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Gauge Coupling Sum Rule as a Unification Barometer

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We study high energy corrections to gauge couplings due to a threshold effect of heavy particles, higher dimensional operators in a gauge kinetic term and Kaluza-Klein excitation modes in the bulk. A specific sum rule among gauge couplings is derived including those corrections, and a magnitude of each correction is estimated.

§ 1. Introduction

It is one of important subjects to explore the physics beyond the Standard Model (SM). The most hopeful candidate is the minimal supersymmetric standard model (MSSM). It is expected that the gauge interactions in the MSSM are unified at a certain high-energy scale and then the theory is described as a supersymmetric grand unified theory (SUSY GUT). Although there is no direct evidence of SUSY, the prediction of $\alpha_3(M_Z)$ from the precision data α and $\sin\theta_W$ is one of the strong motivations for SUSY GUTs.[1] There is no powerful guiding principle to specify a model in SUSY GUTs theoretically. The understanding of the structure of SUSY GUT is influential in the study of ultimate theory in nature. To select a realistic unified model in the future, it is important to derive characteristic relations between the experimental data and the physical quantities which reflect the structure of a model. Several attractive relations among parameters have been proposed, e.g., relations among gauge couplings,[2] a relation among gaugino masses,[3] relations among scalar masses.[4] Particularly the gauge coupling constants are suitable input parameters because they can be measured precisely.

It is pointed out that there is a tiny discrepacy between the value of $\alpha_3(M_Z)$ obtained by using a naive unification scenario based on the MSSM and the experimental value of $\alpha_3(M_Z)$. There are several ways to reconcile the difference, e.g., threshold corrections[5),2)] due to superparticles at the weak scale and heavy particles around the GUT scale, corrections which come from non-renormalizable interactions in gauge kinetic terms suppressed by the reduced Planck mass M as an effect of quantum gravity[6),7)] and corrections in the presence of physics at an intermediate scale.[8)]

In order to test a unification scenario, it is important to utilize a formula of gauge couplings which reflects a feature of unification. As discussed in Ref.9), the quantity $\vec{\alpha}^{-1}(\mu)\cdot(\vec{I}\times\vec{b})$ is useful as a barometer of unification. Here $\vec{\alpha}^{-1}(\mu)$ is a 3-component vector $(\alpha_1^{-1}, \alpha_2^{-1}, \alpha_3^{-1})$ which represents gauge structure constants of the SM, $\vec{I}=(1, 1, 1)$ and $\vec{b}=(b_1, b_2, b_3)$ are coefficients of β functions at one-loop level. For example, for single step unification based on the MSSM, the following relation is derived

$$\Delta\vec{\alpha}_{\mathrm{MSSM}}^{-1}\equiv 5\alpha_1^{-1}(\mu)-12\alpha_2^{-1}(\mu)+7\alpha_3^{-1}(\mu)=0 \tag{1$\cdot$1}$$

for any scale μ in the desert at one-loop level and up to several corrections. By using the experimental values[10)]

$$\alpha_1^{-1}(M_Z)=58.97\pm0.05, \tag{1$\cdot$2}$$
$$\alpha_2^{-1}(M_Z)=29.61\pm0.05, \tag{1$\cdot$3}$$
$$\alpha_3^{-1}(M_Z)=8.44\pm0.05, \tag{1$\cdot$4}$$

the $\Delta\vec{\alpha}_{\mathrm{MSSM}}^{-1}$ is estimated as

$$\Delta\vec{\alpha}_{\mathrm{MSSM}}^{-1}=-1.39\pm1.83. \tag{1$\cdot$5}$$

This shows that the relation (1$\cdot$1) is valid enough as a first approximation and the grand unification scenario is realistic within the framework of the MSSM. Hence the quantity $\vec{\alpha}^{-1}(\mu)\cdot(\vec{I}\times\vec{b})$ can be treated as a barometer of gauge coupling unification. There is a discrepacy between values of $\alpha_3(M_Z)$ in theory and experiment after the two-loop effect is added.[11)] In fact, two-loop correction to the relation (1$\cdot$1) is not negligible and it suggests that there are some corrections to reconcile the discrepacy.

In this paper, we study high energy corrections to the above sum rule among gauge couplings (1$\cdot$1), e.g., two-loop correction, a threshold effect of heavy particles, a correction from higher dimensional operators in a gauge kinetic term and the power-law correction in a higher dimensional theory. A modified version of (1$\cdot$1) is derived including those corrections, and a typical magnitude of each correction is estimated.

In the next section, we explain several corrections to gauge coupling constants. In section 3, we derive a specific gauge coupling sum rule in a framework of SUSY GUTs. We estimate a typical magnitude of each correction in section 4. The final section is devoted to conclusions and discussion.

§ 2. High energy corrections

First we study two-loop corrections to the sum rule (1$\cdot$1). The two-loop renormalization group (RG) equations of gauge couplings are written by

$$\frac{d}{d\ln\mu}\alpha_i^{-1}(\mu)=-\frac{b_i}{2\pi}-\frac{1}{8\pi^2}\Big(\sum_j b_{ij}\alpha_j(\mu)-c_{if}\alpha_f(\mu)\Big) \tag{2$\cdot$1}$$

where α_fs are defined by $\alpha_f\equiv y_f^2/4\pi$, y_fs are Yukawa couplings, μ is a mass scale describing a RG point, and b_{ij}s and c_{if}s are written by group-theoretical factors.

