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LETTER

A Simple Expression of BER Performance in COFDM Systems over Fading Channels*

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SUMMARY Both adaptive modulation and diversity combining are attractive techniques to combat fading and these two can be applicable to each digital-modulated symbol in OFDM transmission. In this letter, aiming to combat severe fading more effectively than the adaptive modulation, we theoretically analyze the benefit of a frequency diversity scheme within one OFDM symbol, which is a simple kind of coded OFDM (COFDM) based on IEEE 802.16 protocols. A simple closed form equation of bit error rate (BER) is derived, and then the advantages of correlated diversity gain and interference suppression by the diversity scheme are verified by both theoretical analysis and Monte Carlo simulation.
key words: *COFDM, diversity combining, fading correlation, eigenvector conversion, closed form equation*

1. Introduction

Orthogonal frequency division multiplexing (OFDM), which has actively been researched, is one of the high-rate wireless data transmission systems and is applied to wireless LAN, terrestrial mobile communication, digital terrestrial TV broadcasting and so on [1], [2]. OFDM has the advantages of high spectrum efficiency and resistance to frequency-selective fading.

Adaptive modulation, in which a transmitter can adapt constellation size depending on the channel condition, and diversity combining, in which a transmitter and/or a receiver can adapt branch size, are both transmission techniques to combat fading. These two techniques are applicable to each digital-modulated symbol in OFDM transmission. The former is attractive in improving throughput performance [3], [4], whereas the latter aims to combat severe fading more effectively than the former.

In this letter, we theoretically analyze the benefit of a frequency diversity scheme within one OFDM symbol, which is a simple kind of coded OFDM (COFDM) based on IEEE 802.16 protocols [5], [6]. For example, changing QPSK into BPSK with constant carrier power and constant symbol rate in the adaptive modulation improves bit error rate (BER) owing to double bit energy, while the information bit rate is reduced by half. In the COFDM systems, that is, the frequency diversity scheme with a repetition code, all subcarriers of the OFDM symbol are divided into two regions (low and high subcarrier frequencies), and the same digital-modulated symbol is transmitted twice with half the channel frequency spacing. The information bit rate is also reduced by half, but by combining the two QPSK symbols at a receiver, the BER is much improved owing to both double bit energy and diversity gain (but not full diversity gain). Performance analysis of COFDM systems over frequency-selective quasi-static fading channels has been proposed on the assumption of indoor wireless environment [7]. But taking account of time-selectivity, that is, mobile wireless environment, the BER is much degraded owing to inter-carrier interference (ICI) and interference to quadrature channel which are caused by Doppler frequency [8]. The diversity scheme can also suppress the interferences, and as a result, the BER degradation can be reduced.

In the proposed analysis, assuming DQPSK/COFDM systems for the purpose of robustness against severe fading, a simple closed form equation of the BER is derived. The advantages of the correlated diversity gain and the interferences suppression are verified by both theoretical analysis and Monte Carlo simulation. Moreover, as an application of the COFDM systems, we evaluate the BER performances of coherent PSK and QAM by the simulation.

2. System Model

Figure 1 shows a block diagram of the COFDM systems in an equivalent low-pass system, where N is the number of subcarriers, M is the division number of all subcarriers and L $(= N/M)$ is the subcarrier spacing. At a transmitter, a binary data sequence is converted to digital-modulated symbols, which are parallelized to L symbols $S_{i,j}$ $(j = 1, 2, \cdots, L)$ by serial-to-parallel (S/P) conversion. Each symbol $S_{i,j}$ is replicated to M subcarriers with L subcarriers spacing, and the following are carried out: OFDM modulation (IFFT), parallel-to-serial (P/S) conversion and addition of a guard interval (GI). T_s is OFDM symbol duration including GI duration T_g and data symbol duration T_d, and α $(= T_g/T_d)$ is called 'GI factor' hereafter. OFDM symbols are subjected to both time- and frequency-selective Rayleigh fading, as well as are added white Gaussian noise (AWGN). The fading channel is simulated using the Jakes' model [9]. It is assumed that inter-symbol interference (ISI) caused by delay spread can be completely avoided with the GI, which is longer than the maximum multipath

