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Electron-Electron Interaction in Refractive Index

Yuji KATO*

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Effects of electron-electron interaction on refractive index in the absence of a constant magnetic field are investigated. A formula for the refractive index is derived on the basis of the microscopic Maxwell equations and by no use of the conventional formulation. It is shown that, in general, the electron-electron interaction has a little effect on the refractive index. In a system of harmonic-oscillator electrons, the electron-electron interaction is exactly ineffective on the refractive index and the force constant in the direction of light is also ineffective on them. A system of anharmonic-oscillator electrons is also investigated and the effects of the electron-electron interaction are discussed.

1. Introduction

Refractive index which is one of familiar optical constants is investigated theoretically by a number of authors in various substances.[1] In most of the theoretical investigations of the refractive index, discussions of the electron-electron interaction have been presented by the perturbation theory.

In the previous papers,[2-4] a general theory of the Faraday effect has been developed[2] and it has been applied to the natural optical activity[3] and the refractive index in the presence of a constant magnetic field.[4] The general formula for the Faraday effect is derived on the besis of the microscopic Maxwell equations and is expressed in terms of a correlation function of the spatial Fourier components of total electric current. By similar method the refractive index can be investigated from the same point of view as is demonstrated in the previous papers.[2-4] The electron-electron interactions are also discussed from the same point of view as is seen in the case of the Faraday effect[3] and the natural optical activity[4] by no use of the conventional formulae. Our theories are different from the conventional ones and the dielectric constant and the magnetic permeability do not appear explicitly.

In the present paper, the effects of the electron-electron interaction on the refractive index are investigated by our general theory presented in the previous paper.[5]

In sec. 2 the theoretical preliminaries for the refractive index and the Hamiltonian of the system are given. In sec. 3 it is shown that the effects of the electron-electron

* Professor, Department of Physics, Faculty of Engineering, Shinshu University, Wakasato, Nagano-shi 380.

interaction on the refractive index have a little in general. In sec. 4 the equations for the Green functions are exactly solved in a system of harmonic oscillators. Section 5 gives also a discussion of the electron-electron interaction in the anharmonic oscillator system. Finally, section 6 is devoted to summary and discussion.

2. Formulation and Hamiltonian

In a previous paper,[5] the theory of refractive index in the presence of a constant magnetic field has been developed on the basis of a standpoint of the microscopic Maxwell equations. It has been shown that, when there is no constant magnetic field, the leading term of the refractive index originates in the zeroth order term in the wave-number q of light. Therefore, when a monochromatic light with the angular frequency ω is propagated through medium in the direction of the z-axis, the refractive index $n(\omega)$ in the absence of the constant magnetic field is expressed in the form

$$\{n(\omega)\}^2 = 1 + \frac{2\pi ie}{Vm\omega}\{G_{xx}(\omega) + G_{yy}(\omega)\}, \tag{1}$$

where m is the mass of an electron, e the charge of the electron and V is the volume of a system. Expressions $G_{xx}(\omega)$ and $G_{yy}(\omega)$ are the Fourier components of the Green functions $G_{xx}(t)$ and $G_{yy}(t)$, respectively, i. e.

$$G_{\mu\mu}(\omega) = \int_{-\infty}^{\infty} \mathrm{d}t\, \mathrm{e}^{-i\omega t}\, G_{\mu\mu}(t). \quad (\mu = x,\ y) \tag{2}$$

The Green function $G_{\mu\mu}(t)$ is defined by

$$G_{\mu\mu}(t) = -\frac{i}{\hbar}\theta(t)\langle[p_\mu(t),\ \mu_\mu(0)]\rangle, \tag{3}$$

where $\boldsymbol{p}(t)$ and $\boldsymbol{\mu}(t)$ denote the momentum operator and the electric dipole moment operator at time t, respectively, the symbol $\theta(t)$ is defined by

$$\theta(t) = \begin{cases} 1 & \text{for } t > 0 \\ 0 & \text{for } t < 0 \end{cases}$$

and triangular brackets denote the canonical ensemble average.

