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An improved formulation for three-charged particles correlations in terms of Coulomb wave functions with degree of coherence

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Abstract

The recent data for Bose-Einstein Correlations (BEC) of three-charged particles obtained by NA44 Collaboration have been analysed using theoretical formula with Coulomb wave functions. It has been recently proposed by Alt et al. It turns out that there are discrepancies between these data and the respective theoretical values. To resolve this problem we seek a possibly improved theoretical formulation of this problem by introducing the degree of coherence for the exchange effect due to the BEC between two-identical bosons. As a result we obtain an improved formulation for the BEC of three-charged particles showing a good agreement with the experimental data of NA44 Collaboration. This indicates that the interaction region in the S+Pb collisions at 200 GeV/c per nucleon is equal to about 1.5 fm.
Key words : Bose-Einstein Correlation, three-charged particles, Coulomb wave functions, high energy heavy-ion collisions

1 Introduction

One of the most interesting subjects in high energy heavy-ion collisions is study of the higher order Bose-Einstein Correlation (BEC) effect [1-6] (known also as the HBT or the GGLP effect, or as the hadron interferometry [7-10]). From data on BEC we can (in principle) infer the size of the interaction region and therefore estimate the energy densities reached in high energy collisions. Such work is a necessary task in the search for the quark-gluon plasma [11,12]-a new, hypothetical form of matter.

To get more precise sizes of the interaction regions, we have to take into account the final state interactions among the charged particles [13,14]. A great advance in this

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direction for the BEC of the three-charged particles has been recently made by Alt et. al. [5]. They have derived a correction formula for the raw data introducing distribution functions of the charged particles. Their formulation is based on the plane wave functions and on the Coulomb wave functions, assuming that produced hadrons are already in the asymptotic region of the Coulomb interactions where the strong interaction already vanishes [15,16]. It amounts in the following correction factor K_{coul} due to the Coulomb effect for identical three-charged particles : [1]

$$K_{coul}=\frac{N_{coul}}{D_{plane}} \tag{1}$$

The denominator D_{plane} is given by ($\rho(\mathrm{x}_i)$ are distribution functions of charged particles):

$$\begin{aligned} D_{plane} \cong & \frac{1}{6}\int d^3\mathrm{x}_1\rho(\mathrm{x}_1)d^3\mathrm{x}_2\rho(\mathrm{x}_2)d^3\mathrm{x}_3\rho(\mathrm{x}_3) \\ & \cdot\Big| e^{i(\mathrm{k}_1\cdot\mathrm{x}_1+\mathrm{k}_2\cdot\mathrm{x}_2+\mathrm{k}_3\cdot\mathrm{x}_3)}+e^{i(\mathrm{k}_1\cdot\mathrm{x}_2+\mathrm{k}_2\cdot\mathrm{x}_1+\mathrm{k}_3\cdot\mathrm{x}_3)} \\ & +e^{i(\mathrm{k}_1\cdot\mathrm{x}_2+\mathrm{k}_2\cdot\mathrm{x}_3+\mathrm{k}_3\cdot\mathrm{x}_1)}+e^{i(\mathrm{k}_1\cdot\mathrm{x}_1+\mathrm{k}_2\cdot\mathrm{x}_3+\mathrm{k}_3\cdot\mathrm{x}_2)} \\ & +e^{i(\mathrm{k}_1\cdot\mathrm{x}_3+\mathrm{k}_2\cdot\mathrm{x}_1+\mathrm{k}_3\cdot\mathrm{x}_2)}+e^{i(\mathrm{k}_1\cdot\mathrm{x}_3+\mathrm{k}_2\cdot\mathrm{x}_2+\mathrm{k}_3\cdot\mathrm{x}_1)}\Big|^2, \end{aligned} \tag{2}$$