The second term in RHS of (2·1) denotes two-loop contributions. The solutions of RG equations (2·1) are expressed as

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(\Lambda) + \frac{b_i}{2\pi}\ln\frac{\Lambda}{\mu} + \delta\alpha_i^{-1}(\mu) \tag{2·2}$$

where Λ is a high-energy scale and $\delta\alpha_i^{-1}(\mu)$ represents two-loop effects. By the use of the solutions (2·2), the following RG invariant relation at two-loop level is derived

$$(\vec{\alpha}^{-1}(\mu) - \delta\vec{\alpha}^{-1}(\mu)) \cdot (\vec{\alpha}^{-1}(\Lambda) \times \vec{b}) = 0. \tag{2·3}$$

When we take a limit such that $\delta\vec{\alpha}^{-1}(\mu) = 0$ and $\vec{\alpha}^{-1}(\Lambda) = \alpha_U^{-1}\vec{I}$, the relation (2·3) reduces to the sum rule (1·1) in the MSSM.

Second we study a boundary condition of $\vec{\alpha}^{-1}$ at Λ. A source to violate a universality $\vec{\alpha}^{-1}(\Lambda) = \alpha_U^{-1}\vec{I}$ stems from a non-renormalizable gauge kinetic term. The gauge kinetic term is, in general, given by

$$\begin{aligned}\mathscr{L}_{g.k.} &= \sum_{\alpha,\beta}\int d^2\theta f_{\alpha\beta}(\Phi^I) W^\alpha W^\beta + H.c.\\ &= -\frac{1}{4}\sum_{\alpha,\beta} Ref_{\alpha\beta}(\phi^I) F^\alpha_{\mu\nu} F^{\beta\mu\nu}\\ &\quad + \sum_{\alpha,\beta,\alpha',\beta'}\sum_I F^I_{\alpha'\beta'}\frac{\partial f_{\alpha\beta}(\phi^I)}{\partial\phi^I_{\alpha'\beta'}}\lambda^\alpha\lambda^\beta + H.c. + \cdots\end{aligned} \tag{2·4}$$

where α,β are indices related to gauge generators, Φ^Is are chiral superfields and λ^αs are the gaugino fields. The scalar and F-components of Φ^I are denoted by ϕ^I and F^I, respectively. The gauge multiplet is in the adjoint representation $R_{\rm ad}$ and the symmetric product of $R_{\rm ad} \times R_{\rm ad}$ is, in general, decomposed as

$$(R_{\rm ad} \times R_{\rm ad})_{\rm sym} = \sum_k R_k. \tag{2·5}$$

The gauge kinetic function $f_{\alpha\beta}(\Phi^I)$ is also decomposed as

$$f_{\alpha\beta}(\Phi^I) = \sum_{R_k} f^{R_k}_{\alpha\beta}(\Phi^I) \tag{2·6}$$

where $f^{R_k}_{\alpha\beta}(\Phi^I)$ is a part of gauge kinetic functions which transforms as the R_k-representation. After a breakdown of GUT symmetry, a boundary condition of α_i^{-1} is given by

$$\alpha_i^{-1}(\Lambda) = \alpha_U^{-1}(1 + C_i) \tag{2·7}$$

where C_is are non-universal factors which parametrize gravitational corrections generally. The R_k includes a singlet representation which gives a universal contribution on $\alpha_i^{-1}(\Lambda)$. For example, $SU(5)$ breaking induces the following factors*)

$$(C_1, C_2, C_3) = \frac{x_{24}}{2\sqrt{15}}(-1,-3,2) + \frac{x_{75}}{6}(-5,3,1) + \frac{x_{200}}{2\sqrt{21}}(10,2,1). \tag{2·8}$$

*)The factors $1/2\sqrt{15}$, $1/6$ and $1/2\sqrt{21}$ come from that the noremalization $Tr(T^aT^b) = \delta^{ab}/2$ of the 5×5, 10×10 and 15×15 matrices representing **24**, **75** and **200**.

Here x_Rs are model-dependent quantities including the VEV of Higgs fields whose representations are Rs, and their order is supposed to be $O(\Lambda/M)$ or less. (Λ is a unification scale.)

Next we discuss threshold corrections due to superheavy particles with mass of order GUT scale. There are three types of superheavy fields. The first ones are heavy vector multiplets denoted by V_A, which is a part of full GUT vector multiplet V. The second ones are heavy chiral multiplets denoted by $\Phi^{B(I)}$, which is a part of full GUT chiral multiplets Φ^I and all multiplets which comprise a part of the MSSM particles and Nambu-Goldstone multiplets are omitted. The last ones are linear combinations orthogonal to the MSSM particles and Nambu-Goldstone multiplets, which are denoted by Φ^C. A generic formula for threshold corrections $\delta\alpha_{i\mathrm{GUT}}^{-1}$ is given by

