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Gauge Couplings and Soft Masses in Supersymmetric E_6 Grand Unified Theories

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Abstract

We study the gauge couplings and the soft mass spectrum within supersymmetric E_6 grand unified theory. We examine how they are useful as the probe of physics at the higher energy scale than the weak scale.

1 Introduction

The supersymmetric grand unified theory (SUSY-GUT) [1] has been hopeful as a realistic theory beyond standard model. In fact, SUSY $SU(5)$ GUT has been taken interest by the LEP experiments [2] and predicts the long lifetime of nucleon consistent with the present data [3]. However, it is difficult to regard SUSY-GUT as the final theory because there are still several problems. First it does not describe gravity, while the Planck scale M_{Pl} is around the corner. Second there exists a great deal of arbitrariness on the model building. That is, a lot of freedom is left over on the choice of matter multiplets and parameters. In fact, SUSY-GUT does not explain the values of observed quark and lepton mass and the family number. Third it is not known how to break supersymmetry (SUSY) and to get the desired low energy physics yet. Last the fine-tuning is needed to keep the weak Higgs doublets light but to make the colored Higgs triplets superheavy in the minimal SUSY-GUT.

It is expected that they are solved in more fundamental theories. Supergravity theory (SUGRA) [4] is regarded as an attractive candidate. When we take SUGRA as an effective theory at M_{Pl}, some of the above problems can be solved. For example, there exists such a scenario [5] that the spontaneous SUSY breaking occurs in the hidden sector and the effect is meditated through the gravitational interaction and the soft SUSY breaking terms appear in the observable sector. Then the form of soft SUSY breaking terms is determined by the structure of SUGRA. As usual, the model with the minimal Kähler function is chosen. Then the soft parameters take universal values at the gravitational scale $M \equiv M_{Pl}/\sqrt{8\pi}$, i. e., the global SUSY model derived from SUGRA with the minimal Kähler function has the universal scalar mass m_0, the universal scalar trilinear coupling constant A and the universal mixing mass parameter B given by $B = A - m_0$. (The universal gaugino mass $M_{1/2}$ is derived from SUGRA with the simplest non-minimal gauge kinetic function $f_{\alpha\beta} = S\ \delta_{\alpha\beta}$.) The values at the low energy scale are obtained by the use of renormalization group equations. The analyses based on the minimal SUGRA are quite interesting because it has high predictabilities and testable enough, but it is difficult to say that this approach is natural from the following reasons. First there is no strong reason that realistic SUGRA takes the minimal structure. In fact, the effective SUGRA derived from superstring theories (SSTs) have, in general, non-minimal structures and they can lead to the effective theories with non-universal type of soft parameters in the flat limit [6]. Second the effects of intermediate physics are ignored in the analysis of the running of parameters. For example, the discrete change can occur at the symmetry breakings scale for the models with certain unified gauge symmetry. Therefore it is generally natural that we expect that measurements of soft parameters at TeV region give useful informations for the GUT scale physics rather the Planck scale one. (They will give a clue to resolve the Planck scale physics only in the case that the effects of intermediate physics are absent or neglected.)

In this paper, we study the utility of the gauge coupling constants and soft masses as the probe of GUT scale physics based on SUSY E_6 GUTs. The first half of our strategy is the same as the analyses before [7] [8]. That is, on the postulation that the minimal supersymmetric standard model (MSSM) is established as an effective theory which describes the physics of $O(1)$ TeV and the precise measurements of the gauge couplings and soft masses can be carried out, we examine how they are useful as the probe of GUT scale physics. In the second half, we investigate the scalar masses based on a gauge coupling unification scenario with chain breakings. And we get new type of scalar mass relations which can be useful to select further the pattern of gauge symmetry breakings.

The content of this paper is as follows. In section 2, we review the unification of gauge coupling constants and the soft masses in the framework of MSSM and the minimal SUSY-GUT. In section 3, we give three examples of E_6 gauge unification scenario with intermediate gauge symmetry breaking consistent with the LEP data. In section 4, we derive the characteristic relation among scalar masses for these examples. It is also given all the tables with the particle assignments and the scalar mass relations for other E_6 breaking patterns. In section 5, we obtain new type of scalar mass relations by adopting the gauge coupling unification scenario discussed in section 3. The conclusions are given in section 6.

2 Gauge couplings and soft masses

Let us first give a brief review on the usual unification scenario of gauge coupling constants [9] and the renormalization flow of soft masses [10] based on MSSM.

The renormalization group equations (RGEs) of gauge couplings α_i are given as

$$\mu\frac{d}{d\mu}\alpha_i^{-1}(\mu) = -\frac{b_i}{2\pi} \tag{1}$$

at one-loop level. Here b_i are the coefficients of the beta function and μ is a energy scale. If the particle contents of MSSM and the precise measurements at LEP are used, the structure constants, α_3, α_2 and α_1 $(\equiv\frac{5}{3}\alpha_Y)$ of the 'standard model gauge group' $G_{SM} \equiv SU(3)_C\times SU(2)_L\times U(1)_Y$ meet at the scale

$$M_X \sim 2.1\times 10^{16}\text{GeV} \tag{2}$$

and the value at M_X is obtained as

$$\alpha_{GUT} \equiv \alpha_i(M_X) \sim \frac{1}{24.6} \quad (i=1,\ 2,\ 3). \tag{3}$$

This fact suggests that G_{SM} is possibly unified in $SU(5)$ gauge group economically. (It is regarded as one of the indirect evidence for SUSY.) We call the above scenario the 'minimal unification scenario'.

Next the running of gaugino masses $M_i(\mu)$ yields to the following RGEs at one-loop level [10],

$$\mu\frac{d}{d\mu}\left(\frac{M_i(\mu)}{\alpha_i(\mu)}\right) = 0 \tag{4}$$

and this equation is easily solved as $M_i(\mu)/\alpha_i(\mu) = const.$ In the 'minimal unification scenario', the following boundary conditions are imposed on

$$M_{GUT} \equiv M_3(M_X) = M_2(M_X) = M_1(M_X) \tag{5}$$

and

$$\alpha_{GUT} \equiv \alpha_3(M_X) = \alpha_2(M_X) = \alpha_1(M_X) \tag{6}$$

because of the unified gauge symmetry at M_X. So we get the interesting relation

$$\frac{M_3(\mu)}{\alpha_3(\mu)} = \frac{M_2(\mu)}{\alpha_2(\mu)} = \frac{M_1(\mu)}{\alpha_1(\mu)} = \frac{M_{GUT}}{\alpha_{GUT}}. \tag{7}$$

This relation is called the 'GUT relation'*.

Last we consider the running of masses m_a of scalar fields $\phi_a(x)$ in MSSM. The one-loop level RGEs of them are as follows [10],

$$\mu\frac{d}{d\mu}m_a(\mu)^2=-\frac{2}{\pi}\sum_i C_2(R_i^a)\alpha_i(\mu)M_i(\mu)^2+\frac{3}{10\pi}Y_a\alpha_1(\mu)S(\mu), \tag{8}$$

$$\mu\frac{d}{d\mu}S(\mu) \quad =\frac{b_1}{2\pi}\alpha_1(\mu)S(\mu), \tag{9}$$

$$S(\mu) \quad \equiv\sum_a Y_a n_a m_a(\mu)^2 \tag{10}$$

where i represents the gauge group, a the species of the scalar, $C_2(R_i^a)$ the second-order Casimir invariant of the gauge group i for the species a, Y_a the hypercharge and n_a the multiplicity of the species a. In Eq. (8), we have neglected the Yukawa coupling contribution. This approximation should be valid for the first-and the second-generation fields. It is straightforward to generalize our results to the third-generation fields by considering the effects of Yukawa couplings. The contribution from S is usually ignored since it is absent under the assumption of the universal scalar mass. For MSSM, it is†

$$S=m_2^2-m_1^2+\sum_{generations}(m_{\tilde{q}}^2-2m_{\tilde{u}}^2+m_{\tilde{e}}^2-m_{\tilde{l}}^2+m_{\tilde{d}}^2). \tag{11}$$

Solving the above RGEs (8)～(10), we obtain

$$m_a(\mu)^2=m_a(\mu_0)^2-\sum_i\frac{2}{b_i}C_2(R_i^a)(M_i(\mu)^2-M_i(\mu_0)^2)$$
$$+\frac{3}{5b_1}Y_a(S(\mu)-S(\mu_0)), \tag{12}$$

$$S(\mu)=\frac{\alpha_1(\mu)}{\alpha_1(\mu_0)}S(\mu_0). \tag{13}$$

Once we obtain the value of S at the weak scale by measurements, we can easily take its contribution to the scalar masses into account. Therefore the uncertainty on S does not prevent us from going further. If we observe the gauge couplings and the soft masses and measure their values precisely, we can obtain the values of scalar masses at certain higher energy scale, e.g. M_X, by using the above solutions.

We comment on an assumption of the universal soft SUSY breaking terms briefly. As usual the following universal type of boundary condition at M (or M_X) is taken,

$$m_0^2=m_a^2 \tag{14}$$

for scalar masses and

* We neglect both the threshold effects [11] and the 'gravitational' corrections [12] which can violate this relation. And two-loop effects violate this relation, but it is small [13].