The numerator N_{coul} has the following form :

$$\begin{aligned} N_{coul} \cong & \frac{1}{6}\int d^3\mathrm{x}_1\rho(\mathrm{x}_1)d^3\mathrm{x}_2\rho(\mathrm{x}_2)d^3\mathrm{x}_3\rho(\mathrm{x}_3) \\ & \cdot\Big| \psi^C_{\mathrm{k}_1\mathrm{k}_2}(\mathrm{x}_1,\mathrm{x}_2)\psi^C_{\mathrm{k}_2\mathrm{k}_3}(\mathrm{x}_2,\mathrm{x}_3)\psi^C_{\mathrm{k}_3\mathrm{k}_1}(\mathrm{x}_3,\mathrm{x}_1) \\ & +\psi^C_{\mathrm{k}_1\mathrm{k}_2}(\mathrm{x}_1,\mathrm{x}_3)\psi^C_{\mathrm{k}_2\mathrm{k}_3}(\mathrm{x}_3,\mathrm{x}_2)\psi^C_{\mathrm{k}_3\mathrm{k}_1}(\mathrm{x}_2,\mathrm{x}_1) \\ & +\psi^C_{\mathrm{k}_1\mathrm{k}_2}(\mathrm{x}_2,\mathrm{x}_1)\psi^C_{\mathrm{k}_2\mathrm{k}_3}(\mathrm{x}_1,\mathrm{x}_3)\psi^C_{\mathrm{k}_3\mathrm{k}_1}(\mathrm{x}_3,\mathrm{x}_2) \\ & +\psi^C_{\mathrm{k}_1\mathrm{k}_2}(\mathrm{x}_2,\mathrm{x}_3)\psi^C_{\mathrm{k}_2\mathrm{k}_3}(\mathrm{x}_3,\mathrm{x}_1)\psi^C_{\mathrm{k}_3\mathrm{k}_1}(\mathrm{x}_1,\mathrm{x}_2) \\ & +\psi^C_{\mathrm{k}_1\mathrm{k}_2}(\mathrm{x}_3,\mathrm{x}_1)\psi^C_{\mathrm{k}_2\mathrm{k}_3}(\mathrm{x}_1,\mathrm{x}_2)\psi^C_{\mathrm{k}_3\mathrm{k}_1}(\mathrm{x}_2,\mathrm{x}_3) \\ & +\psi^C_{\mathrm{k}_1\mathrm{k}_2}(\mathrm{x}_3,\mathrm{x}_2)\psi^C_{\mathrm{k}_2\mathrm{k}_3}(\mathrm{x}_2,\mathrm{x}_1)\psi^C_{\mathrm{k}_3\mathrm{k}_1}(\mathrm{x}_1,\mathrm{x}_3)\Big|^2. \end{aligned} \tag{3}$$

Here $\psi^C_{\mathrm{k}_i\mathrm{k}_j}(\mathrm{x}_i,\mathrm{x}_j)$ are the Coulomb wave functions of the respective 2-body collision expressed as,

$$\psi^C_{\mathrm{k}_i\mathrm{k}_j}(\mathrm{x}_i,\mathrm{x}_j) = \Gamma(1+i\eta_{ij})e^{\pi\eta_{ij}/2}e^{i\mathrm{k}_{ij}\cdot\mathrm{r}_{ij}}F[-i\eta_{ij},\ 1\ ;\ i(k_{ij}r_{ij}-\mathrm{k}_{ij}\cdot\mathrm{r}_{ij})], \tag{4}$$

with $\mathrm{r}_{ij}=(\mathrm{x}_i-\mathrm{x}_j)$, $\mathbf{k}_{ij}=(\mathrm{k}_i-\mathrm{k}_j)/2$ and $\eta_{ij}=m\alpha/k_{ij}$. $F[a,\ b;\ x]$ and $\Gamma(x)$ are the confluent hypergeometric function and the Gamma function, respectively. In order to use Eqs. (1), (2) and (3), one has to assume first some shapes and sizes for the source functions. In

