

## 複合遊星歯車装置の速比と効率の計算式

第2報(その2), 内歯車式単純遊星歯車装置と複式外歯車遊星歯車装置の組合せからなる複合遊星歯車装置

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(昭和58年5月27日受理)

### Speed-Ratio and Efficiency Formulas for Combined Planetary Gear Systems

2nd Report (Part 2). Combined Planetary Gear Systems Composed of a Simple Planetary Train with a Ring Gear and a Planetary Train with Compound Planets

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The combined-planetary-gear transmission was subjected to an analysis with a view to deriving formulas for speed ratio and efficiency. The analysis has revealed that the transmitted power in the transmission in question may be split or may allow a great circulation to occur, thus leading to a successful formulation. The efficiency formulas obtained require as parameters only two quantities, the number of teeth and the efficiency of stationary gear trains.

### 1 緒 言

前報において内歯車式単純遊星歯車装置と複式外歯車遊星歯車装置の組合せからなる複合遊星歯車装置の速比と効率の理論計算式を与えたが<sup>1)</sup>, その場合, 複式外歯車遊星歯車装置に用いる2ヶの太陽外歯車の歯数間の大小関係とは無関係に, 動力分流型または動力循環型と定まってしまう複合遊星歯車装置について論じた. 本報においては複式外歯車遊星歯車装置に用いられている2ヶの太陽外歯車の歯数間の大小関係により, 動力分流型となったり動力循環型に変ったりする複合遊星歯車装置の速比と効率の理論計算式を求め, 装置の歯車の歯数と基準効率から直ちに装置の効率を計算することができるようにした.

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## 2 複合遊星歯車機構の効率計算式の誘導

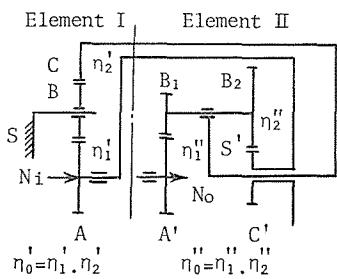


Fig.1 Combined Planetary Gear with Power Circulation

図1に示すごとく、内歯車式単純遊星歯車装置Iと複式外歯車遊星歯車装置II(要素I, IIと呼ぶ)のそれぞれの3本の基本軸のうち2本の基本軸同志を結合し、残りの基本軸の1本を固定した機構を複合遊星歯車機構と呼ぶが、結合させる2本の基本軸の選び方や複式外歯車遊星歯車装置に用いる2ヶの太陽外歯車の歯数間の大小関係により、動力分流型となったり動力循環型になったりする。図1において要素Iと要素IIを連結するのに、外歯太陽歯車AとC'を連結し、内歯太陽歯車CとキャリヤS'を連結し、要素IのキャリヤSを固定して、基本軸Aを入力軸とし、要素IIの基本軸A'を出力軸とするときの効率計算式を誘導する。要素IとIIを切り離したときの要素IとIIの角速度の関係はそれぞれ次式のようになる。

$$\omega_A = \left(1 + \frac{Z_C}{Z_A}\right)\omega_S - \frac{Z_C}{Z_A}\omega_C, \quad (1)$$

$$\omega_{A'} = \left(1 - \frac{Z_{B_1}Z_{C'}}{Z_{B_2}Z_{A'}}\right)\omega_{S'} + \frac{Z_{B_1}Z_{C'}}{Z_{B_2}Z_{A'}}\omega_{C'}. \quad (2)$$

ただし  $Z_A, Z_C, Z_{A'}, Z_{C'}, Z_{B_1}, Z_{B_2}$  はそれぞれ歯車A, C, A', C', B<sub>1</sub>, B<sub>2</sub>の歯数、  $\omega_A, \omega_C, \omega_{A'}, \omega_{C'}$  はそれぞれ歯車A, C, A', C'の角速度、  $\omega_S, \omega_{S'}$  はキャリヤSとS'の角速度とする。図1のように要素IとIIを連結すると  $\omega_A = \omega_{C'}$ ,  $\omega_C = \omega_{S'}$  となり、  $\omega_S = 0$  とすると、式(1)より

$$\omega_C = -\frac{Z_A}{Z_C}\omega_A \quad (3)$$

を得る。これを式(2)に代入すると

$$\omega_{A'} = \frac{Z_{B_1}Z_{C'}(Z_A + Z_C) - Z_{B_2}Z_{A'}Z_A}{Z_{B_2}Z_{A'}Z_C}\omega_A. \quad (4)$$

これより速比uは

$$u = \frac{\omega_{A'}}{\omega_A} = \frac{Z_{B_1}Z_{C'}(Z_A + Z_C) - Z_{B_2}Z_{A'}Z_A}{Z_{B_2}Z_{A'}Z_C} \quad (5)$$

そしてこの場合要素IIは歯車C' とキャリヤS' が入力軸で、歯車A' が出力軸の差動歯車機構である。そこでこの差動歯車機構をつぎのごとき2つの成分の遊星歯車装置に分けて考える、すなわち歯車C'を固定し、キャリヤS'のみの回転によって歯車A'に  $\omega_{A1'}$  の角速度と、No<sub>1</sub>の出力を伝達する第1成分遊星歯車装置と、キャリヤS'を固定し、歯車

$C'$  のみの回転によって歯車  $A'$  に  $\omega_{A2}'$  の角速度と、 $N_{o2}$  の出力を伝達する第2成分遊星歯車装置からなると考える。いま歯車  $A'$  への全出力を  $N_o$ 、出力トルクを  $T_o$  とすると

$$N_o = T_o \omega_A' = T_o (\omega_{A1}' + \omega_{A2}') = N_{o1} + N_{o2}$$

が成立し、これより

$$T_o = \frac{N_o}{\omega_A'} = \frac{N_{o1}}{\omega_{A1}'} = \frac{N_{o2}}{\omega_{A2}'}$$

を得る。これより次式を得る。

$$N_{o1} = \frac{\omega_{A1}'}{\omega_A'} N_o, \quad (6)$$

$$N_{o2} = \frac{\omega_{A2}'}{\omega_A'} N_o. \quad (7)$$

### 第1成分遊星歯車装置

この第1成分では、歯車  $C'$  を固定するから、式(2)より

$$\omega_{A1}' = \frac{Z_{B1}Z_C'Z_A - Z_{B2}Z_A'Z_A}{Z_{B2}Z_A'Z_C} \omega_A. \quad (8)$$

式(6)に式(4)と(8)を代入して次式

$$N_{o1} = \frac{(Z_{B1}Z_C' - Z_{B2}Z_A')Z_A}{Z_{B1}Z_C'(Z_A + Z_C) - Z_{B2}Z_A'Z_A} N_o \quad (9)$$

を得る。

### 第2成分遊星歯車装置

この第2成分では、キャリヤ  $S'$  を固定するから、式(2)より

$$\omega_{A2}' = \frac{Z_{B1}Z_C'}{Z_{B2}Z_A'} \omega_A. \quad (10)$$

式(7)に式(4)と(10)を代入して次式

$$N_{o2} = \frac{Z_{B1}Z_C'Z_C}{Z_{B1}Z_C'(Z_A + Z_C) - Z_{B2}Z_A'Z_A} N_o \quad (11)$$

を得る。

(a)  $Z_A' > Z_C'$  の場合

式(9)と(11)より

$$\frac{N_{o2}}{N_{o1}} = \frac{Z_{B1}Z_C'Z_C}{(Z_{B1}Z_C' - Z_{B2}Z_A')Z_A} < 0$$

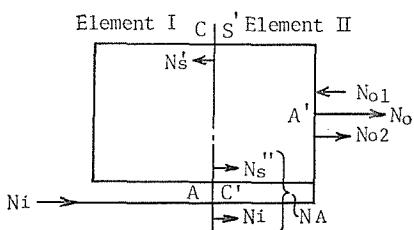


Fig. 2 Power Circulation

となり、式(9)から  $N_{o1} < 0$ 、式(11)から  $N_{o2} > 0$  となるから、要素Ⅱで歯車  $C'$  を固定した第1成分遊星歯車装置において、図2に示すごとく、動力  $(-N_{o1})$  が歯車  $A'$  からキャリヤ  $S'$  に逆に入力として入り、この動力は

$$S' \rightarrow C \rightarrow B \rightarrow A \rightarrow C' \rightarrow B_2 \rightarrow S'$$

の流れ方で動力が循環する。そして  $A'$  から  $S'$  に入いった動力  $(-N_{o1})$  が要素Ⅰの歯車  $C$  に

$N_{S'}$  なる動力となって流れ、これが歯車  $B$ 、 $A$  を経て  $N_{S''}$  となって要素Ⅱの歯車  $C'$  に流れ、この  $N_{S''}$  は、要素Ⅰの歯車  $A$  から  $C'$  に流れる  $N_i$  と合流して  $N_A$  となって流れる。要素Ⅱのキャリヤ  $S'$  を固定した第2成分遊星歯車装置においては、歯車  $C'$  に  $N_A$  が入力として入り、 $A'$  から出力  $N_{o2}$  として出るから次式が成立する。

$$N_A = N_i + N_{S''}, \quad (12)$$

$$N_{S'} = \eta_1(-N_{o1}), \quad (13)$$

$$N_{o2} = \eta_2 N_A, \quad (14)$$

$$N_{S''} = \eta_3 N_{S'}. \quad (15)$$

故に

$$N_i = \frac{N_{o2}}{\eta_2} - \eta_1 \eta_3 (-N_{o1}). \quad (16)$$

ここで  $\eta_1$  は要素Ⅱで歯車  $C'$  を固定し、歯車  $A'$  を入力側、キャリヤ  $S'$  を出力側としたときの効率で

$$\eta_1 = \frac{\eta_o'' - i_o'^{-2}}{\eta_o''(1 - i_o')} . \quad (17)$$

ただし  $0 < i_o' \left( = \frac{Z_{B1} Z_{C'}}{Z_{B2} Z_{A'}} \right) < 1$ .

