Thick Plate Subjected to Three Pairs of External Forces on its Bounding Planes

Bennosuke TANIMOTO*), Dr. Eng.

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Synopsis. The boundary-value problem here treated is that an elastic thick plate of uniform thickness is subjected to three pairs of external forces, consisting of one kind of normal pressure and two kinds of shearing forces, applied on its bounding planes; these forces being supposed to be of any distribution, and, for the present, of extending over a rectangular area.

1. Introductory Remarks

The procedure of solving the present boundary-value problem is due to my proposed one in three dimensions. (Cf. "Potentials of Certain Simultaneous Equations," Bulletin of the Earthquake Research Institute, University of Tokyo, vol. 26 (1948); also "The Derivation of the Proposed Stress-Functions in Three Dimensions," Journal of the Faculty of Engineering, Shinshu University, Nagano, No. 5 (1955).)

The first application of this procedure was made to "The Solution of the Generalized Boussinesq's Problem for Elastic Foundation." (Cf. Proceedings of the Japan Academy, vol. 31 (1955), No. 8, Art. Nos. 129 and 130.) The present work may therefore be stated as the second application of the proposed procedure, and the solution obtained is much similar to the previous one, for, from the analytical standpoint, the former solution can be included in the present one as a special case of the latter.

2. Boundary Conditions

The plane $z=0$ is taken to be the middle plane of the plate, so that the axis of z is perpendicular to the bounding planes $z=\pm c$; $2c$ being the thickness of the plate.

The boundary conditions are then to be written

*) Professor of Civil Engineering, Faculty of Engineering, Shinshu University, Nagano, Japan.

$$\left.\begin{array}{ll} 1) & (\widehat{zz})_{z=\pm c} = F_1(x, y), \\ 2) & (\widehat{xz})_{z=\pm c} = F_2(x, y), \\ 3) & (\widehat{yz})_{z=\pm c} = F_3(x, y). \end{array}\right\} \tag{1}$$

$F_1(x, y)$ represents given distribution of pairs of the normal pressure, while $F_2(x, y)$ and $F_3(x, y)$ represent given distributions of pairs of the two kinds of shearing forces. These functions are valid within a given domain around the z-axis, and vanish outside of the domain, so that they may be expanded in terms of Fourier integrals (cf. Figs. 1 and 2).

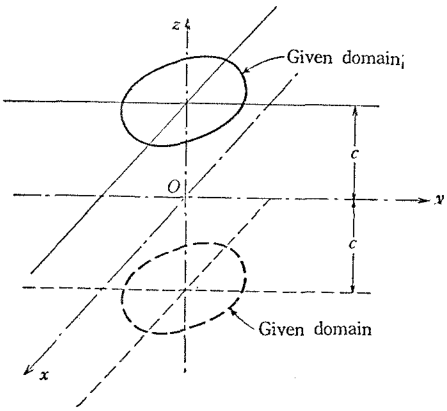

Fig. 1. Coordinates

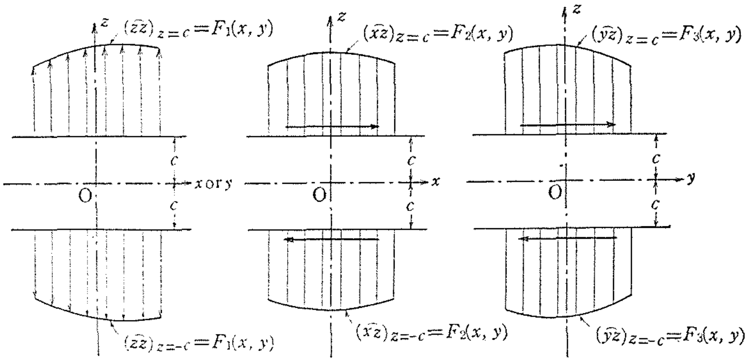

Fig. 2. Applied forces

3. Fundamental Equations and Typical Solutions

The well-known three-dimensional displacement-equations

$$(\lambda+\mu)\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)\Delta+\mu\nabla^2(u, v, w)=0$$

may be replaced by

$$\left.\begin{aligned}
\widehat{xx} &= \frac{\partial^2\phi}{\partial x^2}+\left(\frac{\partial^2}{\partial x\partial y}-\frac{\partial^2}{\partial x\partial z}\right)\psi+\left\{\left(\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\nabla^2-(1-\nu)\frac{\partial^2}{\partial y\partial z}\bar{\nabla}^2\right\}\chi, \\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots; \\
\widehat{yz} &= \frac{\partial^2\phi}{\partial y\partial z}+\frac{1}{2}\left(\frac{\partial^2}{\partial x\partial y}-\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}-\frac{\partial^2}{\partial x\partial z}\right)\psi \\
&\quad +\left\{-\frac{\partial^2}{\partial y\partial z}\nabla^2+\frac{1-\nu}{2}\frac{\partial}{\partial x}\left(-\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)\bar{\nabla}^2\right\}\chi, \\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots;
\end{aligned}\right\} \quad (2)$$

in which

$$\nabla^2\phi=0, \qquad \nabla^2\psi=0, \qquad \text{and} \qquad \nabla^4\chi=0, \qquad (3)$$

$$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}, \qquad \bar{\nabla}^2=\frac{\partial^2}{\partial y\partial z}+\frac{\partial^2}{\partial z\partial x}+\frac{\partial^2}{\partial x\partial y},$$

so long as the present boundary-value problem is concerned.