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(\Lambda) + \frac{b_i}{2\pi}\ln\frac{\Lambda}{\mu} + \delta\alpha_{i\mathrm{GUT}}^{-1} \tag{2·9}$$

$$\delta\alpha_{i\mathrm{GUT}}^{-1} \equiv \sum_A \frac{\xi_i^{V_A}}{2\pi}\ln\frac{\Lambda}{M_{V_A}} + \sum_{B(I)} \frac{\xi_i^{\Phi^{B(I)}}}{2\pi}\ln\frac{\Lambda}{M_{\Phi^{B(I)}}} + \sum_C \frac{\xi_i^{\Phi^C}}{2\pi}\ln\frac{\Lambda}{M_{\Phi^C}}$$

where M_{V_A}, $M_{\Phi^{B(I)}}$ and M_{Φ^C} are masses of heavy gauge multiplet V_A, heavy chiral multiplets $\Phi^{B(I)}$ and Φ^C, and the coefficients $\xi_i^{V_A}$, $\xi_i^{\Phi^{B(I)}}$ and $\xi_i^{\Phi^C}$ are coefficients of one-loop β function from V_A, $\Phi^{B(I)}$ and Φ^C.

Last we discuss a peculiar correction in models with extra space-time dimensions. If a theory has extra δ dimensions other than 4-dimensional Minkowski space-time and any of the MSSM particles live in the bulk, there appear the towers of Kaluza-Klein (KK) excitation modes. It is known that they give non-trivial contributions to the flow of gauge couplings which describes as a power-law behavior, [12)*)]

$$\begin{aligned}\alpha_i^{-1}(\mu) = \alpha_i^{-1}(\Lambda) &+ \frac{b_i}{2\pi}\ln\frac{\Lambda}{\mu} + \frac{b_i^{\mathrm{KK}}}{2\pi}\ln\frac{\Lambda}{M_C} \\ &+ \frac{b_i^{\mathrm{KK}}}{2\pi}\frac{X_\delta}{\delta}\left(\left(\frac{\Lambda}{M_C}\right)^\delta - 1\right),\end{aligned} \tag{2·10}$$

$$X^\delta \equiv \frac{\pi^{\delta/2}}{\Gamma\left(1+\frac{\delta}{2}\right)} \tag{2·11}$$

where $M_C = 1/R, R$ is a radius of compact space and b_i^{KK}s are coefficients of one-loop β functions from KK excitations.

§ 3. Gauge coupling sum rule

We derive a gauge coupling sum rule modified by gravitational corrections and GUT scale threshold corrections.**) We assume that a certain grand unification group G_U is broken down to that in the Standard Model G_{SM} at Λ directly, particles are

*)In Ref. 13), X_δ is defined as $X_\delta = \pi^{\delta/2}/\Gamma(2+\delta/2)$.The difference can be absorbed by the redefinition of the cutoff Λ.

**)The incorportion of other corrections is staghtforward and it will be discussed in the next section.

classified into two categories, superheavy particles whose masses are of $O(\Lambda)$ and light particles whose masses are of $O(1)$TeV and the low energy spectrum at $O(1)$TeV consists of the MSSM particles. After the breakdown of GUT symmetry, the threshold corrections are given as follows. The contributions of heavy vector multiplets are given by

$$\sum_A \xi_i^{V_A} \ln \frac{\Lambda}{M_{V_A}} = \xi_i^V \ln \frac{\Lambda}{M_V} + \Delta_i^V \tag{3·1}$$

where $\xi_i^V = \sum_A \xi_i^{V_A} = (\xi^V, \xi^V+4, \xi^V+6), \Delta_i^V \equiv \sum_A \xi_i^{V_A} \ln M_V/M_{V_A}, \xi^V \equiv -2Tr(T^a(G_U))^2$ and M_V is an averaged mass of V_A. In the same way, the contributions of $\Phi^{B(I)}$'s are given by

$$\sum_{B(I)} \xi_i^{\Phi^{B(I)}} \ln \frac{\Lambda}{M_{\Phi^{B(I)}}} = \sum_I \xi_i^{\Phi^I} \ln \frac{\Lambda}{M_{\Phi^I}} + \Delta_i^\Phi \tag{3·2}$$

where $\xi_i^{\Phi^I}$ and $\sum_{B(I)} \xi_i^{\Phi^{B(I)}}$, $\Delta_i^\Phi \equiv \sum_{B(I)} \xi_i^{\Phi^{B(I)}} \ln M_{\Phi^I}/M_{\Phi^{B(I)}}$ and M_{Φ^I} is an averaged mass of $\Phi^{B(I)}$. For example, in the minimal SUSY $SU(5)$ GUT, heavy supermultiplets are X and Y gauge multiplets, Higgs multiplets with SM quantum numbers $(\mathbf{8},\mathbf{1},0) + (\mathbf{1},\mathbf{3},0) + (\mathbf{1},\mathbf{1},0)$ come from adjoint Higgs multiplets Σ in $SU(5)$ and a pair of colored Higgs multiplets $H=(H_C,\bar{H}_C)$. The coefficients are calculated as $\xi_i^V=(-10,-6,-4)$ with $\xi^V=-10$, $\xi_i^\Sigma=(0,2,3)$ with $\xi^\Sigma=5$ and $\xi_i^H=(2/5,0,1)$ with $\xi^H=1$.