[1]The correlation functions for two and three-identical particles are given as usual by

$$\frac{N^{(2+\mathrm{or}2-)}}{N^{BG}}=\frac{P(\mathrm{k}_1,\mathrm{k}_2)}{P(\mathrm{k}_1)P(\mathrm{k}_2)} \text{ and } \frac{N^{(3+\mathrm{or}3-)}}{N^{BG}}=\frac{P(\mathrm{k}_1,\mathrm{k}_2,\mathrm{k}_3)}{P(\mathrm{k}_1)P(\mathrm{k}_2)P(\mathrm{k}_3)},$$

where k_i is the momentum of particle i, and $P(\mathrm{k}_1,\mathrm{k}_2)$ and $P(\mathrm{k}_1,\mathrm{k}_2,\mathrm{k}_3)$ are two and three particles probability densities, respectively. The probability densities for two-identical particles case can be written as,

$$\iint|\psi^{\mathrm{BE}}_{1,2}(\mathrm{k}_1,\mathrm{k}_2,\mathbf{x}_1,\mathbf{x}_2)|^2\rho(\mathbf{x}_1)\rho(\mathbf{x}_2)d^3\mathrm{x}_1d^3\mathrm{x}_2,$$

where $\rho(\mathbf{x}_i)$ stand for the source functions of particle i.

fact, this is the procedure already used in Ref. [17] by NA44 Collaboration:

$$\text{Corrected data} = (\text{raw data}) \times K_{spc} \times K_{acceptance} \times K_{coul},$$

where K_{spc} and $\boldsymbol{K}_{\rm acceptance}$ denote the effect of multiparticle production in the single particle spectra and the acceptance effect in the experiment.

In this paper, we would like to adopt a different point of view for Eq. (3). As is seen in Ref. [14], the BEC of identical two-charged pions can also be analysed by the Coulomb wave functions. It is therefore reasonable to expect that the numerator N_{coul} is the main theoretical ingredient in analysis of the BEC of three-charged particles. We argue therefore that

$$N^{(3+\text{or}3-)}/N^{BG} \equiv C \times N_{coul}, \tag{5}$$

where we have introduced the normalization factor C, which corresponds to the asymptotic value of the BEC. Using Eq. (5) we can now (with the help of the CERN-MINUIT program) analyse data of Ref. [17] using Gaussian source distributions of radii R, $\rho(\mathrm{x}) = \frac{1}{(2\pi R^2)^{3/2}} \exp\left[-\frac{\mathrm{x}^2}{2R^2}\right]^2$.

In the next paragraph, we analyse the data of NA44 Collaboration [17] by Eq. (5). In the third paragraph we shall derive an improved theoretical formula for 3-particle BEC introducing the degree of coherence parameter into Eq. (5). This formula will be then used in the 4th paragraph for the re-analyses of the experimental data [17]. Concluding remarks are given in the final paragraph.

2 Application of Eq. (5) to the data by NA44 Collaboration

Here we analyse the data by Eq. (5). As can be seen in Fig.1 and Table 1, there are some discrepancies between the data points and theoretical values calculated by means of Eq. (5). Thus we would like to know why this equation cannot explain the data [17]. One of the probable reasons is the possible partial coherent of produced pions. In fact, authors of Ref. [17] have used not the equivalence of Eq. (5) but the following formula instead (cf., Ref. [9,18]):

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[2]It should be remembered that NA44 Collaboration data are for the variable

$$Q_3^2 = (k_1 - k_2)^2 + (k_2 - k_3)^2 + (k_3 - k_1)^2$$

where k_i are four-momentum of charged partices. $Q_3 = \sqrt{Q_3^2}$. However, in our calculations we assume that $q_{0,ij}^2 = (\mathrm{k}_{0i} - \mathrm{k}_{0j})^2 \approx 0$ and use, instead,

$$Q_3^2 = (\mathbf{k}_1 - \mathbf{k}_2)^2 + (\mathbf{k}_2 - \mathbf{k}_3)^2 + (\mathbf{k}_3 - \mathbf{k}_1)^2.$$

As matter of fact, the follwing more general procedure should be used:

$$C \int \prod_{i=1}^{3} d\mathrm{k}_{0i} N_{coul} \cdot \delta\left(Q_3 - \sqrt{(k_1 - k_2)^2 + (k_2 - k_3)^2 + (k_3 - k_1)^2} \right).$$

It turn out, however, to be too much CPU time consuming to be applicable in present calculations.