† We refer to the chiral multiplets as q for left-handed quark, l left-handed lepton, u right-handed up type quark, d right-handed down type quark and e for right-handed charged lepton. The tilde represents their scalar component. m_1^2 and m_2^2 stand for the soft SUSY breaking mass terms of the Higgs with hypercharge $-1/2$ and $+1/2$, respectively.

$$M_{1/2}=M_i \tag{15}$$

for gaugino masses. The condition (14) is motivated by SUGRA with the minimal Kähler function. And the condition (15) is motivated by SUGRA with the simplest non-minimal gauge kinetic function or certain unified gauge symmetry. The various predictions at low-energy physics are derived by the use of RGEs of MSSM and the constraints in SUSY-GUT, the particle cosmology and so on based on (14) and (15). But the assumption (14) is not necessarily proper as described in the introduction. Hence we do not impose it in our analysis.

3 Chain breaking scenario

In the last section, we have explained that the 'minimal unification scenario' is supported by the LEP experiments. There exist, however, many scenarios based on the various models of SUSY-GUT consistent with the LEP data if the concept of 'simplicity' is put aside. For example, there exist the following three types of them where the unification scale is the same as that of the 'minimal' one.

1. The direct breaking of the larger group than $SU(5)$, e.g. $SO(10)$ and E_6, down to G_{SM}.
2. The models with extra heavy generations.
3. The models of SUSY $SO(10)$ GUT with the chain breaking [14] [7].

In this section, we discuss the unification scenario with chain breakings based on SUSY E_6 GUT [15]. There are so many possibilities that E_6 breaks down to G_{SM} such as

$$E_6 \xrightarrow{M_U} \cdots\cdots \longrightarrow G_n \xrightarrow{M_{SB}} G_{SM}. \tag{16}$$

which cannot be selected unless the dynamics of symmetry breakings are clarified. The chief subgroups of E_6 are listed in Table 1. Here we shall exemplify the following three breaking patterns

$$\text{(Ex. 1)} \qquad E_6 \xrightarrow{M_U} SU(5)_F \times U(1)_2 \times U(1)_1 \xrightarrow{M_{SB}} G_{SM} \tag{17}$$

$$\text{(Ex. 2)} \qquad E_6 \xrightarrow{M_U} SU(6) \times SU(2)_I \xrightarrow{M_{SB}} G_{SM} \tag{18}$$

and

$$\text{(Ex. 3)} \qquad E_6 \xrightarrow{M_U} SU(5)_F \times U(1)_{3'} \times SU(2)_I \xrightarrow{M_{SB}} G_{SM} \tag{19}$$

and obtain the particle contents consistent with the LEP data. It turns out that the final breaking scale M_{SB} agrees with the unification scale M_X in the 'minimal scenario' for

Table 1. The chief subgroups of E_6

(0) $E_2 \supset$		$SO(10)\times U(1)_1$	(1)
		$SU(6)\times SU(2)_R$	$(2)_R$
		$SU(6)\times SU(2)_I$	$(2)_I$
		$SU(6)\times SU(2)_J$	$(2)_J$
		$SU(6)\times SU(2)_L$	$(2)_L$
		$SU(3)_C\times SU(3)_L\times SU(3)_R$	$(3)_R$
		$SU(3)_C\times SU(3)_L\times SU(3)_I$	$(3)_I$
		$SU(3)_C\times SU(3)_L\times SU(3)_J$	$(3)_J$
(1)	$SO(10)\times U(1)_1$	$\supset SU(5)\times U(1)_2\times U(1)_1$	$(1-1)$
		$SU(5)_F\times U(1)_2\times U(1)_1$	$(1-1)_F$
		$SU(4)\times SU(2)_L\times SU(2)_R\times U(1)_1$	$(1-2)_R$
		$SU(4)\times SU(2)_L\times SU(2)_I\times U(1)_1$	$(1-2)_I$
		$SU(4)\times SU(2)_L\times SU(2)_J\times U(1)_1$	$(1-2)_J$
$(2)_R$	$SU(6)\times SU(2)_R$	$\supset SU(5)_{F'}\times U(1)_3\times SU(2)_R$	$(2-1)_R$
		$SU(4)\times SU(2)_L\times SU(2)_R\times U(1)_1$	$(2-2)_R$
		$SU(3)_C\times SU(3)_L\times SU(2)_R\times U(1)_R$	$(2-3)_R$
$(2)_I$	$SU(6)\times SU(2)_I$	$\supset SU(5)_F\times U(1)_{3'}\times SU(2)_I$	$(2-1)_I$
		$SU(4)\times SU(2)_L\times SU(2)_I\times U(1)_1$	$(2-2)_I$
		$SU(3)_C\times SU(3)_L\times SU(2)_I\times U(1)_I$	$(2-3)_I$
$(2)_J$	$SU(6)\times SU(2)_J$	$\supset SU(5)\times U(1)_{3''}\times SU(2)_J$	$(2-1)_J$
		$SU(4)\times SU(2)_L\times SU(2)_J\times \mathrm{U}(1)_1$	$(2-2)_J$
		$SU(3)_C\times SU(3)_L\times SU(2)_J\times U(1)_J$	$(2-3)_J$
$(2)_L$	$SU(6)\times SU(2)_L$	$\supset SU(5)_{F''}\times U(1)_{3'''}\times SU(2)_L$	$(2-1)_L$
		$SU(4)\times SU(2)_R\times SU(2)_L\times U(1)_1$	$(2-2)_L$
		$SU(4)\times SU(2)_I\times SU(2)_L\times U(1)_1$	$(2-3)_L$
		$SU(3)_C\times SU(3)_R\times SU(2)_L\times U(1)_L$	$(2-4)_L$
		$SU(3)_C\times SU(3)_I\times SU(2)_L\times U(1)_L$	$(2-5)_L$
$(3)_R$	$SU(3)_C\times SU(3)_L\times SU(3)_R$	$\supset SU(3)_C\times SU(3)_L\times SU(2)_R\times U(1)_R$	
		$SU(3)_C\times SU(2)_L\times U(1)_L\times SU(3)_R$	
		$SU(3)_C\times SU(2)_L\times U(1)_L\times SU(2)_R\times U(1)_R$	
$(3)_I$	$SU(3)_C\times SU(3)_L\times SU(3)_I$	$\supset SU(3)_C\times SU(3)_L\times SU(2)_I\times U(1)_I$	
		$SU(3)_C\times SU(2)_L\times U(1)_L\times SU(3)_I$	
		$SU(3)_C\times SU(2)_L\times U(1)_L\times SU(2)_I\times U(1)_I$	
$(3)_J$	$SU(3)_C\times SU(3)_L\times SU(3)_J$	$\supset SU(3)_C\times SU(3)_L\times SU(2)_J\times U(1)_J$	
		$SU(3)_C\times SU(2)_L\times U(1)_L\times SU(3)_J$	
		$SU(3)_C\times SU(2)_L\times U(1)_L\times SU(2)_J\times U(1)_J$	

all the above examples on the postulation that the physics below M_{SB} is described by MSSM.

For (Ex.1), we have a relation among the structure constants α_1 $(\equiv\frac{5}{3}\alpha_Y)$, α_{5F} and $\alpha_{1(2)}$ at M_{SB} such as

$$\frac{25}{\alpha_1}=\frac{1}{\alpha_{5F}}+\frac{24}{\alpha_{1(2)}} \tag{20}$$

where we denote the structure constants of $SU(5)_F$ and $U(1)_2$ as α_{5F} and $\alpha_{1(2)}$, respectively. Here the gauge group $SU(5)_F\times U(1)_2$ corresponds to that of the flipped $SU(5)$ model [16]. Provided by the unificaion of the gauge couplings of G_{SM} is realized at M_X from the LEP data, the relation $\alpha_{5F}(M_X)=\alpha_1(M_X)$ holds and so the accidental relation $\alpha_{5F}(M_X)=\alpha_{1(2)}(M_X)$ is derived. On the other hand, the relation $\alpha_{GUT}\equiv\alpha_{5F}(M_U)=\alpha_{1(2)}(M_U)$ holds because of E_6 gauge symmetry‡. There exists a scenario that the relation $\alpha_{5F}(\mu)=\alpha_{1(2)}(\mu)$ holds from M_U to M_X. In this case, we have a constraint $b_5=b_{1(2)}$. As an example of the anomaly free particle contents which satisfy $b_{5F}=b_{1(2)}$, we can find the following one, $3((\overline{\mathbf{5}}, \frac{3}{2\sqrt{10}}, \frac{1}{2\sqrt{6}})+(\mathbf{10}, -\frac{1}{2\sqrt{10}}, \frac{1}{2\sqrt{6}})+(\mathbf{1}, -\frac{5}{2\sqrt{10}}, \frac{1}{2\sqrt{6}}))$ for matter multiplets, $(\mathbf{24}, 0, 0)+(\mathbf{1}, 0, 0)+(\mathbf{1}, 0, 0)$ for gauge multiples and $3((\overline{\mathbf{5}}, -\frac{1}{\sqrt{10}}, -\frac{1}{\sqrt{6}})+(\mathbf{5}, \frac{1}{\sqrt{10}}, -\frac{1}{\sqrt{6}})+(\mathbf{1}, 0, \frac{2}{\sqrt{6}}))+6((\overline{\mathbf{10}}, \frac{1}{2\sqrt{10}}, \frac{1}{2\sqrt{6}})+(\mathbf{10}, -\frac{1}{2\sqrt{10}}, \frac{1}{2\sqrt{6}}))$ for Higgs multiplets under $SU(5)_F\times U(1)_2\times U(1)_1$. In such particle contents, we have $b_{5F}=b_{1(2)}=12$.

