J. FAC. SCI., SHINSHU UNIVERSITY, Vol. 33, No. 1, pp.9-16 1998

Analysis of Bose-Einstein correlations in $e^+e^- \rightarrow W^+W^-$ events including final state interactions

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(Received July 30, 1998)

Abstract

Recently DELPHI Collaboration reported new data on Bose-Einstein correlations (BEC) measured in $e^+e^- \rightarrow W^+W^-$ events. Apparently no enhancement has been observed. We have analyzed these data including final state interactions (FSI) of both Coulomb and strong (s-wave) origin and found that there is enhancement in BEC but it is overshadowed by the FSI which are extremely important for those events. We have found the following values for the size of the interaction range β and the degree of coherence λ, $\beta=0.87\pm0.31$ fm and $\lambda=1.19\pm0.48$, respectively.

1. Introduction

Recently DELPHI Collaboration has reported an interesting result on the Bose-Einstein correlations (BEC) in $e^+e^- \rightarrow W^+W^-$ events at $\sqrt{s}=172\text{GeV}$ [1]. Namely, analyzing their data by the following formula

$$N^{\pm\pm}/N^{BG(+-)}=c[1+\lambda\exp(-\beta^2Q^2/2)], \tag{1}$$

where the denominator $N^{BG(+-)}$ denotes the contribution of the different charge pair, (c and λ are the normalization factor and the degree of coherence, respectively), authors of Ref. [1] found that

$$c=1.0\ ,\ \lambda=-0.22\pm0.2(\text{statics}), \quad \text{and} \quad \beta=0.5\ \text{fm},$$

i.e., apparently there is no enhancement due to the Bose-Einstein effect in $e^+e^- \rightarrow W^+W^-$ events, most probably because measured pions come from different $W's$ (i.e., different "sources").

On the other hand, a different approach (weight function method) to the BEC in

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Refs.	β [fm]	λ	χ^2/NDF	FSI
Eq.(1)	0.50	-0.11 ± 0.2	$\sim$14/14	no-FSI
Event weighting [2]	0.322$\sim$0.459 (depending on weight function)	0.032$\sim$0.146	—	no-FSI
Eq.(12)				
$\pi^\pm\pi^\pm/\pi^+\pi^-$	0.86 ± 0.31	1.19 ± 0.48	14.3/13	Coulomb+strong

Table I : Effect of FSI on BEC in $e^+e^-\rightarrow W^+W^-$ events.

$e^+e^-\rightarrow W^+W^-$ events used recently in Ref.[2] leads to slightly different results, namely λ was estimated to be equal 0.032$\sim$0.146[1] (See Table I).

Because in these reactions the space-time distances between $W's$ are very small (of the order 0.5 fm) the usual Coulombic and strong final state interactions(FSI) between $\pi^\pm$-$\pi^\pm$ pairs should be specially important here and should be taken into account in order to estimate correctly and precisely β and λ. Therefore, we would like to re-analyze data of [1] by formula containing the FSI (of both the Coulombic and strong origin, the later in s-wave) proposed by us in Ref. [4].

In the next section, a theoretical formula for correlation of $\pi^\pm$-$\pi^\pm$ pairs are derived. In Sec. 3 we derive also a theoretical formula for π^+-π^- pairs. Using these two formulas we analyze in Sec. 4 $e^+e^-\rightarrow W^+W^-$ events showing clearly the roles of strong and Coulomb final state interactions in the BEC. We also predict in this Section correlation functions for $N^{\pm\pm}/N^{BG}$ and N^{+-}/N^{BG}. Our concluding remarks are presented in the last Section.

2. A theoretical formula for $\pi^\pm\pi^\pm$ pairs including the FSI

To describe correlations of pair of identical bosons we need to symmetrize the corresponding total production amplitude, which reads now:

$$A_{12}=\frac{1}{\sqrt{2}}[\Psi_C(\mathbf{k},\mathbf{r})+\Psi_C^S(\mathbf{k},\mathbf{r})+\Phi_{\rm st}(\mathbf{k},\mathbf{r})+\Phi_{\rm st}^S(\mathbf{k},\mathbf{r})], \tag{2}$$

where the superscript S denotes the symmetrization of the wave functions. The functions $\Psi_C(\mathbf{k},\mathbf{r})$ and $\Phi_{\rm st}(\mathbf{k},\mathbf{r})$ stand for the Coulomb wave function and the wave function induced by the strong interactions, respectively.