The system under consideration is a polymer composed of similar monomers. The Hamiltonian $\mathscr{H}$ of the system is expressed as

$$\begin{aligned} \mathscr{H} = \sum_n\sum_i \left\{\frac{1}{2m}\boldsymbol{p}_{in}^2 + v_n(\boldsymbol{r}_{in})\right\} + \frac{1}{2}\sum_n\sum_{i\neq j}\sum v_n'(\boldsymbol{r}_{in} - \boldsymbol{r}_{jn}) \\ + \frac{1}{2}\sum_{n\neq m}\sum\sum_i\sum_j V_{nm}(\boldsymbol{r}_{in} - \boldsymbol{r}_{jm}), \end{aligned} \tag{4}$$

where $\boldsymbol{p}_{in}$ is the momentum operator of the ith electron in the nth monomer in the polymer, $v_n(\boldsymbol{r}_{in})$ the interaction between the ith electron in the nth monomer and the nucleus in the same nth monomer, $v_n'(\boldsymbol{r}_{in} - \boldsymbol{r}_{jn})$ the interaction between the ith and the jth electrons in the same nth monomer and $V_{nm}(\boldsymbol{r}_{in} - \boldsymbol{r}_{jm})$ is the interaction between the ith electron in the nth monomer and the jth electron in the mth monomer.

3. Green Functions

A set of equations of the Green functions of various types can be calculated by differentiating with respect to time t as is seen in the previous paper.[5] The simultaneous equations for the Green functions are found to be

$$-\frac{\hbar}{\mathrm{i}}\frac{\mathrm{d}G_{\mu\mu}(t)}{\mathrm{d}t}=\frac{N\hbar e}{\mathrm{i}}\delta(t)+G^{1}_{\mu\mu}(t), \tag{5}$$

$$-\frac{\hbar}{\mathrm{i}}\frac{\mathrm{d}G^{1}_{\mu\mu}(t)}{\mathrm{d}t}=-\frac{1}{2m}G^{21}_{\mu\mu}(t)-\frac{1}{m}G^{22}_{\mu\mu}(t), \tag{6}$$

$$-\frac{\hbar}{\mathrm{i}}\frac{\mathrm{d}G^{21}_{\mu\mu}(t)}{\mathrm{d}t}=-\frac{1}{2m}G^{311}_{\mu\mu}(t)-\frac{1}{m}G^{312}_{\mu\mu}(t), \tag{7}$$

$$-\frac{\hbar}{\mathrm{i}}\frac{\mathrm{d}G^{22}_{\mu\mu}(t)}{\mathrm{d}t}=\frac{N\hbar e}{\mathrm{i}}C_{\mu}\delta(t)-\frac{1}{2m}G^{312}_{\mu\mu}(t)-\frac{1}{m}G^{321}_{\mu\mu}(t)+G^{322}_{\mu\mu}(t)$$
$$+\frac{1}{2}G^{323}_{\mu\mu}(t)+\frac{1}{2}G^{324}_{\mu\mu}(t), \tag{8}$$
$$(\mu=x,\ y)$$

where N is the number of electrons is the system and

$$C_{\mu}=\langle\frac{1}{N}\sum_{n}\sum_{i}(p_{\mu in}\,p_{\mu in}\,v_{n}(\boldsymbol{r}_{in}))\rangle, \tag{9}$$

$$G^{1}_{\mu\mu}(t)=-\frac{\mathrm{i}}{\hbar}\theta(t)\langle[\sum_{n}\sum_{i}(p_{\mu in}\,v_{n}(\boldsymbol{r}_{in}(t))),\ \mu_{\mu}(0)]\rangle, \tag{10}$$

$$G^{21}_{\mu\mu}(t)=-\frac{\mathrm{i}}{\hbar}\theta(t)\langle[\sum_{n}\sum_{i}\sum_{\nu}(p_{\mu in}\,p_{\nu in}\,p_{\nu in}\,v_{n}(\boldsymbol{r}_{in}(t))),\ \mu_{\mu}(0)]\rangle, \tag{11}$$

$$G^{22}_{\mu\mu}(t)=-\frac{\mathrm{i}}{\hbar}\theta(t)\langle[\sum_{n}\sum_{i}\sum_{\nu}(p_{\mu in}\,p_{\nu in}\,v_{n}(\boldsymbol{r}_{in}(t)))\,p_{\nu in}(t),\ \mu_{\mu}(0)]\rangle, \tag{12}$$