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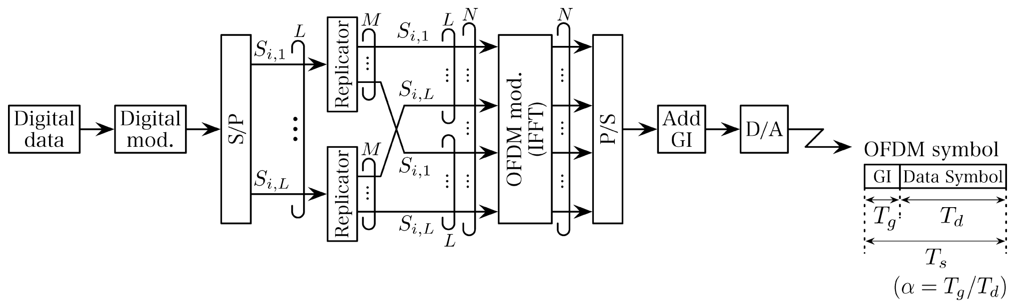
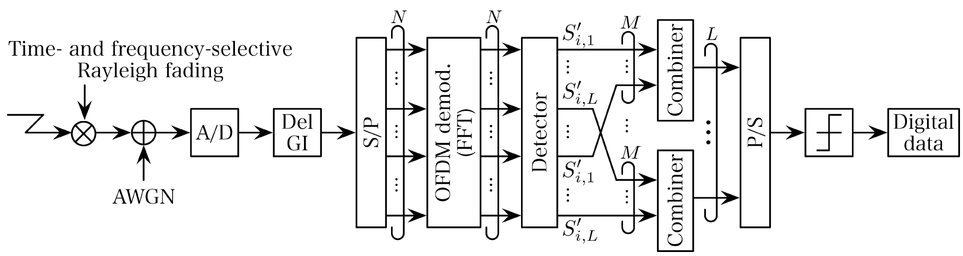


(a) Transmitter



(b) Receiver

Fig. 1 Block diagram of COFDM systems.

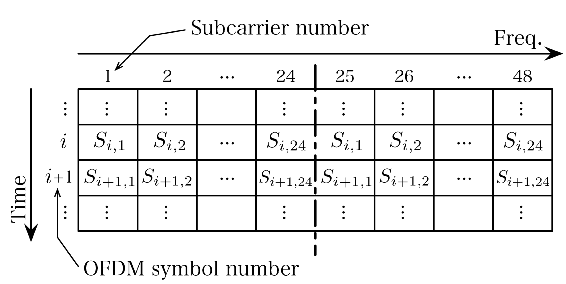


Fig. 2 Concept of diversity scheme (N = 48, M = 2).

delay. A receiver is constructed by reversing the process of the transmitter as follows: GI deletion, S/P conversion, OFDM demodulation (FFT), detection, diversity combining for M symbols $S'_{i,j}$ with L subcarriers spacing, P/S conversion and data decision. Post-detection diversity combining is assumed because the diversity gain can be simply but effectively achieved [10].

Figure 2 illustrates a concept of the diversity scheme with a repetition code in the case of $N = 48$ and $M = 2$. In the i-th OFDM symbol, digital-modulated symbols $S_{i,j}$ ($j = 1, 2, \cdots, 24$), such as PSK and QAM, are replicated to $(j+24)$-th subcarrier. Full diversity gain cannot be achieved because of frequency correlation, but we will theoretically demonstrate the improvement of the BER by the effect of correlated (imperfect) diversity gain in Sect. 3. It should be noted that the scheme in Fig. 2 can adapt the division number of all subcarriers M and the constellation size depending on the fading channel condition. Moreover, the frequency diversity could be replaced by the time diversity with a time interval if transmission delay could not be taken into account.

3. Theoretical Analysis for Correlated Diversity Gain

In the theoretical analysis, for the purpose of robustness against severe fading, we assume DQPSK/COFDM systems considering the correlated diversity gain between two symbols ($M = 2$). It is noted that differential encoding in DQPSK can be applied either in the time or frequency domain [8], [11]. The main purpose in this section is to verify the influence of correlated diversity gain upon the BER, then only the time-domain differential encoding is assumed.