The Coulomb wave function of the identical π-π scattering with momenta p_1 and p_2 is given as [5, 6]:

$$\Psi_C(\mathbf{k},\mathbf{r})=\Gamma(1+i\eta)e^{-\pi\eta/2}e^{i\mathbf{k}\cdot\mathbf{r}}F(-i\eta;1;ikr(1-\cos\theta)), \tag{3}$$

where F denotes the confluent hypergeometric function, $2k=Q=p_1-p_2$, and $\eta=m_\pi\alpha/$

[1]Another scheme (shift of the final state momentum) is recently argued in Ref.[3] in order to also determine the W-mass.

$2k$. For calculations of the confluent hypergeometric function F in Eq. (3), there are two methods. One is to use formulas given in Ref.[7]. The other is to use the subroutine program 'WWHITM' in the CERN program library (See Refs.[7, 8]).

The strong interaction in the Coulomb field is given by

$$\Phi_{\rm st}(\mathbf{k},\mathbf{r}) = \sqrt{G(\eta)}\,\frac{f_0(\theta)\exp[i(kr-\eta\ln(2kr))]}{r},$$
$$f_0(\theta) = \frac{1}{2ik}\exp[2i\ \arg\Gamma(1+i\eta)][\exp(2i\delta_0^{(2)})-1]. \tag{4}$$

Here $\sqrt{G(\eta)} = \sqrt{\dfrac{2\pi\eta}{\exp[2\pi\eta]-1}}$ is the square root of the Gamow factor which is assumed to be a normalization factor of the strong wave function[2] and $\delta_0^{(2)}$ is the the phase shift for the π-$\pi(I=2)$ interaction parametrized as[3][9]

$$\delta_0^{(2)} = \frac{1}{2}\frac{a_0^{(2)}Q}{1.0+0.5Q^2} \tag{5}$$

We obtain therefore

$$N^{\pm\pm}/N^{BG(mix)} = \int\rho(r)d^3r|A_{12}|^2,$$
$$= G(\eta)[(1+\Delta_{\rm 1C})+(E_{\rm 2B}+\Delta_{\rm EC})+I_{\rm Cst}+I_{\rm st}], \tag{6}$$

where

$$(1+\Delta_{\rm 1C})+(E_{\rm 2B}+\Delta_{\rm EC}) = \frac{1}{G(k)}\int d^3r\rho(r)\left|\frac{1}{\sqrt{2}}[\Psi_C(\mathbf{k},\mathbf{r})+\Psi_C^S(\mathbf{k},\mathbf{r})]\right|^2,$$
$$E_{\rm 2B} = \int d^3r\rho(r)\exp[-iQ\cdot r],$$
$$I_{\rm st} = \frac{1}{G(k)}\int d^3r\rho(r)\left|\frac{1}{\sqrt{2}}[\Phi_{\rm st}(\mathbf{k},\mathbf{r})+\Phi_{\rm st}^S(\mathbf{k},\mathbf{r})]\right|^2,$$
$$I_{\rm Cst} = \frac{1}{G(k)}{\rm Re}\int d^3r\rho(r)[\{\Psi_C(\mathbf{k},\mathbf{r})+\Psi_C^S(\mathbf{k},\mathbf{r})\}\times\{\Phi_{\rm st}(\mathbf{k},\mathbf{r})+\Phi_{\rm st}^S(\mathbf{k},\mathbf{r})\}^*], \tag{7}$$

where $N^{BG(mix)}$ is the number of $\pi^+\pi^-$ pairs obtained by event mixing method. In present calculations we use Gaussian source function, $\rho(r) = \left(\frac{1}{\sqrt{2\pi}\beta}\right)^3\exp\left(\frac{-r^2}{2\beta^2}\right)$, the Fourier transform of which is given by:

$$E_{2B} = \exp(-\beta^2Q^2/2).$$

To obtain the ratio $N^{\pm\pm}/N^{BG(mix)}$ the parameter λ should be introduced into it in the usual way. Notice that one more parameter, the additional normalization factor c, is also introduced by hand. Our final formula with these parameters is then given as:

$$N^{\pm\pm}/N^{BG(mix)}(Q=2k) = G(\eta)\times c(1+\Delta_{\rm 1C}+\Delta_{\rm EC}+I_{\rm Cst}+I_{\rm st})$$
$$\times\left[1+\lambda\frac{E_{\rm 2B}}{1+\Delta_{\rm 1C}+\Delta_{\rm EC}+I_{\rm Cst}+I_{\rm st}}\right]. \tag{8}$$

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[2]This normalization factor is introduced to remove divergence of $f_0(\theta)$ at $k=0$.

