Tension Buckling of Plate Having a Hole

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Abstract

Generally, it is believed that buckling of plates never arises under tensile load. However, when a plate has a hole, compression stresses appear locally near the hole under a tensile load, and the compression stress may cause local buckling - so called tension buckling - of the plate. In this paper, some results of numerical analysis on the tension buckling are presented, and basic behaviour on the tension buckling is discussed.

Keywords: tension buckling, plate, hole

1. Introduction

This paper presents some numerical results of the "tension buckling" in plates having a hole.

As illustrated in **Fig.1**, sometimes steel structures have holes, which are often arranged in the part subjected to tension, to avoid buckling. Generally, no attention is paid to the buckling of plates if they are subjected to a tensile load. However, when a plate has a hole, the compressive stress arises near the hole under a tensile load, and the stress may cause local buckling of the plate. **Fig.2** shows a demonstrative model of the tension buckling. The left part of this figure indicates a simple frame structure composed of 8 members. When this structure is subjected to a set of tensile loading

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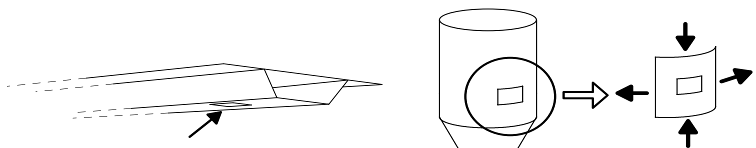
at the nodes A and A’, compression shall arise in the members B-B’ and C-C’, and this compression may cause buckling of members B-B’ and C-C’. Similar phenomenon occurs in a plate having a hole shown on the right of **Fig.2**. That is, when this plate is subjected to the tensile load for the vertical direction in the figure, compressive stress for the horizontal direction arises just above and below the hole (hatched zones in the figure), and a local buckling may be caused by the compressive stress. This phenomenon is called “tension buckling”.



(a) Manhole on Box Girder Flange (b) Manhole on Silo

Fig. 1. Example of Holes under Tension

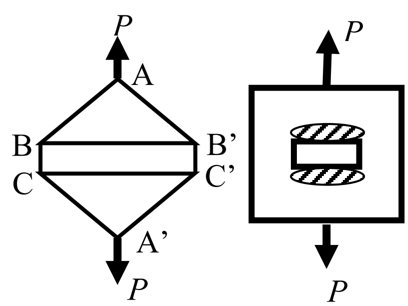


Fig. 2. Demonstrative Model

A very limited number of studies have been made on the tension buckling problem. Fujimoto et al. carried out an experiment and analysed the tension buckling of plates with a crack [1]-[3]. In the In his papers Fujimoto studies among other things the contribution of aspect ratios and crack length to the buckling load. Fujimoto et al. also proposed a formula for estimating the approximate tension buckling load of such plates. Bamberger et al. made a numerical analysis on tension buckling of plates having a hole [4].

The author of this paper has also made a series of studies on the tension buckling [5]-[9]. Paper [5] provides the elastic buckling analysis as a bifurcation problem and the study of the basic behaviour of the tension buckling. The papers [6] [7] and [9] present the contribution of the initial out-of-plane deflection to the tension buckling with the elasto-plastic large deflection analysis, and in paper [8] tension buckling strength under combined loadings such as in-plane bending or compression with tension is studied.

The current paper gives an overview of the basic behaviour of the tension buckling. Section 3 of this paper presents review of the elastic buckling behaviour as the bifurcation problem. Section 4 shows a review of the large deflection elasto-plastic analysis. Numerical models used for the review are described in section 2.