$$G^{311}_{\mu\mu}(t)=-\frac{\mathrm{i}}{\hbar}\theta(t)\langle[\sum_{n}\sum_{i}\sum_{\nu}\sum_{\nu'}(p_{\mu in}\,p_{\nu in}\,p_{\nu in}\,p_{\nu' in}\,p_{\nu' in}\,v_{n}(\boldsymbol{r}_{in}(t))),\ \mu_{\mu}(0)]\rangle, \tag{13}$$

$$G^{312}_{\mu\mu}(t)=-\frac{\mathrm{i}}{\hbar}\theta(t)\langle[\sum_{n}\sum_{i}\sum_{\nu}\sum_{\nu'}(p_{\mu in}\,p_{\nu in}\,p_{\nu' in}\,p_{\nu' in}\,v_{n}(\boldsymbol{r}_{in}(t)))p_{\nu in}(t),\ \mu_{\mu}(0)]\rangle, \tag{14}$$

$$G^{321}_{\mu\mu}(t)=-\frac{\mathrm{i}}{\hbar}\theta(t)\langle[\sum_{n}\sum_{i}\sum_{\nu}\sum_{\nu'}(p_{\mu in}\,p_{\nu in}\,p_{\nu' in}\,v_{n}(\boldsymbol{r}_{in}(t)))\,p_{\nu in}(t)\,p_{\nu' in}(t),\ \mu_{\mu}(0)]\rangle, \tag{15}$$

$$G^{322}_{\mu\mu}(t)=-\frac{\mathrm{i}}{\hbar}\theta(t)\langle[\sum_{n}\sum_{i}\sum_{\nu}(p_{\mu in}\,p_{\nu in}\,v_{n}(\boldsymbol{r}_{in}(t)))(p_{\nu in}\,v_{n}(\boldsymbol{r}_{in}(t))),\ \mu_{\mu}(0)]\rangle, \tag{16}$$

$$G^{323}_{\mu\mu}(t)=-\frac{\mathrm{i}}{\hbar}\theta(t)\langle[\sum_{n}\sum_{i\neq j}\sum_{\nu}\{(p_{\mu in}\,p_{\nu in}\,v_{n}(\boldsymbol{r}_{in}(t)))-(p_{\mu jn}\,p_{\nu jn}\,v_{n}(\boldsymbol{r}_{jn}(t)))\}$$
$$\times(p_{\nu in}\,v'_{n}(\boldsymbol{r}_{in}(t)-\boldsymbol{r}_{jn}(t))),\ \mu_{\mu}(0)]\rangle, \tag{17}$$

$$G^{324}_{\mu\mu}(t)=-\frac{\mathrm{i}}{\hbar}\theta(t)\langle[\sum_{n\neq m}\sum_{i}\sum_{j}\sum_{\nu}\{(p_{\mu in}\,p_{\nu in}\,v_{n}(\boldsymbol{r}_{in}(t)))-(p_{\mu jm}\,p_{\nu jm}\,v_{m}(\boldsymbol{r}_{jm}(t)))\}$$

$$\times (p_{\nu in} V_{nm}(\boldsymbol{r}_{in}(t) - \boldsymbol{r}_{jm}(t))),\ \mu_\mu(0)]\rangle. \tag{18}$$

$$(\nu,\ \nu' = x,\ y,\ z)$$

The electron-electron interactions appear for the first time only in the Green functions $G^{323}_{\mu\mu}(t)$ and $G^{324}_{\mu\mu}(t)$ on the right-hand side of eq. (8), which are defined by eqs. (17) and (18) with terms of the form $(p_{\nu in} v'_n(\boldsymbol{r}_{in}(t) - \boldsymbol{r}_{jn}(t)))$ and $(p_{\nu in} V_{nm}(\boldsymbol{r}_{in}(t) - \boldsymbol{r}_{jm}(t)))$, respectively. This states that the electron-electron interaction has a little effect on the refractive index. Similarly, this interaction effect on the Faraday rotation is also a little one and there is a relation between the Faraday rotation and refractive index as is shown in the previous papers.[2,3]