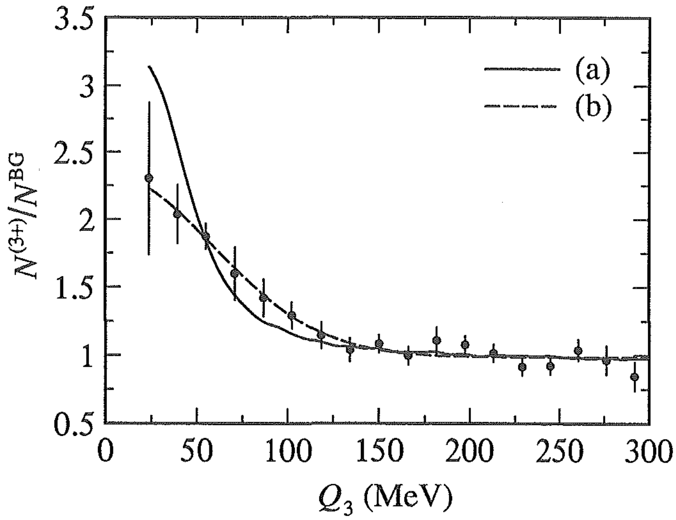


Figure 1: Analysis of $3\pi^+$ BEC in S+Pb collision [17].(a) and (b) are results of Eqs. (5) and (6),respectively. The error bars are systematic errors.

$$\frac{N^{(3+)}}{N^{BG}} = C(1+\lambda_3 e^{-R_3^2 Q_3^2}). \tag{6}$$

It contains one more parameter, λ_3, which can be regarded as a kind of effective degree of coherence and which, in our opinion, should therefore occur also somehow in Eq. (5).

Table 1: Estimated values for the data [17] by Eqs. (5) and (6) using CERN-MINUIT program.

Formulas	C	R[fm]	λ	χ^2/N_{dof}
Eq.(5)	0.941 ± 0.026	2.47 ± 0.14	—	17.6/16
Eq.(6)	0.986 ± 0.028	2.36 ± 0.26	1.37 ± 0.19	7.8/15

3 Diagram Decomposition of Eq. (5)

First of all, we have to find a possible way for the introduction of the degree of coherence parameter λ into Eq. (5). Let us therefore examine the plane wave (PW) approximations of the Coulomb wave functions ($\mathbf{k}_{12}=\mathbf{k}_1-\mathbf{k}_2$ and $\mathbf{r}_{12}=\mathbf{r}_1-\mathbf{r}_2$):

$$A(1) = \psi^C_{\mathrm{k}_{12}}(\mathrm{r}_{12})\psi^C_{\mathrm{k}_{23}}(\mathrm{r}_{23})\psi^C_{\mathrm{k}_{31}}(\mathrm{r}_{31}) \xrightarrow{\mathrm{PW}} e^{i\mathrm{k}_{12}\cdot\mathrm{r}_{12}}e^{i\mathrm{k}_{23}\cdot\mathrm{r}_{23}}e^{i\mathrm{k}_{31}\cdot\mathrm{r}_{31}} = e^{(3/2)i(\mathrm{k}_1\cdot\mathrm{x}_1+\mathrm{k}_2\cdot\mathrm{x}_2+\mathrm{k}_3\cdot\mathrm{x}_3)}, \tag{7a}$$

$$A(2) = \psi^C_{\mathrm{k}_{12}}(\mathrm{r}_{13})\psi^C_{\mathrm{k}_{23}}(\mathrm{r}_{32})\psi^C_{\mathrm{k}_{31}}(\mathrm{r}_{21}) \xrightarrow{\mathrm{PW}} e^{i\mathrm{k}_{12}\cdot\mathrm{r}_{13}}e^{i\mathrm{k}_{23}\cdot\mathrm{r}_{32}}e^{i\mathrm{k}_{31}\cdot\mathrm{r}_{21}} = e^{(3/2)i(\mathrm{k}_1\cdot\mathrm{x}_1+\mathrm{k}_2\cdot\mathrm{x}_3+\mathrm{k}_3\cdot\mathrm{x}_2)}, \tag{7b}$$