[3]It is numerically confirmed that calculations for the BEC using Eqs. (3) and (4) is equivalent to that using a solution of Schrödinger equation with Coulomb and ρ-meson exchange potential[4].

It should be noted that the normalization parameter c and an effective degree of coherence, i.e., the denominator of the ratio $E_{2B}/(1+\Delta_{1C}+\Delta_{EC}+I_{Cst}+I_{st})$, are related to each other.

3. A theoretical formula for $\pi^+\pi^-$ pairs including the FSI

Using the following amplitudes of the strong interaction for the $\pi^+\pi^-$ scattering ($I=2$ and 0) and changing the sign of the factor $\eta=m\alpha/2q$, one can estimate correlations for the $\pi^+\pi^-$ pairs. The strong amplitude for $\pi^+\pi^-\to\pi^+\pi^-$ scattering is decomposed as

$$\phi_0^{(2,0)}=\frac{1}{3}f_0^{(2)}(\theta)+\frac{2}{3}f_0^{(0)}(\theta). \tag{9}$$

Here, we have assumed that contribution of $\pi^0\pi^0\to\pi^+\pi^-$ channels is small. The phase shifts $\delta_0^{(0)}$ for $I=0$ which are obtained in Refs. [9]-[16] are parameterized by the polynomials (cf., Fig. 1):

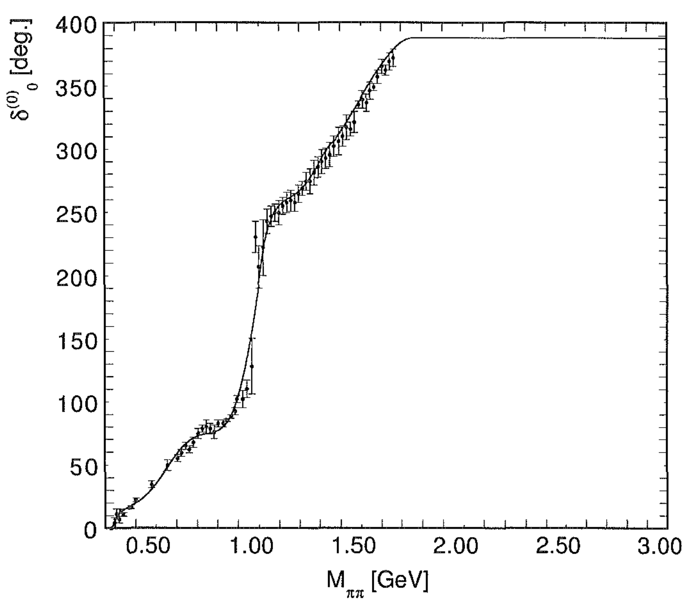

Fig. 1: The phase shifts of $\delta_0^{(0)}$ parameterized by Eq.(10). See Refs. [9]-[16].

$$\begin{aligned}
\delta_0^{(0)}(Q)&=-75.63\ Q+2345.\ Q^2-12588.\ Q^3+23872.\ Q^4+7431.5\ Q^5\\
&\quad -77590.\ Q^6+87705.\ Q^7-30866.\ Q^8\,[\text{deg}]\ \ (\text{for } Q\le 0.985\text{ GeV})\\
\delta_0^{(0)}(Q)&=-53061.+179950.\ Q-227210.\ Q^2+127110.\ Q^3\\
&\quad -26558.\ Q^4\,[\text{deg}]\ \ (\text{for } 0.985<Q\le 1.350\text{ GeV})\\
\delta_0^{(0)}(Q)&=+3978.5-7988.6\ Q+5608.9\ Q^2\\
&\quad -1269.3\ Q^3\,[\text{deg}]\ \ (\text{for } 1.350<Q\le 1.750\text{ GeV})\\
\delta_0^{(0)}(Q)&=388.05\,[\text{deg}]\qquad (\text{for } 1.750\text{ GeV}<Q)
\end{aligned} \tag{10}$$