3.1 Fading Correlation between Two Branches [12]

We utilize an eigenvector transform which can decouple correlated signals into uncorrelated ones. The outline is as follows. The average signal power P_s and noise power P_n at each branch are assumed to be equal, respectively. A correlation matrix C can be expressed as

$$C = \begin{bmatrix} P_s + P_n & \rho_B P_s \\ \rho_B^* P_s & P_s + P_n \end{bmatrix}, \tag{1}$$

where ρ_B is the fading correlation between two signals. Eigenvalues of the matrix C become

$$\lambda_{\pm} = (1 \pm |\rho_B|)P_s + P_n. \tag{2}$$

From (2), the average SNR Γ_0 (Signal to Noise power Ratio) before eigenvector conversion is decoupled into two uncorrelated SNRs $\Gamma_{\pm}$ which can be expressed as

$$\Gamma_{\pm} = (1 \pm |\rho_B|)\Gamma_0. \tag{3}$$

The concept mentioned above can be utilized for the theoretical analysis of the BER.

3.2 Closed Form Equation of BER

We have already derived a closed form equation of the BER in DPSK/OFDM systems with post-detection diversity combining for two independent branches of the same SNR, which is approximately given by [8], [13]

$$P_e = \frac{3}{\left(\frac{\sigma_{r+}^2}{\sigma_{r-}^2} + 1\right)^2}, \tag{4}$$

where

$$\frac{\sigma_{r+}^2}{\sigma_{r-}^2} = \frac{\{1 + J_0(2\pi f_D T_s)\} \frac{\Gamma'_{EN}}{(1+\alpha)} + 1}{\{1 - J_0(2\pi f_D T_s)\} \frac{\Gamma'_{EN}}{(1+\alpha)} + 1}. \tag{5}$$

In (5), $f_D T_s$ is the maximum Doppler frequency normalized by the OFDM symbol frequency, and $J_0(\cdot)$ is a Bessel function of the first kind of zeroth order which means the fading correlation for differential detection between adjacent symbols in the time domain. Γ'_{EN} means the ratio of energy per bit to the spectral noise density (E_b/N_0) including interferences as follows:

$$\begin{aligned} \Gamma'_{EN} &= \frac{1}{\frac{1}{\Gamma_{EN}} + 2(I_a + I_d)} \\ I_a &= \frac{(\pi f_D T_s)^2}{3(1+\alpha)^2} \\ I_d &= \frac{(\pi f_D T_s)^2}{2}, \end{aligned} \tag{6}$$

where Γ_{EN} means E_b/N_0 without interferences, I_a and I_d are inter-carrier interference (ICI) and interference to quadrature channel, respectively. Finally, $(1 + \alpha)$ in (5) means the loss of energy when removing GI.

Now we focus attention to the fact that a joint probability of statistically independent events is equal to the product of each probability. By the eigenvector conversion in Sect. 3.1, two dependent subcarriers of the same E_b/N_0 in Fig. 2 can be converted into two independent ones of the different E_b/N_0. Then a simple closed form equation of the BER in the DQPSK/COFDM systems can be derived by rewriting the product of the terms in (4) as follows:

$$P'_e = \frac{3}{\left(\left[\frac{\sigma_{r+}^2}{\sigma_{r-}^2}\right]_+ + 1\right)\left(\left[\frac{\sigma_{r+}^2}{\sigma_{r-}^2}\right]_- + 1\right)}, \tag{7}$$

where

$$\left[\frac{\sigma_{r+}^2}{\sigma_{r-}^2}\right]_\pm = \frac{\{1 + J_0(2\pi f_D T_s)\} \frac{(1\pm|\rho_B|)\Gamma'_{EN}}{(1+\alpha)} + 1}{\{1 - J_0(2\pi f_D T_s)\} \frac{(1\pm|\rho_B|)\Gamma'_{EN}}{(1+\alpha)} + 1}. \tag{8}$$

When assuming a multipath power delay profile as an exponential decay model, the frequency correlation $|\rho_B|$ between two symbols with L subcarriers spacing can be expressed as [8]

$$|\rho_B| = \frac{1}{\sqrt{1 + \left\{2\pi(1+\alpha)\frac{\sigma_\tau}{T_s} L\right\}^2}}, \tag{9}$$

where σ_τ/T_s is the rms delay spread normalized by the OFDM symbol duration. In the case that the frequency diversity is replaced by the time diversity with an interval of L' OFDM symbols, $|\rho_B|$ in (8) means the time correlation as follows:

$$|\rho_B| = J_0\left(2\pi f_D T_s L'\right). \tag{10}$$

4. Numerical Evaluation

We verify the advantage of the COFDM systems with frequency-correlated (or time-correlated) diversity gain. In order to confirm the theoretical analysis in Sect. 3, the results obtained from Monte Carlo simulation are also evaluated. It is noted in the simulation that a sufficient number of trials are conducted in order to meet the condition of random process.