For (Ex.2), the relation $\alpha_6(M_X)=\alpha_{2I}(M_X)$ is derived by the use of the LEP data where α_6 and α_{2I} are the structure constants of $SU(6)$ and $SU(2)_I$, respectively. And E_6 gauge symmetry yields to the relation $\alpha_{GUT}\equiv\alpha_6(M_U)=\alpha_{2I}(M_U)$. Hence there exists a constraint $b_6=b_{2I}$ in the scenario that the equality $\alpha_6(\mu)=\alpha_{2I}(\mu)$ is kept from M_U to M_X. As an example of the particle contents which satisfy $b_6=b_{2I}$, we can find the following one, $3((\overline{\mathbf{6}}, \mathbf{2})+(\mathbf{15}, \mathbf{1}))$ for matter multiplets, $(\mathbf{35}, \mathbf{1})+(\mathbf{1}, \mathbf{3})$ for gauge multiplets and $4((\mathbf{15}, \mathbf{1})+(\overline{\mathbf{15}}, \mathbf{1}))+(\overline{\mathbf{6}}, \mathbf{2})+(\mathbf{6}, \mathbf{2})$ for Higgs multiplets under $SU(6)\times SU(2)_I$. In such particle contents, we have $b_6=b_{2I}=9$.

We consider the final example. In the same way, the relation

$$\frac{1}{\alpha_{5F}(M_X)}=\frac{3}{8}\frac{1}{\alpha_{1(3')}(M_X)}+\frac{5}{8}\frac{1}{\alpha_{2I}(M_X)} \tag{21}$$

is derived phenomenologically by the use of the LEP data and the relation among charges

‡ Here we ignore the effects of higher dimensional operators which split the values of $\alpha_{5F}(M_U)$ and $\alpha_{1(2)}(M_U)$ on the E_6 breaking. For other two examples, we ignore the same effects, too.

$$Y \equiv -\frac{1}{\sqrt{15}} T_{5F}^{24} + \frac{3}{\sqrt{15}} Q_{3'} + T_{2I}^{3} \qquad (22)$$

where we denote the structure constants of $U(1)_{3'}$ and $SU(2)_I$ as $\alpha_{1(3)'}$ and α_{2I}, respectively. If the group is further embedded into the E_6 group, then there exists a constraint $b_{5F} = \frac{3}{8} b_{1(3')} + \frac{5}{8} b_{2I}$ in the scenario that the equality $\alpha_{5F}(\mu)^{-1} = \frac{3}{8} \alpha_{1(3')}(\mu)^{-1} + \frac{5}{8} \alpha_{2I}(\mu)^{-1}$ is kept from M_U to M_X. If we choose the particle contents as follows, $3((\overline{\mathbf{5}}, -\frac{1}{2\sqrt{15}}, \mathbf{2}) + (\mathbf{5}, -\frac{2}{\sqrt{15}}, \mathbf{1}) + (\mathbf{10}, \frac{1}{\sqrt{15}}, \mathbf{1}) + (\mathbf{1}, \frac{5}{2\sqrt{15}}, \mathbf{2}))$ for matter multiplets, $(\mathbf{24}, 0, \mathbf{1}) + (\mathbf{1}, 0, \mathbf{1}) + (\mathbf{1}, 0, \mathbf{3})$ for gauge multiplets and $(\overline{\mathbf{5}}, -\frac{1}{2\sqrt{15}}, \mathbf{2}) + (\mathbf{5}, \frac{1}{2\sqrt{15}}, \mathbf{2}) + 5((\mathbf{10}, \frac{1}{\sqrt{15}}, \mathbf{1}) + (\overline{\mathbf{10}}, -\frac{1}{\sqrt{15}}, \mathbf{1}))$ for Higgs multiplets under $SU(5)_F \times U(1)_{3'} \times SU(2)_I$, we have $b_{5F} = \frac{3}{8} b_{1(3')} + \frac{5}{8} b_{2I} = 11$.

We can construct the similar unification scenarios for other breaking patterns [17].

It is difficult to distinguish these scenarios by the use of the precise measurements of gauge couplings alone. Hence it is an important task to study how we can discriminate among them by other experimental methods.

It is expected that the soft SUSY-breaking mass parameters can be novel probes of physics at higher energy scales. In fact, their utility has been examined in Ref. [7]. Let us recall the results again. The gaugino mass spectrum satisfies the 'GUT-relation' as far as 'standard model gauge group' is embedded into a simple group, irrespective of the symmetry breaking pattern, while the squark and slepton mass spectrum carries the information on the breaking pattern of the gauge symmetry. Therefore, the gaugino and the scalar mass spectrum play a complementary role to select among the models of SUSY-GUT experimentally. As an explicit example, it is demonstrated how the scalar mass spectrum distinguishes various $SO(10)$ breaking patterns from each other. Furthermore the scalar mass relations for E_6 gauge symmetry breakings are given in Ref. [8].

We shall explain the above-mentioned consequence of the gaugino masses by using the third example. The $SU(3)_C$ and $SU(2)_L$ gauginos come from the $SU(5)_F$ gaugino and remain unbroken at M_X and so the following equalities hold

$$\frac{M_3(\mu)}{\alpha_3(\mu)} = \frac{M_2(\mu)}{\alpha_2(\mu)} = \frac{M_{5F}(M_X)}{\alpha_{5F}(M_X)} = \frac{M_{GUT}}{\alpha_{GUT}} \qquad (23)$$

for the $SU(3)_C$ gaugino and the $SU(2)_{2L}$ gaugino. Here M_{GUT} and M_{5F} represents the E_6 and $SU(5)_F$ gaugino mass, respectively. There is a complication for the $U(1)_Y$ gaugino because it is a mixture of $SU(5)_F$, $U(1)_{3'}$ and $SU(2)_I$ gauginos. The gauge fields A^μ mix as $g_1^{-1}A_1^\mu=-g_{5F}^{-1}\frac{1}{5}A_{5F}^{24\ \mu}+g_{1(3')}^{-1}\frac{3}{5}A_{1(3')}^\mu+g_{2I}^{-1}\sqrt{\frac{3}{5}}A_{2I}^{3\ \mu}$, and the gaugino fields λ mix correspondingly as

$$\frac{1}{g_1}\lambda_1=-\frac{1}{g_{5F}}\frac{1}{5}\lambda_{5F}^{24}+\frac{1}{g_{1(3')}}\frac{3}{5}\lambda_{1(3')}+\frac{1}{g_{2I}}\sqrt{\frac{3}{5}}\lambda_{2I}^{3} \tag{24}$$

as required from SUSY. Thus the relation for the $U(1)_Y$ gaugino mass is given as

$$\frac{M_1(\mu)}{\alpha_1(\mu)}=\frac{M_1(M_X)}{\alpha_1(M_X)}=\frac{1}{25}\frac{M_{5F}(M_X)}{\alpha_{5F}(M_X)}+\frac{9}{25}\frac{M_{1(3')}(M_X)}{\alpha_{1(3')}(M_X)}+\frac{3}{5}\frac{M_{2I}(M_X)}{\alpha_{2I}(M_X)}=\frac{M_{GUT}}{\alpha_{GUT}} \tag{25}$$

where $M_{1(3')}$ is the $U(1)_{3'}$ gaugino mass and the solution to the RGEs for gaugino masses is used. From Eqs. (23) and (25), the gaugino masses M_3, M_2 and M_1 satisfy the 'GUT-relation' (7). Exactly the same argument applies to the first, second examples and other breaking patterns as well.

Hence the gaugino masses give no information of gauge symmetry breaking pattern and we need the other probes. In the next section, we show that the scalar masses can be useful.

4 Scalar mass relations

In this section, we examine how the scalar masses are useful to select the pattern of gauge symmetry breakings.

Suppose scalar species a, b, c,$\cdots$ belong to a single multiplet R under G_n. One naively expects a kind of 'unification' of the scalar masses at M_{SB} as

$$m_a(M_{SB})^2=m_b(M_{SB})^2=m_c(M_{SB})^2=\cdots\cdots=m_R^2. \tag{26}$$

There are some factors that the above 'unification' is violated.

First the threshold effects due to the heavy particle loops can give further corrections to the Eq. (26). However, it is expected to be of the order of O (α/π) just like in the case of the gauge coupling constants [18] or gaugino masses [11]. It can be important only when there are large representations in the loops or large splitting among the heavy multiplets producing large logarithms.

Second effect may come from the 'gravitational' corrections, like higher dimensional non-renormalizable interactions. Such corrections are suppressed by powers of M_{SB}/M_{Pl}. They can be important if M_{SB} is close to M_{Pl}.

