The Rate of Turbulent Spot Formation in a Boundary-Layer Transition Region

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Synopsis

The distribution of the rate of turbulent spot formation in the boundary layer transition region along a flat plate is estimated, in a phenomenological manner, on the basis of available experimental results.

The distribution shows an unexpected feature; i. e., the gradual decrement in the latter part of the transition region.

This implies the possible existence of some effects suppressing the formation of spots in the latter part of the region.

1. Introduction

Generally the boundary layer that grows on the surface of any body is laminar for some distance downstream from the leading edge.

This is followed by more or less extensive region, i. e. transition region, throughout which the mean characteristics of the layer change gradually from those characterizing laminar flow to those characterizing fully developed turbulent flow.

This nonstationary region is too complicated to be suscptible to theoretical analysis.

Hence present knowledge of the detailed nature of the region consists primarily of experimental investigations.

Through experipental works of many authors much has been revealed in regard to the structure of transition region and it is now generally accepted that the natural transition from laminar to turbulent flow occurs as a succession of turbulent spots which grow more or less independently as they move downstream.

On the rate of formation of these turbulent spots, however, no definite information has been obtained.

Direct measurment, as well as purely theoretical approach, seems to be formidably difficult.

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In this paper, therfore, we attemp to estimate it in a phenomenological manner on the basis of available results of experiments.

2. Intermittency factor

In the transition region, isolated spots within which a fully developed turbulent flow exists appear successivelly in a random fashion in time and space and grow as they are washed downstream.

Thus the flow at any point in the region becomes turbulent during those periods of time during which a spot moves over it and is laminar for the remainder.

Although the transition is a random phenomenon, it may be possible to determine the fraction of the total time in which the flow is turbulent as an average taken over an appropriate interval of time.

In fact this fraction, which is called intermittency factor, has been determined by Schubauer & Klebanoff from their experimental data.

Similarly it may be reasonable to assume the existence of a sort of probability function specifying the rate of spot formation per unit area.

Obviously the intermittency factor depends on the rate of spot formation and that of its subsequent growth.

The form of this dependence can be derived merely from the fact that transition is the process of formation and growth of turbulent spots.

Now it is convenient to introduce an x, y, t space, in which x, y denote the coordinates on the body surface—the coordinate x to be taken in the di-

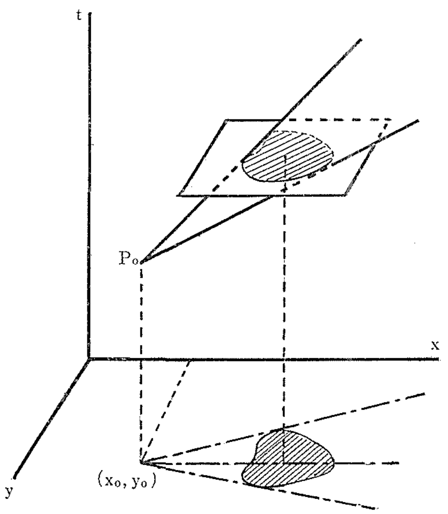

Fig. 1 Propagation cone

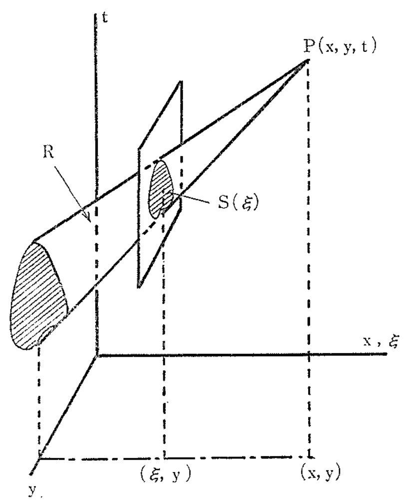

Fig. 2 Retrograde cone

rection of the stream—and t does the time.

If the history of a spot originating at P_0 (x_0, y_0, t_0) is expressed in this space, it will sweep out a volume of a cone-like shape as shown in Fig. 1.

Then, a given point P (x, y, t) will be covered by this spot if the point is located in this propagation cone from P_0.

It can be easily seen that the locus of all points P_0 which can influence the state of turbulence at P becomes a retrograde cone R as shown in Fig. 2.

The probability of turbulence at P due to a volume element dV_0 $(=dx_0, dy_0, dt_0)$ at point P_0 is given by $g(P_0)dV_0$ if P_0 is in the volume R, and is zero if P_0 is not in R, where $g(P_0)$ denotes the rate of spot formation at P_0.