要素Ⅱの複式遊星歯車装置において、歯車  $A'$  と  $B_1$  の噛合効率を  $\eta_1''$ 、歯車  $C'$  と  $B_2$  の噛合効率を  $\eta_2''$  とすると、キャリヤ  $S'$  を固定したときの要素Ⅱの基準効率  $\eta_o''$  は  $\eta_o'' = \eta_1'' \cdot \eta_2''$ 。つぎに  $\eta_2$  は要素Ⅱでキャリヤ  $S'$  を固定し、歯車  $C'$  を入力側、歯車  $A'$  を出力側としたときの効率で

$$\eta_2 = \eta_o''. \quad (18)$$

$\eta_3$  は要素Ⅰでキャリヤ  $S$  を固定し、歯車  $C$  を入力側、歯車  $A$  を出力側としたときの効率で

$$\eta_3 = \eta_o'. \quad (19)$$

ただし要素 I の単純遊星歯車装置において、歯車 A と B の噛合効率を  $\eta_1'$ 、歯車 B と C との噛合効率を  $\eta_2'$  とすると、キャリヤ S を固定したときの要素 I の基準効率  $\eta_o'$  は  $\eta_o' = \eta_1' \cdot \eta_2'$ 。式(16)に式(9), (11), (17), (18), (19)を代入すれば

$$N_i = \frac{\eta_o' Z_{B1} Z_C' Z_A + Z_{B1} Z_C' Z_C - \eta_o' \eta_o'' Z_{B2} Z_A' Z_A}{\eta_o'' (Z_{B1} Z_C' Z_A + Z_{B1} Z_C' Z_C - Z_{B2} Z_A' Z_A)} N_o. \quad (20)$$

したがって装置全体の効率  $\eta$  は

$$\eta = \frac{N_o}{N_i} = \frac{\eta_o'' (Z_{B1} Z_C' Z_A + Z_{B1} Z_C' Z_C - Z_{B2} Z_A' Z_A)}{\eta_o' Z_{B1} Z_C' Z_A + Z_{B1} Z_C' Z_C - \eta_o' \eta_o'' Z_{B2} Z_A' Z_A}. \quad (21)$$

表 1 の一番上の欄の式が式(21)であり、他の場合についても同様な計算を行なって表 1を得た。

(b)  $Z_A' < Z_C'$  の場合

式(9)と(11)より

$$\frac{N_{o2}}{N_{o1}} = \frac{Z_{B1} Z_C' Z_C}{(Z_{B1} Z_C' - Z_{B2} Z_A') Z_A} > 0.$$

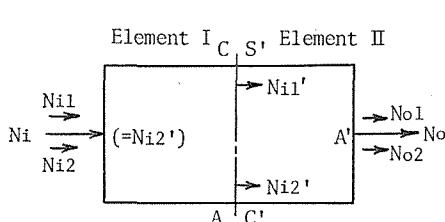


Fig. 3 Power Division

そして式(9)から  $N_{o1} > 0$ 、式(11)から  $N_{o2} > 0$  となるから、この機構では図 3 に示すごとく、動力は分流し、次式が成立する。

$$N_i = N_{i1} + N_{i2}, \quad (22)$$

$$N_{o1} = \eta_1 N_{i1}', \quad (23)$$

$$N_{o2} = \eta_2 N_{i2}', \quad (24)$$

$$N_{i1}' = \eta_3 N_{i1}, \quad (25)$$

$$N_{i2}' = N_{i2}. \quad (26)$$

式(22), (23), (24), (25), (26) から

$$N_i = \frac{N_{o1}}{\eta_1 \eta_3} + \frac{N_{o2}}{\eta_2} \quad (27)$$

を得る。ここで  $\eta_1$  は要素 II で歯車  $C'$  を固定し、キャリヤ  $S'$  を入力側、歯車  $A'$  を出力側としたときの効率で、次式から計算される。

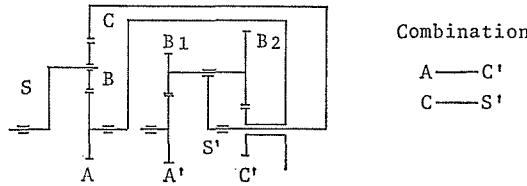
$$\eta_1 = \frac{\eta_o'' (1 - i_o')^2}{\eta_o'' - i_o'}. \quad (28)$$

$$\text{ただし } 1 < i_o' \left( = \frac{Z_{B1} Z_C'}{Z_{B2} Z_A'} \right).$$

$\eta_2$  は要素 II でキャリヤ  $S'$  を固定し、歯車  $C'$  を入力側、歯車  $A'$  を出力側としたときの

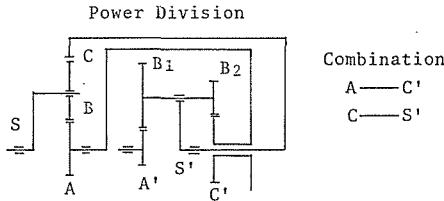
Table 1 Speed Ratio and Efficiency Formula for Combined Planetary Gears with Power Circulation

## Power Circulation



	Driver	Follower	Fixed	Speed Ratio	Efficiency of Combined Planetary Gears
$Z_A > Z_C$	A	A'	S	$U = \frac{\omega_{A'}}{\omega_A} = \frac{Z_{B1}Z_C(Z_A+Z_C) - Z_{B2}Z_AZ_A}{Z_{B2}Z_AZ_C}$	$\eta = \frac{\eta_o(Z_{B1}Z_CZ_A + Z_{B1}Z_CZ_C - Z_{B2}Z_AZ_A)}{\eta_oZ_{B1}Z_CZ_A + Z_{B1}Z_CZ_C - \eta_o\eta_oZ_{B2}Z_AZ_A}$
	C	A'	S	$U = \frac{\omega_{A'}}{\omega_C} = \frac{Z_{B2}Z_AZ_A - Z_{B1}Z_C(Z_A+Z_C)}{Z_{B2}Z_AZ_A}$	$\eta = \frac{\eta_o\eta_o(Z_{B1}Z_CZ_A + Z_{B1}Z_CZ_C - Z_{B2}Z_AZ_A)}{\eta_oZ_{B1}Z_CZ_A + Z_{B1}Z_CZ_C - \eta_o\eta_oZ_{B2}Z_AZ_A}$
	S	C'	A'	$U = \frac{\omega_{C'}}{\omega_S} = \frac{(Z_{B1}Z_C - Z_{B2}Z_A)(Z_A+Z_C)}{Z_{B1}Z_C(Z_A+Z_C) - Z_{B2}Z_AZ_A}$	$\eta = \frac{(Z_A+Z_C)(Z_{B2}Z_A - Z_{B1}Z_C)(\eta_oZ_{B1}Z_CZ_A + \eta_o\eta_oZ_{B1}Z_CZ_C - Z_{B2}Z_AZ_A)}{(Z_A+\eta_oZ_C)(Z_{B2}Z_A - \eta_oZ_{B1}Z_C)(Z_{B1}Z_CZ_A + Z_{B1}Z_CZ_C - Z_{B2}Z_AZ_A)}$
	S	S'	A'	$U = \frac{\omega_{S'}}{\omega_S} = \frac{Z_{B1}Z_C(Z_A+Z_C)}{Z_{B1}Z_C(Z_A+Z_C) - Z_{B2}Z_AZ_A}$	$\eta = \frac{(Z_A+Z_C)(\eta_oZ_{B1}Z_CZ_A + \eta_o\eta_oZ_{B1}Z_CZ_C - Z_{B2}Z_AZ_A)}{\eta_o(Z_A+\eta_oZ_C)(Z_{B1}Z_CZ_A + Z_{B1}Z_CZ_C - Z_{B2}Z_AZ_A)}$
	A'	A	S	$U = \frac{\omega_A}{\omega_{A'}} = \frac{Z_{B2}Z_AZ_C}{Z_{B1}Z_C(Z_A+Z_C) - Z_{B2}Z_AZ_A}$	$\eta = \frac{(Z_{B2}Z_A - \eta_oZ_{B1}Z_C)Z_A - \eta_o\eta_oZ_{B1}Z_CZ_C}{\eta_o(Z_{B2}Z_AZ_A - Z_{B1}Z_CZ_A - Z_{B1}Z_CZ_C)}$
	A'	C	S	$U = \frac{\omega_C}{\omega_{A'}} = \frac{Z_{B2}Z_AZ_A}{Z_{B2}Z_AZ_A - Z_{B1}Z_C(Z_A+Z_C)}$	$\eta = \frac{(Z_{B2}Z_A - \eta_oZ_{B1}Z_C)Z_A - \eta_o\eta_oZ_{B1}Z_CZ_C}{Z_{B2}Z_AZ_A - Z_{B1}Z_CZ_A - Z_{B1}Z_CZ_C}$
	C'	S	A'	$U = \frac{\omega_S}{\omega_C} = \frac{Z_{B1}Z_C(Z_A+Z_C) - Z_{B2}Z_AZ_A}{(Z_{B1}Z_C - Z_{B2}Z_A)(Z_A+Z_C)}$	$\eta = \frac{(\eta_oZ_A + Z_C)(\eta_oZ_{B2}Z_A - Z_{B1}Z_C)\{(Z_{B2}Z_A - Z_{B1}Z_C)Z_A - Z_{B1}Z_CZ_C\}}{(Z_A+Z_C)(Z_{B2}Z_A - Z_{B1}Z_C)\{\eta_o(\eta_oZ_{B2}Z_A - Z_{B1}Z_C)Z_A - Z_{B1}Z_CZ_C\}}$
	S'	S	A'	$U = \frac{\omega_S}{\omega_{S'}} = \frac{Z_{B1}Z_C(Z_A+Z_C) - Z_{B2}Z_AZ_A}{Z_{B1}Z_C(Z_A+Z_C)}$	$\eta = \frac{(\eta_oZ_A + Z_C)(Z_{B2}Z_AZ_A - Z_{B1}Z_CZ_A - Z_{B1}Z_CZ_C)}{(\eta_oZ_A + Z_C)\{\eta_o(\eta_oZ_{B2}Z_A - Z_{B1}Z_C)Z_A - Z_{B1}Z_CZ_C\}}$