4. Harmonic Oscillator Model

To understand the effects of the electron-electron interaction on the refractive index, it is useful to investigate a few model systems. Let us consider a system composed of the three-dimensional harmonic-oscillator electrons. For this system one may obtain a rigorous solution of simultaneous equations for the Green functions. The Hamiltonian $\mathscr{H}$ of the system is now of the form

$$\mathscr{H} = \sum_n \sum_i \sum_\mu \left\{ \frac{1}{2m} \boldsymbol{p}^2_{\mu in} + k_\mu (\mu_{in} - M_{in})^2 \right\} + \frac{1}{2} \sum_n \sum_{i \neq j} v'_n(\boldsymbol{r}_{in} - \boldsymbol{r}_{jn}) + \frac{1}{2} \sum_{n \neq m} \sum_i \sum_j V_{nm}(\boldsymbol{r}_{in} - \boldsymbol{r}_{jm}),$$

$$(\mu = x,\ y,\ z \text{ and } M = X,\ Y,\ Z) \tag{19}$$

where the term $\sum_\mu k_\mu (\mu_{in} - M_{in})^2$ is the potential of the ith electron oscillator in the nth monomer located at $\boldsymbol{R}_{in}(X_{in},\ Y_{in},\ Z_{in})$.

By differentiating the Green function $G_{\mu\mu}(t)$ defined by eq. (3) with respect to time t and by making use of Hamiltonian (19), one obtains an equation for $G_{\mu\mu}(t)$ in the form

$$-\frac{\hbar}{\mathrm{i}} \frac{\mathrm{d}G_{\mu\mu}(t)}{\mathrm{d}t} = \frac{N\hbar e}{\mathrm{i}} \delta(t) + 2\frac{\hbar}{\mathrm{i}} k_\mu G^1_{\mu\mu}(t), \tag{20}$$

where $G^1_{\mu\mu}(t)$ is a new Green function defined by

$$G^1_{\mu\mu}(t) = -\frac{\mathrm{i}}{\hbar} \theta(t) \langle [\sum_n \sum_i (\mu_{in}(t) - M_{in}),\ \mu_\nu(0)]\rangle. \tag{21}$$

From the similar calculation the equation for $G^1_{\mu\mu}(t)$ becomes

$$-\frac{\hbar}{\mathrm{i}} \frac{\mathrm{d}G^1_{\mu\mu}(t)}{\mathrm{d}t} = -\frac{\hbar}{\mathrm{i}} \frac{1}{m} G_{\mu\mu}(t). \tag{22}$$

Then, a set of two equations $G_{\mu\mu}(t)$ and $G^1_{\mu\mu}(t)$ is obtained and the exact solutions of these two simultaneous equations can be found. The Fourier component $G_{\mu\mu}(\omega)$ of the Green function $G_{\mu\mu}(t)$ takes the simple form

$$G_{\mu\mu}(\omega) = -\frac{N\hbar e}{\mathrm{i}} \frac{\hbar\omega}{(\hbar\omega)^2 - \dfrac{2\hbar^2}{m}k_\mu}. \qquad (\mu = x,\ y) \tag{23}$$

From eq. (1) the refractive index $n(\omega)$ is found to be

$$\{n(\omega)\}^2 = 1 - \frac{2\pi N\hbar^2 e^2}{Vm}\left\{\frac{1}{(\hbar\omega)^2 - \dfrac{2\hbar^2}{m}k_x} + \frac{1}{(\hbar\omega)^2 - \dfrac{2\hbar^2}{m}k_y}\right\}. \tag{24}$$

The expression $n(\omega)$ is exact one and is independent of the force constant $2k_z$. Furthermore, it should be noted that the electron-electron interaction is exactly ineffective on the refractive index. It is very important that the results are exact in regard to the electron-electron interactions. Similarly, the effect of the electron-electron interaction on the Faraday effect has been discussed in the previous paper[6] and similar results have been obtained.