However, we cannot find correctly the intermittency factor $\gamma(P)$ at P by simply integrating $g(P_0)\,dV_0$ over all the volume R, since there may be more than one spot in R and the resultant turbulence at P would be counted twice.

In order to avoid these overlaps, we have to pick up only those spots which have traveled furthest and are most dominant.

Then we have, after a little manipulation (see Emmons), the following relation

$$\gamma(P) = 1 - \exp\left\{ -\int_R g(P_0)\, d\, V_0 \right\}. \tag{1}$$

In what follows, we restrict our considerations to the transition region of the boundary layer on a semi-infinite flat plate in a uniform steady stream.

In this case the turbulent spots are known to propagate at a constant rate with their shape approximately preserved.

Then it can be seen that the retrograde cone in Fig. 2 as well as the propagation cone in Fig. 1 is true cone with straight generators, in which the area $S(\xi)$ of the $\xi =$ constant cross-section is proportional to the square of the distance from the apex.

That is,

$$S(\xi) = \lambda (x - \xi)^2,$$

where λ is a constant to be determined from the shape of turbulent spot, the rate of growth and the streamwise propagation velocity.

In case of two-dimensional steady flow, both $\gamma(P)$ and $g(P_0)$ are considered to be independent of y and t.

After all Eq. (1) can be written as

$$\gamma(x) = 1 - \exp\left\{ -\lambda \int_{x_0}^{x} (x - \xi)^2\, g(\xi)\, d\xi \right\}, \tag{2}$$

in which we consider, for convenience, the origin of the coordinate x to be located in the point where $\gamma = 0.5$.

The lower limit of the integral on the right hand side of Eq. 2 can be safely extended to $-\infty$, since the rate of spot formation $g(\xi)$ vanishes upstream from the transition front.

On the other hand, Schubauer & Klebanoff measured the details of the transition region of a boundary layer on a flat plate and determined the intermittency factor γ for several cases in which conditions leading to transition were varied.

The results are seen plotted in Fig. 3, where the streamwise distance is straind linearly by a parameter σ proportional to the length of the transition region.

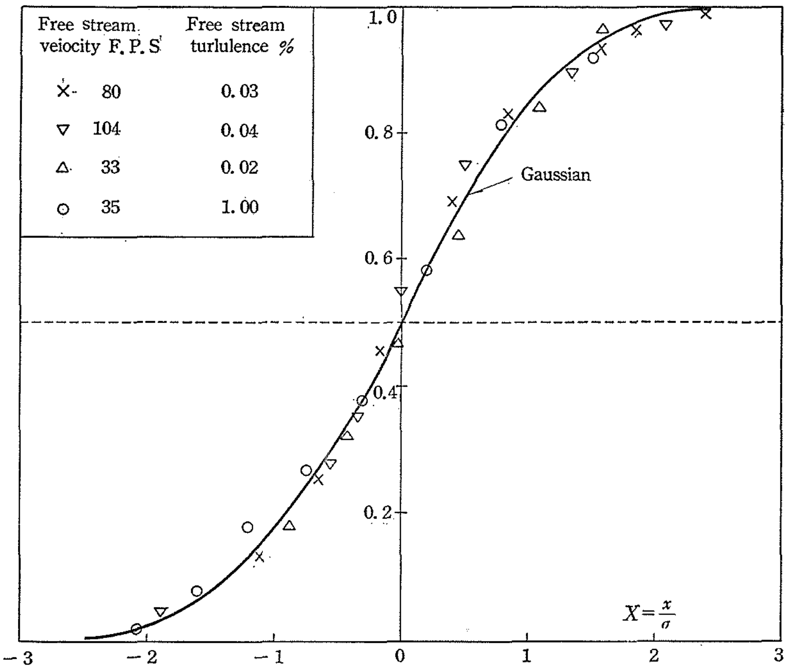

Fig. 3 Intermittency factor (Schubauer & Klebanoff)

It will be noticed that the distributions are similar in all of those cases and are represented by one curve (Gaussian integral curve) within the error of experiment.

Then we have as a sort of empirical formula

$$\gamma(x) = \frac{1}{\sqrt{2\pi}\,\sigma} \int_{-\infty}^{x} e^{-\frac{\xi^2}{2\sigma^2}}\, d\xi, \tag{3}$$

or in alternative form

$$\gamma(x) = 1 - \frac{1}{\sqrt{2\pi}\,\sigma} \int_x^\infty e^{-\frac{\xi^2}{2\sigma^2}} d\xi \,, \tag{3)'$$

where σ is a sort of standard deviation, representing the characteristic length of transition region.