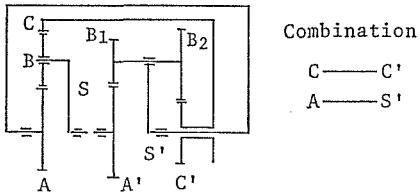
Table 2 Speed Ratio and Efficiency Formula for Combined Planetary Gears with Power Division



	Driver	Follower	Fixed	Speed Ratio	Efficiency of Combined Planetary Gears
$Z_A < Z_C$	A	A'	S	$U = \frac{\omega A'}{\omega A} = \frac{Z_{B1}Z'_C(Z_A+Z_C)-Z_{B2}Z'_A Z_A}{Z_{B2}Z'_A Z_C}$	$\eta = \frac{\eta'_o(Z_{B1}Z'_C Z_A + Z_{B1}Z'_C Z_C - \eta''_o Z_{B2}Z'_A Z_A)}{Z_{B1}Z'_C Z_A + \eta'_o Z_{B1}Z'_C Z_C - \eta''_o Z_{B2}Z'_A Z_A}$
	C	A'	S	$U = \frac{\omega A'}{\omega C} = \frac{Z_{B2}Z'_A Z_A - Z_{B1}Z'_C(Z_A+Z_C)}{Z_{B2}Z'_A Z_A}$	$\eta = \frac{\eta'_o(Z_{B1}Z'_C Z_A + Z_{B1}Z'_C Z_C - \eta''_o Z_{B2}Z'_A Z_A)}{\eta'_o Z_{B1}Z'_C Z_A + Z_{B1}Z'_C Z_C - \eta''_o Z_{B2}Z'_A Z_A}$
	S	C'	A'	$U = \frac{\omega C'}{\omega S} = \frac{(Z_{B1}Z'_C - Z_{B2}Z'_A)(Z_A+Z_C)}{Z_{B1}Z'_C(Z_A+Z_C) - Z_{B2}Z'_A Z_A}$	$\eta = \frac{\eta'_o(Z_A+Z_C)(Z_{B2}Z'_A - Z_{B1}Z'_C)\{(Z_A+\eta'_o Z_C)Z_{B1}Z'_C Z_A + (\eta'_o Z_A+Z_C)Z_{B1}Z'_C Z_C - \eta''_o(Z_A+\eta'_o Z_C)Z_{B2}Z'_A Z_A\}}{(Z_A+\eta'_o Z_C)(\eta'_o Z_A+Z_C)(\eta''_o Z_{B2}Z'_A - Z_{B1}Z'_C)\{(Z_{B1}Z'_C Z_A + Z_{B1}Z'_C Z_C - Z_{B2}Z'_A Z_A)\}}$
	S	S'	A'	$U = \frac{\omega S'}{\omega S} = \frac{Z_{B1}Z'_C(Z_A+Z_C)}{Z_{B1}Z'_C(Z_A+Z_C) - Z_{B2}Z'_A Z_A}$	$\eta = \frac{\eta'_o(Z_A+Z_C)\{(Z_A+\eta'_o Z_C)Z_{B1}Z'_C Z_A + \eta'_o(\eta'_o Z_A+Z_C)Z_{B1}Z'_C Z_C - (Z_A+\eta'_o Z_C)Z_{B2}Z'_A Z_A\}}{\eta''_o(Z_A+\eta'_o Z_C)(\eta'_o Z_A+Z_C)(Z_{B1}Z'_C Z_A + Z_{B1}Z'_C Z_C - Z_{B2}Z'_A Z_A)}$
	A'	A	S	$U = \frac{\omega A}{\omega A'} = \frac{Z_{B2}Z'_A Z_C}{Z_{B1}Z'_C(Z_A+Z_C) - Z_{B2}Z'_A Z_A}$	$\eta = \frac{\eta'_o(Z_{B2}Z'_A - \eta''_o Z_{B1}Z'_C)Z_A - \eta''_o Z_{B1}Z'_C Z_C}{Z_{B2}Z'_A Z_A - Z_{B1}Z'_C Z_A - Z_{B1}Z'_C Z_C}$
	A'	C	S	$U = \frac{\omega C}{\omega A'} = \frac{Z_{B2}Z'_A Z_A}{Z_{B2}Z'_A Z_A - Z_{B1}Z'_C(Z_A+Z_C)}$	$\eta = \frac{\eta'_o(Z_{B2}Z'_A - \eta''_o Z_{B1}Z'_C)Z_A - \eta''_o Z_{B1}Z'_C Z_C}{Z_{B2}Z'_A Z_A - Z_{B1}Z'_C Z_A - Z_{B1}Z'_C Z_C}$
	C'	S	A'	$U = \frac{\omega S}{\omega C'} = \frac{Z_{B1}Z'_C(Z_A+Z_C) - Z_{B2}Z'_A Z_A}{(Z_{B1}Z'_C - Z_{B2}Z'_A)(Z_A+Z_C)}$	$\eta = \frac{(\eta'_o Z_A+Z_C)(Z_A+\eta'_o Z_C)(Z_{B2}Z'_A - \eta''_o Z_{B1}Z'_C)(Z_{B2}Z'_A Z_A - Z_{B1}Z'_C Z_A - Z_{B1}Z'_C Z_C)}{(Z_A+Z_C)(Z_{B2}Z'_A - Z_{B1}Z'_C)\{(\eta'_o Z_A+Z_C)(Z_{B2}Z'_A - \eta''_o Z_{B1}Z'_C)Z_A - \eta''_o(Z_A+\eta'_o Z_C)Z_{B1}Z'_C Z_C\}}$
	S'	S	A'	$U = \frac{\omega S}{\omega S'} = \frac{Z_{B1}Z'_C(Z_A+Z_C) - Z_{B2}Z'_A Z_A}{Z_{B1}Z'_C(Z_A+Z_C)}$	$\eta = \frac{(\eta'_o Z_A+Z_C)(Z_A+\eta'_o Z_C)(Z_{B2}Z'_A Z_A - Z_{B1}Z'_C Z_A - Z_{B1}Z'_C Z_C)}{(Z_A+Z_C)\{(\eta'_o Z_A+Z_C)(\eta''_o Z_{B2}Z'_A - Z_{B1}Z'_C)Z_A - (Z_A+\eta'_o Z_C)Z_{B1}Z'_C Z_C\}}$

Table 3 Speed Ratio and Efficiency Formula for Combined Planetary Gears with Power Circulation