Our formula for the $\pi^+\pi^-$ pairs is given as

$$\Psi_{\rm total}(\mathbf{k}, \mathbf{r}) = \Psi_C(\mathbf{k}, \mathbf{r}) + \Phi_0^{(2,0)}(\mathbf{k}, \mathbf{r}),$$
$$\Phi_0^{(2,0)}(\mathbf{k}, \mathbf{r}) = \sqrt{G(-\eta)}\,\frac{\phi^{(2,0)}\exp[i(kr+\eta\ln(2kr))]}{r},$$
$$N^{+-}/N^{BG(mix)} = \int d^3r\rho(r)|\Psi_{\rm total}(\mathbf{k}, \mathbf{r})|^2, \tag{11}$$

It should be noted here that Eq.(11) gives a prediction for $\pi^+\pi^-$ correlation function caused by the FSI. Using now Eqs. (8) and (11), one obtains the following theoretical formula of the BEC with unlike-particle reference [1] as

$$\pi^{\pm\pm}/\pi^+\pi^- = \frac{N^{\pm\pm}/N^{BG(mix)}}{N^{+-}/N^{BG(mix)}}. \tag{12}$$

4. Analyses of $e^+e^- \to W^+W^-$ events by means of Eq.(12)

We have now analyzed data on $e^+e^- \to W^+W^-$ events at $\sqrt{s} = 172$ GeV [1] using Eq.(12). A result of minimum-χ^2 fit is shown in Fig.2. Because the scattering length parameter in Eq.(5) was fixed as $a_0 = -1.20\text{GeV}^{-1}$[9] and the normalization parameter c was fixed as $c=1$, we can estimate the remaining two parameters as:

$$\lambda = 1.19 \pm 0.48, \quad \beta = 0.87 \pm 0.31 \text{ fm}, \quad (\chi^2/\text{NDF} = 14.3/13).$$

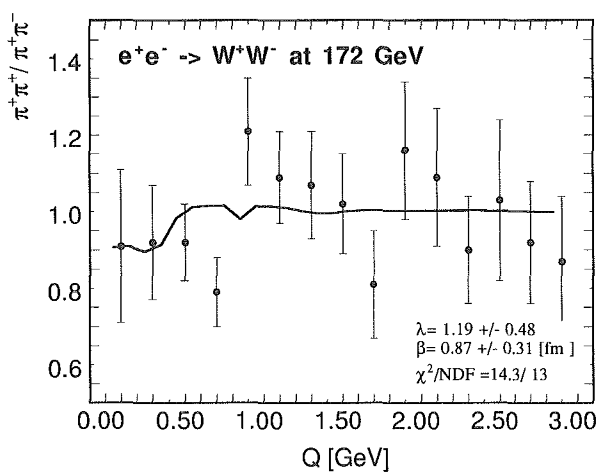

Fig. 2 : Our result of analysis for the BEC data by DELPHI Collab.[1] using Eq. (12).

It should be noted that the value of the λ parameter obtained by Eq.(12) is considerably different from one estimated in Ref.[1, 2]. See Table I.

In order to expose the cause this difference in a more clear way, we also analyze the data by the following two formulas:

$$\pi^{\pm\pm}/\pi^{+-}(\text{strong corr. only}) = \left.\frac{N^{\pm\pm}/N^{BG(mix)}}{N^{+-}/N^{BG(mix)}}\right|_{\alpha=0}, \tag{13}$$

$$\pi^{\pm\pm}/\pi^{+-}(\text{Coulomb corr. only}) = \left.\frac{N^{\pm\pm}/N^{BG(mix)}}{N^{+-}/N^{BG(mix)}}\right|_{f_0(\theta)=0}. \tag{14}$$

Eq.	FSI	β[fm]	λ	χ^2/NDF
Eq.(13)	Coulomb corr.	0.35 ± 0.17	-0.15 ± 0.13	11.5/13
Eq.(14)	strong corr.	0.87 ± 0.32	1.16 ± 0.48	14.4/13

Table II : Influences of the Coulomb and strong final state interactions for the β and λ parameters.

