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Coulomb wave function correction including momentum resolution for charged hadron pairs: Analysis of data of $\pi^+\pi^-$ pair in $p+$ Ta reaction at 70 GeV/c

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Abstract

We propose a new method for the Coulomb wave function correction including the momentum resolution for charged hadron pairs and apply it to the precise data on $\pi^+\pi^-$ correlations obtained in $p+$ Ta reaction at 70 GeV/c. It is found that interaction regions of this reaction (assuming Gaussian source function) are 5.6 ± 3.0 and 4.4 ± 2.6 fm for the thicknesses of the target 8 and 1.4 microns, respectively. The physical picture of the source size obtained in this way is discussed.

1. Introduction. Recently we have obtained the new formulae for the Coulomb wave function correction for charged hadron pairs [1,2]. In particular we have applied them (in [2]) to data on $\pi^+\pi^-$ correlation obtained in $p+$ Ta reaction at 70 GeV/c [3] (which were originally corrected by usual Gamow factor only). However, as it was pointed out to us by one of the authors of [3], we did not take into account their published finite momentum resolution [4]. In fact, our formulae cannot be applied directly to experimental data in which such momentum resolution is accounted for. Therefore in the present paper we would like to extend our method for the Coulomb wave correction provided in [1,2] to include also the momentum resolution case and to re-analyse data of [3] and also to analyse the new, preliminary data of [5] obtained with two kinds of thickness of Ta target: 8 microns (8 mkm) and 1.4 microns (1.4 mkm).

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In the next paragraph we first reconstruct (for the sake of completeness) the analysis performed in Ref. [3] and then, in the third paragraph, we propose our new method for the Coulomb wave correction including this time also the momentum resolution. The final part contains our concluding remarks.

2. Reconstruction of the analysis of $\pi^+\pi^-$ correlation data performed by using the Gamow factor with momentum resolution. It has been stressed in [3] that relative momenta of $\pi^+\pi^-$ pairs observed by them have some finite resolutions. The averaged correlation function, defined as

$$R(\pi^+\pi^-)=\left\langle \frac{1}{\sigma}\frac{d^2\sigma}{dp_1dp_2}\Big/\left(\frac{1}{\sigma}\frac{d\sigma}{dp_1}\frac{1}{\sigma}\frac{d\sigma}{dp_2}(1+B_{pr})\right)\right\rangle \tag{1}$$

depends therefore on this momentum resolution, where $(1+B_{pr})$ stands for non-Coulomb correlation factor. To account for it the following random number method has been proposed in Ref. [3] in order to obtain the corresponding averaged quantities in analysis of the correlation data.

First of all, the relative momentum of the measured pair, $q=p_1-p_2$, is decomposed into its longitudinal and transverse components, q_L and q_T respectively, by making use of the uniform random number $u\in(0,1)$ (it is worthwhile to notice at this point that the transverse components q_T in data of [3] are smaller than 10 MeV/c). One uses the following scheme here:

$$q_T=\begin{cases}10\sqrt{u} & \text{for } q\geq 10\text{MeV/c},\\ q\sqrt{u} & \text{for } q\leq 10\text{MeV/c},\end{cases} \tag{2}$$
$$q_L=\sqrt{q^2-q_T^2}.$$

At the next step, the Gaussian random numbers for q_L and q_T are generated in the following way:

$$q_{L(\text{random})}=\sigma_L X+q_L, \tag{3}$$
$$q_{T(\text{random})}=\sigma_T X+q_T. \tag{4}$$

In the above equations X stands for the standard Gaussian random number[1] whereas σ_L and σ_T are longitudinal and transverse setup resolutions for the corresponding components, which are equal to (values used in [5]): $\sigma_L=1.3\,\text{MeV/c}$, $\sigma_T=0.6\,\text{MeV/c}$ (for target of the thickness 8 mkm) and $\sigma_T=0.4\,\text{MeV/c}$ (for the 1.4 mkm target).