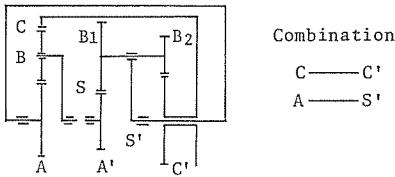
## Power Circulation



Driver	Follower	Fixed	Speed Ratio	Efficiency of Combined Planetary Gears
$Z_A > Z_C$	A	$A'$	$U = \frac{\omega_A}{\omega_{A'}} = \frac{Z_{B2}Z_AZ_C - (Z_A + Z_C)Z_{B1}Z_C}{Z_{B2}Z_AZ_C}$	$n = \frac{n_o(Z_{B1}Z_CZ_A + Z_{B1}Z_CZ_C - Z_{B2}Z_AZ_C)}{Z_{B1}Z_CZ_A + n_oZ_{B1}Z_CZ_C - n_oZ_{B2}Z_AZ_C}$
	C	$A'$	$U = \frac{\omega_A}{\omega_C} = \frac{(Z_A + Z_C)Z_{B1}Z_C - Z_{B2}Z_AZ_C}{Z_{B2}Z_AZ_A}$	$n = \frac{n_o(Z_{B1}Z_CZ_A + Z_{B1}Z_CZ_C - Z_{B2}Z_AZ_C)}{Z_{B1}Z_CZ_A + n_oZ_{B1}Z_CZ_C - n_oZ_{B2}Z_AZ_C}$
	S	$C'$	$U = \frac{\omega_C}{\omega_S} = \frac{(Z_{B1}Z_C - Z_{B2}Z_A)(Z_A + Z_C)}{(Z_A + Z_C)(Z_{B1}Z_C - Z_{B2}Z_AZ_C)}$	$n = \frac{(Z_A + Z_C)(Z_{B2}Z_A - Z_{B1}Z_C)(n_oZ_{B1}Z_CZ_A + n_oZ_{B1}Z_CZ_C - Z_{B2}Z_AZ_C)}{(n_oZ_A + Z_C)(Z_{B2}Z_A - n_oZ_{B1}Z_C)(Z_{B1}Z_CZ_A + Z_{B1}Z_CZ_C - Z_{B2}Z_AZ_C)}$
	S	$S'$	$U = \frac{\omega_S}{\omega_{S'}} = \frac{Z_{B1}Z_C(Z_A + Z_C)}{Z_{B1}Z_C(Z_A + Z_C) - Z_{B2}Z_AZ_C}$	$n = \frac{(Z_A + Z_C)(n_oZ_{B1}Z_CZ_A + n_oZ_{B1}Z_CZ_C - Z_{B2}Z_AZ_C)}{n_o(n_oZ_A + Z_C)(Z_{B1}Z_CZ_A + Z_{B1}Z_CZ_C - Z_{B2}Z_AZ_C)}$
	$A'$	A	$U = \frac{\omega_A}{\omega_{A'}} = \frac{Z_{B2}Z_AZ_C}{Z_{B2}Z_AZ_C - (Z_A + Z_C)Z_{B1}Z_C}$	$n = \frac{n_o(n_oZ_{B2}Z_A - Z_{B1}Z_C)Z_C - Z_{B1}Z_CZ_A}{n_o(n_oZ_{B2}Z_AZ_C - Z_{B1}Z_CZ_C - Z_{B1}Z_CZ_A)}$
	$A'$	C	$U = \frac{\omega_C}{\omega_{A'}} = \frac{Z_{B2}Z_AZ_A}{(Z_A + Z_C)Z_{B1}Z_C - Z_{B2}Z_AZ_C}$	$n = \frac{n_o(n_oZ_{B2}Z_A - Z_{B1}Z_C)Z_C - Z_{B1}Z_CZ_A}{n_o(Z_{B2}Z_AZ_C - Z_{B1}Z_CZ_C - Z_{B1}Z_CZ_A)}$
	$C'$	S	$U = \frac{\omega_S}{\omega_{C'}} = \frac{(Z_A + Z_C)Z_{B1}Z_C - Z_{B2}Z_AZ_C}{(Z_{B1}Z_C - Z_{B2}Z_A)(Z_A + Z_C)}$	$n = \frac{(n_oZ_A + Z_C)(Z_{B2}Z_A - n_oZ_{B1}Z_C)(Z_{B2}Z_AZ_C - Z_{B1}Z_CZ_C - Z_{B1}Z_CZ_A)}{(Z_A + Z_C)(Z_{B2}Z_A - Z_{B1}Z_C)\{(Z_{B2}Z_A - n_oZ_{B1}Z_C)Z_C - n_oZ_{B1}Z_CZ_A\}}$
	$S'$	S	$U = \frac{\omega_S}{\omega_{S'}} = \frac{Z_{B1}Z_C(Z_A + Z_C) - Z_{B2}Z_AZ_C}{Z_{B1}Z_C(Z_A + Z_C)}$	$n = \frac{n_o(n_oZ_A + Z_C)(Z_{B2}Z_AZ_C - Z_{B1}Z_CZ_C - Z_{B1}Z_CZ_A)}{(Z_A + Z_C)\{(Z_{B2}Z_A - n_oZ_{B1}Z_C)Z_C - n_oZ_{B1}Z_CZ_A\}}$

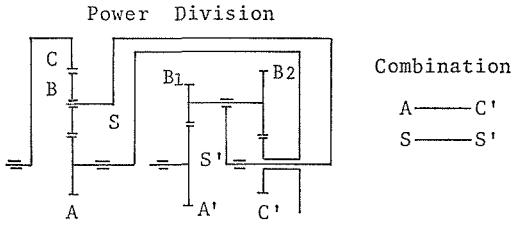
Table4 Speed Ratio and Efficiency Formula for Combined Planetary Gears with Power Division

## Power Division



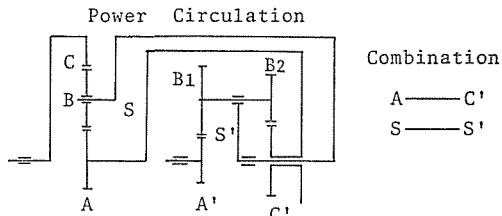
	Driver	Follower	Fixed	Speed Ratio	Efficiency of Combined Planetary Gears
$Z_A < Z_C$	A	A'	S	$U = \frac{\omega A'}{\omega A} = \frac{Z_{B2}Z_AZ_C - (Z_A + Z_C)Z_{B1}Z_C}{Z_{B2}Z_AZ_C}$	$\eta = \frac{\eta_o \eta_o (Z_{B1}Z_C Z_A + Z_{B1}Z_C Z_C - Z_{B2}Z_A Z_C)}{Z_{B1}Z_C Z_A + \eta_o Z_{B1}Z_C Z_C - \eta_o \eta_o Z_{B2}Z_A Z_C}$
	C	A'	S	$U = \frac{\omega A'}{\omega C} = \frac{(Z_A + Z_C)Z_{B1}Z_C - Z_{B2}Z_A Z_C}{Z_{B2}Z_A Z_A}$	$\eta = \frac{\eta_o \eta_o (Z_{B1}Z_C Z_A + Z_{B1}Z_C Z_C - Z_{B2}Z_A Z_C)}{\eta_o Z_{B1}Z_C Z_A + Z_{B1}Z_C Z_C - \eta_o Z_{B2}Z_A Z_C}$
	S	C'	A'	$U = \frac{\omega C'}{\omega S} = \frac{(Z_{B1}Z_C - Z_{B2}Z_A)(Z_A + Z_C)}{(Z_A + Z_C)(Z_{B1}Z_C - Z_{B2}Z_A Z_C)}$	$\eta = \frac{\eta_o (Z_A + Z_C)(Z_{B2}Z_A - Z_{B1}Z_C) \{ (Z_A + \eta_o Z_C)Z_{B1}Z_C Z_A + (\eta_o Z_A + Z_C)(Z_{B1}Z_C Z_C - \eta_o Z_{B2}Z_A Z_C) \}}{(Z_A + \eta_o Z_C)(\eta_o Z_A + Z_C)(\eta_o Z_B Z_A - Z_{B1}Z_C)(Z_{B1}Z_C Z_A + Z_{B1}Z_C Z_C - Z_{B2}Z_A Z_C)}$
	S	S'	A	$U = \frac{\omega S'}{\omega S} = \frac{Z_{B1}Z_C(Z_A + Z_C)}{Z_{B1}Z_C(Z_A + Z_C) - Z_{B2}Z_A Z_C}$	$\eta = \frac{\eta_o (Z_A + Z_C) \{ \eta_o (Z_A + \eta_o Z_C)Z_{B1}Z_C Z_A + (\eta_o Z_A + Z_C)(\eta_o Z_B Z_C - Z_{B2}Z_A Z_C) \}}{\eta_o (Z_A + \eta_o Z_C)(\eta_o Z_A + Z_C)(Z_{B1}Z_C Z_A + Z_{B1}Z_C Z_C - Z_{B2}Z_A Z_C)}$
	A'	A	S	$U = \frac{\omega A}{\omega A'} = \frac{Z_{B2}Z_A Z_C}{Z_{B2}Z_A Z_C - (Z_A + Z_C)Z_{B1}Z_C}$	$\eta = \frac{(Z_{B2}Z_A - \eta_o Z_{B1}Z_C)Z_C - \eta_o \eta_o Z_{B1}Z_C Z_A}{Z_{B2}Z_A Z_C - Z_{B1}Z_C Z_C - Z_{B1}Z_C Z_A}$
	A'	C	S	$U = \frac{\omega C}{\omega A'} = \frac{Z_{B2}Z_A Z_A}{(Z_A + Z_C)Z_{B1}Z_C - Z_{B2}Z_A Z_C}$	$\eta = \frac{\eta_o (Z_{B2}Z_A - \eta_o Z_{B1}Z_C)Z_C - \eta_o Z_{B1}Z_C Z_A}{Z_{B2}Z_A Z_C - Z_{B1}Z_C Z_C - Z_{B1}Z_C Z_A}$
	C'	S	A'	$U = \frac{\omega S}{\omega C'} = \frac{(Z_A + Z_C)Z_{B1}Z_C - Z_{B2}Z_A Z_C}{(Z_{B1}Z_C - Z_{B2}Z_A)(Z_A + Z_C)}$	$\eta = \frac{(\eta_o Z_A + Z_C)(Z_A + \eta_o Z_C)(Z_{B2}Z_A - \eta_o Z_{B1}Z_C)(Z_{B2}Z_A Z_C - Z_{B1}Z_C Z_C - Z_{B1}Z_C Z_A)}{(Z_A + Z_C)(Z_{B2}Z_A - Z_{B1}Z_C) \{ (Z_A + \eta_o Z_C)(Z_{B2}Z_A - \eta_o Z_{B1}Z_C)Z_C - \eta_o (\eta_o Z_A + Z_C)Z_{B1}Z_C Z_A \}}$
	S'	S	A'	$U = \frac{\omega S}{\omega S'} = \frac{Z_{B1}Z_C(Z_A + Z_C) - Z_{B2}Z_A Z_C}{Z_{B1}Z_C(Z_A + Z_C)}$	$\eta = \frac{(\eta_o Z_A + Z_C)(Z_A + \eta_o Z_C)(Z_{B2}Z_A Z_C - Z_{B1}Z_C Z_C - Z_{B1}Z_C Z_A)}{(Z_A + Z_C) \{ (Z_A + \eta_o Z_C)(\eta_o Z_B Z_A - Z_{B1}Z_C)Z_C - (\eta_o Z_A + Z_C)Z_{B1}Z_C Z_A \}}$

Table 5 Speed Ratio and Efficiency Formula for Combined Planetary Gears with Power Division



