

A Note on Right Locally Finite Simple Ring Extensions

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Throughout A will represent an (Artinian) simple ring, B a simple subring of A containing 1 of A , and V the centralizer of B in A . A ring extension A/B' is said to be right locally finite if for any finite subset F' of A' the subring $B'[F']$ is right finite over B' . In [1], S. Takamatsu and the second author dealt with a right locally finite extension A/B such that V is simple and $A = BN$ with the normalizer N of B in A , and proved that A/BV is right locally finite, which played an important role in the proof of [1, Theorem]. In this note, we shall prove the same without any restriction.

Theorem. If A/B is right locally finite, then so is A/BV .

Proof. Let F be an arbitrary finite subset of A , and choose an intermediate ring B' of $A/B[F]$ such that ${}_A A_{B'}$ is irreducible and the right rank $[B' : B]_R$ is finite. Then by [2, Proposition 5.4 (b)] the centralizer V' of B' in A is a division ring and $m = [V : V']_R \leq [B' : B]_R$. Let $\{v_1, v_2, \dots, v_m\}$ be a right V' -basis of V and set $B'' = B[F, v_1, \dots, v_m] = \sum_{j=1}^m b''_j B$. Since every element of V' commutes with all the elements of $B[F]$, we see that $B''V' \supset V'B''$, namely, $B''V'$ is a subring of A . Hence, $(BV)[F] = B''V' = \sum_{j=1}^m b''_j (BV)$, which proves the right local finiteness of A/BV .

References

- [1] S. TAKAMATSU and H. TOMINAGA : On normalizers of simple ring extensions, Proc. Japan Acad., 49 (1973), 678-680.
- [2] H. TOMINAGA and T. NAGAHARA : Galois Theory of Simple Rings, Okayama Math. Lectures, 1970.