Finally, using the randomized number $q_{(\text{random})}=\sqrt{q^2_{L(\text{random})}+q^2_{T(\text{random})}}$ one calculates the corresponding randomized Gamow factor correction:

[1] The probability density of such random numbers is given by

$$f(x)=\frac{1}{\sqrt{2\pi}}\exp(-x^2/2)\ (-\infty<x<\infty).$$

$$G(-\eta_{(\mathrm{random})}) = \frac{-2\pi\eta_{\mathrm{random}}}{\exp(-2\pi\eta_{\mathrm{random}})-1}, \tag{5}$$

where $\eta_{(\mathrm{random})} = m\alpha/q_{(\mathrm{random})}$. The full flow chart for this procedure is shown in Fig. 1. Calculating now the average value of $G(-\eta_{(\mathrm{random})})$ in 100 k events one can estimate the Gamow factor with this finite momentum resolution,

$$R(q) = \tilde{G}(-\eta)b + (1-b), \tag{6}$$

where b is a free parameter. It is understood (or, rather, implicitly assumed) that essentially all unlike sign pions one deals with here originate from decays of long lived particles like η, K_0^S, Λ, and so on[2]. Figure 2 shows the results of analysis of new data (for 8 mkm target) [5] for region q > 3 MeV/c using this method.

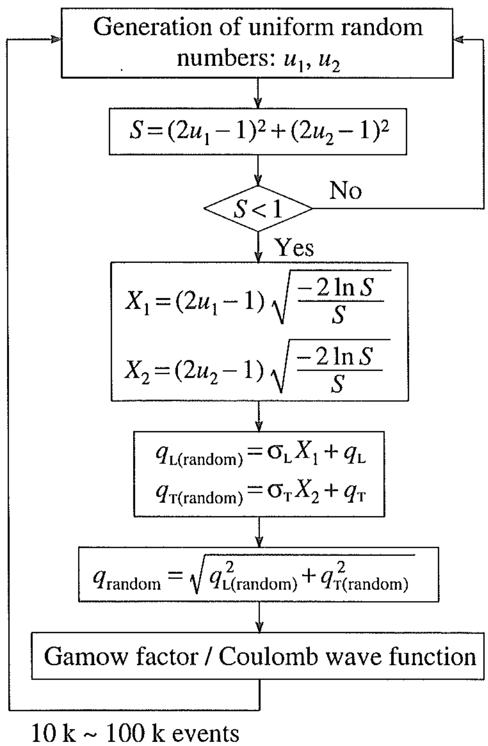

Fig. 1. Flow chart of the present procedure for Gamow factor and/or Coulomb wave function, which includes momentum resolution by generating the Gaussian random numbers.

[2] This fact has some consequences on the determination of the radius of the interaction region as will be shown later on and makes it different from that obtained from the Bose-Einstein correlations.

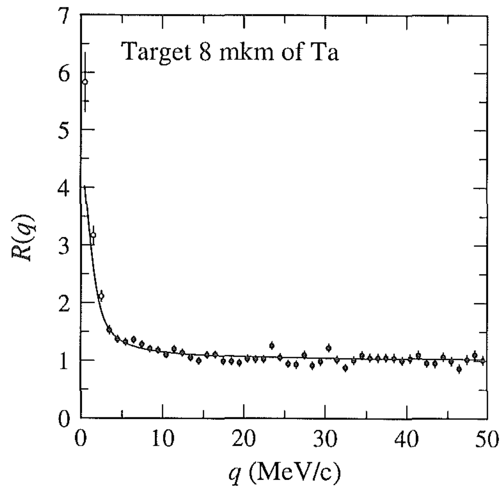

Fig. 2. Results of χ^2 fit for 8 mkm target of $p+\mathrm{Ta}\rightarrow\pi^+\pi^-+X$ reaction with $q>3$ MeV/c by eq.(6).