	Driver	Follower	Fixed	Speed Ratio	Efficiency of Combined Planetary Gears
Z <sub>A</sub> '>Z <sub>C</sub> '	A	A'	C	$U = \frac{\omega A'}{\omega A} = \frac{Z_{B2}Z_AZ_A + Z_{B1}Z_CZ_C}{Z_{B2}Z_A'(Z_A + Z_C)}$	$\eta = \frac{\eta_o(Z_A + \eta_oZ_C)(Z_{B2}Z_AZ_A + Z_{B1}Z_CZ_C)}{(Z_A + Z_C)\{\eta_oZ_{B2}Z_AZ_A + (1 - \eta_o^2)Z_{B1}Z_CZ_A + \eta_oZ_{B1}Z_CZ_C\}}$
	S	A'	C	$U = \frac{\omega A'}{\omega S} = \frac{Z_{B2}Z_AZ_A + Z_{B1}Z_CZ_C}{Z_{B2}Z_AZ_A}$	$\eta = \frac{\eta_o\eta_o(Z_{B2}Z_AZ_A + Z_{B1}Z_CZ_C)}{\eta_o\eta_oZ_{B2}Z_AZ_A + \eta_o(1 - \eta_o^2)Z_{B1}Z_CZ_A + Z_{B1}Z_CZ_C}$
	C	C'	A'	$U = \frac{\omega C'}{\omega C} = \frac{(Z_{B1}Z_C' - Z_{B2}Z_A')Z_C}{Z_{B2}Z_AZ_A + Z_{B1}Z_CZ_C}$	$\eta = \frac{(Z_{B2}Z_A' - Z_{B1}Z_C')(Z_BZ_2Z_A + \eta_oZ_{B1}Z_CZ_C)}{(Z_{B2}Z_A' - \eta_oZ_{B1}Z_C')(Z_{B2}Z_AZ_A + Z_{B1}Z_CZ_C)}$
	C	S'	A'	$U = \frac{\omega S'}{\omega C} = \frac{Z_{B1}Z_CZ_C}{Z_{B2}Z_AZ_A + Z_{B1}Z_CZ_C}$	$\eta = \frac{\eta_o\eta_oZ_{B2}Z_AZ_A + Z_{B1}Z_CZ_C}{Z_{B2}Z_AZ_A + Z_{B1}Z_CZ_C}$
	A'	A	C	$U = \frac{\omega A}{\omega A'} = \frac{Z_{B2}Z_A'(Z_A + Z_C)}{Z_{B2}Z_AZ_A + Z_{B1}Z_CZ_C}$	$\eta = \frac{(Z_A + Z_C)\{\eta_o(\eta_oZ_{B2}Z_A - Z_{B1}Z_C)Z_A + (\eta_o)^2(\eta_oZ_A + Z_C)Z_{B1}Z_C\}}{\eta_o(\eta_oZ_A + Z_C)(Z_{B2}Z_AZ_A + Z_{B1}Z_CZ_C)}$
	A'	S	C	$U = \frac{\omega S}{\omega A'} = \frac{Z_{B2}Z_AZ_A}{Z_{B2}Z_AZ_A + Z_{B1}Z_CZ_C}$	$\eta = \frac{(\eta_oZ_{B2}Z_A - Z_{B1}Z_C)Z_A + (\eta_o)^2(Z_A + \eta_oZ_C)Z_{B1}Z_C}{\eta_o(Z_{B2}Z_AZ_A + Z_{B1}Z_CZ_C)}$
	C'	C	A'	$U = \frac{\omega C}{\omega C'} = \frac{Z_{B2}Z_AZ_A + Z_{B1}Z_CZ_C}{(Z_{B1}Z_C' - Z_{B2}Z_A')Z_C}$	$\eta = \frac{\eta_o(\eta_oZ_{B2}Z_A - Z_{B1}Z_C)(Z_{B2}Z_AZ_A + Z_{B1}Z_CZ_C)}{(Z_{B2}Z_A' - Z_{B1}Z_C)(\eta_oZ_{B2}Z_AZ_A + \eta_oZ_{B1}Z_CZ_C)}$
	S'	C	A'	$U = \frac{\omega C}{\omega S'} = \frac{Z_{B2}Z_AZ_A + Z_{B1}Z_CZ_C}{Z_{B1}Z_CZ_C}$	$\eta = \frac{\eta_o\eta_o(Z_{B2}Z_AZ_A + Z_{B1}Z_CZ_C)}{\eta_o\eta_oZ_{B1}Z_CZ_C + Z_{B2}Z_AZ_A}$

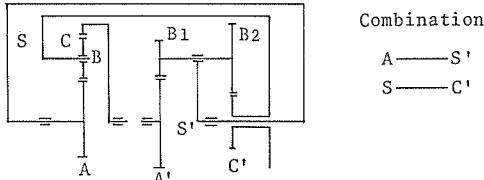
Table 6 Speed Ratio and Efficiency Formula for Combined Planetary Gears with Power Circulation



	Driver	Follower	Fixed	Speed Ratio	Efficiency of Combined Planetary Gears
$Z_A < Z_C$	A	A'	C	$U = \frac{\omega_A' Z_{B2} Z_A^C Z_A + Z_{B1} Z_C^C Z_C}{\omega_A Z_{B2} Z_A^C (Z_A + Z_C)}$	$\eta = \frac{n_o^n (n_o^o Z_A + Z_C) (Z_{B2} Z_A^C Z_A + Z_{B1} Z_C^C Z_C)}{(Z_A + Z_C) \{ n_o^n Z_{B2} Z_A^C Z_A + n_o^o (1 - n_o^{o2}) Z_{B1} Z_C^C Z_A + Z_{B1} Z_C^C Z_C \}}$
	S	A'	C	$U = \frac{\omega_A' Z_{B2} Z_A^C Z_A + Z_{B1} Z_C^C Z_C}{\omega_S Z_{B2} Z_A^C Z_A}$	$\eta = \frac{n_o^n (Z_{B2} Z_A^C Z_A + Z_{B1} Z_C^C Z_C)}{n_o^n Z_{B2} Z_A^C Z_A + n_o^o (1 - n_o^{o2}) Z_{B1} Z_C^C Z_A + Z_{B1} Z_C^C Z_C}$
	C	C'	A'	$U = \frac{\omega_C' (Z_{B1} Z_C^C - Z_{B2} Z_A^C) Z_C}{\omega_C Z_{B2} Z_A^C Z_A + Z_{B1} Z_C^C Z_C}$	$\eta = \frac{(Z_{B2} Z_A^C - Z_{B1} Z_C^C) \{ n_o^n Z_{B2} Z_A^C Z_A + (n_o^{o2} - 1) Z_{B1} Z_C^C Z_A + n_o^o Z_{B1} Z_C^C Z_C \}}{n_o^o (n_o^n Z_{B2} Z_A^C - Z_{B1} Z_C^C) (Z_{B2} Z_A^C Z_A + Z_{B1} Z_C^C Z_C)}$
	C	S'	A'	$U = \frac{\omega_S' Z_{B1} Z_C^C Z_C}{\omega_C Z_{B2} Z_A^C Z_A + Z_{B1} Z_C^C Z_C}$	$\eta = \frac{n_o^n Z_{B2} Z_A^C Z_A + (n_o^{o2} - 1) Z_{B1} Z_C^C Z_A + n_o^o Z_{B1} Z_C^C Z_C}{Z_{B2} Z_A^C Z_A + Z_{B1} Z_C^C Z_C}$
	A'	A	C	$U = \frac{\omega_A Z_{B2} Z_A^C (Z_A + Z_C)}{\omega_A' Z_{B2} Z_A^C Z_A + Z_{B1} Z_C^C Z_C}$	$\eta = \frac{(Z_A + Z_C) \{ n_o^{o2} (Z_A + n_o^o Z_C) Z_{B1} Z_C^C + (n_o^n Z_{B2} Z_A^C - Z_{B1} Z_C^C) Z_A \}}{n_o^n (Z_A + n_o^o Z_C) (Z_{B2} Z_A^C Z_A + Z_{B1} Z_C^C Z_C)}$
	A'	S	C	$U = \frac{\omega_S Z_{B2} Z_A^C Z_A}{\omega_A' Z_{B2} Z_A^C Z_A + Z_{B1} Z_C^C Z_C}$	$\eta = \frac{n_o^{o2} (Z_A + n_o^o Z_C) Z_{B1} Z_C^C + (n_o^n Z_{B2} Z_A^C - Z_{B1} Z_C^C) Z_A}{n_o^n (Z_{B2} Z_A^C Z_A + Z_{B1} Z_C^C Z_C)}$
	C'	C	A'	$U = \frac{\omega_C Z_{B2} Z_A^C Z_A + Z_{B1} Z_C^C Z_C}{\omega_C' (Z_{B1} Z_C^C - Z_{B2} Z_A^C) Z_C}$	$\eta = \frac{n_o^o (Z_{B2} Z_A^C - n_o^n Z_{B1} Z_C^C) (Z_{B2} Z_A^C Z_A + Z_{B1} Z_C^C Z_C)}{(Z_{B2} Z_A^C - Z_{B1} Z_C^C) \{ n_o^n (Z_A + n_o^o Z_C) Z_{B1} Z_C^C + n_o^{o2} (Z_{B2} Z_A^C - n_o^n Z_{B1} Z_C^C) Z_A \}}$
	S'	C	A'	$U = \frac{\omega_C Z_{B2} Z_A^C Z_A + Z_{B1} Z_C^C Z_C}{\omega_S' Z_{B1} Z_C^C Z_C}$	$\eta = \frac{n_o^n (Z_{B2} Z_A^C Z_A + Z_{B1} Z_C^C Z_C)}{n_o^n (Z_A + n_o^o Z_C) Z_{B1} Z_C^C + n_o^{o2} (Z_{B2} Z_A^C - n_o^n Z_{B1} Z_C^C) Z_A}$

