200 GeV which are presented in Fig. 8. The

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[2]

$$y=\frac{1}{2}\ln\frac{E+p_z}{E-p_z}=\frac{1}{2}\ln\left[\frac{\sqrt{1+m^2/p_t^2+\sinh^2\eta}+\sinh\eta}{\sqrt{1+m^2/p_t^2+\sinh^2\eta}-\sinh\eta}\right]=\tanh^{-1}\left(\frac{p_z}{E}\right)\approx-\ln\tan(\theta/2)\equiv\eta.$$

$$\eta=\frac{1}{2}\ln\frac{p+p_z}{p-p_z}\ \text{and}\ \frac{dn}{dy}=\frac{dn}{d\eta}\frac{d\eta}{dy},\ \text{where}\ \eta(y)=\mathrm{arcsinh}(\sqrt{M}\sinh y).$$

For $dn/d\eta=(p/E)dn/dy$, we have $p/E=\cosh\eta/\sqrt{1+m^2/p_t^2+\sinh^2\eta}$. Moreover, we have confirmed that $\int_{-\infty}^{\infty}(dn/dy)dy=1$ and $\int_{-\infty}^{\infty}(dn/d\eta)d\eta=1$.

[3]We have estimated the normalization factor c_N as follows

$$c_N=\int_{-\infty}^{\infty}\frac{(1+e^{-y_0/d})^2}{(1+e^{(-y-y_0)/d})(1+e^{(y-y_0)/d})}dy=6.68.$$

[4]Notice that a similar expression with its symmetrization can be seen in Ref. [16]. A different expression based on the non-linear Fokker-Planck equation for dn/dy is found in Ref. [17]. Both are proposed for analyses of pp (or $\bar{p}p$) collisions.

coincidences between data and theory are very well, when y_0 and d are treated as free parameters.

6 Concluding Remarks

1) We have observed that the behaviors of η scaling of $dn/d\eta$ by BRAHMS Collaboration hold fairly well among the various centrality cuts at $\sqrt{s_{NN}}=130$ GeV and 200 GeV.
2) To explain those scaling behaviors, we have assumed that $dn/d\eta$ is governed by the O-U stochastic process with two sources at $\pm y_{\max}(\cong \ln\sqrt{s_{NN}}/m_N)$. The intercept of $dn/d\eta$ at $\eta=0$ is expressed by Eq. (7). See Tables 3 and 4. The constant c's are reflecting the scaling property relating to the O-U process.
3) From the evolution parameter γt and the assumed size of the interaction region of Au+Au collision (10 fm), we have obtained the following value, $\gamma\approx 0.1$ fm^{-1}, which

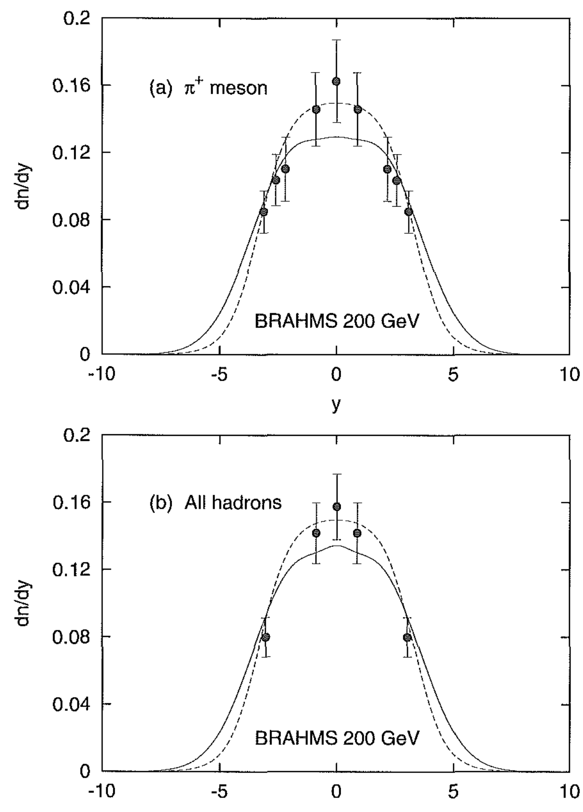

Figure 7: p and $V^2(t)$ are adopted from Tables 3 and 4. The averaged parameters $p=0.865$ (fixed) and $V^2(t)=3.138$ (fixed) are used. (a) dn/dy of all π meson. $m/p_t=0.4$ (fixed), $N_{ch}=1503\pm 77$. (b) dn/dy of all hadrons ($\pi^\pm$, $K^\pm$, p and $\bar{p}$). $m/p_t=0.5$ (fixed) and $N_{ch}=3915\pm 234$. The dashed lines are obtained from Eq. (12) with $1/c_N=0.149$.