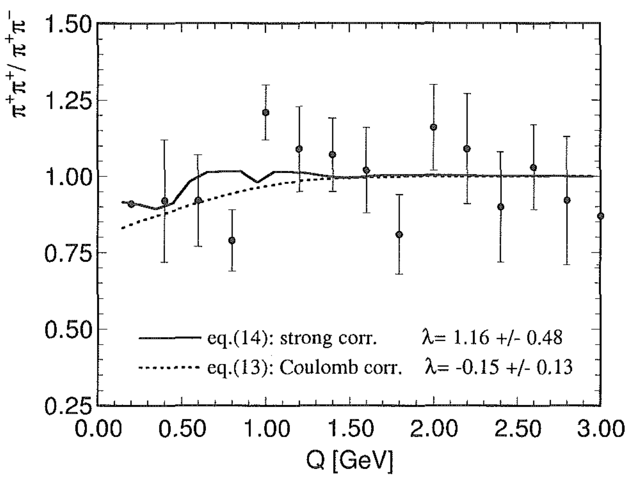

Fig. 3 : Illustration of the role of the Coulomb and strong final state interaction in $\pi^\pm\pi^\pm/\pi^+\pi^-$.

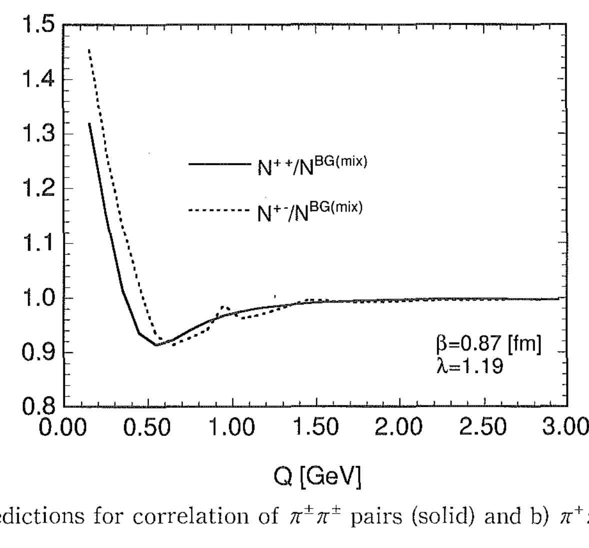

Fig. 4 : Our predictions for correlation of $\pi^\pm\pi^\pm$ pairs (solid) and b) $\pi^+\pi^-$ pairs (dotted).

Results of minimum-χ^2 fit to the data by means of Eqs. (13) and (14) are summarized in Table II. We find that the correction caused by the strong FSI change drastically the value of the λ parameter. The fitting results by means of Eqs.(13) and (14) are also shown in Fig.3. Since the Coulomb correction plays important role only in small Q region ($Q \leq 0.2$ GeV), strong interaction is dominant in the Q region observed by authors of Ref. [1][4]. (cf. Figs.2 and 3.)

Finally, we also predict the correlation functions, $N^{\pm\pm}/N^{BG(mix)}$ and $N^{+-}/N^{BG(mix)}$ in Fig. 4.

5. Concluding remarks

We have analyzed the data of the BEC in $e^+e^- \to W^+W^-$ events by means of Eq. (12), i.e., including full FSI structure (both Coulombic and strong). In this way we have found that there is normal Bose-Einstein effect in the correlation data because the degree of coherence parameter λ included in Eq.(12) comes out to be approximately unity. However, this expected enhancement is dissolved by the complicated structure of the FSI (especially important here due to the small space-time distances involved in this particular reaction).

Acknowledgments: We are grateful to Dr. A. Tomaradze for providing us with the experimental data. One of us (M. B.) thanks the Exchange Program between the JSPS and the Polish Academy of Sciences and Jagellonian University, Cracow, for financial support and acknowledges various conversations with A. Bialas, J. Bartke, A. Krzywicki and G. Wilk. This work is partially supported by Japanese Grant-in-Aid for Scientific Research from the Ministry of Education, Science, Sports and Culture (# 09440103). Finally the authors are also indebted to G. Wilk for his reading the manuscript.

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