3. Proposal of the new method. We would like to propose now a new method of Coulomb wave function correction with a source function $\rho(r)$ instead of the Gamow factor, in order to analyse the same data. As usual we decompose the wave function of unlike charged bosons with momenta p_1 and p_2 into the wave function of the center-of-mass system (c.m.) with total momentum $P=\frac{1}{2}(p_1+p_2)$ and the inner wave function with relative momentum $q=(p_1-p_2)$. This allows us to express Coulomb wave function $\Psi(\mathbf{q},\mathbf{r})$ in terms of the confluent hypergeometric function Φ [7]:

$$\Psi(\boldsymbol{q},\boldsymbol{r})=\Gamma(1-i\eta)e^{\pi\eta/2}e^{i\mathbf{q}\cdot\mathbf{r}/2}\Phi(i\eta;1;iqr(1-\cos\theta)/2), \tag{7}$$

where $r=x_1-x_2$ and the parameter $\eta=ma/q$. Assuming factorization in the source functions, $\rho(r_1,r_2)=\rho(r_1)\,\rho(r_2)=\rho(R)\,\rho(r)\left(\text{here } R=\frac{1}{2}(x_1+x_2)\right)$, we obtain the expression for Coulomb correction for the system of $\pi^+\pi^-$ pairs identical (modulo the sign) as in [6,1,2]:

$$\begin{aligned}C_c(-\eta)&=\int\rho(R)d^3R\int\rho(r)d^3r|\Psi(\mathbf{q},\mathbf{r})|^2\\&=G(-\eta)\sum_{n=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-i)^n(i)^m}{n+m+1}q^{n+m}\,I_{\mathrm{R}}(n,m)A_nA_m^*\\&=G(-\eta)[1+\Delta_{1\mathrm{C}}(-\eta)],\end{aligned} \tag{8}$$

where

$$I_{\mathrm{R}}(n,m)=4\pi\int dr\;r^{2+n+m}\,\rho(r),\qquad A_n=\frac{\Gamma(-i\eta+n)}{\Gamma(-i\eta)}\frac{1}{(n!)^2}.$$

For the specific choice of Gaussian source distribution, $\rho(r)=\frac{\beta^3}{\sqrt{\pi^3}}\exp(-\beta^2r^2)$, we have

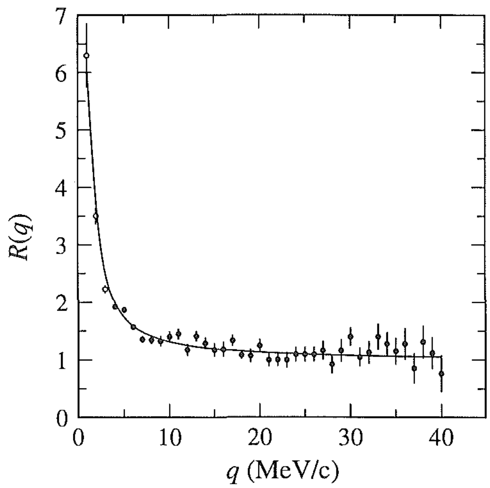

Fig. 3. Results of the χ^2 fits for Ref. [3] of $p+\mathrm{Ta}\rightarrow\pi^+\pi^-+X$ reaction with $q>3$ MeV/c by eq. (11).