Table 1 shows the simulation parameters and Fig. 3 shows the influence of frequency-correlated diversity gain upon the BER in the case of E_b/N_0 (Γ_{EN}) = 30 [dB]. When the rms delay spread σ_τ/T_s becomes over 10^{-4}, the BER can be improved, but the performance improvement is saturated around $\sigma_\tau/T_s = 10^{-2}$ ($|\rho_B| = 0.47$). The BER degradation in

Table 1 Simulation parameters (1).

Modulation	DQPSK
Detection	Differential
FFT point size	64
Number of subcarriers N	48
Division number of subcarriers M	2
Subcarrier spacing L	24
GI factor α	1/4
Noise	AWGN
Fading	Time- and frequency-selective
Channel model	Exponential decay

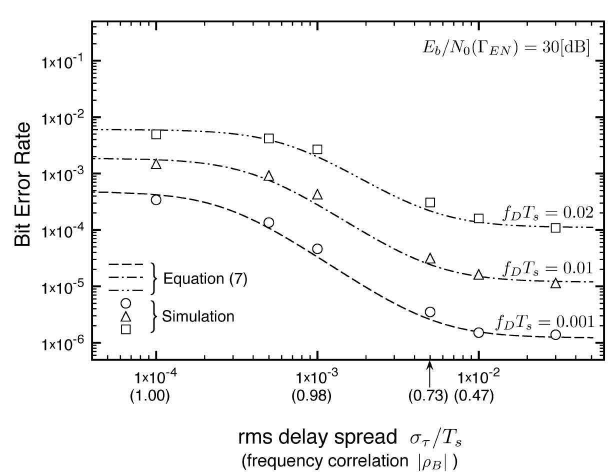


Fig. 3 Influence of frequency-correlated diversity gain.

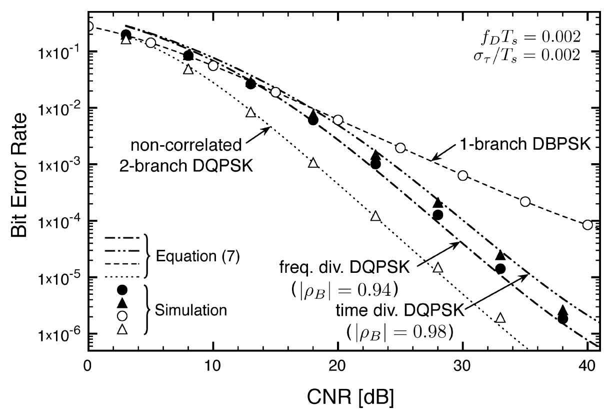


Fig. 4 Comparison on the same conditions of carrier power and information bit rate.

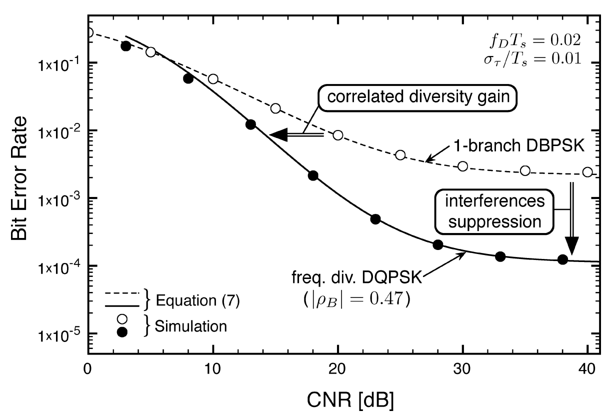


Fig. 5 Influences of correlated diversity gain and interferences suppression.