2. Numerical Models

For the purpose of the numerical analysis as a tension buckling problem the paper investigates among other thingd the effect of the shapes and dimensions of the hole as well as the plate length and the width of loading. An example of the analysed model is shown in **Fig.3.**

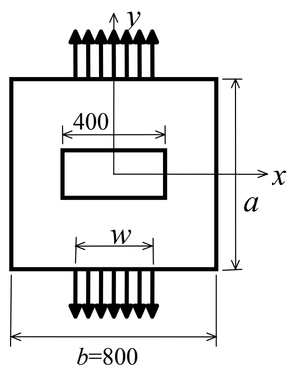


Fig. 3. A Typical Model

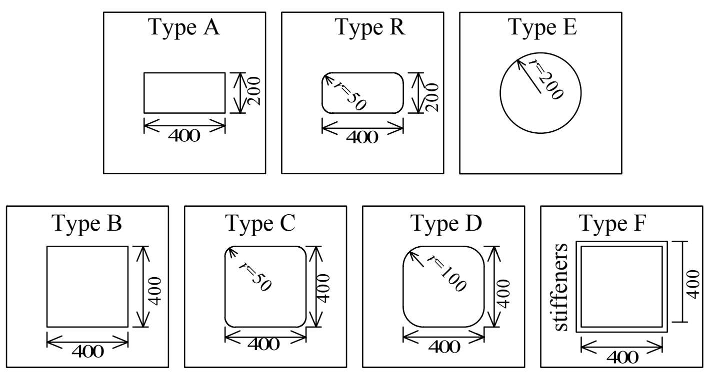


Fig. 4 Hole Types

The example model of a general width b where b=800 mm has a hole of 40 mm width in the center, as showm in figure 4. Plate thickness t varies from t=4 mm to 8 mm, although in the elastic buckling analysis only t=6 mm is considered. These values of plate thickness are corresponding to the width-thickness ratio of the plate b/t where b/t=200, 133 or 100 respectively.

The length of the plate is a, and the length-width ratio (the aspect ratio) $\alpha=a/b$ is considered as one of the parameters in the elastic buckling analysis, and varied within $0.5 \le \alpha \le 4.0$. In the elasto-plastic analysis, aspect ratio is fixed as α=1.0.

The plate is subjected to a set of tensile loadings P with the width w at its top and bottom edges as illustrated in the figure. The loading width w is selected as w/b=0.0, 0.5, 1.0, and expressed as the undimensioned form $\beta=w/b$. Here the undimensioned loading width β=0.0 means that the plate is subjected to a set of concentrated loads at the centre of the top and the bottoms edges, and β=1.0 indicates fully distributed load.

The shapes of the hole are illustrated in **Fig.4**. In this paper 7 types of the hole shapes are considered. The hole **Type A** is a rectangular hole with the dimensions of 400 x 200 mm, and the hole **Type B** has square shape of 400 x 400 mm. The **Type C** and the **Type D** are similar to the **Type B**, but the corners of the holes of these types are rounded off as its radius r=50 mm or r=100 mm respectively. In **Type E** the plate has a circular hole of the radius r=200 mm. The hole **Type F** is the same hole as the **Type B** with square shape of 400 x 400 mm, but this type has stiffeners with their section of 30 x 6 mm or 10 x 6 mm attached at the four edges of the hole. The **Type R** is the same as the **Type A** but the corners of the holes of these types are rounded off as its radius r=50 mm as similar to the **Type C**.

In the elastic buckling analysis, all these hole types except **type R** are dealt with, and in the elasto-plastic problem, only the hole **types A** and **R** are analysed.

In the elasto-plastic large deflection analysis the shape and the magnitude of the out-of-plane initial deflection are also considered as parameters. Details of the initial deflection shall be described in the later section.

In the analysis the plates are assumed to be simply supported at their four edges. The material properties used in the elastic buckling analysis are Young's modulus E = 206GPa and Poisson's ratio ν= 0, 3. The yield stress of the material used in the elasto-plastic analysis σ_y is σ_y=312 MPa.