Now we introduce two kinds of 3-dimensional vectors such that $\vec{u}=(5,-3,-2)$ and $\vec{v}=(-1,3,-2)$. The vector $\vec{u}$ is orthogonal to the vectors $\vec{I}=(1,1,1)$ and $\xi_i^H=(2/5,0,1)$, and the vector $\vec{v}$ is orthogonal to the vectors $\vec{I}$ and ξ_i^V. By using the RG equations in the MSSM including gravitational corrections and GUT scale threshold corrections, we obtain the following relations,

$$\begin{aligned}\vec{u}\cdot\vec{\alpha}^{-1}(\mu) &= (5\alpha_1^{-1} - 3\alpha_2^{-1} - 2\alpha_3^{-1})(\mu) \\ &= \alpha_U^{-1}(5C_1 - 3C_2 - 2C_3) + \frac{18}{\pi}\ln\frac{M_U}{\mu} \\ &\quad + \frac{1}{2\pi}(5\Delta_1 - 3\Delta_2 - 2\Delta_3),\end{aligned} \tag{3·3}$$

$$\begin{aligned}\vec{v}\cdot\vec{\alpha}^{-1}(\mu) &= (-\alpha_1^{-1} + 3\alpha_2^{-1} - 2\alpha_3^{-1})(\mu) \\ &= \alpha_U^{-1}(-C_1 + 3C_2 - 2C_3) + \frac{6}{5\pi}\ln\frac{M_H}{\mu} \\ &\quad + \frac{1}{2\pi}(-\Delta_1 + 3\Delta_2 - 2\Delta_3)\end{aligned} \tag{3·4}$$

where $\Delta_i \equiv \Delta_i^V + \Delta_i^\Phi$, and the mass parameters M_U and M_H are defined by

$$M_U \equiv \left(\frac{M_V^2 \prod_I M_{\Phi^I}^{n_I}}{\prod_C M_C^{n_C}}\right)^{1/3}, M_H \equiv \frac{\prod_I M_{\Phi^I}^{\hat{n}_I}}{\prod_C M_C^{\hat{n}_C}} \tag{3·5}$$

where $n_I, n_C, \hat{n}_I$ and $\hat{n}_C$ are some rational numbers calculated by the formula $n_I = -\vec{u}\cdot\xi^{\Phi^I}/12, n_C = \vec{u}\cdot\xi^{\Phi^C}/12, \hat{n}_I = -5\vec{v}\cdot\xi^{\Phi^I}/12$ and $\hat{n}_C = 5\vec{v}\cdot\xi^{\Phi^C}/12$. There are relations among $n_I, n_C, \hat{n}_I$ and $\hat{n}_C$ such that $\sum_I n_I = \sum_C n_C + 1$ and $\sum_I \hat{n}_I = \sum_C \hat{n}_C + 1$. (In the minimal model, M_H is the colored Higgs mass M_{H_C} itself.) By eliminating the parameter μ in

RHSs of relations (3·3) and (3·4), we obtain a specific relation such that

$$\begin{aligned}\Delta\vec{\alpha}^{-1}_{\rm MSSM} &\equiv 5\alpha_1^{-1}(\mu)-12\alpha_2^{-1}(\mu)+7\alpha_3^{-1}(\mu)\\ &=\alpha_U^{-1}(5C_1-12C_2+7C_3)+\frac{9}{2\pi}\ln\frac{M_U}{M_H}\\ &\quad+\frac{1}{2\pi}(5\Delta_1-12\Delta_2+7\Delta_3).\end{aligned}\tag{3·6}$$

This is a modified version of gauge coupling sum rule (1·1) in the MSSM. The RHS of Eq. (3·6) is expressed by physical quantities which depend on high energy physics. On the other hand, the value of $\Delta\vec{\alpha}^{-1}_{\rm MSSM}$ is estimated by the experimental data. Hence this type of relation is useful to select a model in high energy physics.

§ 4. Estimation of magnitude

The modified gauge coupling sum rule is, in general, written down by

$$\begin{aligned}\Delta\vec{\alpha}^{-1}_{\rm MSSM} &\equiv 5\alpha_1^{-1}(\mu)-12\alpha_2^{-1}(\mu)+7\alpha_3^{-1}(\mu)\\ &=\delta\vec{\alpha}^{-1}_{\rm ext}\end{aligned}\tag{4·1}$$

where extra corrections are denoted by $\delta\vec{\alpha}^{-1}_{\rm ext}$ in total. In the following, we estimate a magnitude of each correction using typical values for model-dependent parameters. The estimation is carried out without specifying a SUSY-GUT model. The ranges are summarized in Fig. 1.