$$A(3) = \psi^C_{\mathrm{k}_{12}}(\mathrm{r}_{21})\psi^C_{\mathrm{k}_{23}}(\mathrm{r}_{13})\psi^C_{\mathrm{k}_{31}}(\mathrm{r}_{32}) \xrightarrow{\mathrm{PW}} e^{i\mathrm{k}_{12}\cdot\mathrm{r}_{21}}e^{i\mathrm{k}_{23}\cdot\mathrm{r}_{13}}e^{i\mathrm{k}_{31}\cdot\mathrm{r}_{32}} = e^{(3/2)i(\mathrm{k}_1\cdot\mathrm{x}_2+\mathrm{k}_2\cdot\mathrm{x}_1+\mathrm{k}_3\cdot\mathrm{x}_3)}, \tag{7c}$$

$$A(4) = \psi^C_{\mathrm{k}_{12}}(\mathrm{r}_{23})\psi^C_{\mathrm{k}_{23}}(\mathrm{r}_{31})\psi^C_{\mathrm{k}_{31}}(\mathrm{r}_{12}) \xrightarrow{\mathrm{PW}} e^{i\mathrm{k}_{12}\cdot\mathrm{r}_{23}}e^{i\mathrm{k}_{23}\cdot\mathrm{r}_{31}}e^{i\mathrm{k}_{31}\cdot\mathrm{r}_{12}} = e^{(3/2)i(\mathrm{k}_1\cdot\mathrm{x}_2+\mathrm{k}_2\cdot\mathrm{x}_3+\mathrm{k}_3\cdot\mathrm{x}_1)}, \tag{7d}$$

$$A(5) = \psi^C_{k_{12}}(r_{31})\psi^C_{k_{23}}(r_{12})\psi^C_{k_{31}}(r_{23}) \stackrel{\rm PW}{\longrightarrow} e^{ik_{12}\cdot r_{31}}e^{ik_{23}\cdot r_{12}}e^{ik_{31}\cdot r_{23}} = e^{(3/2)i(k_1\cdot x_3+k_2\cdot x_1+k_3\cdot x_2)}, \tag{7e}$$

$$A(6) = \psi^C_{k_{12}}(r_{32})\psi^C_{k_{23}}(r_{21})\psi^C_{k_{31}}(r_{13}) \stackrel{\rm PW}{\longrightarrow} e^{ik_{12}\cdot r_{32}}e^{ik_{23}\cdot r_{21}}e^{ik_{31}\cdot r_{13}} = e^{(3/2)i(k_1\cdot x_3+k_2\cdot x_2+k_3\cdot x_1)}, \tag{7f}$$

Notice that, except for the factor 3/2, exponential functions are the same expressions as those present in the integrand of Eq. (2). This difference is attributed to the fact that Coulomb wave function used here describes two-charged particles collisions, therefore factor 3/2 appears because there are relevant two-particle three combinations among three-charged particles.

Combining Eqs. (7) and Figs.2, we obtain the following three sets of equations:

$$F_1 = \frac{1}{6}\sum_{i=1}^{6} A(i)A^*(i) \stackrel{\rm PW}{\longrightarrow} 1, \tag{8a}$$

$$\begin{aligned} F_2 = \frac{1}{6}[&A(1)A^*(2)+A(1)A^*(3)+A(1)A^*(6)+A(2)A^*(4)+A(2)A^*(5) \\ &+A(3)A^*(4)+A(3)A^*(5)+A(4)A^*(6)+A(5)A^*(6)+\text{c. c.}] \\ &\stackrel{\rm PW}{\longrightarrow} \text{BEC between two-charged particles (See Figs. 2(b)}\sim\text{(d)),} \end{aligned} \tag{8b}$$

$$\begin{aligned} F_3 = \frac{1}{6}[&A(1)A^*(4)+A(1)A^*(5)+A(2)A^*(3)+A(2)A^*(6)+A(3)A^*(6) \\ &+A(4)A^*(5)+\text{c. c.}] \\ &\stackrel{\rm PW}{\longrightarrow} \text{BEC among three-charged particles (See Figs. 2(e) and (f)).} \end{aligned} \tag{8c}$$