3. Elastic Buckling Behaviour

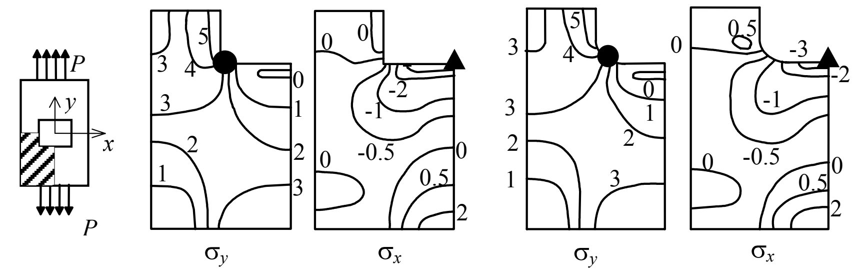
3.1. Analysis and Result Expressions

The elastic buckling analysis is made as an eigen-value problem. For the analysis, at first, stress distribution of the plate is calculated under the tensile load of P=9,8 kN, then the eigen-value analysis is performed by using the stresses. The buckling load P_{cr} is obtained by multiplying the eigen-value to the initial load of P=9,8 kN.

The buckling strength of the plate is expressed with the buckling stress σ_{cr} estimated from the buckling load P_{cr} and the effective section area of plates A as $\sigma_{cr} = P_{cr}/A$.,here the effective area is the plate section area subtracted by the area of the hole. In this study, the width of a hole is assumed to be half of the plate width, therefore the effective section area A is half of the full section area, $A = t(b/2)$. The results of the analysis in this section are discussed with the buckling coefficient k, i.e. $\sigma_{cr} = k \times \sigma_e$, here σ_e is Euler's buckling stress defined as $\sigma_e = \frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2$, where b denotes the plate width defined as in **Fig.1**, and t is plate thickness.

3.2. Stresses of Plates

Fig.5 shows the stress distributions of **type B** and **type D** models, with their aspect ratio α=2.0, the loading width β= 0.5 and plate thickness t= 6 mm, under the load of P = 9,8 kN. In this figure, because of the symmetry, only quarter parts indicated in **Fig. 5 (a)** are shown.



a) Layout   b) Hole Type B   c) Hole Type D

Fig. 5 Stress Contours (MPa)

In this figure it is found that both types have very similar stress contour patterns. Although the figure is not shown in this paper, these stress contour patterns are true for all other model types. This figure indicates that the tensile stress is dominant for the vertical direction (y-direction), and the relatively larger tensile stress is observed in the left-adjacent zone of the hole. The magnitude of the tensile stress at the left zone of the hole is over 5 MPa in all models. On the other hand, the compressive stress of over -3 MPa arises for the horizontal direction (x-direction) just under the hole in both types. As it is not clear in this figure, the tensile stress for the vertical direction σ_y at the corner of the hole (marked as a black point ● in the figure) of the model **type B** is 3.7 MPa, and in the **type D**, which has the corners of rounded off, 3.0 MPa, and in **the type E** with a circular hole, the stress σ_y at the at the lower left part is 2.0 MPa. This value of 2.0 MPa is smaller by about 46% than that of the type B. The compressive stress at the point ▲ shown in the figure is –3.4 MPa for the type B and 3.2 MPa for the **type D**. In the type E with a circular hole, σ_x at the bottom part of the hole is –3.0 MPa, smaller by 12% than that of the **type B**. Thus, it is obvious that the rounded corners of the hole reduce the magnitude of the tensile stress for the loading direction (y-direction) at the corner effectively, although the compressive stress is not reduced so much by the shape of the hole.

3.3. Buckling Behaviour

Fig.6 shows the out-of-plane deformation patterns due to the 'tension buckling' for the model **type A**, which has a hole of 400 x 200 mm, with β=1.0 and t=6 mm and the aspect ratio α=0.75 or 1.5. The deformation patterns shown in this figure are obtained as an eigen-vector of the eigen-value problem of the elastic buckling analysis. Therefore, it should be noted that only the deformation patterns are discussed and the magnitude (or the degree) of the deformation in the figure is meaningless. Because of