4. 1. *Two-loop correction*

The two-loop correction to the sum rule is given by

$$\delta\vec{\alpha}^{-1}_{\rm ext}\supset\delta\vec{\alpha}^{-1}_{(2)}\equiv 5\delta\alpha_1^{-1}-12\delta\alpha_2^{-1}+7\delta\alpha_3^{-1}.\tag{4·2}$$

We estimate the magnitude of $\delta\vec{\alpha}^{-1}_{(2)}$ in the MSSM. In the case where the only top Yukawa coupling y_t is not negligible, $\delta\alpha_i^{-1}(\mu)$ is given by

$$\begin{aligned}\delta\alpha_i^{-1}(\mu) &=\frac{1}{4\pi}\sum_j\frac{b_{ij}}{b_j}\ln(1+b_j\alpha_j(\Lambda)t_\mu)\\ &\quad-\frac{c_{it}}{4\pi}\int_0^{t_\mu}\frac{\alpha_t(\Lambda)F'(t')}{1+6\alpha_t(\Lambda)F(t')}dt'\end{aligned}\tag{4·3}$$

where $t_\mu\equiv\frac{1}{2\pi}\ln\frac{\Lambda}{\mu}$, and b_{ij} and c_{it} are

$$b_{ij}=\begin{pmatrix}\frac{199}{25} & \frac{27}{5} & \frac{88}{5}\\ \frac{9}{5} & 25 & 24\\ \frac{11}{5} & 9 & 14\end{pmatrix},\quad c_{it}=(\frac{26}{5},6,4).\tag{4·4}$$

Here the function $F(t')$ is given by

$$F(t')=\int_0^{t'}\prod_i(1+b_{\alpha_i}(\Lambda)t'')^{p_i^t/b_i}dt''\tag{4·5}$$

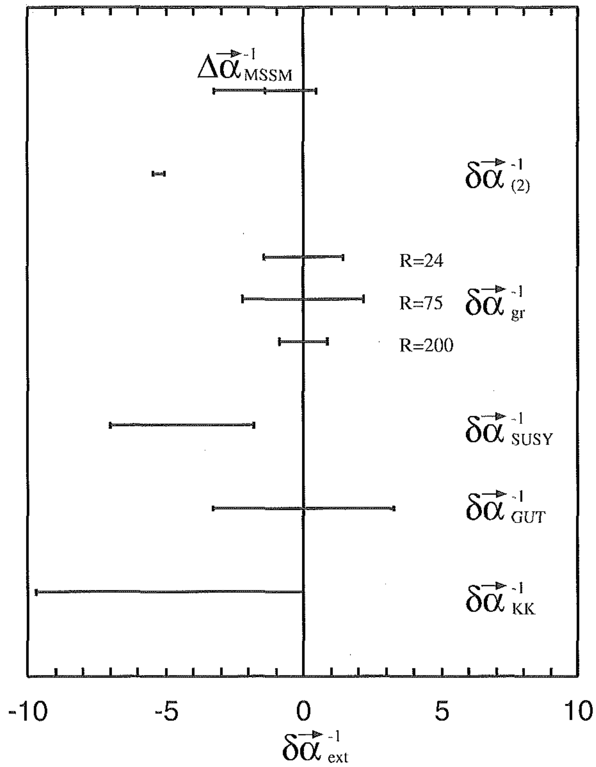

Fig. 1. The typical values of each correction to a gauge coupling sum rule. Each estimation is carried out in the following case, i. e., $\delta\vec{\alpha}^{-1}_{(2)}$ is a two-loop correction with a small $\tan\beta=2\sim10$, $\delta\vec{\alpha}^{-1}_{\rm gr}$ is a gravitational correction with $a=1$, $\delta\vec{\alpha}^{-1}_{\rm SUSY}$ is a threshold correction of superparticles at $O(1)$TeV with some assumptions on SUSY breaking parameters, $\delta\vec{\alpha}^{-1}_{\rm GUT}$ is a threshold correction at the GUT scale with $M_U/M_H=0.1\sim10$ and $\Delta_i=0$, and $\delta\vec{\alpha}^{-1}_{\rm KK}$ is a power-law correction of KK excitations with $b_i^{\rm KK}=(3/5,-3,-6)$, $\delta=1$ and $X_\delta(\Lambda/M_C)^\delta/\delta=2\sim20$.

where p_i^ts are constants, e.g., $(p_1^t,p_2^t,p_3^t)=(13/15,3,16/3)$ in the MSSM. The $F'(t)$ is a derivative of $F(t)$ by t. For $\alpha_i^{-1}(\Lambda)=\alpha_U^{-1}=24.45$ and $\Lambda=1.7\times10^{16}$GeV, the range of $\delta\vec{\alpha}^{-1}_{(2)}$ are $-5.01\sim-5.46$ with a small $\tan\beta=2\sim10$and $m_t=175$GeV. This correction is significantly large and it must be compensated by other corrections.

4. 2. *Gravitational correction*

The gravitational corrections to the sum rule are given by

$$\delta\vec{\alpha}^{-1}_{\rm ext}\supset\delta\vec{\alpha}^{-1}_{\rm gr}\equiv\alpha_U^{-1}(5C_1-12C_2+7C_3). \qquad (4\cdot6)$$

When we take $\alpha_U^{-1}=24.45$ and $|x_R|=a/100$ in SUSY $SU(5)$ GUT, typical values are given by $|\delta\vec{\alpha}^{-1}_{\rm gr}|=1.42a$ for $R=\mathbf{24}$, $2.20a$ for $R=\mathbf{75}$. $0.88a$ for $R=\mathbf{200}$. Here a is a model-dependent parameter whose magnitude would be $O(1)$ or less than that. To cancel two-loop correction, x_R should take the following values for each case, $a\sim-4$ for $R=\mathbf{24}$, 2.4 for $R=\mathbf{75}$, -6 for $R=\mathbf{200}$. If Λ is close to M, this contribution can be more sizable. In such a case, we need a mechanism to reduce this correction. If the vector $\vec{C}$ is given as a linear combination of $\vec{I}$ and $\vec{b}$, we have $\delta\vec{\alpha}_{\mathrm{gr}}^{-1}=0$. When the vector multiplet does not couple to the Higgs multiplets which break a unified symmetry, $\vec{C}$ is proportional to $\vec{I}$. For example, there is no non-universal gravitational correction at the lowest order in $SO(10)$ GUT where **45** and $\mathbf{16}(\overline{\mathbf{16}})$ break $SO(10)$. If $\vec{C}\propto\vec{b}$, this case is regarded as a kind of mirage unification suggested in superstring theory.[14]