3. Estimation of the rate of spot formation

For the fraction of the total time in which the flow is turbulent at any point in the transition region we have now two different expressions (2) and (3).

By equating (2) and (3), we have

$$\exp\left\{-\lambda \int_{-\infty}^{x} (x-\xi)^2 g(\xi)\, d\xi\right\} = \frac{1}{\sqrt{2\pi}\,\sigma} \int_x^\infty e^{-\frac{\xi^2}{2\sigma^2}} d\xi \,. \tag{4}$$

This may be written in a more familiar form as

$$\lambda \int_{-\infty}^{x} (x-\xi)^2 g(\xi)\, d\xi = -\log_e \left\{\frac{1}{\sqrt{2\pi}\,\sigma} \int_x^\infty e^{-\frac{\xi^2}{2\sigma^2}} d\xi\right\}. \tag{4)'$$

Thus, we have a Volterra equation of the first kind for determining unknown function $g(x)$ –the rate of spot formation. The solution of the integral equation ((4))′ can be easily found as

$$g(x) = \frac{1}{2\lambda\sigma^3} G(X), \tag{5}$$

where

$$G(X) = \frac{2e^{-\frac{3}{2}X^2}}{\left(\int_X^\infty e^{-\frac{\xi^2}{2}} d\xi\right)^3} - \frac{3Xe^{-X^2}}{\left(\int_X^\infty e^{-\frac{\xi^2}{2}} d\xi\right)^2} + \frac{(X^2-1)\,e^{-\frac{X^2}{2}}}{\int_X^\infty e^{-\frac{\xi^2}{2}} d\xi} \,, \tag{6}$$

with the abbreviation

$$X = x/\sigma \,.$$

From the expression (5) it can be seen that the function $G(X)$ gives a non-dimensional expression for the rate of spot formation.

The behaviour of this function is shown in Fig. 4.

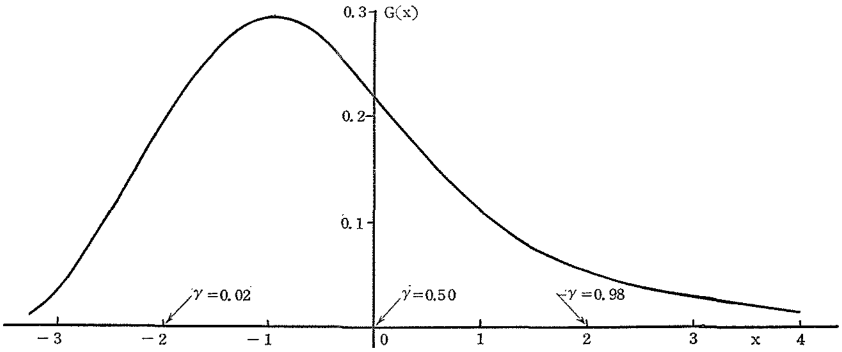

Fig. 4 Rate of spot formation in nondimentional form

In view of the fact that the amplitude and frequency of disturbances in an unstable region of a boundary layer increase as they proceed downstream, it may as well be anticipated that the rate of spot formation also increases as the distance increases downstream from the transition front.

Fig. 4, however, indicates that this is not the case, but that the rate of spot formation reaches its maximum at some distance upstream from the middle of the transition region and then decreases gradually downstream from the maximum point.

This fact implies that there may exist some effects which suppress the formation of spots in the latter part of the transition region.

In connection with this, it may be of some interest to refer to a phenomenon of calming effect proposed by Schubauer & Klebanoff.

They made a close observation respecting the behaviour of an artificially initiated turbulent spot and found that the laminar layer following the passage of turbulence remained in a highly stable state for a short interval during which no breakdown was likely to occur.

This phenomenon associated with receding turbulence was termed calming effect.

Although calming effect was observed originally concerning an artificially initiated turbulent spot, our result presented above suggests the possible existence of calming effect also in natural transition region.

4. Concluding remarks

In this paper, the distiribution of the rate of spot formation in transition region has been estimated in a limitted case of the flat plate boundary layer without pressure gradient.

It does not follow that the rate of spot formation would have the same distribution in more general cases with pressure gradient. However, the essential feature of the distridution, i. e. the gradual decrement in the latter part of transition region, would be true also in the presence of pressure gradient.

References

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