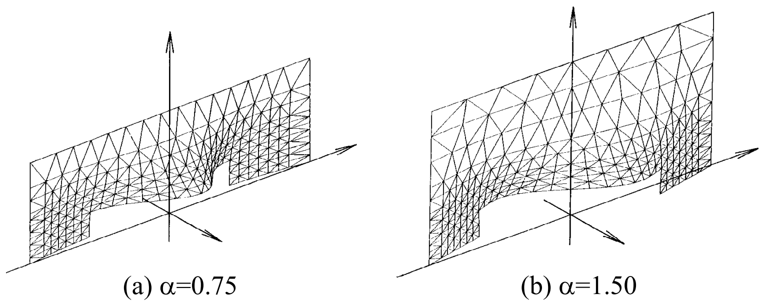


Fig. 6. Buckling Mode of the Hole Type A

the symmetry, the upper half of the plate is displayed in this figure.

In this figure, two types of the buckling mode are found. The buckling mode in **Fig.6(a)** for α=0.75 is symmetrical with respect to both x and y axes. In this model, the plate deforms with almost one half-wave along the x-axis.

In **Fig. 6(b)** with the model of α=1.5, two half waves are observed along the x-axis, and the plate deforms for the alternative direction.

The similar buckling mode like the case with α=1.5 is obtained for models with the aspect ratio α greater than approximately 1.4 under the load width β=1.0, and the models with the smaller aspect ratio than 1.4 has the buckling mode similar to one of the model of α=0.75.

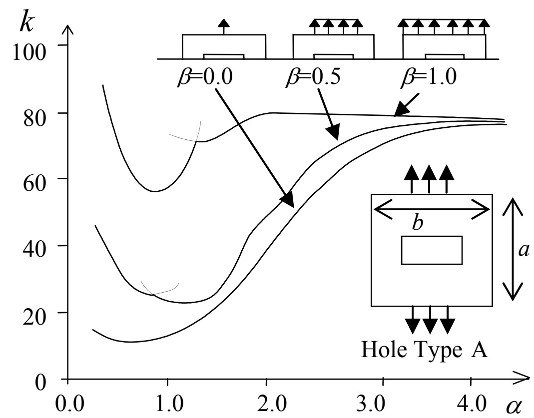


Fig. 7 Buckling Coefficients-Aspect Ratio Curves of Hole Types A

Hereafter the buckling mode as shown in **Fig. 6(a)** is denominated as the mode type 1 and the mode in **Fig. 6(b)** as the mode type 2.

The buckling coefficient-aspect ratio curves of the hole type A are plotted in **Fig. 7** for the load width β=0.0, β=0.5 and β=1.0. The vertical axis of this figure denotes the buckling coefficient k and the horizontal axis the aspect ratio $\alpha=a/b$ of the plate. This figure indicates that the curves of the load width β=0.5 and 1.0 consist of two parts, and the transition arises at α=0.9 for β=0.5 and α=1.4 for β=1.0. In these two cases, the buckling mode type 1 is observed in the smaller aspect ratio than the transition, and the buckling mode type 2 for the larger aspect ratio. On the model with the load width β=0.0, the buckling mode type 2 is obtained for the whole range of aspect ratios analysed in this study.

Fig. 7 also indicates that a smaller load width entails a smaller buckling coefficient, i.e. a smaller buckling strength. The minimum buckling coefficient $k_{\min}$ for the case β=0.0 is $k_{\min}\cong 10.0$ at $\alpha\cong 0.65$. The model with β=0.5 has $k_{\min}\cong 23.5$ at $\alpha\cong 1.25$ and β=1.0 has $k_{\min}\cong 56.0$ at $\alpha\cong 0.90$. That is, the case with β=0.0 has the minimum buckling strength 60% smaller than β=0.5, and β=1.0 has the strength of 2.4 times of the case of β=0.5. In all three cases in this figure, the buckling coefficient is increased with the aspect ratio, and the value of k is converged to $k\cong 75.0$. In the range with the larger aspect ratio, for all three cases above buckling coefficient is almost equal. This is due to the fact that these three cases of the load width have very similar stress distribution near the hole when their aspect ratio is larger.