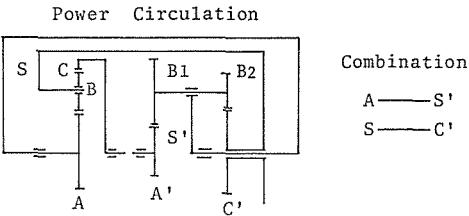
Table 7 Speed Ratio and Efficiency Formula for Combined Planetary Gears with Power Division

## Power Division



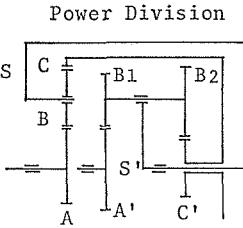
Driver	Follower	Fixed	Speed Ratio	Efficiency of Combined Planetary Gears
$Z_A > Z_C$	A	$A'$	$C$	$\eta = \frac{\eta_o(Z_A + \eta_o Z_C)(Z_B2Z_A - Z_B2Z_C - Z_B1Z_C)}{(Z_A + Z_C)\{\eta_o Z_B2Z_A + \eta_o Z_B2Z_C + (1 - \eta_o^2)Z_B1Z_C\} - \eta_o \eta_o^2 Z_B1Z_C}$
	S	$A'$	$C$	$\eta = \frac{\eta_o \eta_o^o (Z_B2Z_A - Z_B1Z_C)}{\eta_o \eta_o^o Z_B2Z_A + \eta_o^o Z_B2Z_C + \eta_o^o (1 - \eta_o^2)Z_B1Z_C - \eta_o^o Z_B1Z_C}$
	C	$C'$	$A'$	$\eta = \frac{(Z_B2Z_A - Z_B1Z_C)(\eta_o Z_B2Z_A + Z_B2Z_C - \eta_o Z_B1Z_C)}{(Z_B2Z_A - \eta_o Z_B1Z_C)(Z_B2Z_A + Z_B2Z_C - Z_B1Z_C)}$
	C	$S'$	$A'$	$\eta = \frac{\eta_o \eta_o^o Z_B2Z_A + \eta_o^o Z_B2Z_C - Z_B1Z_C}{Z_B2Z_A + Z_B2Z_C - Z_B1Z_C}$
	$A'$	A	$C$	$\eta = \frac{(Z_A + Z_C)\{(\eta_o Z_A + Z_C)(\eta_o Z_B2Z_A - Z_B1Z_C) + \eta_o^2 \eta_o Z_B1Z_C\}}{\eta_o^2 (\eta_o Z_A + Z_C)(Z_B2Z_A + Z_B2Z_C - Z_B1Z_C)}$
	$A'$	S	$C$	$\eta = \frac{(Z_A + \eta_o Z_C)(\eta_o Z_B2Z_A - Z_B1Z_C) + (\eta_o)^2 Z_B1Z_C}{\eta_o^2 (Z_B2Z_A + Z_B2Z_C - Z_B1Z_C)}$
	$C'$	C	$A'$	$\eta = \frac{\eta_o^o (\eta_o Z_B2Z_A - Z_B1Z_C)(Z_B2Z_A + Z_B2Z_C - Z_B1Z_C)}{(Z_B2Z_A - Z_B1Z_C)\{(\eta_o Z_A + \eta_o Z_C)(\eta_o Z_B2Z_A - Z_B1Z_C) + Z_B1Z_C\}}$
	$S'$	C	$A'$	$\eta = \frac{\eta_o \eta_o^o (Z_B2Z_A + Z_B2Z_C - Z_B1Z_C)}{\eta_o^o (Z_B2Z_A - \eta_o Z_B1Z_C)Z_C + Z_B2Z_A}$

Table 8 Speed Ratio and Efficiency Formula for Combined Planetary Gears with Power Circulation



Driver	Follower	Fixed	Speed Ratio	Efficiency of Combined Planetary Gears
ZA < ZC	A	A'	C	$\eta = \frac{\omega_A' Z_{B2} Z_A (Z_A + Z_C) - Z_{B1} Z_C' Z_C}{\omega_A Z_{B2} Z_A (Z_A + Z_C)}$
	S	A'	C	$\eta = \frac{\omega_A' Z_{B2} Z_A (Z_A + Z_C) - Z_{B1} Z_C' Z_C}{\omega_S Z_{B2} Z_A (Z_A + Z_C)}$
	C	C'	A'	$\eta = \frac{\omega_C' (Z_{B2} Z_A - Z_{B1} Z_C') Z_C}{\omega_C Z_{B2} Z_A (Z_A + Z_C) - Z_{B1} Z_C' Z_C}$
	C	S'	A'	$\eta = \frac{\omega_S' Z_{B1} Z_C' Z_C}{\omega_C Z_{B1} Z_C' Z_C - Z_{B2} Z_A (Z_A + Z_C)}$
	A'	A	C	$\eta = \frac{\omega_A Z_{B2} Z_A (Z_A + Z_C)}{\omega_A' Z_{B2} Z_A (Z_A + Z_C) - Z_{B1} Z_C' Z_C}$
	A'	S	C	$\eta = \frac{\omega_S Z_{B2} Z_A (Z_A + Z_C)}{\omega_A' Z_{B2} Z_A (Z_A + Z_C) - Z_{B1} Z_C' Z_C}$
	C'	C	A'	$\eta = \frac{\omega_C Z_{B2} Z_A (Z_A + Z_C) - Z_{B1} Z_C' Z_C}{\omega_C' (Z_{B2} Z_A - Z_{B1} Z_C') Z_C}$
	S'	C	A'	$\eta = \frac{\omega_C Z_{B1} Z_C' Z_C - Z_{B2} Z_A (Z_A + Z_C)}{\omega_S' Z_{B1} Z_C' Z_C}$

Table 9 Speed Ratio and Efficiency Formula for Combined Planetary Gears with Power Division

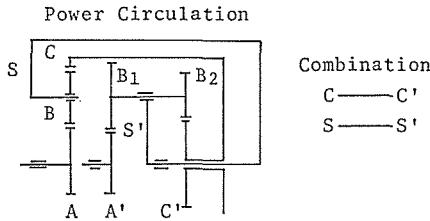


Combination

$C \longrightarrow C'$   
 $S \longrightarrow S'$

	Driver	Follower	Fixed	Speed Ratio	Efficiency of Combined Planetary Gears
$Z_A > Z_C$	C	A'	A	$U = \frac{\omega_{A'} (Z_B 2 Z_A Z_C + Z_B 1 Z_C' Z_A)}{\omega_C (Z_B 2 Z_A (Z_A + Z_C))}$	$\eta = \frac{\eta_o (Z_A + Z_C) (Z_B 2 Z_A Z_C + Z_B 1 Z_C' Z_A)}{(Z_A + Z_C) \{ \eta_o Z_B 2 Z_A Z_C + (1 - \eta_o^2) Z_B 1 Z_C' Z_C + \eta_o Z_B 1 Z_C' Z_A \}}$
	S	A'	A	$U = \frac{\omega_{A'} (Z_B 2 Z_A' Z_C + Z_B 1 Z_C' Z_A)}{\omega_S (Z_B 2 Z_A Z_C)}$	$\eta = \frac{\eta_o \eta_o (Z_B 2 Z_A' Z_C + Z_B 1 Z_C' Z_A)}{\eta_o \eta_o Z_B 2 Z_A Z_C + \eta_o (1 - \eta_o^2) Z_B 1 Z_C' Z_C + Z_B 2 Z_A' Z_A}$
	A	C'	A'	$U = \frac{\omega_{C'} (Z_B 1 Z_C' - Z_B 2 Z_A') Z_A}{\omega_A (Z_B 2 Z_A' Z_C + Z_B 1 Z_C' Z_A)}$	$\eta = \frac{(Z_B 2 Z_A' - Z_B 1 Z_C') (Z_A + Z_B 2 Z_A' Z_C + \eta_o Z_B 1 Z_C' Z_A)}{(Z_B 2 Z_A' - \eta_o Z_B 1 Z_C') (Z_B 2 Z_A' Z_C + Z_B 1 Z_C' Z_A)}$
	A	S'	A	$U = \frac{\omega_{S'} (Z_B 1 Z_C' Z_A)}{\omega_A (Z_B 2 Z_A' Z_C + Z_B 1 Z_C' Z_A)}$	$\eta = \frac{\eta_o \eta_o (Z_B 2 Z_A' Z_C + Z_B 1 Z_C' Z_A)}{Z_B 2 Z_A' Z_C + Z_B 1 Z_C' Z_A}$
	A'	C	A	$U = \frac{\omega_C (Z_B 2 Z_A' (Z_A + Z_C))}{\omega_{A'} (Z_B 2 Z_A' Z_C + Z_B 1 Z_C' Z_A)}$	$\eta = \frac{(Z_A + Z_C) \{ \eta_o (Z_B 2 Z_A' - Z_B 1 Z_C') Z_C + \eta_o^2 (Z_A + \eta_o Z_C) Z_B 1 Z_C' \}}{\eta_o (Z_A + \eta_o Z_C) (Z_B 2 Z_A' Z_C + Z_B 1 Z_C' Z_A)}$
	A'	S	A	$U = \frac{\omega_S (Z_B 2 Z_A' Z_C)}{\omega_{A'} (Z_B 2 Z_A' Z_C + Z_B 1 Z_C' Z_A)}$	$\eta = \frac{(\eta_o Z_B 2 Z_A' - Z_B 1 Z_C') Z_C + (\eta_o)^2 (Z_A + Z_C) Z_B 1 Z_C'}{\eta_o (Z_B 2 Z_A' Z_C + Z_B 1 Z_C' Z_A)}$
	C'	A	A'	$U = \frac{\omega_A (Z_B 2 Z_A' Z_C + Z_B 1 Z_C' Z_A)}{\omega_{C'} (Z_B 1 Z_C' - Z_B 2 Z_A') Z_A}$	$\eta = \frac{\eta_o (\eta_o Z_B 2 Z_A' - Z_B 1 Z_C') (Z_B 2 Z_A' Z_C + Z_B 1 Z_C' Z_A)}{(Z_B 2 Z_A' - Z_B 1 Z_C') (\eta_o Z_B 2 Z_A' Z_C + \eta_o Z_B 1 Z_C' Z_A)}$
	S'	A	A'	$U = \frac{\omega_A (Z_B 2 Z_A' Z_C + Z_B 1 Z_C' Z_A)}{\omega_{S'} (Z_B 1 Z_C' Z_A)}$	$\eta = \frac{\eta_o \eta_o (Z_B 2 Z_A' Z_C + Z_B 1 Z_C' Z_A)}{\eta_o \eta_o Z_B 1 Z_C' Z_A + Z_B 2 Z_A' Z_A}$