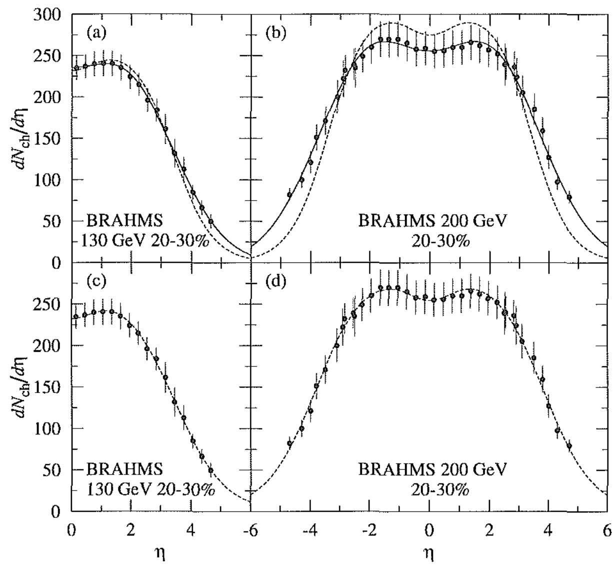

Figure 8 : Using Eqs. (11) and (12), we calculate $dN_{ch}/d\eta$ (centrality cut 20–30%) at 130 GeV and 200 GeV. (a) Dashed line is obtained by $y_0=3.3$, $d=0.65$, $N_{ch}^{(Th)}=1731\pm34$, $m/p_t=0.5$ and χ^2/n.d.f.$=12.1/15$. Solid line is obtained by O-U process [χ^2/n.d.f.$=0.43/13$ from Table 3], (b) Dashed line is obtained by $y_0=3.3$, $d=0.65$, $N_{ch}^{Th}=2054\pm31$, $m/p_t=0.5$ and χ^2/n.d.f.$=134/35$. Solid line is obtained by O-U process [χ^2/n.d.f.$=4.3/33$ from Table 4]. When y_0 and d are treated as free parameters, the following sets of parameters are obtained. (c) $y_0=3.32$, $d=0.83$, χ^2/n.d.f.$=0.40/13$. (d) $y_0=3.72$, $d=0.83$, χ^2/n.d.f.$=4.4/33$.

is almost the same value as that estimated in Ref. [11].

4) From Fig. 6, it can be said that the z_r scaling holds at 130 GeV and 200 GeV. It is difficult to distinguish them, as compared both data without the labels of incident energies.
5) Using Eq. (11) with η distributions at 200 GeV, we have calculated the y distributions which explain the data of Ref. [3]. The comparison with different approach given in Ref. [15] is also shown. In a future both approaches can be distinguished by the existence of a projection (small peak) at $y\approx0$.

Finally, it can be concluded that the O-U process is one of possible explanations for the scaling property of $dn/d\eta$ at $\sqrt{s_{NN}}=130$ GeV and 200 GeV by BRAHMS Collaboration [1, 2] as well as distributions by PHOBOS Collaboration [14].

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References

[1] I.G. Bearden *et al.* [BRAHMS Collaborations], Phys. Lett. B **523**, 227 (2001).

[2] I.G. Bearden *et al.* [BRAHMS Collaboration], Phys. Rev. Lett. 88, 202301 (2002).

[3] D. Ouerdane [BRAHMS Collaboration], nucl-ex/0212001.

[4] M. Biyajima, M. Ide, T. Mizoguchi and N. Suzuki, Prog. Theor. Phys. **108**, 559 (2002) and Addenda-*ibid.* **109**, 151 (2003). See nucl-th/0207002 and also hep-ph/0110305.

[5] M. Biyajima, M. Ide, T. Mizoguchi and N. Suzuki, "Analyses of $(0.5\langle N_{\rm part}\rangle)^{-1}dN_{\rm ch}/d\eta$ distributions of PHOBOS and BRAHMS Collaborations by means of a stochastic process" in Proceedings of the 4th symposium on science of hadrons under extreme conditions (March 4-6, 2002, JAERI, TOKAI, JAPAN), edited by S. Chiba and T. Maruyama, JAERI-Conf, 2002-11, p. 153.

[6] N. S. Goel and N. Richter-Dyn, *Stochastic Models in Biology* (Academic Press, New York, 1974).

[7] N. G. van Kampen, *Stochastic Processes in Physics and Chemistry* (North-Holland Publ., Amsterdam, 1981).

[8] K. Saitou, *Probability and Stochastic Process for Engineers* (in Japanese), (Saiensu-Sha, Tokyo, 1980).

[9] J. Hori, *Langevin Equation* (in Japanese), (Iwanami-Shoten, Tokyo, 1982).

[10] G. Wolschin, Eur. Phys. J. A **5**, 85 (1999) : His Fokker-Planck equation is given with a replacing $R(Y,t)$ by $P(y,t)$ as

$$\frac{\partial P(y,t)}{\partial t}=\frac{1}{\tau_y}\frac{\partial}{\partial y}[(y-y_{eq})P(y,t)]+D_y\frac{\partial^2}{\partial y^2}P(y,t)$$

where y_{eq} is relating to the rapidity of the colliding energy.

[11] M. M. Aggarwal *et al.* [WA98 Collaboration], Eur. Phys. J. C **18**, 651 (2001).

[12] K. Morita, S. Muroya, C. Nonaka and T. Hirano, Phys. Rev. C **66**, 054904 (2002).

[13] M. Biyajima and T. Mizoguchi, nucl-th/0209004, Prog. Theor. Phys. **109** (2003) 483.

[14] R. Nouicer *et al.* [PHOBOS Collaboration], nucl-ex/0208003. See also, B. B. Back et al. [PHOBOS Collaboration], Phys. Rev. Lett. **87** (2001), 102303, and B. B. Back et al. [PHOBOS Collaboration], Phys. Rev. Lett. **88** (2002), 022302.

[15] K. J. Eskola, K. Kajantie, P. V. Ruuskanen and K. Tuominen, Phys. Lett. B **543**, 208 (2002).

[16] A. Ohsawa, Prog. Theor. Phys. **92**, 1005 (1994).

[17] M. Rybczynski, Z. Wlodarczyk and G. Wilk, hep-ph/0206157.