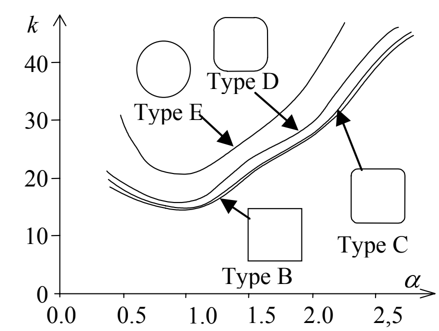


Fig. 8 Buckling Coefficients of Hole Types B-E

In **Fig. 8**, buckling coefficient-aspect ratio curves of hole **types B-E** are plotted under the load width β=0.0. This figure indicate that the 4 curves in this figure are almost parallel to each other, and the curve of hole type C is very close to that of **type B**. The buckling coefficients of hole **types B, C, D** and E at α=1.0 are 14.2, 14.5, 15.6 and 20.1, respectively. The buckling coefficients of hole types C and D are 2% and 10% larger respectively at α=1.0 than the buckling coefficients of **type B**. This fact suggests that a rounded corner of a hole has a small effect in improving the tension buckling strength of a plate.

When the stiffeners are arranged at the edges of the hole like the hole **type F**, the buckling strength of the plate is expected to be improved considerably. Although figures are not shown in this paper, the buckling coefficient k of the case of hole **type F** with stiffeners of 6 x 30 mm is k=63.6, and with stiffeners of 6 x 10 mm is k=53.9 at the aspect ratio α=1.0. These values are 4.5 and 3.8 times larger respectively than the corresponding model of the hole **type B** which has k=14.2. Thus, when the stiffeners are arranged, the possibility of the collapse due to the tension buckling becomes small.

3.4. Remarks on the Elastic Buckling Analysis

In a plate having a hole subjected to a tensile load compressive stress arises locally near the hole for the perpendicular direction to the loading, and this compressive stress may cause local buckling of the plate. In this case a smaller load width entails a smaller buckling strength. However, when the plate has the larger aspect ratio (i.e. larger length), the influence of the load width becomes smaller and the plate shall have almost equal buckling strength regardless of the load width. This is due to the fact that in the plate with larger aspect ratio, the force in the plate is transmitted for the whole plate width near the hole and therefore all cases have the similar stress distribution near the hole regardless of the load width.

When the corners of the hole are rounded the tensile stress near the hole corner is reduced effectively. However, the rounded corner has small contribution to the tension buckling strength because the tension buckling is caused not by the tensile stress near the corner but by the compressive stress above and below the hole.

4. Elasto-Plastic Behaviour with Initial Deflection

4.1 Models and Analysis

As described in section 2, elasto-plastic large deflection analysis is made for the models with the aspect ratio α=1.0. In addition to the parameters described in the previous section, the shapes and magnitude of the initial out-of-plane (hereafter abbreviated as o-o-p) deflection of the plate are adopted as parameters. **Fig. 9** shows the initial o-o-p deflection patterns. The pattern **Type G1** in Fig. 9 has the cosines shaped o-o-p deflection extended for the whole plate. The o-o-p deflection pattern **Types H1, H2** and **H3** have initial deflections appeared only in the adjacent zones (above and below) of the hole. Within these patterns, **H1** has its initial deflection deformed for the same direction, **H2** has deflection for the alternate direction, and Model **H3** has the initial deflection with two half-waves on each edge of the hole. **Types L1** and **L2** have locally arranged initial deflection shapes at the centre of the hole edges as illustrated in the figure.

The initial deflection is indicated with dimensionless out-of-plane deflection δ_0/t where δ_0 denotes the magnitude of initial out-of-plane deflection and t the thickness of the plate, and assumed as δ_0/t=1/2, 1/10, 1/50 or 1/100.

In the analysis the plate elements are divided into 5 layers to practice the elasto-plastic bending, and the load-increment procedure is applied for the analysis shown in this paper.