the large Doppler frequency f_DT_s is caused by the interferences I_a and I_d in (6). Figure 4 shows an example of comparison between the adaptive modulation and the diversity scheme aiming not to improve throughput performance but to combat severe fading. Changing QPSK into BPSK with constant carrier power and constant symbol rate in the adaptive modulation ("1-branch DBPSK" in the figure) improves the BER owing to double bit energy, while the information bit rate is reduced by half. In the frequency diversity scheme ("freq. div. DQPSK" in the figure), the information bit rate is also reduced by half, but by combining the two QPSK symbols at a receiver, the BER is much improved owing to both double bit energy and correlated diversity gain ($|\rho_B| = 0.94$). Just for reference, the BER performances with the time diversity scheme ($L' = 24$, $|\rho_B| = 0.98$) and non-correlated 2-branch diversity scheme for DQPSK are presented in Fig. 4. The BER performance over time- and frequency-selective fading channels in the case of $f_DT_s = 0.02$ and $\sigma_\tau/T_s = 0.01$ is shown in Fig. 5. It is found from Figs. 3–5 that the simple closed form equation in (7) can quantitatively express the influences of the correlated diversity gain and the interferences suppression.

As an application of the COFDM systems, we evaluate the BER performances of coherent PSK and QAM over typical fading channel models by Monte Carlo simulation. Table 2 shows the additional simulation parameters, in which the channel models including f_DT_s and σ_τ/T_s are the same as [8]. Figures 6 and 7 show the BER performances over ETSI/BRAN channel-A model [14] and IMT-2000 vehicular-B model [15], respectively. The same shape of plot in Fig. 6 indicates the same information bit rate. In Fig. 7, both frequency and time diversity schemes (like 4-branch diversity) are applied in the case of 16QAM, then three methods are all the same rate. It is found from the figures that the diversity schemes outperform the non-diversity schemes because of the correlated diversity gain, and that the error floor caused by the severe Doppler frequency in Fig. 7 can be also reduced.

Table 2 Simulation parameters (2).

Modulation	PSK, QAM	
Detection	Coherent	
Channel Model	ETSI/BRAN channel-A	IMT-2000 vehicular-B
Doppler frequency f_DT_s	7×10^{-5}	0.025
rms Delay spread σ_τ/T_s	0.007	0.012

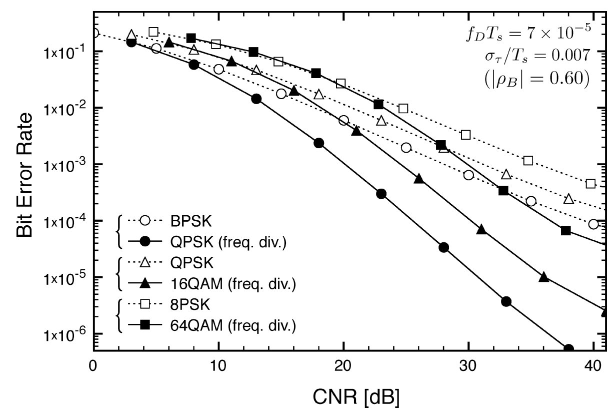


Fig. 6 BER Performances over ETSI/BRAN channel-A model.

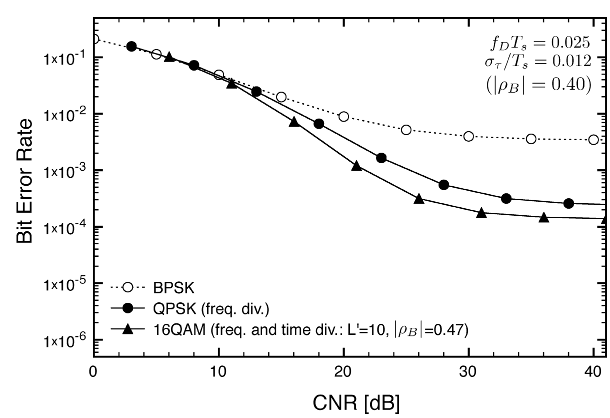


Fig. 7 BER Performances over IMT-2000 vehicular-B model.

5. Conclusion

In this letter, aiming to combat severe fading, we have theoretically analyzed and derived a simple closed form equation of the BER in a simple kind of COFDM systems. The advantages of the correlated diversity gain and interferences suppression have been confirmed by both theoretical analysis and Monte Carlo simulation. We will expand the theoretical analysis for more than two symbols combiner in future work.

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