§ Historically, it was demonstrated that the D-term contribution occurs when the gauge symmetry is broken at an intermediate scale due to the soft SUSY breaking terms in Refs. [20] and its existence in a more general situation was suggested in Ref. [21].

¶ It is known that the non-universal soft SUSY breaking parameters emerge in the effective theory derived from superstring theory [6]. Even if they are universal at the Planck scale M_{Pl} as in the minimal supergravity or SUSY breaking by dilaton F-term, the radiative corrections between M_{Pl} and M_{SB} generally induce non-universality.

Table 2. The list of scalar mass relations

Here g_{NX} is the gauge coupling constant of $SU(N)_X$. The gauge group $SU(5)_{F''}\times U(1)_{3'''}\times SU(2)_L$ is one of subgroups of $SU(6)\times SU(2)_L$ and the two kinds of particle assignments exist corresponding to the choice of $SU(2)_R$ or $SU(2)_I$ as the subgroup of $SU(5)_{F''}$. We assume that the Higgs doublets H_1 and H_2 belong to a multiplet of another **27** (or $\overline{\mathbf{27}}$) of E_6. The asterisk (*) represents the scalar mass relation derived under the assumption of 'flavor' universality at M_{SB}. Here the assumption of 'flavor' universality means that $m_{R(a)}$'s take the same value for the same type of representation.

G_n	*Scalar Masses*
E_6	$m_{\tilde d}^2=m_{\tilde l}^2$, $m_{\tilde u}^2=m_{\tilde q}^2=m_{\tilde e}^2$, $m_2^2-m_1^2=m_{\tilde d}^2-m_{\tilde u}^2$
$SO(10)\times U(1)_1$	$m_{\tilde d}^2=m_{\tilde l}^2$, $m_{\tilde u}^2=m_{\tilde q}^2=m_{\tilde e}^2$, (a) $m_2^2-m_1^2=m_{\tilde d}^2-m_{\tilde u}^2$
	$m_{\tilde d}^2=m_{\tilde l}^2$, $m_{\tilde u}^2=m_{\tilde q}^2=m_{\tilde e}^2$, (b) $m_2^2-m_1^2=m_{\tilde d}^2-m_{\tilde u}^2$ (*)
$SU(5)\times U(1)_2\times U(1)_1$	$m_{\tilde d}^2=m_{\tilde l}^2$, $m_{\tilde u}^2=m_{\tilde q}^2=m_{\tilde e}^2$
$SU(5)_F\times U(1)_2\times U(1)_1$	$m_{\tilde q}^2-m_{\tilde d}^2=m_{\tilde u}^2-m_{\tilde l}^2$
$SU(4)\times SU(2)_L\times SU(2)_R\times U(1)_1$	$m_{\tilde l}^2-m_{\tilde q}^2=m_{\tilde d}^2-m_{\tilde e}^2$, $g_{2R}^2(m_{\tilde l}^2-m_{\tilde q}^2)=g_4^2(m_{\tilde d}^2-m_{\tilde u}^2)$, $m_2^2-m_1^2=m_{\tilde d}^2-m_{\tilde u}^2$
$SU(3)_C\times SU(2)_L\times SU(2)_R\times U(1)_{B-L}\times U(1)_1$	$m_2^2-m_1^2=m_{\tilde d}^2-m_{\tilde u}^2$
$SU(4)\times SU(2)_L\times SU(2)_I\times U(1)_1$	$g_{2I}^2(m_{\tilde q}^2-m_1^2)=g_4^2(m_{\tilde l}^2-m_2^2)$, (*) $g_{2I}^2(m_{\tilde e}^2-m_{\tilde u}^2)=(g_4^2-g_{2I}^2)\ (m_{\tilde l}^2-m_1^2)$ (*)
$SU(6)\times SU(2)_R$	$m_{\tilde l}^2-m_{\tilde q}^2=m_{\tilde d}^2-m_{\tilde e}^2$, $g_{2R}^2(m_{\tilde l}^2-m_{\tilde q}^2)=g_6^2(m_{\tilde d}^2-m_{\tilde u}^2)$, $m_2^2-m_1^2=m_{\tilde d}^2-m_{\tilde u}^2$
$SU(5)_{F'}\times U(1)_3\times SU(2)_R$	$m_2^2-m_1^2=m_{\tilde d}^2-m_{\tilde u}^2$
$SU(6)\times SU(2)_I$	$m_{\tilde q}^2-m_{\tilde d}^2=m_{\tilde u}^2-m_{\tilde l}^2$
$SU(5)_F\times U(1)_{3'}\times SU(2)_I$	$m_{\tilde q}^2-m_{\tilde d}^2=m_{\tilde u}^2-m_{\tilde l}^2$
$SU(6)\times SU(2)_J$	$m_{\tilde d}^2=m_{\tilde l}^2$, $m_{\tilde u}^2=m_{\tilde q}^2=m_{\tilde e}^2$
$SU(5)\times U(1)_{3''}\times SU(2)_J$	$m_{\tilde d}^2=m_{\tilde l}^2$, $m_{\tilde u}^2=m_{\tilde q}^2=m_{\tilde e}^2$
$SU(6)\times SU(2)_L$	$m_2^2-m_1^2=m_{\tilde d}^2-m_{\tilde u}^2=m_{\tilde l}^2-m_{\tilde q}^2$, $m_{\tilde u}^2=m_{\tilde e}^2$
$SU(5)_{F''}\times U(1)_{3'''}\times SU(2)_L$	$m_2^2-m_1^2=m_{\tilde d}^2-m_{\tilde u}^2$, $(SU(2)_R)$ or $m_{\tilde d}^2-m_{\tilde e}^2=m_{\tilde l}^2-m_{\tilde q}^2$, $(SU(2)_I)$
$SU(3)_C\times SU(3)_L\times SU(3)_R$	$m_2^2-m_1^2=m_{\tilde d}^2-m_{\tilde u}^2$, $g_{3R}^2(m_2^2-m_{\tilde e}^2)=g_{3L}^2(m_{\tilde d}^2-m_{\tilde u}^2+m_2^2-m_{\tilde l}^2)$ (*)
$SU(3)_C\times SU(3)_L\times SU(2)_R\times U(1)_R$	$m_2^2-m_1^2=m_{\tilde d}^2-m_{\tilde u}^2$
$SU(3)_C\times SU(2)_L\times U(1)_L\times SU(3)_R$	$m_2^2-m_1^2=m_{\tilde d}^2-m_{\tilde u}^2$
$SU(3)_C\times SU(2)_L\times U(1)_L\times SU(2)_R\times U(1)_R$	$m_2^2-m_1^2=m_{\tilde d}^2-m_{\tilde u}^2$
$SU(3)_C\times SU(3)_L\times SU(3)_I$	$m_2^2-m_1^2=m_{\tilde d}^2-m_{\tilde u}^2$, $g_{3I}^2(m_2^2-m_{\tilde e}^2)=g_{3L}^2(m_{\tilde d}^2-m_{\tilde u}^2+m_2^2-m_{\tilde l}^2)$ (*)
$SU(3)_C\times SU(2)_L\times U(1)_L\times SU(3)_I$	$m_2^2-m_1^2=m_{\tilde d}^2-m_{\tilde u}^2$
$SU(3)_C\times SU(3)_L\times SU(3)_J$	$m_2^2-m_1^2=m_{\tilde d}^2-m_{\tilde u}^2$, $g_{3J}^2(m_2^2-m_{\tilde e}^2)=g_{3L}^2(m_{\tilde d}^2-m_{\tilde u}^2+m_2^2-m_{\tilde l}^2)$ (*)
$SU(3)_C\times SU(2)_L\times U(1)_L\times SU(3)_J$	$m_2^2-m_1^2=m_{\tilde d}^2-m_{\tilde u}^2$

We neglect the above two effects in this paper.

Third effect is significant. There can exist additional tree level contributions to scalar masses from F-term and D-term after the heavy fields are integrated out [19]§ and the scalar mass formula is given as

$$m_a(M_{SB})^2 = m_{R(a)}{}^2 + \sum_I g_I{}^2 Q_I(\phi_a) D_I + (F-terms). \tag{27}$$

Here the $m_{R(a)}$'s represent the soft mass parameters of the scalar fields ϕ_a included in $R(a)$ representation of G_n and show a kind of 'unification' in the unified theory based on the gauge group G_n. (Note that the assumption that the soft mass parameters have a universal structure is not imposed on. It is only assumed that the $m_{R(a)}$'s respect the gauge symmetries.) The second term on the right-hand side of Eq. (27) represents the D-term contributions to scalar masses on the symmetry breaking which violate the 'unification'. The g_I's and $Q_I(\phi_a)$'s are the gauge coupling constants and the diagonal charges related to the broken gauge symmetry respectively, and the D_I's are the quantities which depend on the heavy field condensations. One can show that the sizable D-term contributions generally exist [19] when the soft SUSY breaking terms in the scalar potential are non-universal ¶ and the rank of the group is reduced due to the gauge symmetry breakings. When E_6 breaks down to G_{SM}, the rank is reduced by two and the D-term contributions are expressed by two parameters. The third term (F-*terms*) represents the contributions from F-term. We assume that they are negligible. This assumption is justified for the unified theory with a certain type of non-universal soft SUSY breaking terms when Yukawa couplings with heavy fields are negligible and there exists no heavy field with the same quantum number as usual matter fields.