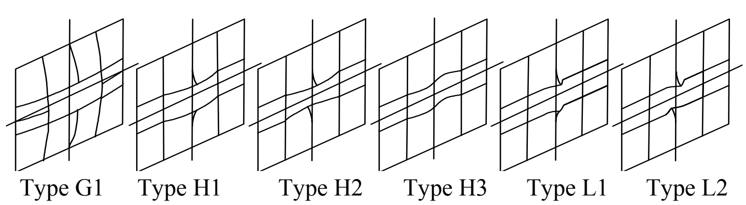


Fig. 9 Initial Out-of-Plane Deformation Patterns

4.2. Contribution of Initial Deflection

Fig. 10 shows the load-deflection relations (P-δ relations) of the plate with the initial deflection type **H1**, plate thickness t=6 mm and the load width β=0.5 for the various values of the initial deflection. In this figure, the vertical axis is the load applyed to the plate, and the horizontal axis the out-of-plane (o-o-p) deflection of the plate. The o-o-p deflection in this figure is measured at the centre of the hole edge shown as black point ● in the figure. This figure indicates that the numerical model with a small initial deflection has the curve which is almost vertical at the earlier stage, and then the deflection increases rapidly after the load reaches approximately 300 kN. On the other hand, the curves of the models with the larger initial deflection are increased gradually. This behaviour is similar to the "normal" buckling (buckling under the compression).

In all the numerical models shown in this figure, yielding is initiated when the load reaches about 275 kN at the adjacent zone of the hole corner. After this stage the yielded zone is extended for both sides of the hole at the load of P≅290 kN for the model with δ_0/t=1/2, at P≅330 kN for the model with δ_0/t=1/100 and at P≅330 for all other models.

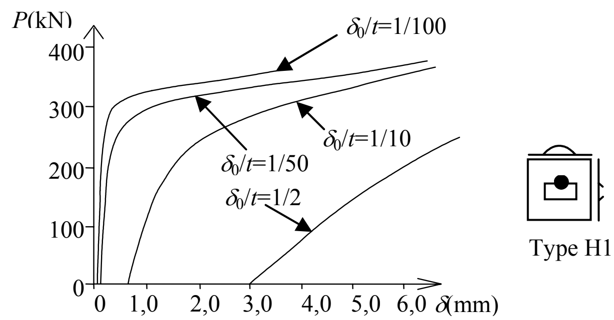


Fig. 10. Load-Deflection Relations according to Initial Deflection

4.3 Initial Deflection Patterns

In **Fig. 11**, the P-δ relation of the plate with the initial deflection type **G1**, having cosines shaped initial o-o-p deflection extended for the whole plate with the load width β=0.5, the plate thickness t=6 mm and δ_0/t=1/50, is plotted. The o-o-p deflection is also measured at the centre of the hole edge.

This figure indicates that the load-deflection relation consists of 3 stages shown as (1)-(3) in the figure. In stage (1) the o-o-p deflection of the plate is decreased slightly from the initial deflection. This fact means that the initial o-o-p deflection extended for the whole plate vanishes due to the tensile load. This phenomenon is one of the characteristics of tension buckling. After this stage, the load is increased without the increase of the deformation in stage (2). Beyond this the deflection begins to increase slightly, then it increases very rapidly. Thus, in the tension buckling problem with the initial o-o-p deflection extended for the whole plate, the plate behaves as if one with a small initial deflection.