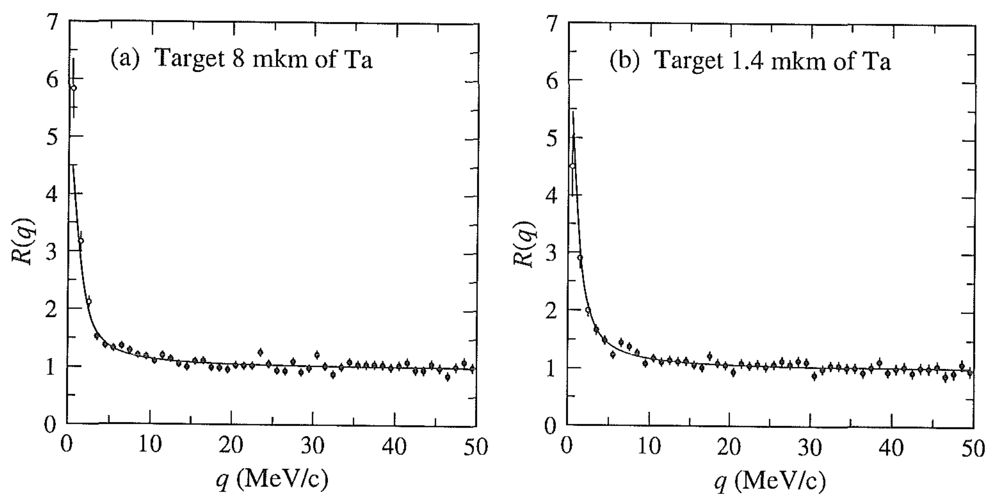

Fig. 4. Results of the χ^2 fits for $p+\mathrm{Ta}\rightarrow\pi^+\pi^-+X$ reaction with $q>3$ MeV/c by eq.(11) : (a) 8 mkm target ; (b) 1.4 mkm target.

$$I_{\mathrm{R}}^{\mathrm{G}}(n,\, m)=\frac{2}{\sqrt{\pi}}\left(\frac{1}{\beta}\right)^{n+m}\Gamma\left(\frac{n+m+3}{2}\right), \tag{9}$$

whereas exponential source function, $\rho(r)=\frac{\beta^3}{8\pi}\exp(-\beta r)$, leads to

$$I_{\mathrm{R}}^{\mathrm{E}}(n,\, m)=\left(\frac{1}{\beta}\right)^{n+m}\frac{(n+m+2)!}{2}. \tag{10}$$

Using now the same method of Gaussian random numbers as in the previous

Table I: Results of the χ^2 fits of $R(q)$ for Gaussian source by eqs. (6) and (11).

Reaction	Formula	$1/2\beta$[fm]	b	χ^2/NDF
data of Ref. [3] cf. Ref. [2] (without momentum resolution)	Gamow factor	——	——	57.8/40
	eq. (8)	2.30±0.88	——	51.0/39
data of Ref. [3]	eq. (6)	——	b=1 (fixed)	53.9/37
	eq. (11)	2.96±1.02	b=1 (fixed)	44.9/36
data of Ref. [5] 8 mkm	eq. (6)	——	0.43±0.03	55.0/46
	eq. (11)	5.62±3.03	0.53±0.07	51.5/45
data of Ref. [5] 1.4 mkm	eq. (6)	——	0.51±0.04	37.9/46
	eq. (11)	4.44±2.63	0.60±0.07	35.1/45

paragraph, we can analyse the old and the new data on $\pi^+\pi^-$ pairs [3,5] using the following formula:

$$R(q)\tilde{C}_c(-\eta)b+(1-b). \tag{11}$$

Figs. 3 and 4 show results of our analysis of the old and the new data, respectively. Table I shows our results obtained using eq. (8) applied to old and new data with $q >$ 3 MeV/c.

4. Concluding remarks. We have proposed the new method for the Coulomb wave function correction with momentum resolution and applied it to the analysis of the precise data provided by [3,5]. Authors of Ref. [3] have analysed their $\pi^+\pi^-$ correlation data using Gamow factor for Coulomb corrections together with the random numbers method to account for final momentum resolution. We have repeated this analysis replacing Gamow factor by the Coulomb wave function but following the same method for correction for the momentum resolution effect (cf. eq. (8)). As a result we have obtained the following ranges of interaction for the Gaussian source function:

$$\begin{aligned} r(p+\mathrm{Ta}) = \frac{1}{2\beta} &= 5.6 \pm 3.0 \ \text{fm} \quad \text{for 8 mkm,} \\ &= 4.4 \pm 2.6 \ \text{fm} \quad \text{for 1.4 mkm.} \end{aligned} \tag{12}$$