Table 10 Speed Ratio and Efficiency Formula for Combined Planetary Gears with Power Circulation



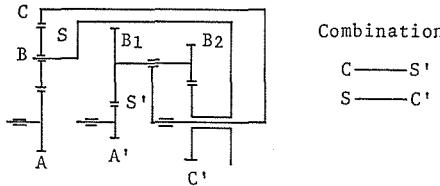
	Driver	Follower	Fixed	Speed Ratio	Efficiency of Combined Planetary Gears
$Z_A < Z_C$	C	A'	A	$U = \frac{\omega_A'}{\omega_C} = \frac{Z_B 2 Z_A' Z_C + Z_B 1 Z_C' Z_A}{Z_B 2 Z_A (Z_A + Z_C)}$	$\eta = \frac{\eta_0'' (Z_A + \eta_0' Z_C) (Z_B 2 Z_A' Z_C + Z_B 1 Z_C' Z_A)}{(Z_A + Z_C) \{ \eta_0' \eta_0'' Z_B 2 Z_A' Z_C + \eta_0' (1 - \eta_0''^2) Z_B 1 Z_C' Z_C + Z_B 1 Z_C' Z_A \}}$
	S	A'	A	$U = \frac{\omega_A'}{\omega_S} = \frac{Z_B 2 Z_A' Z_C + Z_B 1 Z_C' Z_A}{Z_B 2 Z_A' Z_C}$	$\eta = \frac{\eta_0'' (Z_B 2 Z_A' Z_C + Z_B 1 Z_C' Z_A)}{\eta_0' \eta_0'' Z_B 2 Z_A' Z_C + \eta_0' (1 - \eta_0''^2) Z_B 1 Z_C' Z_C + Z_B 1 Z_C' Z_A}$
	A	C'	A'	$U = \frac{\omega_C'}{\omega_A} = \frac{Z_B 2 Z_A' (Z_B 1 Z_C - Z_B 2 Z_A) Z_A}{Z_B 2 Z_A' Z_C + Z_B 1 Z_C' Z_A}$	$\eta = \frac{(Z_B 2 Z_A' - Z_B 1 Z_C) \{ \eta_0'' Z_B 2 Z_A' Z_C + (\eta_0' - 1) Z_B 1 Z_C' Z_C + \eta_0' Z_B 1 Z_C' Z_A \}}{\eta_0' ( \eta_0'' Z_B 2 Z_A' - Z_B 1 Z_C ) (Z_B 2 Z_A' Z_C + Z_B 1 Z_C' Z_A)}$
	A	S'	A'	$U = \frac{\omega_{S'}}{\omega_A} = \frac{Z_B 1 Z_C' Z_A}{Z_B 2 Z_A' Z_C + Z_B 1 Z_C' Z_A}$	$\eta = \frac{\eta_0'' Z_B 2 Z_A' Z_C + (\eta_0' - 1) Z_B 1 Z_C' Z_C + \eta_0' Z_B 1 Z_C' Z_A}{\eta_0' (Z_B 2 Z_A' Z_C + Z_B 1 Z_C' Z_A)}$
	A'	C	A	$U = \frac{\omega_C}{\omega_{A'}} = \frac{Z_B 2 Z_A' (Z_A + Z_C)}{Z_B 2 Z_A' Z_C + Z_B 1 Z_C' Z_A}$	$\eta = \frac{(Z_A + Z_C) \{ (\eta_0')^2 (\eta_0' Z_A + Z_C) Z_B 1 Z_C' + (\eta_0'' Z_B 2 Z_A' - Z_B 1 Z_C) Z_C \}}{\eta_0' (\eta_0' Z_A + Z_C) Z_B 2 Z_A' Z_C + Z_B 1 Z_C' Z_A}$
	A'	S	A	$U = \frac{\omega_S}{\omega_{A'}} = \frac{Z_B 2 Z_A' Z_C}{Z_B 2 Z_A' Z_C + Z_B 1 Z_C' Z_A}$	$\eta = \frac{\eta_0'^2 (\eta_0' Z_A + Z_C) Z_B 1 Z_C' + (\eta_0'' Z_B 2 Z_A' - Z_B 1 Z_C) Z_C}{\eta_0' (Z_B 2 Z_A' Z_C + Z_B 1 Z_C' Z_A)}$
	C'	A	A'	$U = \frac{\omega_A}{\omega_{C'}} = \frac{Z_B 2 Z_A' Z_C + Z_B 1 Z_C' Z_A}{(Z_B 1 Z_C - Z_B 2 Z_A) Z_A}$	$\eta = \frac{\eta_0' (Z_B 2 Z_A' - \eta_0'' Z_B 1 Z_C) (Z_B 2 Z_A' Z_C + Z_B 1 Z_C' Z_A)}{(Z_B 2 Z_A' - Z_B 1 Z_C) \{ \eta_0' (\eta_0' Z_A + Z_C) Z_B 1 Z_C' + \eta_0'^2 (Z_B 2 Z_A' - \eta_0'' Z_B 2 Z_C) Z_C \}}$
	S'	A	A'	$U = \frac{\omega_A}{\omega_{S'}} = \frac{Z_B 2 Z_A' Z_C + Z_B 1 Z_C' Z_A}{Z_B 1 Z_C' Z_A}$	$\eta = \frac{\eta_0'' (Z_B 2 Z_A' Z_C + Z_B 1 Z_C' Z_A)}{\eta_0' (\eta_0' Z_A + Z_C) Z_B 1 Z_C' + (\eta_0')^2 (Z_B 2 Z_A' - \eta_0'' Z_B 1 Z_C) Z_C}$

Table 11 Speed Ratio and Efficiency Formula for Combined Planetary Gears with Power Division

Power Division					Combination
	Driver	Follower	Fixed	Speed Ratio	Efficiency of Combined Planetary Gears
$Z_A > Z_C$	C	A'	A	$U = \frac{\omega A' Z_{B2} Z_A (Z_A + Z_C) - Z_{B1} Z_C Z_A}{\omega C Z_{B2} Z_A (Z_A + Z_C)}$	$\eta = \frac{n_o (n_o Z_A + Z_C) (Z_{B2} Z_A Z_A + Z_{B2} Z_A Z_C - Z_{B1} Z_C Z_A)}{(Z_A + Z_C) \{ n_o n_o Z_{B2} Z_A Z_A + n_o Z_{B2} Z_A Z_C + (1 - n_o^2) Z_{B1} Z_C Z_C - n_o n_o^2 Z_{B1} Z_C Z_A \}}$
	S	A'	A	$U = \frac{\omega A' Z_{B2} Z_A (Z_A + Z_C) - Z_{B1} Z_C Z_A}{\omega S Z_{B2} Z_A Z_C}$	$\eta = \frac{n_o n_o (Z_{B2} Z_A Z_A + Z_{B2} Z_A Z_C - Z_{B1} Z_C Z_A)}{n_o n_o Z_{B2} Z_A Z_A + n_o n_o Z_{B2} Z_A Z_C + n_o (1 - n_o^2) Z_{B1} Z_C Z_C - n_o^2 Z_{B1} Z_C Z_A}$
	A	C'	A'	$U = \frac{\omega C' (Z_{B2} Z_A - Z_{B1} Z_C) Z_A}{\omega A Z_{B2} Z_A (Z_A + Z_C) - Z_{B1} Z_C Z_A}$	$\eta = \frac{(Z_{B2} Z_A - Z_{B1} Z_C) (Z_{B2} Z_A Z_A + n_o Z_{B2} Z_A Z_C - n_o^2 Z_{B1} Z_C Z_A)}{(Z_{B2} Z_A - n_o Z_{B1} Z_C) (Z_{B2} Z_A Z_A + Z_{B2} Z_A Z_C - Z_{B1} Z_C Z_A)}$
	A	S'	A'	$U = \frac{\omega S' Z_{B1} Z_C Z_A}{\omega A Z_{B1} Z_C Z_A - Z_{B2} Z_A (Z_A + Z_C)}$	$\eta = \frac{n_o Z_{B2} Z_A Z_A + n_o n_o Z_{B2} Z_A Z_C - Z_{B1} Z_C Z_A}{Z_{B2} Z_A Z_A + Z_{B2} Z_A Z_C - Z_{B1} Z_C Z_A}$
	A'	C	A	$U = \frac{\omega C Z_{B2} Z_A (Z_A + Z_C)}{\omega A' Z_{B2} Z_A (Z_A + Z_C) - Z_{B1} Z_C Z_A}$	$\eta = \frac{(Z_A + Z_C) \{ (Z_A + n_o Z_C) (n_o Z_{B2} Z_A - Z_{B1} Z_C) + n_o n_o^2 Z_{B1} Z_C Z_C \}}{n_o (Z_A + n_o Z_C) (Z_{B2} Z_A Z_A + Z_{B2} Z_A Z_C - Z_{B1} Z_C Z_A)}$
	A'	S	A	$U = \frac{\omega S Z_{B2} Z_A Z_C}{\omega A' Z_{B2} Z_A (Z_A + Z_C) - Z_{B1} Z_C Z_A}$	$\eta = \frac{(n_o Z_A + Z_C) (n_o Z_{B2} Z_A - Z_{B1} Z_C) + n_o^2 Z_{B1} Z_C Z_C}{n_o (Z_{B2} Z_A Z_A + Z_{B2} Z_A Z_C - Z_{B1} Z_C Z_A)}$
	C'	A	A'	$U = \frac{\omega A Z_{B2} Z_A (Z_A + Z_C) - Z_{B1} Z_C Z_A}{\omega C' (Z_{B2} Z_A - Z_{B1} Z_C) Z_A}$	$\eta = \frac{n_o (n_o Z_{B2} Z_A - Z_{B1} Z_C) (Z_{B2} Z_A Z_A + Z_{B1} Z_C Z_A - Z_{B1} Z_C Z_A)}{(Z_{B2} Z_A - Z_{B1} Z_C) \{ n_o (n_o Z_{B2} Z_A - Z_{B1} Z_C) Z_A + n_o^2 Z_{B2} Z_A Z_C \}}$
	S'	A	A'	$U = \frac{\omega A Z_{B1} Z_C Z_A - Z_{B2} Z_A (Z_A + Z_C)}{\omega S' Z_{B1} Z_C Z_A}$	$\eta = \frac{n_o n_o (Z_{B2} Z_A Z_A + Z_{B2} Z_A Z_C - Z_{B1} Z_C Z_A)}{(n_o Z_A + Z_C) Z_{B2} Z_A - n_o n_o Z_{B1} Z_C Z_A}$

