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A Proof of Cauchy's Integral Theorem

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In this note, we derive Cauchy (-Goursat)'s Integral theorem from Stokes' theorem of Alexander -Spanier cochain ([1], theorem 4).

Let f be a complex valued complex variable function, then we know that to set

$$F_0(z_0,\ z_1)=f(z_0)(z_0-z_1),\quad F_1(z_0, z_1)=f(z_1)(z_0-z_1),$$

we get

(1) $$\int_\gamma F_0=\int_\gamma F_1=\int_\gamma f(z)dz,$$

if γ is a Lipschitz continuous curve and f is Riemannian integrable on γ ([1], § 2). On the other hand, by Stokes' theorem, we have

(2) $$\int_{\partial D} F_i=\int_D \delta F_i,\quad i=0,\ 1.$$

In this right hand side, if $f'(z)$ exists for any $z\in\overline{D}$, then we obtain

$$\begin{aligned}\delta F_0(z_0,\ z_1,\ z_2)&=\{f(z_1)-f(z_0)\}(z_1-z_2)\\&=f'(z_0)(z_1-z_0)(z_1-z_2)+o(|z_1-z_0|)(z_1-z_2).\\\delta F_1(z_0,\ z_1,\ z_2)&=-f'(z_1)(z_1-z_0)(z_1-z_2)+o(|z_1-z_2|)(z_1-z_0).\end{aligned}$$

But, since $f'(z)$ is continuous on the set of the second category by Baire's theorem and $|f'(z)|$ is bounded on $\overline{D}$ by lemma 2 of [2] in this case, we have

(3) $$\int_D \delta F_i=0,\quad i=0,\ 1.$$

By (1), (2) and (3), we obtain Cauchy's integral theorem (without the assumption about the continuity of $f'(z)$).

References

[1] ASADA, A.: Integration of Alexander -Spanier cochains, J. of Fac. Sci. Shinshu Univ., 5 (1970), 79–106.

[2] ASADA, A.: Generalized integral curves of generalized vector fields, J. Fac. Sci. Shinshu Univ., 7 (1972), 59–118.