In Eq.(27), the free parameters are $m_{R(a)}$'s, D_I's and M_{SB}. And if the number of independent equations is more than that of unknown parameters, the non-trivial relations among scalar masses exsist. The scalar mass relations for E_6 gauge symmetry breaking patterns have been already obtained [8]. We give the result in Table 2 again for a completeness.

Here we shall explain how they are obtained for (Ex. 1) where $G_n = SU(5)_F \times U(1)_2 \times U(1)_1$. The particle assignment and quantum numbers are shown in Table 3-2. The scalar masses satisfy

$$m_{\tilde{d}}(M_X)^2 = m_{10}{}^2 + D_1 + \left(-\frac{2}{5} g_{5F}{}^2 + \frac{1}{40} g_{1(2)}{}^2\right) \cdot D', \tag{28}$$

$$m_{\tilde{l}}(M_X)^2 = m_{\bar{5}}{}^2 + D_1 - \left(\frac{3}{10} g_{5F}{}^2 + \frac{3}{40} g_{1(2)}{}^2\right) \cdot D', \tag{29}$$

$$m_{\tilde{u}}(M_X)^2 = m_{\bar{5}}{}^2 + D_1 + \left(\frac{1}{5} g_{5F}{}^2 - \frac{3}{40} g_{1(2)}{}^2\right) \cdot D', \tag{30}$$

$$m_{\tilde{q}}(M_X)^2 = m_{10}^2 + D_1 + (\frac{1}{10}g_{5F}{}^2 + \frac{1}{40}g_{1(2)}{}^2) \cdot D', \tag{31}$$

$$m_{\tilde{e}}(M_X)^2 = m_1^2 + D_1 + \frac{1}{8}g_{1(2)}{}^2 \cdot D^1, \tag{32}$$

$$m_1(M_X)^2 = m_{\bar{5}H}{}^2 - 2D_1 - (\frac{3}{10}g_{5F}{}^2 - \frac{1}{20}g_{1(2)}{}^2) \cdot D', \tag{33}$$

$$m_2(M_X)^2 = m_{5H}{}^2 - 2D_1 - (\frac{3}{10}g_{5F}{}^2 + \frac{1}{20}g_{1(2)}{}^2) \cdot D', \tag{34}$$

where $m_{10}{}^2$ $m_{\bar{5}}^2$ and $m_1{}^2$ are the soft SUSY breaking masses for sfermion fields with the representations **10**, $\overline{\mathbf{5}}$ and **1** under $SU(5)_F$ and $m_{\bar{5}H}{}^2$ and $m_{5H}{}^2$ are those for Higgs fields with the representations $\overline{\mathbf{5}}$ and **5** under $SU(5)_F$. The following relation is derived by the elimination of $m_{10}{}^2$, $m_{\bar{5}}^2$ and D',

$$m_{\tilde{q}}(M_X)^2 - m_{\tilde{d}}(M_X)^2 = m_{\tilde{u}}(M_X)^2 - m_{\tilde{l}}(M_X)^2. \tag{35}$$

We get the same relation (35) for the second and third examples. In the same way, we can obtain specific relations among scalar masses at M_{SB} in other breaking patterns by using the particle assignments under E_6 subgroups given in Table 3-1~3-22. Therefore we can get the information on the final stage of pattern of gauge symmetry breaking by measuring the scalar masses precisely and checking the scalar mass relations.

We give two comments.

1. The same results hold for its $U(1)$ subgroup in place of $SU(2)_{R(I,J)}$.
2. We notice that the common relations appear in the wide class of E_6 breakings. This fact originates from the G_n gauge symmetry and the matter assignment. Here we explain it by taking an example. The relations $m_{\tilde{d}} = m_{\tilde{l}}$ and $m_{\tilde{u}} = m_{\tilde{q}} = m_{\tilde{e}}$ are obtained for $G_n = E_6$, $SO(10) \times U(1)_1$, $SU(5) \times U(1)_2 \times U(1)_1$, $SU(6) \times SU(2)_J$ and $SU(5) \times U(1)_{3''} \times SU(2)_J$. This is due to the fact that the above groups include $SU(5)$ as a subgroup, and $(\tilde{d}, \tilde{l})$ and $(\tilde{u}, \tilde{q}, \tilde{e})$ belong to $\overline{\mathbf{5}}$ and **10** of the $SU(5)$, respectively.

5 Additional scalar mass relations

As pointed out in the last section, we find that the same relations hold for the different chain breaking patterns. In this section, we show that the additional scalar mass relations are derived for some breaking patterns by adopting a scenario of the gauge coupling unification discussed in section 3 and they can be useful informations to select the breaking patterns further.

First we write down the RGEs for scalar masses $m_{R(a)}$ above M_{SB},

$$\mu\frac{d}{d\mu}m_{R(a)}(\mu)^2 = -\frac{2}{\pi}\sum_i C_2(R_i^a)\alpha_i(\mu)M_i(\mu)^2$$

$$+\frac{1}{2\pi}\sum_j Q^{(j)}_{R(a)}\alpha_j(\mu)S_j(\mu), \tag{36}$$

$$\mu\frac{d}{d\mu}S_j(\mu)=\frac{b_j}{2\pi}\alpha_j(\mu)S_j(\mu), \tag{37}$$

$$S_j=\sum_{R(a)} Q^{(j)}_{R(a)}n_{R(a)}m_{R(a)}{}^2, \tag{38}$$

where i runs all the gauge groups, but j runs only $U(1)$ gauge groups whose charges are $Q^{(j)}_{R(a)}$. Here we used the anomaly cancellation condition $\sum_{R(a)}C_2(R^a_i)Q^{(j)}_{R(a)}n_{R(a)}=0$ and the relation of orthgonality $\sum_{R(a)}Q^{(j)}_{R(a)}Q^{(j')}_{R(a)}n_{R(a)}=0$ for $j\neq j'$. The solutions of the above RGEs are given as

$$m_{R(a)}(\mu)^2=m_R(\mu_0)^2-\sum_i\frac{2}{b_i}C_2(R^a_i)(M_i(\mu)^2-M_i(\mu_0)^2)$$
$$+\sum_j\frac{1}{b_j}Q^{(j)}_{R(a)}(S_j(\mu)^2-S_j(\mu_0)^2), \tag{39}$$

$$S_j(\mu)=\frac{\alpha_j(\mu)}{\alpha_j(\mu_0)}S_j(\mu_0). \tag{40}$$

For the breaking pattern

$$E_6\xrightarrow{M_U}G_n\xrightarrow{M_{SB}}G_{SM}, \tag{41}$$

the mass formula at M_{SB} is

$$m_{R(a)}(M_{SB})^2=m_{27}{}^2-\sum_i\frac{2}{b_i}C_2(R^a_i)(M_i(M_{SB})^2-M_{GUT}{}^2)$$
$$+\sum_j\frac{1}{b_j}Q^{(j)}_{R(a)}S_j(M_{SB})^2 \tag{42}$$

where we use the relation $m_R(M_U)^2=m_{27}{}^2$, $M_i(M_U)^2=M_{GUT}{}^2$ and $S_j(M_U)=m_{27}{}^2\sum_{R(a)}Q^{(j)}_{R(a)}n_{R(a)}=0$. Here we impose the condition that the decouplings of particles occur keeping the relation $\sum_{R(a)}Q^{(j)}_{R(a)}n_{R(a)}=0$ ‖. Then we have the further relation $S_j(M_{SB})=0$.

Now we take the breaking pattern (Ex. 1) as an example and derive the scalar mass relations. The values of second-order Casimir operator are given as $C_2(R^{10}_{5F})=\frac{18}{5}$, $C_2(R^{\bar{5}}_{5F})=\frac{12}{5}$ and $C_2(R^1_{5F})=0$ for $SU(5)_F$. And those are $C_2(R^{10}_{1(2)})=\frac{1}{40}$, $C_2(R^{\bar{5}}_{1(2)})=\frac{9}{40}$, $C_2(R^1_{1(2)})=\frac{5}{8}$ for $U(1)_2$ and $C_2(R^{10}_{1(1)})=\frac{1}{24}$, $C_2(R^{\bar{5}}_{1(1)})=\frac{1}{24}$, $C_2(R^1_{1(1)})=\frac{1}{24}$ for $U(1)_1$. We consider the model with $b_{5F}=b_{1(2)}$. This model has the relations $\alpha_{5F}(\mu)=\alpha_{1(2)}(\mu)$ and $M_{5F}(\mu)=M_{1(2)}(\mu)$.