5. Anharmonic Oscillator Model

To understand contributions of the electron-electron interaction to the refractive index, a system composed of the three-dimensional anharmonic-oscillator electrons is considered as a more complicated model. The Hamiltonian $\mathscr{H}$ of the system is of the form

$$\mathscr{H} = \sum_n\sum_i\sum_\mu \left\{\frac{1}{2m}\boldsymbol{p}_{\mu in}^2 + k_\mu(\mu_{in} - M_{in})^4\right\} + \frac{1}{2}\sum_n\sum_{i\neq j} v_n'(\boldsymbol{r}_{in} - \boldsymbol{r}_{jn}) + \frac{1}{2}\sum_{n\neq m}\sum_i\sum_j V_{nm}(\boldsymbol{r}_{in} - \boldsymbol{r}_{jm}). \tag{25}$$

By calculating in the similar fashion as is seen in the preceding section one may write down a series of equations for the Green functions. The expressions are more complicated than eqs. (20) and (22):

$$-\frac{\hbar}{\mathrm{i}}\frac{\mathrm{d}G_{\mu\mu}(t)}{\mathrm{d}t} = \frac{N\hbar e}{\mathrm{i}}\delta(t) + 4\frac{\hbar}{\mathrm{i}}k_\mu G_{\mu\mu}^1(t), \tag{26}$$

$$-\frac{\hbar}{\mathrm{i}}\frac{\mathrm{d}G_{\mu\mu}^1(t)}{\mathrm{d}t} = 3\frac{\hbar^2}{m}G_{\mu\mu}^{21}(t) - 3\frac{\hbar}{\mathrm{i}}\frac{1}{m}G_{\mu\mu}^{22}(t), \tag{27}$$

$$-\frac{\hbar}{\mathrm{i}}\frac{\mathrm{d}G_{\mu\mu}^{21}(t)}{\mathrm{d}t} = -\frac{\hbar}{\mathrm{i}}\frac{1}{m}G_{\mu\mu}(t), \tag{28}$$

$$-\frac{\hbar}{\mathrm{i}}\frac{\mathrm{d}G_{\mu\mu}^{22}(t)}{\mathrm{d}t} = \frac{N\hbar e}{\mathrm{i}}C_\mu\delta(t) + \frac{\hbar^2}{m}G_{\mu\mu}(t) - 2\frac{\hbar}{\mathrm{i}}\frac{1}{m}G_{\mu\mu}^{321}(t) + 4\frac{\hbar}{\mathrm{i}}k_\mu G_{\mu\mu}^{322}(t) + \frac{1}{2}G_{\mu\mu}^{323}(t) + \frac{1}{2}G_{\mu\mu}^{324}(t), \tag{29}$$

where

$$C_\mu = \langle \frac{1}{N} \sum_n \sum_i (\mu_{in} - M_{in})^2 \rangle, \tag{30}$$

$$G^1_{\mu\mu}(t) = -\frac{\mathrm{i}}{\hbar} \theta(t) \langle [\sum_n \sum_i (\mu_{in}(t) - M_{in})^3,\ \mu_\mu(0)] \rangle, \tag{31}$$

$$G^{21}_{\mu\mu}(t) = -\frac{\mathrm{i}}{\hbar} \theta(t) \langle [\sum_n \sum_i (\mu_{in}(t) - M_{in}),\ \mu_\mu(0)] \rangle, \tag{32}$$

$$G^{22}_{\mu\mu}(t) = -\frac{\mathrm{i}}{\hbar} \theta(t) \langle [\sum_n \sum_i (\mu_{in}(t) - M_{in})^2 p_{\mu in}(t),\ \mu_\mu(0)] \rangle, \tag{33}$$

$$G^{321}_{\mu\mu}(t) = -\frac{\mathrm{i}}{\hbar} \theta(t) \langle [\sum_n \sum_i (\mu_{in}(t) - M_{in}) p_{\mu in}(t) p_{\mu in}(t),\ \mu_\mu(0)] \rangle, \tag{34}$$

$$G^{322}_{\mu\mu}(t) = -\frac{\mathrm{i}}{\hbar} \theta(t) \langle [\sum_n \sum_i (\mu_{in}(t) - M_{in})^5,\ \mu_\mu(0)] \rangle, \tag{35}$$

$$G^{323}_{\mu\mu}(t) = -\frac{\mathrm{i}}{\hbar} \theta(t) \langle [\sum_n \sum_{i \neq j} \sum \{(\mu_{in}(t) - M_{in})^2 - (\mu_{jn}(t) - M_{jn})^2\} \times (p_{\mu in} v'_n(\boldsymbol{r}_{in}(t) - \boldsymbol{r}_{jn}(t))),\ \mu_\mu(0)] \rangle, \tag{36}$$