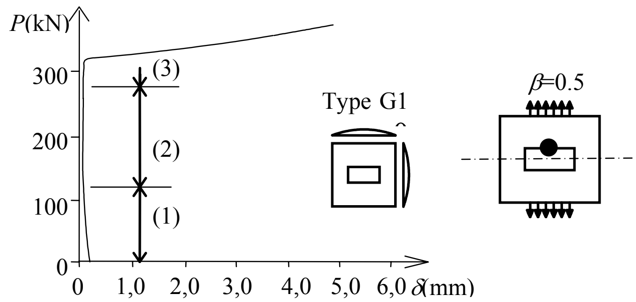


Fig. 11. Typical Load-Deflection curves

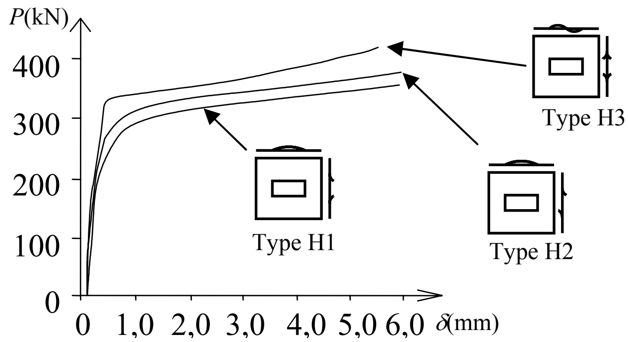


Fig. 12 Load-Deflection Curves of Initial Deflection Type H

In **Fig. 12**, P-δ relations of the plates with the initial deflection arranged in the zone very close to the hole. Such an initial deflection may be caused by the error during fabricating the structure. The o-o-p deflection in these figures is also measured at the centre of the hole edge as in **Fig. 10**. These two figures show that the increasing rate becomes larger gradually as the load reaches 300 kN. However, in the initial deflection type **H3**, which has the o-o-p deflection for the alternative direction near the hole, the inclination of the curve is changed rapidly and behaves similarly to the case with a smaller initial deflection.

Thus, in the tension buckling problem, the initial o-o-p deflection arranged locally near the hole has a larger influence on the behaviour of the plate than the initial deflection extended for the whole plate.

4.4. Effect of Load Width

As described in section 3, in the elastic buckling analysis, the load width influences the buckling strength of the plate, and a larger load width generally entails a larger buckling strength. **Fig. 13** shows the P-δ relations of the cases with the load width

β=1.0, 0.5 and 0.0. As it is clear from the figure, on the model with the load width β=0.0 the slope of the curves change at the load level of 200 kN. This is about 37 % smaller than the model with β=0.5. The model with the load width of β=1.0 has a load level of 500 kN, and 60% larger than the model of β=0.5. Beyond these load levels deformation increases as a faster rate in all models. In the elastic buckling analysis the buckling strength of the case with β=1.0 is 2.4 times larger than that of the case with β=0.5 so in the elasto-plastic analysis the load width has a smaller effect on improving the strength. This is due to the fact that in the elasto-plastic analysis the strength of the plate is influenced not only by the buckling but also the yielding of the plate near the hole.

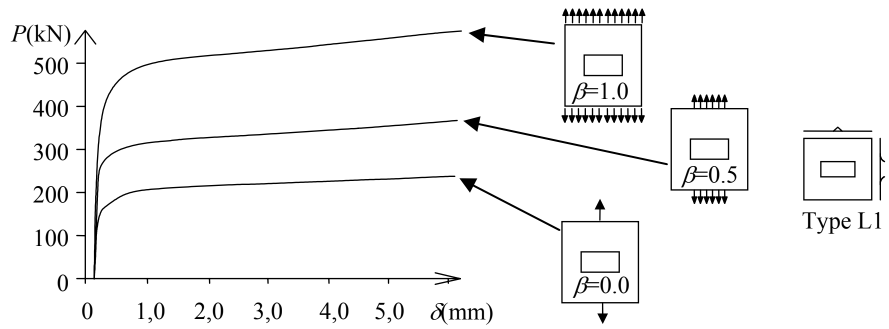


Fig. 13 Contribution of Load Width

4.5. Remarks on the Elasto-Plastic Analysis

Generally, when the corners of the hole are rounded stress intensity shall be defused. In the elastic buckling analysis, as described in section 3, the rounded corners have a small effect on the tension buckling strength, because the tension buckling is caused by the local compression, and not by the stress near the hole corners. In **Fig. 14** P-δ curves of plates with the hole **type A** and the **type R** are plotted. The initial deflection type **H1** and the load width β=0.5 are used for this figure. As described in the earlier section, the hole **type R** is similar to the **type A**, but the corners of this type are rounded by the radius of 5 mm. This figure indicates that on the **type R** the slope of the curve is changed at the load level of approximately 340 kN, and this about 13 % larger than the **type A**.