The solutions of scalar masses are as follows,

‖This relation agrees with the gravitational anomaly cancellation condition.

$$m_{10F}(\mu)^2 = m_{27}{}^2 + (C_2(R^{10}_{5F}) + C_2(R^{10}_{1(2)}))\widehat{M}^2_{5F} + C_2(R^{10}_{1(1)})\widehat{M}^2_{1(1)}, \tag{43}$$

$$m_{\bar{5}F}(\mu)^2 = m_{27}{}^2 + (C_2(R^{\bar{5}}_{5F} + C_2(R^{\bar{5}}_{1(2)}))\widehat{M}^2_{5F} + C_2(R^{\bar{5}}_{1(1)})\widehat{M}^2_{1(1)} \tag{44}$$

and

$$m_{1F}(\mu)^2 = m_{27}{}^2 + (C_2(R^{1}_{5F}) + C_2(R^{1}_{1(2)}))\widehat{M}^2_{5F} + C_2(R^{1}_{1(1)})\widehat{M}^2_{1(1)} \tag{45}$$

where $\widehat{M}_i(\mu)^2 \equiv \frac{2}{b_i}(M_{GUT}{}^2 - M_i(\mu)^2)$.

By the use of the above solutions, the scalar masses at M_X are given as

$$m_{\tilde{d}}(M_X)^2 = \hat{m}^2_{27} + \frac{29}{8}\widehat{M}^2_{5F} - 3D_F, \tag{46}$$

$$m_{\tilde{l}}(M_X)^2 = \hat{m}^2_{27} + \frac{21}{8}\widehat{M}^2_{5F} - 3D_F, \tag{47}$$

$$m_{\tilde{u}}(M_X)^2 = \hat{m}^2_{27} + \frac{21}{8}\widehat{M}^2_{5F} + D_F, \tag{48}$$

$$m_{\tilde{q}}(M_X)^2 = \hat{m}^2_{27} + \frac{29}{8}\widehat{M}^2_{5F} + D_F, \tag{49}$$

$$m_{\tilde{e}}(M_X)^2 = \hat{m}^2_{27} + \frac{5}{8}\widehat{M}^2_{5F} + D_F, \tag{50}$$

where $\hat{m}^2_{27} \equiv m_{27}{}^2 + D_1 + \frac{1}{24}\widehat{M}^2_{1(1)}$ and $D_F \equiv \frac{1}{8}g_{5F}{}^2 \cdot D'$. The number of unknown parameters is three, i. e. $(\hat{m}^2_{27}, \widehat{M}^2_{5F}, D_F)$, and the number of independent equations, i. e. (46)~(50), is five, so there must exist two relations. In fact, we get the following relations,

$$m_{\tilde{q}}(M_X)^2 - m_{\tilde{d}}(M_X)^2 = m_{\tilde{u}}(M_X)^2 - m_{\tilde{l}}(M_X)^2 \tag{51}$$

and

$$2(m_{\tilde{d}}(M_X)^2 - m_{\tilde{l}}(M_X)^2) = m_{\tilde{u}}(M_X)^2 - m_{\tilde{e}}(M_X)^2. \tag{52}$$

The Eq. (52) is a new relation. Note that only the relation $b_{5F} = b_{1(2)}$ is used in the derivation of the above equations and we do not have to know the particle contents above M_X.

For (Ex. 2) we obtain the relation,

$$m_{\tilde{u}}(M_X)^2 = m_{\tilde{e}}(M_X)^2 \tag{53}$$

by adopting the unification scenario discussed in section 3.

When the same method is applied for (Ex.3), no additional relation is derived.

So we can discriminate among three examples by checking the relation (52) and (53). The same method is applied for other breaking patterns and some new type of relations can be derived [17].

6 Conclusions

We have studied how the gauge couplings and soft mass spectrum are useful to probe the physics at higher energy scales within supersymmetric E_6 grand unified theory. We have given a scenario of gauge coupling unification based on three types of E_6 chain breakings (Ex. 1), (Ex. 2) and (Ex. 3). It has been pointed out that there exist many gauge coupling unification scenarios based on SUSY E_6 GUT consistent with the LEP data. Since the 'GUT relation' holds for all chain breakings in the grand unified theories, the gaugino masses give no information of gauge symmetry breaking pattern. On the other hand, the scalar masses can give a useful information. We have obtained the scalar mass relations specific to the E_6 breaking patterns. It is important that the specific relations hold without specifying the particle content above the symmetry breaking scale from the group theoretical reason. We can select the final stage of some chain breakings by checking scalar mass relations. But it is not easy to carry out the complete selection of gauge symmetry breakings since the same relations hold in the wide class of SUSY E_6 GUTs. The other powerful information is needed to specify the pattern of symmetry breakings further. Additional relations derived by the consideration of the physics beyond M_{SB} can be the candidate. In fact, we have derived the new type of relations specific to the breaking pattern $E_6 \to SU(5)_F \times U(1)_2 \times U(1)_1 \to G_{SM}$ and $E_6 \to SU(6) \times SU(2)_I \to G_{SM}$. It is important that they are derived not by specifying the particle content above M_X but based on a scenario of the gauge coupling unification.

In conclusion, the measurements of the scalar masses will give a big impact on high energy physics in the future because it is expected that scalar mass spectrum owns a useful information on the pattern of gauge symmetry breaking in SUSY-GUTs.

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Table 3-1～3-22 The particle assignments and quantum numbers under E_6 subgroups

We refer to the chiral multiplets as q for left-handed quark, l left-handed lepton, u right-handed up type quark, d right-handed down type quark, e right-handed charged lepton and ν right-handed neutrino. The 'exotics' are denoted as D, D^c, L, L^c and N^c whose quantum numbers under G_{SM} can be read through the Tables. The superscript c represents their charge conjugated states. We take the following normalization for the $U(1)_i$ charges Q_i,

$$\sum_{27} Q_i^2 = 3$$

and for the $U(1)_Y$ charge Y,

$$\sum_{27} Y^2 = 5\,.$$

The assignment that d^c and l lie in **16** of $SO(10)$ is case (a) and assignment that they lie **10** of $SO(10)$ is case (b) in Table 3-1 and 3-2.

Table 3-1 (1-1) (a) ((b))

E_6	$SO(10)$	$SU(5)$	$SU(3)_C$	$SU(2)_L$	$U(1)_h$ $2\sqrt{15}Q_h$	$U(1)_2$ $2\sqrt{10}Q_2$	$U(1)_1$ $2\sqrt{6}Q_1$	*Species*
			$\overline{3}$	1	2			$d^c(D^c)$
		$\overline{5}$	1	2	−3	3		$l(L)$
			3	2	1			q
	16	10	$\overline{3}$	1	−4	−1	1	u^c
			1	1	6			e^c
27		1	1	1	0	−5		$\nu^c(N^c)$
			3	1	−2			D
		5	1	2	3	2		L^c
	10		$\overline{3}$	1	2		−2	$D^c(d^c)$
		$\overline{5}$	1	2	−3	−2		$L(l)$
	1	1	1	1	0	0	4	$N^c(\nu^c)$

$$Y \equiv \sqrt{\frac{5}{3}}Q_h$$

Table 3-2 $(1\text{-}1)_F$ (a) ((b))

E_6	$SO(10)$	$SU(5)_F$	$SU(3)_C$	$SU(2)_L$	$U(1)_F$ $2\sqrt{15}Q_F$	$U(1)_2$ $2\sqrt{10}Q_2$	$U(1)_1$ $2\sqrt{6}Q_1$	$U(1)_Y$ $6Y$	*Species*
27	16	$\overline{5}$	$\overline{3}$	1	2	3	1	−4	u^c
			1	2	−3			−3	$l(L)$
		10	3	2	1	−1		1	q
			$\overline{3}$	1	−4			2	$d^c(D^c)$
			1	1	6			0	$\nu^c(N^c)$
		1	1	1	0	−5		6	e^c
	10	5	3	1	−2	2	−2	−2	D
			1	2	3			3	L^c
		$\overline{5}$	$\overline{3}$	1	2	−2		2	$D^c(d^c)$
			1	2	−3			−3	$L(l)$
	1	1	1	1	0	0	4	0	$N^c(\nu^c)$

$$Y \equiv -\frac{1}{\sqrt{15}}Q_F - \frac{4}{\sqrt{10}}Q_2$$

Table 3-3 $(1\text{-}2)_R$

E_6	$SO(10)$	$SU(4)$	$SU(3)_C$	$SU(2)_L$	$SU(2)_R$	$U(1)_{B-L}$ $\frac{2}{3}\sqrt{6}Q_{B-L}$	$U(1)_1$ $2\sqrt{6}Q_1$	$U(1)_Y$ $6Y$	*Species*
27	16	4	3	2	1	$-\frac{1}{3}$	1	1	q
			1			1		−3	l
		$\overline{4}$	$\overline{3}$	1	2	$\frac{1}{3}$		2	d^c
								−4	u^c
			1			−1		6	e^c
								0	ν^c
	10	1	1	2	2	0	−2	3	L^c
								−3	L
		6	3	1	1	$\frac{2}{3}$		−2	D
			$\overline{3}$			$-\frac{2}{3}$		2	D^c
	1	1	1	1	1	0	4	0	N^c