Combining now Eqs. (8) and the concept of partial coherence for the BEC [17,18], we can introduce a coherence parameter $\sqrt{\lambda}$ for the single mark (×) in Fig.2[3]. Taking into account the strength of the degree of coherence λ between two-identical bosons and $\lambda^{3/2}$ among three-identical bosons in Figs.2, we can finally express the BEC for three identical charged particles as:

$$\frac{N^{(3+\text{or}3-)}}{N^{BG}} \cong C\int d^3x_1\rho(x_1)d^3x_2\rho(x_2)d^3x_3\rho(x_3)[F_1+\lambda F_2+\lambda^{3/2}F_3]. \tag{9}$$

Equation (9) is the improved theoretical formula we were looking for. It differs from Eq. (5) originally proposed by Alt et. al. in [5] by the presence of the degree of coherence λ and in the limit of $\eta_{ij}\to 0$ it becomes

$$\text{Eq.(9)} \stackrel{\eta_{ij}\to 0}{\longrightarrow} C(1+3\lambda e^{-R^2Q_3^2}+2\lambda^{3/2}e^{-\frac{3}{2}R^2Q_3^2}), \tag{10}$$

which is the extended formula proposed some time ago by Deutschmann et al. [18].

It should be noticed that Eq. (9) can be applied to data corrected only by the Gamow factor $G(\eta_{12})G(\eta_{23})G(\eta_{31})$ in an ideal case [19], because Eq. (9) is described by the Coulomb wave functions including the Gamow factors (see Ref. [20])[4].

[3]The $\lambda=1$ corresponds to the totally chaotic source, which is the assumption behind Eq. (5).

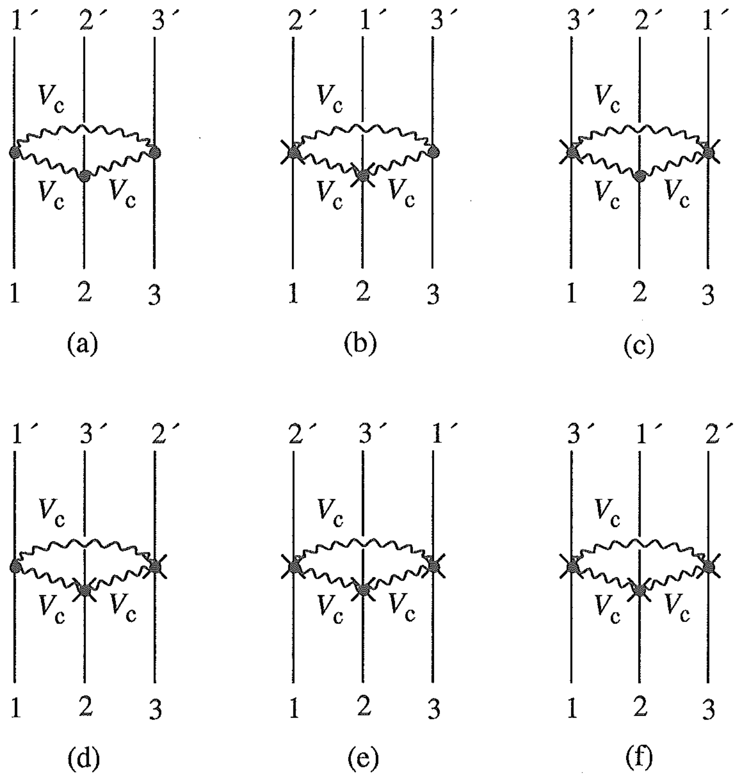


Figure 2 : Diagram reflecting three-charged particles Bose-Einstein Correlation (BEC) and Coulombic potential (Vc). × means the exchange effect of BEC.

4 Reanalyses of NA44 Collaboration data by means of Eq. (9)

At present we have no data corrected only by the Gamow factors, therefore we apply Eq. (9) to the analysis of NA44 Collaboration data [17] using the CERN-MINUIT program. Our results are shown in Fig. 3 and Table 2. Comparing them with those of Table 1, it can be said that the x^2-value becomes smaller, i.e., the agreement is now improved. The range of interaction becomes also smaller. For the sake of reference we present in Table 2 also results obtained by using Eq. (10).