To get a correct physical picture of the source size, we should calculate the root mean squared size, which is equal to:

$$\begin{aligned} r_{\mathrm{rms}} = \frac{\sqrt{3}}{2\beta} &= 9.7 \pm 5.3 \ \text{fm} \quad \text{for 8 mkm,} \\ &= 7.7 \pm 4.5 \ \text{fm} \quad \text{for 1.4 mkm.} \end{aligned} \tag{13}$$

The present study of $\pi^+\pi^-$ pair correlations has shown therefore that one can estimate the interaction region even from the $\pi^+\pi^-$ correlation data. It can be compared with the size of the Ta nucleus, which is given by:

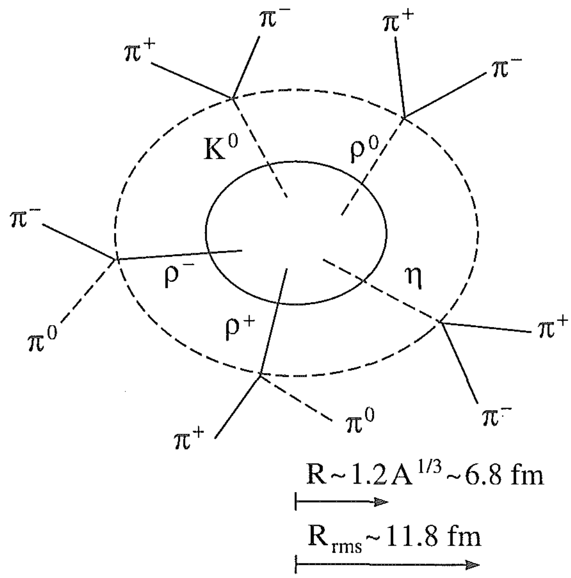

Fig. 5. Physical picture of the obtained source size based on the assumed resonance effects.

Table II: Results of the χ^2 fits of $R(q)$ for exponential source by eq. (11).

	$q_{\rm lim}$[MeV]	$1/\beta$[fm]	b	χ^2/NDF
8 mkm	25	3.19±3.55	0.51±0.08	24.0/20
1.4 mkm	25	3.00±3.88	0.59±0.10	20.3/20
	30	0.65±2.62	0.54±0.08	23.1/25
	35	2.08±2.77	0.57±0.08	27.2/30
	40	2.60±2.85	0.58±0.08	30.3/35

$$r_{\rm Ta} = 1.2 \times A^{1/3} = 1.2 \times (181)^{1/3} = 6.8 \text{ fm}. \tag{14}$$

As one can see, $r_{\rm rms}$ is significantly bigger than $r_{\rm Ta}$. We attribute this difference to a physical picture shown in Fig. 5, i.e., to the fact that unlike-sign pions are mostly (if not totally) emerging from the long-lived resonances shown there. In a future one should consider also a possibility of more direct estimation of the parameter b and its role in determining the source size parameter[3].

[3] In an analysis of $\pi^0\pi^0$ correlation data, a similar function $f(q)$ is introduced:

$$R(\pi^0\pi^0) = f(q) + (1-f(q))\,[1+\lambda E_{\rm 2B}^2].$$

where λ and $E_{\rm 2B}$ are the degree of coherence and an exchange function due to the Bose-Einstein effect. $f(q)$ is attributed to the resonances effect; $f(q) \approx 0.9 \sim 0.7$ depends on the Monte Carlo programs [8].

For completeness we have also tried to analyse the same data using exponential source function instead of Gaussian. As is shown in Table II this leads to errors on $r=\frac{1}{2\beta}$ of the order of 100%, i.e., with this type of source function we cannot estimate the source size (therefore it has to be discarded).

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