Table 12 Speed Ratio and Efficiency Formula for Combined Planetary Gears with Power Circulation

## Power Circulation



Driver	Follower	Fixed	Speed Ratio	Efficiency of Combined Planetary Gears
$Z_A < Z_C$	C	A'	$U = \frac{\omega_A' Z_{B2} Z_A (Z_A + Z_C) - Z_{B1} Z_C Z_A}{\omega_C Z_{B2} Z_A (Z_A + Z_C)}$	$\eta = \frac{n_o^o (n_o^o Z_A + Z_C) (Z_{B2} Z_A Z_A + Z_{B2} Z_A Z_C - Z_{B1} Z_C Z_A)}{(Z_A + Z_C) \{ n_o^o n_o^o Z_{B2} Z_A Z_A + n_o^o Z_{B2} Z_A Z_C + (1 - n_o^o)^2 Z_{B1} Z_C Z_C - n_o^o n_o^o Z_{B1} Z_C Z_A \}}$
	S	A'	$U = \frac{\omega_A' Z_{B2} Z_A (Z_A + Z_C) - Z_{B1} Z_C Z_A}{\omega_S Z_{B2} Z_A Z_C}$	$\eta = \frac{n_o^o (Z_{B2} Z_A Z_A + Z_{B2} Z_A Z_C - Z_{B1} Z_C Z_A)}{n_o^o n_o^o Z_{B2} Z_A Z_A + n_o^o Z_{B2} Z_A Z_C + (1 - n_o^o)^2 Z_{B1} Z_C Z_C - n_o^o n_o^o Z_{B1} Z_C Z_A}$
	A	C'	$U = \frac{\omega_C' (Z_{B2} Z_A - Z_{B1} Z_C) Z_A}{\omega_A Z_{B2} Z_A (Z_A + Z_C) - Z_{B1} Z_C Z_A}$	$\eta = \frac{(Z_{B2} Z_A - Z_{B1} Z_C) \{ n_o^o n_o^o Z_{B2} Z_A Z_A + n_o^o Z_{B2} Z_A Z_C + (n_o^o)^2 - 1 \} Z_{B1} Z_C Z_C - n_o^o Z_{B1} Z_C Z_A}{n_o^o (n_o^o Z_{B2} Z_A - Z_{B1} Z_C) (Z_{B2} Z_A Z_A + Z_{B2} Z_A Z_C - Z_{B1} Z_C Z_A)}$
	A	S'	$U = \frac{\omega_S' Z_{B1} Z_C Z_A}{\omega_A Z_{B1} Z_C Z_A - Z_{B2} Z_A (Z_A + Z_C)}$	$\eta = \frac{n_o^o n_o^o Z_{B2} Z_A Z_A + n_o^o Z_{B2} Z_A Z_C + (n_o^o)^2 - 1 Z_{B1} Z_C Z_C - n_o^o Z_{B1} Z_C Z_A}{n_o^o (Z_{B2} Z_A Z_A + Z_{B2} Z_A Z_C - Z_{B1} Z_C Z_A)}$
	A'	C	$U = \frac{\omega_C Z_{B2} Z_A (Z_A + Z_C)}{\omega_A' Z_{B2} Z_A (Z_A + Z_C) - Z_{B1} Z_C Z_A}$	$\eta = \frac{(Z_A + Z_C) \{ n_o^o (n_o^o)^2 Z_{B1} Z_C Z_C + (Z_A + n_o^o Z_C) (n_o^o Z_{B2} Z_A - Z_{B1} Z_C) \}}{n_o^o (Z_A + n_o^o Z_C) (Z_{B2} Z_A Z_A + Z_{B2} Z_A Z_C - Z_{B1} Z_C Z_A)}$
	A'	S	$U = \frac{\omega_S Z_{B2} Z_A Z_C}{\omega_A' Z_{B2} Z_A (Z_A + Z_C) - Z_{B1} Z_C Z_A}$	$\eta = \frac{n_o^o n_o^o Z_{B1} Z_C Z_C + (n_o^o Z_{B2} Z_A - Z_{B1} Z_C) (Z_A + n_o^o Z_C)}{n_o^o n_o^o (Z_{B2} Z_A Z_A + Z_{B2} Z_A Z_C - Z_{B1} Z_C Z_A)}$
	C'	A	$U = \frac{\omega_A Z_{B2} Z_A (Z_A + Z_C) - Z_{B1} Z_C Z_A}{\omega_C' (Z_{B2} Z_A - Z_{B1} Z_C) Z_A}$	$\eta = \frac{n_o^o (Z_{B2} Z_A - n_o^o Z_{B1} Z_C) (Z_{B2} Z_A Z_A + Z_{B2} Z_A Z_C - Z_{B1} Z_C Z_A)}{(Z_{B2} Z_A - Z_{B1} Z_C) \{ n_o^o Z_{B1} Z_C Z_C + n_o^o (Z_A + n_o^o Z_C) (Z_{B2} Z_A - n_o^o Z_{B1} Z_C) \}}$
	S'	A	$U = \frac{\omega_A Z_{B1} Z_C Z_A - Z_{B2} Z_A (Z_A + Z_C)}{\omega_S' Z_{B1} Z_C Z_A}$	$\eta = \frac{n_o^o n_o^o (Z_{B2} Z_A Z_A + Z_{B2} Z_A Z_C - Z_{B1} Z_C Z_A)}{n_o^o Z_{B1} Z_C Z_C + n_o^o (Z_A + n_o^o Z_C) (Z_{B2} Z_A - n_o^o Z_{B1} Z_C)}$

効率で

$$\eta_2 = \eta_o'' (= \eta_1'' \cdot \eta_2''). \quad (29)$$

$\eta_3$  は要素 I でキャリヤ S を固定し、歯車 A を入力側、歯車 C を出力側とするときの効率で

$$\eta_3 = \eta_o' (= \eta_1' \cdot \eta_2'). \quad (30)$$

式(27)に式(9), (11), (28), (29), (30)を代入すれば

$$N_i = \frac{Z_{B1} Z_C' Z_A + \eta_o' Z_{B1} Z_C' Z_C - \eta_o'' Z_{B2} Z_A' Z_A}{\eta_o' \eta_o'' (Z_{B1} Z_C' Z_A + Z_{B1} Z_C' Z_C - Z_{B2} Z_A' Z_A)} N_o. \quad (31)$$

したがって装置全体の効率  $\eta$  は

$$\eta = \frac{N_o}{N_i} = \frac{\eta_o' \eta_o'' (Z_{B1} Z_C' Z_A + Z_{B1} Z_C' Z_C - Z_{B2} Z_A' Z_A)}{Z_{B1} Z_C' Z_A + \eta_o' Z_{B1} Z_C' Z_C - \eta_o'' Z_{B2} Z_A' Z_A}. \quad (32)$$

表 2 の一番上の欄の式が式(32)である。他の場合についても同様な計算を行なって表 2 を得た。なお他の組合せの複合遊星歯車装置についても同様な計算を行ない、表 3, 表 4, 表 5, 表 6, 表 7, 表 8, 表 9, 表 10, 表 11, 表 12を得た。

### 3 結 言

内歯車式の単純遊星歯車装置と複式外歯車遊星歯車装置の組合せからなる複合遊星歯車装置で、それぞれの基本軸の連結の仕方と複式外歯車遊星歯車装置に用いられている 2 ケの太陽外歯車の歯数間の大小関係により動力循環型となったり動力分流型となったりする場合を明らかにし、それぞれの場合の速比と理論効率計算式を求め、これらの表を作成した。この場合キャリヤを連結軸としないで、どちらかのキャリヤを固定させると、いわゆる閉路式遊星歯車機構となる。最後に本研究に助力された西村信治、田中徳雄の両君に感謝の意を表します。

### 文 献

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