$$Y \equiv -\frac{\sqrt{6}}{3}Q_{B-L} + T^3_{2R}$$

Table 3-4 $(1\text{-}2)_I$

E_6	$SO(10)$	$SU(4)$	$SU(3)_C$	$SU(2)_L$	$SU(2)_I$	$U(1)_{B-L'}$ $\frac{2}{3}\sqrt{6}Q_{B-L'}$	$U(1)_1$ $2\sqrt{6}Q_1$	$U(1)_Y$ $6Y$	Species
27	16	4	3	2	1	$-\frac{1}{3}$	1	1	q
			1			1		-3	L
		$\overline{4}$	$\overline{3}$	1	2	$\frac{1}{3}$		2	D^c
								-4	u^c
			1			-1		6	e^c
								0	N^c
	10	1	1	2	2	0	-2	3	L^c
								-3	l
		6	3	1	1	$\frac{2}{3}$		-2	D
			$\overline{3}$			$-\frac{2}{3}$		2	d^c
	1	1	1	1	1	0	4	0	ν^c

$$Y \equiv -\frac{\sqrt{6}}{3}Q_{B-L'} + T_{2I}^3$$

Table 3-5 $(1\text{-}2)_J$

E_6	$SO(10)$	$SU(4)$	$SU(3)_C$	$SU(2)_L$	$SU(2)_J$	$U(1)_{B-L''}$ $\frac{2}{3}\sqrt{6}Q_{B-L''}$	$U(1)_1$ $2\sqrt{6}Q_1$	$U(1)_Y$ $6Y$	Species
27	16	4	3	2	1	$-\frac{1}{3}$	1	1	q
			1			1		3	L^c
		$\overline{4}$	$\overline{3}$	1	2	$\frac{1}{3}$		2	(d^c, D^c)
			1			-1		0	(ν^c, N^c)
	10	1	1	2	2	0	-2	-3	(l, L)
		6	3	1	1	$\frac{2}{3}$		-2	D
			$\overline{3}$			$-\frac{2}{3}$		-4	u^c
	1	1	1	1	1	0	4	6	e^c

$$Y \equiv \frac{1}{\sqrt{6}}Q_{B-L''} + \frac{\sqrt{6}}{2}Q_1$$

Table 3-6 $(2\text{-}1)_R$

E_6	$SU(6)$	$SU(5)_{F'}$	$SU(3)_C$	$SU(2)_L$	$U(1)_{F'}$ $2\sqrt{15}Q_{F'}$	$U(1)_3$ $2\sqrt{15}Q_3$	$SU(2)_R$	$U(1)_Y$ $6Y$	*Species*
27	15	5	3	1	-2	-4	1	-2	D
			1	2	3			-3	l
		10	3	2	1	2		1	q
			$\overline{3}$	1	-4			2	D^c
			1	1	6			0	N^c
	$\overline{6}$	$\overline{5}$	$\overline{3}$	1	2	-1	2	2	d^c
								-4	u^c
			1	2	-3			3	L^c
								-3	L
		1	1	1	0	5		6	e^c
								0	ν^c

$$Y \equiv -\frac{1}{\sqrt{15}}Q_{F'} + \frac{3}{\sqrt{15}}Q_3 + T^3_{2R}$$

Table 3-7 $(2\text{-}2)_R$

E_6	$SU(6)$	$SU(4)$	$SU(3)_C$	$U(1)_{B-L}$ $\frac{2}{3}\sqrt{6}Q_{B-L}$	$SU(2)_L$	$U(1)_1$ $2\sqrt{6}Q_1$	$SU(2)_R$	$U(1)_Y$ $6Y$	*Species*
27	15	6	3	$\frac{2}{3}$	1	-2	1	-2	D
			$\overline{3}$	$-\frac{2}{3}$				2	D^c
		4	3	$-\frac{1}{3}$	2	1		1	q
			1	1				-3	l
		1	1	0	1	4		0	N^c
	$\overline{6}$	$\overline{4}$	$\overline{3}$	$\frac{1}{3}$	1	1	2	2	d^c
								-4	u^c
			1	-1				6	e^c
								0	ν^c
		1	1	0	2	-2		3	L^c
								-3	L

$$Y \equiv -\frac{\sqrt{6}}{3}Q_{B-L} + T^3_{2R}$$

Table 3-8 $(2\text{-}3)_R$

E_6	$SU(6)$	$SU(3)_C$	$SU(3)_L$	$SU(2)_L$	$U(1)_L$ $2\sqrt{3}Q_L$	$U(1)_R$ $2\sqrt{3}Q_R$	$SU(2)_R$	$U(1)_Y$ $6Y$	*Species*
27	15	$\overline{3}$	1	1	0	2	1	2	D^c
		1	$\overline{3}$	2	-1	-2		-3	l
				1	2			0	N^c
		3	3	2	1	0		1	q
				1	-2			-2	D
	$\overline{6}$	$\overline{3}$	1	1	0	-1	2	2 -4	d^c u^c
		1	$\overline{3}$	2	-1	1		3 -3	L^c L
				1	2			6 0	e^c ν^c

$$Y \equiv \frac{1}{\sqrt{3}}Q_L + \frac{1}{\sqrt{3}}Q_R + T^3_{2R}$$

Table 3-9 $(2\text{-}1)_I$

E_6	$SU(6)$	$SU(5)_F$	$SU(3)_C$	$SU(2)_L$	$U(1)_F$ $2\sqrt{15}Q_F$	$U(1)_{3'}$ $2\sqrt{15}Q_{3'}$	$SU(2)_I$	$U(1)_Y$ $6Y$	*Species*
27	15	5	3	1	-2	-4	1	-2	D
			1	2	3			-3	L
		10	3	2	1	2		1	q
			$\overline{3}$	1	-4			2	d^c
			1	1	6			0	ν^c
	$\overline{6}$	$\overline{5}$	$\overline{3}$	1	2	-1	2	2 -4	D^c u^c
			1	2	-3			3 -3	L^c l
		1	1	1	0	5		6 0	e^c N^c

$$Y \equiv -\frac{1}{\sqrt{15}}Q_F + \frac{3}{\sqrt{15}}Q_{3'} + T^3_{2I}$$

Table 3-10 $(2\text{-}2)_I$

E_6	$SU(6)$	$SU(4)$	$SU(3)_C$	$U(1)_{B-L'}$ $\frac{2}{3}\sqrt{6}Q_{1B-L'}$	$SU(2)_L$	$U(1)_1$ $2\sqrt{6}Q_1$	$SU(2)_I$	$U(1)_Y$ $6Y$	*Species*
			3	$\frac{2}{3}$				-2	D
		6	$\overline{3}$	$-\frac{2}{3}$	1	-2		2	d^c
	15		3	$-\frac{1}{3}$			1	1	q
		4	1	1	2	1		-3	L
		1	1	0	1	4		0	ν^c
27								2	D^c
			$\overline{3}$	$\frac{1}{3}$				-4	u^c
		$\overline{4}$			1	1		6	e^c
	$\overline{6}$		1	-1			2	0	N^c
								3	L^c
		1	1	0	2	-2		-3	l

$$Y \equiv -\frac{\sqrt{6}}{3}Q_{B-L'} + T^3_{2I}$$

Table 3-11 $(2\text{-}3)_I$

E_6	$SU(6)$	$SU(3)_C$	$SU(3)_L$	$SU(2)_L$	$U(1)_L$ $2\sqrt{3}Q_L$	$U(1)_I$ $2\sqrt{3}Q_I$	$SU(2)_I$	$U(1)_Y$ $6Y$	*Species*
		$\overline{3}$	1	1	0	2		2	d^c
				2	-1			-3	L
	15	1	$\overline{3}$	1	2	-2	1	0	ν^c
				2	1			1	q
		3	3	1	-2	0		-2	D
27								2	D^c
		$\overline{3}$	1	1	0	-1		-4	u^c
								3	L^c
	$\overline{6}$			2	-1		2	-3	l
		1	$\overline{3}$			1		6	e^c
				1	2			0	N^c

$$Y \equiv \frac{1}{\sqrt{3}}Q_L + \frac{1}{\sqrt{3}}Q_I + T^3_{2I}$$

Table 3-12 $(2\text{-}1)_J$

E_6	$SU(6)$	$SU(5)$	$SU(3)_C$	$SU(2)_L$	$U(1)_h$ $2\sqrt{15}Q_h$	$U(1)_{3''}$ $2\sqrt{15}Q_{3''}$	$SU(2)_J$	Species
27	15	5	3	1	-2	-4	1	D
			1	2	3			L^c
		10	3	2	1	2		q
			$\overline{3}$	1	-4			u^c
			1	1	6			e^c
	$\overline{6}$	$\overline{5}$	$\overline{3}$	1	2	-1	2	(d^c, D^c)
			1	2	-3			(l, L)
		1	1	1	0	5		(ν^c, N^c)

$$Y \equiv \sqrt{\frac{5}{3}}Q_h$$

Table 3-13 $(2\text{-}2)_J$

E_6	$SU(6)$	$SU(4)$	$SU(3)_C$	$U(1)_{B-L''}$ $\frac{2}{3}\sqrt{6}Q_{B-L''}$	$SU(2)_L$	$U(1)_1$ $2\sqrt{6}Q_1$	$SU(2)_J$	$U(1)_Y$ $6Y$	Species
27	15	6	3	$\frac{2}{3}$	1	-2	1	-2	D
			$\overline{3}$	$-\frac{2}{3}$				-4	u^c
		4	3	$-\frac{1}{3}$	2	1		1	q
			1	1				3	L^c
		1	1	0	1	4		6	e^c
	$\overline{6}$	$\overline{4}$	$\overline{3}$	$\frac{1}{3}$	1	1	2	2	(d^c, D^c)
			1	-1				0	(ν^c, N^c)
		1	1	0	2	-2		-3	(l, L)