4. 3. *Weak threshold correction*

There are, in general, threshold corrections at $O(1)$TeV due to the appearance of superpartners of SM particles. The correction to the sum rule is given by

$$\delta\vec{\alpha}_{\mathrm{ext}}^{-1}\supset\delta\vec{\alpha}_{\mathrm{SUSY}}^{-1}$$
$$=\frac{1}{2}\sum_{a=1}^{Ng}\ln\frac{m_{\tilde{q}_a}^7 m_{\tilde{l}_a}^3}{m_{\tilde{u}_a}^5 m_{\tilde{d}_a}^3 m_{\tilde{e}_a}^2}+2\ln\frac{m_{\tilde{w}}^8}{m_{\tilde{g}}^7 M_Z}+\frac{3}{2}\ln\frac{m_H m_{\tilde{H}}^4}{M_Z^5} \qquad (4\cdot 7)$$

where N_g is the number of generations, $m_{\tilde{f}}$s are masses of superpartners of chiral fermions f, $m_{\tilde{g}}$ is a gluino mass, $m_{\tilde{w}}$ is a wino mass, m_H is a Higgs mass and $m_{\tilde{H}}$ is a Higgsino mass. Here we assume that one of Higgs doublets is light, and its contribution is omitted in $\delta\vec{\alpha}_{\mathrm{SUSY}}^{-1}$. We neglect a $SU(2)_L\times U(1)_Y$ breaking effect, e.g., a mixing among gauginos and Higgsinos.*) Further we omit the threshold correction of top quark for simplicity. In the case that a GUT-relation among gaugino masses $M_1=M_2=M_3$ holds at a unification scale, the contribution of the second term in RHS of Eq.(4$\cdot$7) is negative and its magnitude can be $O(10)$. The contribution of the last term is also large and positive. Some cancellation is required to make the magnitude of $\delta\vec{\alpha}_{\mathrm{SUSY}}^{-1}$ a suitable value. Let us discuss whether such a cancellation occurs or not naturally under some assumptions on SUSY breaking parameters. We assume that scalar fields and gauginos take a common mass m_0 and $M_{1/2}$ at Λ, respectively. For the first term in (4$\cdot$7), the correction is approximately given by $\frac{3}{2}\ln m_{\tilde{l}}^3/m_{\tilde{q}}m_{\tilde{e}}^2$. This correction is small compared with two-loop effect, e.g., -1.6 for $m_{\tilde{q}}=3m_{\tilde{l}}^3/m_{\tilde{e}}^2$. The magnitude of the second term in (4$\cdot$7) is estimated as $14\ln m_{\tilde{w}}/m_{\tilde{g}}\sim-17.5$ if $m_{\tilde{w}}\sim M_2$ and $m_{\tilde{w}}/m_{\tilde{g}}=\alpha_2/\alpha_3$. The correction of the third term in (4$\cdot$7) is approximately given by $\frac{15}{2}\ln m_{\tilde{H}}/M_Z$ with $m_{\tilde{H}}=m_H$. This contribution is also sizable, e.g., 12.1 for $m_{\tilde{H}}=5M_Z$ and 17.3 for $m_{\tilde{H}}=10M_Z$. The range of total magnitude of $\delta\vec{\alpha}_{\mathrm{SUSY}}^{-1}$ is given $-7.0\sim-1.8$ under the above assumptions. To cancel the two-loop correction, we need a model that the magnitude of M_3 almost equals to that of M_2 at the weak scale or a model with a large mass hierarchy between $m_{\tilde{H}}$ and M_Z. The first situation can be realized in the presence of nonrenormalizable interactions in the gauge kinetic term in SUSY GUTs. If the F-component of the Higgs field has a non-vanishing vacuum expectation value (VEV), gauginos also receive a non-universal correction to their masses.[16] Several

*)The $SU(2)_L\times U(1)_Y$ breaking terms in SUSY threshold effects have been considered in Ref.15).

phenomenological aspects of models with such non-universal gaugino masses have been studied and interesting difference among models have been shown.[17)]

4. 4. *GUT threshold correction*

The threshold correction at the GUT scale is given by

$$\delta\vec{\alpha}_{\rm ext}^{-1} \supset \delta\vec{\alpha}_{\rm GUT}^{-1} \equiv \frac{9}{2\pi}\ln\frac{M_U}{M_H} + \frac{1}{2\pi}(5\Delta_1 - 12\Delta_2 + 7\Delta_3). \tag{4·8}$$

The mass parameters (M_U, M_H) and the correction due to mass splitting Δ_i are model dependent. (Δ_i can be absorbed into M_U and M_H by their definition.) The bigger M_U compared with M_H is favor to cancel the two-loop effect, but, in this case, the constraint from proton decay becomes severe in the minimal SUSY $SU(5)$ GUT. In the minimal model, $\Delta_i=0$ and the range of $\delta\vec{\alpha}_{\rm GUT}^{-1}$ is, for example, $-3.30 \sim 3.30$ for $1/10 \leq M_U/M_H \leq 10$.