$$G^{324}_{\mu\mu}(t) = -\frac{\mathrm{i}}{\hbar} \theta(t) \langle [\sum_{n \neq m} \sum \sum_i \sum_j \{(\mu_{in}(t) - M_{in})^2 - (\mu_{jm}(t) - M_{jm})^2\} \times (p_{\mu in} V_{nm}(\boldsymbol{r}_{in}(t) - \boldsymbol{r}_{jm}(t))),\ \mu_\mu(0)] \rangle. \tag{37}$$

The electron-electron interactions appear for the first time only in the Green functions $G^{323}_{\mu\mu}(t)$ and $G^{324}_{\mu\mu}(t)$ on the right-hand side eq. (29) defined by eqs. (36) and (37) with terms of the form $(p_{\mu in} v'_n(\boldsymbol{r}_{in}(t) - \boldsymbol{r}_{jn}(t)))$ and $(p_{\mu in} V_{nm}(\boldsymbol{r}_{in}(t) - \boldsymbol{r}_{jm}(t)))$, respectively.

If a few approximations are made, one may have a solution $G_{\mu\mu}(\omega)$ from eqs. (26)-(29). The Green functions $G^{321}_{\mu\mu}(t)$, $G^{322}_{\mu\mu}(t)$, $G^{323}_{\mu\mu}(t)$ and $G^{324}_{\mu\mu}(t)$ defined by eqs. (34)-(37) may be approximately replaced by

$$G^{321}_{\mu\mu}(t) \approx \langle p^2_{\mu in} \rangle G^{21}_{\mu\mu}(t), \tag{38}$$

$$G^{322}_{\mu\mu}(t) \approx \langle (\mu_{in} - M_{in})^2 \rangle G^1_{\mu\mu}(t), \tag{39}$$

$$G^{323}_{\mu\mu}(t) \approx \langle (\mu_{in} - M_{in}) \sum_j (p_{\mu in}(v'_n(\boldsymbol{r}_{in} - \boldsymbol{r}_{jn}) + v'_n(\boldsymbol{r}_{jn} - \boldsymbol{r}_{in}))) \rangle G^{21}_{\mu\mu}(t), \tag{40}$$

$$G^{324}_{\mu\mu}(t) \approx \langle (\mu_{in} - M_{in}) \sum_j (p_{\mu in}(V_{nm}(\boldsymbol{r}_{in} - \boldsymbol{r}_{jm}) + V_{mn}(\boldsymbol{r}_{jm} - \boldsymbol{r}_{in}))) \rangle G^{21}_{\mu\mu}(t). \tag{41}$$

From these approximations the simultaneous equations for the Green functions become

$$-\frac{\hbar}{\mathrm{i}} \frac{\mathrm{d}G_{\mu\mu}(t)}{\mathrm{d}t} = \frac{N\hbar e}{\mathrm{i}} \delta(t) + 4\frac{\hbar}{\mathrm{i}} k_\mu G^1_{\mu\mu}(t), \tag{42}$$

$$-\frac{\hbar}{\mathrm{i}} \frac{\mathrm{d}G^1_{\mu\mu}(t)}{\mathrm{d}t} = 3\frac{\hbar^2}{m} G^{21}_{\mu\mu}(t) - 3\frac{\hbar}{\mathrm{i}} \frac{1}{m} G^{22}_{\mu\mu}(t), \tag{43}$$

$$-\frac{\hbar}{\mathrm{i}} \frac{\mathrm{d}G^{21}_{\mu\mu}(t)}{\mathrm{d}t} = -\frac{\hbar}{\mathrm{i}} \frac{1}{m} G_{\mu\mu}(t), \tag{44}$$

$$-\frac{\hbar}{\mathrm{i}} \frac{\mathrm{d}G^{22}_{\mu\mu}(t)}{\mathrm{d}t} = \frac{N\hbar e}{\mathrm{i}} C_\mu \delta(t) + \frac{\hbar^2}{m} G_{\mu\mu}(t) + 4\frac{\hbar}{\mathrm{i}} k_\mu C_\mu G^1_{\mu\mu}(t) - \frac{V_\mu}{\mathrm{i}} G^{21}_{\mu\mu}(t), \tag{45}$$