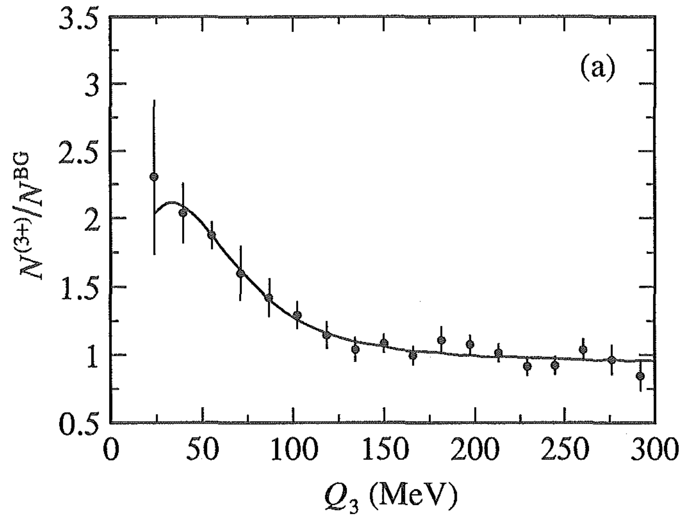
Table 2 : Reanalyses of $3\pi^+$ BEC in S + Pb collision [17] by Eqs. (9) and (10).

Formulas	C	R[fm]	λ	x^2/N_{dof}
Eq.(9)	0.917±0.032	1.53±0.20	0.55±0.07	6.7/15
Eq.(10)	0.984±0.029	2.25±0.24	0.33±0.04	7.7/15

[4]In other words, ideal data sets for Eq. (9) are of the form

$$\text{Corrected data} = (\text{raw data}) \times K_{\text{spc}} \times K_{\text{acceptance}} \times K_{\text{Gamow}},$$

where $K_{\text{Gamow}} = 1/(G(\eta_{12})G(\eta_{23})G(\eta_{31}))$.



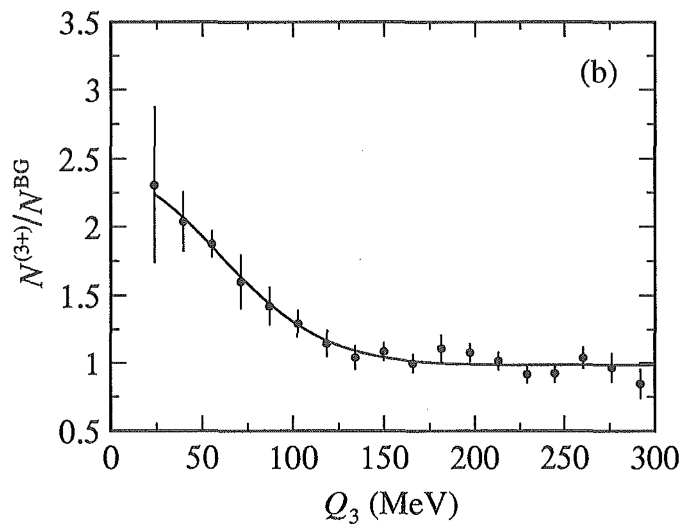


Figure 3 : Reanalyses of $3\pi^+$ BEC in S + Pb collision [17]. (a) is result of Eq. (9). (b) is that of Eq. (10).

5 Concluding remarks

We have derived the theoretical formula for the BEC of three-charged identical particle using both the Coulomb wave functions and the notion of the degree of coherence and compared it with the experimental data[5]. Historically the degree of

[5]For the numerical calculations of Eqs. (5) and (9) (in order to save the CPU-time), we have first calculated 10^3k values of the Coulomb wave functions, which were then used together with some interpolation procedure during the concrete calculations. In this way we could make use of the CERN -MINUIT program in our analyses..

coherence in the BEC of the two-identical bosons has been introduced by experimentalists [18], and theoretical works in this direction have been performed in Ref. [9].

Our present analyses suggest that the degree of coherence λ is a necessary ingredient also for the BEC of three-charged particles, in the same way as it was for the BEC for two-charged identical particles. This fact means that the source producing finally observed particles is not purely chaotic. It should be noticed that also recent data on $3\pi^-$ BEC reported by OPAL Collaboration [21] suggest the necessity of introduction of some degree of coherence[6].

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