4. 5. *Intermediate physics*

There is a correction in the presence of physics at the intermediate scales. Here we discuss a double step unification scenario with one intermediate scale M_I[8)] for simplicity. The solutions of RG equations at one-loop level are expressed as

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(\Lambda) + \frac{b_i}{2\pi}\ln\frac{M_I}{\mu} + \frac{b_i'}{2\pi}\ln\frac{\Lambda}{M_I} \tag{4·9}$$

where b_i's are coefficients of β functions in a model above M_I. From the relation $2\pi\, \vec{\alpha}^{-1}(\mu)\cdot(\vec{\alpha}^{-1}(\Lambda)\times\vec{b}) = \vec{b}'\cdot(\vec{\alpha}^{-1}(\Lambda)\times\vec{b})\ln\Lambda/M_I$, the unificability condition is given by $\vec{b}'\cdot(\vec{\alpha}^{-1}(\Lambda)\times\vec{b})=0$. If the theory below M_I is the MSSM and $\vec{\alpha}^{-1}(\Lambda)=\alpha_U^{-1}\vec{I}$, the condition turns out to be $5b_1'-12b_2'+7b_3'=0$ or $5\Delta b_1-12\Delta b_2+7\Delta b_3=0(\Delta b_i\equiv b_i'-b_i)$. In the following subsubsection, we discuss two interesting topics related to a gauge coupling sum rule in intermediate physics, i.e., power-law correction in a higher dimensional theory and a $U(1)$ kinetic mixing effect.

4.5.1. Power-law correction

The correction to the sum rule due to KK modes is given by

$$\delta\vec{\alpha}_{\rm ext}^{-1} \supset \delta\vec{\alpha}_{\rm KK}^{-1} \equiv \frac{1}{2\pi}\left(\frac{X_\delta}{\delta}\left(\left(\frac{\Lambda}{M_C}\right)^\delta - 1\right) + \ln\frac{\Lambda}{M_C}\right) \times (5b_1^{\rm KK} - 12b_2^{\rm KK} + 7b_3^{\rm KK}). \tag{4·10}$$

The unificability of gauge couplings has been studied in a various class of models with extra dimensions.[18)] The magnitude of $\delta\vec{\alpha}_{\rm KK}^{-1}$ depends on particle contents in the bulk and number of dimension and radii of internal space. In the MSSM where non-chiral states such as gauge multiplets and Higgs multiplets have KK excitations, $b_i^{\rm KK}$ is given by $b_i^{\rm KK}=(3/5,-3,-6)$ and the value of $5b_1^{\rm KK}-12b_2^{\rm KK}+7b_3^{\rm KK}$ is -3. In this case, the magnitude of $\delta\vec{\alpha}_{\rm KK}^{-1}$ is estimated as $\delta\vec{\alpha}_{\rm KK}^{-1} = 0 \sim -9.7$ for $\delta=1$ and $X_\delta(\Lambda/M_C)^\delta/\delta = 2\sim 20$. To cancel the two loop effect, we need a model with a positive value for $5b_1^{\rm KK}-12b_2^{\rm KK}+7b_3^{\rm KK}$.

4.5.2. $U(1)$ kinetic mixing

A $U(1)$ gauge kinetic mixing, in general, occurs at one-loop level if there is a mass-splitting among charged particles under $U(1)s$.[19] We study the symmetry breaking pattern such that $G_U \to G_{SM} \times U(1)_\delta \to G_I \equiv SU(3)_C \times SU(2)_L \times U(1)_\alpha \times U(1)_\beta \to G_{SM}$ as an example. The $U(1)$ gauge kinetic mixing occurs on the decoupling of charged paricles under $U(1)$s, e.g., for colored Higgs multiplets, it occurs at M_H. We assume that the gauge symmetry breaking $U(1)_\alpha \times U(1)_\beta \to U(1)_Y$ occurs at some intermediate scale M_I, the theory becomes the MSSM below M_I and hypercharge Y is given by $\sqrt{\frac{5}{3}}Y = xQ_\alpha + yQ_\beta$. Thus α_1^{-1} at $\mu(<M_I)$ is given by

$$\alpha_1^{-1}(\mu) = \alpha_1^{-1}(\Lambda) + \frac{b_1}{2\pi}\ln\frac{\Lambda}{\mu} + \frac{\Delta b_1}{2\pi}\ln\frac{\Lambda}{M_I} \qquad (4\cdot11)$$

where $\alpha_1^{-1}(\Lambda) \equiv x^2\alpha_\alpha^{-1}(\Lambda) + y^2\alpha_\beta^{-1}(\Lambda)$ and $\Delta b_1 \equiv x^2 b^{(\alpha)} + y^2 b^{(\beta)} - b_1$. The sum rule is given by