where

$$V_\mu = \frac{2\hbar}{m}\langle p_{\mu in}^2\rangle - \mathrm{i}\langle(\mu_{in} - M_{in}) \times \sum_j (p_{\mu in}(v_n'(\boldsymbol{r}_{in} - \boldsymbol{r}_{jn}) + v_n'(\boldsymbol{r}_{jn} - \boldsymbol{r}_{in}) + V_{nm}(\boldsymbol{r}_{in} - \boldsymbol{r}_{jm}) + V_{mn}(\boldsymbol{r}_{jm} - \boldsymbol{r}_{in})))\rangle. \quad (46)$$

Thus, the Fourier components $G_{\mu\mu}(\omega)$ of $G_{\mu\mu}(t)$ can be solved from eqs. (42)-(45), i. e.

$$G_{\mu\mu}(\omega) = -\frac{N\hbar e}{\mathrm{i}} \frac{(\hbar\omega)^3}{(\hbar\omega)^4 - (\hbar\omega)^2\dfrac{12\hbar^2 k_\mu C_\mu}{m} + (\hbar\omega)\dfrac{24\hbar^4 k_\mu}{m^2} + \dfrac{12\hbar^3 k_\mu V_\mu}{m^2}} \quad (47)$$

and the refractive index $n(\omega)$ takes the form

$$\{n(\omega)\}^2 = 1 - \frac{2\pi N\hbar^2 e^2}{Vm} \times \sum_{\mu=x,y} \frac{(\hbar\omega)^2}{(\hbar\omega)^4 - (\hbar\omega)^2\dfrac{12\hbar^2 k_\mu C_\mu}{m} + (\hbar\omega)\dfrac{24\hbar^4 k_\mu}{m^2} + \dfrac{12\hbar^3 k_\mu V_\mu}{m^2}}. \quad (48)$$

In the result, the expression $n(\omega)$ comes to be independent of the force constant in the direction of the z-axis as is similar to the case of the harmonic oscillators shown in the preceding section. The effects of electron-electron interaction are presented by the last term in denominator on the right-hand side of eq. (48).

6. Summary and Discussion

The refractive index in the absence of a constant magnetic field has been formulated on the basis of the microscopic Maxwell equations and has been discussed by no use of the dielectric constant. It has been shown that the leading term of the refractive index originates in the zeroth order in the wave-number of light and the electron-electron interaction has a little effect on the refractive index. Since the Faraday effect also has these features as is shown in the previous paper,[3] there is a relationship between the refractive index and the Faraday rotational angle, which is a generalized Becquerel formula.[8]

As the simplest model, the system composed of the three-dimensional harmonic-oscillator electrons has been investigated. It should be noted that this harmonic oscillator system has a exact solution and the electron-electron interaction is exactly ineffective on the refractive index.

A more complicated model is the system of the anharmonic-oscillator electrons. To find the effects of the electron-electron interaction on the refractive index, a few approximations for the Green functions have been made. Dispersion of the refractive index expressed in terms of the electron-electron interactions can be obtained and the interaction effects can be also discussed.

Furthermore, it should be noted that, in the cases of the harmonic oscillators and the anharmonic oscillators, the force constant in the direction of the light is ineffective on the refractive index.

The interactions between the electron in a monomer and the nucleus in the other monomer have been neglected in comparison with the electron-electron interactions. The electron-phonon interactions or the exciton-phonon interactions have been also neglected in comparison with the electron-electron interactions as has been discussed in the previous paper.[7]

Finally the theory of the Faraday effect,[3,6] the natural optical activity[4] and the refractive index[5] have been developed on the basis of the microscopic Maxwell equations formerly by us. These three phenomena have different features each other, however, can be discussed from the same theoretical viewpoints by our general theory.[2] By our general treory being different from the conventional ones, various properties of these phenomena can be investigated generally, systematically and directly from the first principle without use of the dielectric constant and the magnetic permeability in the conventional formulation. In regard to the electron-electron interaction, in particular, the interaction effects on these phenomena are very similar each other and there are relations between these phenomena. These relations may be considered to be concerned with the Becquerel formula.[8]

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