$$Y \equiv \frac{1}{\sqrt{6}}Q_{B-L''} + \frac{3}{\sqrt{6}}Q_1$$

Table 3-14 $(2\text{-}3)_J$

E_6	$SU(6)$	$SU(3)_C$	$SU(3)_L$	$SU(2)_L$	$U(1)_L$ $2\sqrt{3}Q_L$	$U(1)_J$ $2\sqrt{3}Q_J$	$SU(2)_J$	$U(1)_Y$ $6Y$	Species
27	15	$\overline{3}$	1	1	0	2	1	−4	u^c
		1	$\overline{3}$	2	−1	−2		3	L^c
				1	2			6	e^c
		3	3	2	1			1	q
				1	−2	0		−2	D
	$\overline{6}$	$\overline{3}$	1	1	0	−1	2	2	(d, D^c)
		1	$\overline{3}$	2	−1	1		−3	(l, L)
				1	2			0	(ν^c, N^c)

$$Y \equiv \frac{1}{\sqrt{3}}Q_L - \frac{2}{\sqrt{3}}Q_J$$

Table 3-15 $(2\text{-}1)_L$

E_6	$SU(6)$	$SU(5)_{F''}$	$SU(3)_C$	$SU(2)_{R(I)}$	$U(1)_h$ $2\sqrt{15}Q_h$	$U(1)_{3'''}$ $2\sqrt{15}Q_{3'''}$	$SU(2)_L$	$U(1)_Y$ $6Y$	Species
$\overline{27}$	$\overline{15}$	$\overline{5}$	$\overline{3}$	1	2	4	1	2	$D^c(d^c)$
			1	2	−3			6 0	e^c $\nu^c(N^c)$
		$\overline{10}$	$\overline{3}$	2	−1	−2		2 −4	$d^c(D^c)$ u^c
			3	1	4			−2	D
			1	1	−6			0	$N^c(\nu^c)$
	6	5	3	1	−2	1	2	1	q
			1	2	3			3 −3	L^c $L(l)$
		1	1	1	0	−5		−3	$l(L)$

$$Y \equiv -\frac{1}{\sqrt{15}}Q_h + \frac{3}{\sqrt{15}}Q_{3'''} + T^3_{2R(I)}$$

Table 3-16 $(2\text{-}2)_L$

E_6	$SU(6)$	$SU(4)$	$SU(3)_C$	$U(1)_{B-L}$ $\frac{2}{3}\sqrt{6}Q_{B-L}$	$SU(2)_R$	$U(1)_1$ $2\sqrt{6}Q_1$	$SU(2)_L$	$U(1)_Y$ $6Y$	Species
$\overline{27}$	$\overline{15}$	$\overline{6}$	3	$\frac{2}{3}$	1	2	1	-2	D
			$\overline{3}$	$-\frac{2}{3}$				2	D^c
		$\overline{4}$	$\overline{3}$	$\frac{1}{3}$	2	-1		2	d^c
								-4	u^c
			1	-1				6	e^c
								0	ν^c
		1	1	0	1	-4		0	N^c
	6	4	3	$-\frac{1}{3}$	1	-1	2	1	q
			1	1				-3	l
		1	1	0	2	2		3	L^c
								-3	L

$$Y \equiv -\frac{\sqrt{6}}{3} Q_{B-L} + T^3_{2R}$$

Table 3-17 $(2\text{-}3)_L$

E_6	$SU(6)$	$SU(4)$	$SU(3)_C$	$U(1)_{B-L'}$ $\frac{2}{3}\sqrt{6}Q_{B-L'}$	$SU(2)_I$	$U(1)_1$ $2\sqrt{6}Q_1$	$SU(2)_L$	$U(1)_Y$ $6Y$	Species
$\overline{27}$	$\overline{15}$	$\overline{6}$	3	$\frac{2}{3}$	1	2	1	-2	D
			$\overline{3}$	$-\frac{2}{3}$				2	d^c
		$\overline{4}$	$\overline{3}$	$\frac{1}{3}$	2	-1		2	D^c
								-4	u^c
			1	-1				6	e^c
								0	N^c
		1	1	0	1	-4		0	ν^c
	6	4	3	$-\frac{1}{3}$	1	-1	2	1	q
			1	1				-3	L
		1	1	0	2	2		3	L^c
								-3	l

$$Y \equiv -\frac{\sqrt{6}}{3} Q_{B-L'} + T^3_{2I}$$

Table 3-18 $(2\text{-}4)_L$

E_6	$SU(6)$	$SU(3)_C$	$SU(3)_R$	$SU(2)_R$	$U(1)_R$ $2\sqrt{3}Q_R$	$U(1)_L$ $2\sqrt{3}Q_L$	$SU(2)_L$	$U(1)_Y$ $6Y$	*Species*
$\overline{27}$	$\overline{15}$	3	1	1	0	-2	1	-2	D
		1	3	2	1	2		6 0	e^c ν^c
				1	-2			0	N^c
		$\overline{3}$	$\overline{3}$	2	-1	0		2 -4	d^c u^c
				1	2			2	D^c
	6	3	1	1	0	1	2	1	q
		1	3	2	1	-1		3 -3	L^c L
				1	-2			-3	l

$$Y \equiv \frac{1}{\sqrt{3}}Q_L + \frac{1}{\sqrt{3}}Q_R + T^3_{2R}$$

Table 3-19 $(2\text{-}5)_L$

E_6	$SU(6)$	$SU(3)_C$	$SU(3)_I$	$SU(2)_I$	$U(1)_I$ $2\sqrt{3}Q_I$	$U(1)_L$ $2\sqrt{3}Q_L$	$SU(2)_L$	$U(1)_Y$ $6Y$	*Species*
$\overline{27}$	$\overline{15}$	3	1	1	0	-2	1	-2	D
		1	3	2	1	2		6 0	e^c N^c
				1	-2			0	ν^c
		$\overline{3}$	$\overline{3}$	2	-1	0		2 -4	D^c u^c
				1	2			2	d^c
	6	3	1	1	0	1	2	1	q
		1	3	2	1	-1		3 -3	L^c l
				1	-2			-3	L

$$Y \equiv \frac{1}{\sqrt{3}}Q_L + \frac{1}{\sqrt{3}}Q_I + T^3_{2I}$$

Table 3-20 $(3)_R$

E_6	$SU(3)_C$	$SU(3)_L$	$SU(3)_R$	$SU(2)_L$	$U(1)_L$ $2\sqrt{3}Q_L$	$SU(2)_R$	$U(1)_R$ $2\sqrt{3}Q_R$	$U(1)_Y$ $6Y$	Species
27	3	3	1	2	1	1	0	1	q
				1	-2			-2	D
	$\overline{3}$	1	$\overline{3}$	1	0	2	-1	2 -4	d^c u^c
						1	2	2	D^c
	1	$\overline{3}$	3	2	-1	2	1	3 -3	L^c L
						1	-2	-3	l
				1	2	2	1	6 0	e^c ν^c
						1	-2	0	N^c

$$Y \equiv \frac{1}{\sqrt{3}}Q_L + \frac{1}{\sqrt{3}}Q_R + T^3_{2R}$$

Table 3-21 $(3)_I$

E_6	$SU(3)_C$	$SU(3)_L$	$SU(3)_I$	$SU(2)_L$	$U(1)_L$ $2\sqrt{3}Q_L$	$SU(2)_I$	$U(1)_I$ $2\sqrt{3}Q_I$	$U(1)_Y$ $6Y$	Species
27	3	3	1	2	1	1	0	1	q
				1	-2			-2	D
	$\overline{3}$	1	$\overline{3}$	1	0	2	-1	2 -4	D^c u^c
						1	2	2	d^c
	1	$\overline{3}$	3	2	-1	2	1	3 -3	L^c l
						1	-2	-3	L
				1	2	2	1	6 0	e^c N^c
						1	-2	0	ν^c

$$Y \equiv \frac{1}{\sqrt{3}}Q_L + \frac{1}{\sqrt{3}}Q_I + T^3_{2I}$$

Table 3-22 $(3)_J$

E_6	$SU(3)_C$	$SU(3)_L$	$SU(3)_J$	$SU(2)_L$	$U(1)_L$ $2\sqrt{3}Q_L$	$SU(2)_J$	$U(1)_J$ $2\sqrt{3}Q_J$	$U(1)_Y$ $6Y$	Species
27	3	3	1	2	1	1	0	1	q
				1	-2			-2	D
	$\overline{3}$	1	$\overline{3}$	1	0	2	-1	2	(d^c, D^c)
						1	2	-4	u^c
	1	$\overline{3}$	3	2	-1	2	1	-3	(l, L)
						1	-2	3	L^c
				1	2	2	1	0	(ν^c, N^c)
						1	-2	6	e^c

$$Y \equiv \frac{1}{\sqrt{3}}Q_L - \frac{2}{\sqrt{3}}Q_J$$