$$\begin{aligned}\Delta\tilde{\alpha}_{\mathrm{MSSM}}^{-1} &\equiv 5\alpha_1^{-1}(\mu) - 12\alpha_2^{-1}(\mu) + 7\alpha_3^{-1}(\mu) \\ &= \alpha_U^{-1}(5C_1 - 12C_2 + 7C_3) + \frac{9}{2\pi}\ln\frac{M_H}{M_U} \\ &\quad + \frac{1}{2\pi}(5\Delta_1 - 12\Delta_2 + 7\Delta_3) \\ &\quad + \frac{1}{2\pi}(5\Delta b_1 - 12\Delta b_2 + 7\Delta b_3)\ln\frac{M_H}{M_I} \end{aligned} \qquad (4\cdot12)$$

where $\Delta b_{2,3}$ are differences $(b'_{2,3} - b_{2,3})$ between the coefficients $b'_{2,3}$ of β functions in the theory with G_I and those of the MSSM. The relation (4・12) holds on below the scale M_I.

§ 5. Conclusions and discussion

We have studied high energy corrections to the gauge couplings, e.g., two-loop correction, the threshold effect of heavy particles, corrections from higher dimensional operators in gauge kinetic terms and the power-law correction. A modified version of the gauge coupling sum rule (1・1) in the MSSM has been derived in more generic situation, and a typical magnitude of each correction has been estimated. The estimation has been carried out without specifying a SUSY-GUT model. The two-loop correction is significantly large. In order to cancel this contribution, gravitational corrections, SUSY threshold correction at O(1)TeV and a power-law correction can be essential. The ranges given in Fig.1 vary with values of model-dependent parameters such as $\tan\beta$, a, soft SUSY breaking parameters, M_U, M_H, b_i^{KK}, δ and M_C. The model-dependent analysis is important to know which type of SUSY-GUT model is realistic or not.

The study of other aspects of gauge coupling sum rule is also important to explore physics at higher energy. We have discussed other situations than SUSY-GUT releted to the sum rule, e.g., a model with a non-stardard value of Kac-Moody level and non-SUSY GUT, in the appendix. In future, it is expected that the modified sum rule will be useful to check a realistic unification scenario.

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Appendix A
—— *Kac-Moody level* ——

There is an arbitariness of normalization for gauge couplings if we do not stick the unification scenario based on a simple Lie group. The gauge couplings at Λ are given by

$$\alpha_i^{-1}(\Lambda)=k\alpha_U^{-1}(1+C_i) \qquad (\mathrm{A}\cdot 1)$$

where k_is are Kac-Moody level. The scenario based on a GUT group corresponds to $(k_Y, k_2, k_3) = (5/3, 1, 1)$. Here we consider the case with an arbitrary k_Y fixing $k_2=k_3=1$. For single step unification based on the MSSM, the following relation is derived

$$5\alpha_1^{-1}(\mu)-\frac{3k_Y}{4}\left(3+\frac{11}{k_Y}\right)\alpha_2^{-1}(\mu)-\frac{3k_Y}{4}\left(1-\frac{11}{k_Y}\right)\alpha_3^{-1}(\mu)=0 \qquad (\mathrm{A}\cdot 2)$$

at one-loop level. Using the above relation, the $\Delta\vec{\alpha}^{-1}_{\mathrm{MSSM}}$ is written as

$$\begin{aligned}\Delta\vec{\alpha}^{-1}_{\mathrm{MSSM}}&\equiv 5\alpha_1^{-1}(\mu)-12\alpha_2^{-1}(\mu)+7\alpha_3^{-1}(\mu)\\&=\frac{3}{4}\left(k_Y-\frac{5}{3}\right)(3\alpha_2^{-1}(\mu)+\alpha_3^{-1}(\mu))\\&\equiv\delta\vec{\alpha}^{-1}_{\mathrm{KM}}\end{aligned} \qquad (\mathrm{A}\cdot 3)$$

and, from this relation, we find that a small deviation of k_Y from 5/3 induces a sizable contribution.

Appendix B
—— *Non-SUSY model* ——

For single step unification based on the SM, the following sum rule is derived*)

$$\begin{aligned}\Delta\vec{\alpha}^{-1}_{\mathrm{SM}}&\equiv 115_1^{-1}(\mu)-333\alpha_2^{-1}(\mu)+218\alpha_3^{-1}(\mu)\\&=\delta\vec{\alpha}^{-1}_{(2)}+\alpha_U^{-1}(115C_1-333C_2+218C_3)+\frac{22}{\pi}\ln\frac{M_U}{M_H}\\&+\frac{1}{2\pi}(115\Delta_1-333\Delta_2+218\Delta_3)\end{aligned} \qquad (\mathrm{B}\cdot 1)$$

*)We take a normalization where a fractional number does not appear in coefficients. If one wants to compare the MSSM case with the SM case, we should change a suitable normalization.

up to other contributions. By using the experimental values (1·2)-(1·3), the $\Delta\tilde{\alpha}_{SM}^{-1}$ is estimated as

$$\Delta\tilde{\alpha}_{SM}^{-1} = -1232.12 \pm 90.36. \tag{B·2}$$

This shows that the unification is realized only if there exist sizable corrections. For example, there is a possibility that $\alpha_U^{-1} \sim 45$ and $C_i = O(1/10)$.[7)]

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