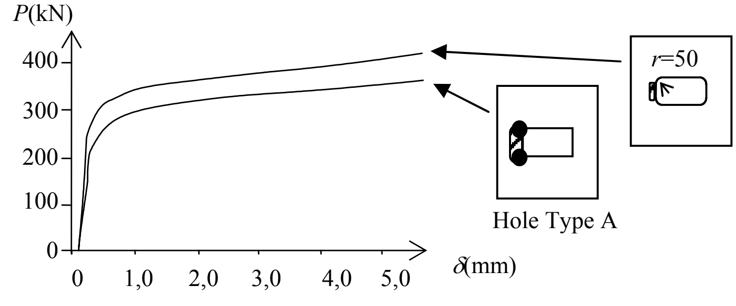


Fig. 14. Effect of Rounded Hole Corner

In the plate with the hole **type A**, yielding is initiated at the corners of the hole (indicated as the black point ● in the figure) at the load of $P\cong275$ kN and the yielded zone is extended for the side of the hole (hatched zone in the figure) at $P\cong320$ kN. In the **Type R** case, the plate begins to be yielded at the load of $P\cong280$ kN at the hatched zone. But the yielded zone in this plate is smaller than the **Type A** case, and is not extended until the load reaches $P\cong400$ kN.

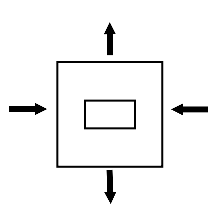
Thus, in the elasto-plastic large deflection behaviour, unlike in the elastic buckling problem, rounded corners of a hole are effective in improving the strength.

In addition, in the tension buckling problem, initial o-o-p deflection occuring very close to the hole has a larger influence than that extended for the whole plate. In the case with the initial o-o-p deflection extended for the whole plate, the initial deflection is reduced at the earlier stage by the tensile load and the plate behaves as if it had a smaller initial deflection.

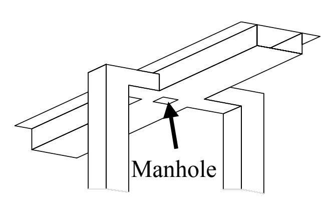
5. Conclusion

In this paper, the basic behaviour of tension buckling is described. Generally, no attention is paid to plate buckling when the plate is subjected to tension. When a plate has a hole and is subjected to the tensile loading, it is often considered as a stress intensity problem. However, as shown in this paper, if the plate has a hole buckling may occur under tension. In this paper, only very basic patterns are shown. But, in the practical structures, a more severe situation may arise. One of the examples is a plate with a hole which is subjected not only a tensile load but also to the perpendicular compression as illustrated in **Fig. 15(a)**. In the case demonstrated in **Fig.15(a)** the compression perpendicular to the tension may reduce the tension buckling strength considerably even if the compression is small[8]. In fact, such a case is not so unusual. For example, a steel box girder is sometimes connected to the pier as illustrated in **Fig.15(b)**. In addition, when a manhole is required for the girder, it is often arranged in the bottom flange which is generally subjected to tension. This arrangement shall result subjecting the plate panel near the hole to the loading shown in **Fig.15(a)**. The plate panel shown in **Fig.15(a)** may also be found in a silo structure having a manhole shown in **Fig.1(b)** in this paper. In such structures, buckling of plate panel may occur as the "tension buckling" under very small compression.

Therefore, the author of this paper believes that in such structures, more attention should be paid to the tension buckling problem and that further studies are required on



(a) severer case of tension buckling



(b) manhole arranged at the bottom flange of a box girder

Fig. 15. Example of Combined Load

this problem.

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