Typical solutions satisfying equations (3) are

$$\left.\begin{aligned}
\phi &= (A_1\cos\alpha x\cos\beta y+A_2\cos\alpha x\sin\beta y+A_3\sin\alpha x\cos\beta y \\
&\quad +A_4\sin\alpha x\sin\beta y)\cosh\gamma z+(A_5\cos\alpha x\cos\beta y+A_6\cos\alpha x\sin\beta y \\
&\quad +A_7\sin\alpha x\cos\beta y+A_8\sin\alpha x\sin\beta y)\sinh\gamma z, \\
\psi &= (B_1\cos\alpha x\cos\beta y+B_2\cos\alpha x\sin\beta y+B_3\sin\alpha x\cos\beta y \\
&\quad +B_4\sin\alpha x\sin\beta y)\cosh\gamma z+(B_5\cos\alpha x\cos\beta y+B_6\cos\alpha x\sin\beta y \\
&\quad +B_7\sin\alpha x\cos\beta y+B_8\sin\alpha x\sin\beta y)\sinh\gamma z, \\
\chi &= (E_1\cos\alpha x\cos\beta y+E_2\cos\alpha x\sin\beta y+E_3\sin\alpha x\cos\beta y \\
&\quad +E_4\sin\alpha x\sin\beta y)\, z\cosh\gamma z+(E_5\cos\alpha x\cos\beta y+E_6\cos\alpha x\sin\beta y \\
&\quad +E_7\sin\alpha x\cos\beta y+E_8\sin\alpha x\sin\beta y)\, z\sinh\gamma z,
\end{aligned}\right\} \quad (4)$$

where α, β, and γ are parameters provided

$$\alpha^2+\beta^2=\gamma^2. \qquad (5)$$

4. Stress-components

For simplicity, we shall frequently use the abbreviation

$$\cos\cos\cosh = \cos\alpha x \cos\beta y \cosh\gamma z, \quad \text{etc.},$$

when confusion does not occur.

Substituting equations (4) into equations (2), we have

$$\begin{aligned}
\widehat{xx} = [&-\alpha^2A_1 + \alpha\beta B_4 - \alpha\gamma B_7 - \{(1-2\nu)\alpha^2\beta + 2(1-\nu)\beta^3\}E_2 \\
&\qquad + \alpha^3E_3 - 2\gamma U_4] \cos\cos\cosh \\
&+ [-\alpha^2A_2 - \alpha\beta B_3 - \alpha\gamma B_8 + \{(1-2\nu)\alpha^2\beta + 2(1-\nu)\beta^3\}E_1 \\
&\qquad + \alpha^3E_4 - 2\gamma R_3] \cos\sin\cosh \\
&+ [-\alpha^2A_3 - \alpha\beta B_2 + \alpha\gamma B_5 - \{(1-2\nu)\alpha^2\beta + 2(1-\nu)\beta^3\}E_4 \\
&\qquad - \alpha^3E_1 - 2\gamma R_2] \sin\cos\cosh \\
&+ [-\alpha^2A_4 + \alpha\beta B_1 + \alpha\gamma B_6 + \{(1-2\nu)\alpha^2\beta + 2(1-\nu)\beta^3\}E_3 \\
&\qquad - \alpha^3E_2 - 2\gamma U_1] \sin\sin\cosh \\
&+ [-\alpha^2A_5 + \alpha\beta B_8 - \alpha\gamma B_3 - \{(1-2\nu)\alpha^2\beta + 2(1-\nu)\beta^3\}E_6 \\
&\qquad + \alpha^3E_7 - 2\gamma R_4] \cos\cos\sinh \\
&+ [-\alpha^2A_6 - \alpha\beta B_7 - \alpha\gamma B_4 + \{(1-2\nu)\alpha^2\beta + 2(1-\nu)\beta^3\}E_5 \\
&\qquad + \alpha^3E_8 - 2\gamma U_3] \cos\sin\sinh \\
&+ [-\alpha^2A_7 - \alpha\beta B_6 + \alpha\gamma B_1 - \{(1-2\nu)\alpha^2\beta + 2(1-\nu)\beta^3\}E_8 \\
&\qquad - \alpha^3E_5 - 2\gamma U_2] \sin\cos\sinh \\
&+ [-\alpha^2A_8 + \alpha\beta B_5 + \alpha\gamma B_2 + \{(1-2\nu)\alpha^2\beta + 2(1-\nu)\beta^3\}E_7 \\
&\qquad - \alpha^3E_6 - 2\gamma R_1] \sin\sin\sinh \\
&- \alpha^2 z [(R_4\cos\cos + U_3\cos\sin + U_2\sin\cos + R_1\sin\sin)\cosh \\
&\qquad + (U_4\cos\cos + R_3\cos\sin + R_2\sin\cos + U_1\sin\sin)\sinh], \qquad (6)
\end{aligned}$$

$$\begin{aligned}
\widehat{yy} = [&-\beta^2A_1 + \beta\gamma B_6 - \alpha\beta B_4 - \{(1-2\nu)\alpha\beta^2 + 2(1-\nu)\alpha^3\}E_3 \\
&\qquad + \beta^3E_2 - 2\gamma U_4] \cos\cos\cosh \\
&+ [-\beta^2A_2 - \beta\gamma B_5 + \alpha\beta B_3 - \{(1-2\nu)\alpha\beta^2 + 2(1-\nu)\alpha^3\}E_4 \\
&\qquad - \beta^3E_1 - 2\gamma R_3] \cos\sin\cosh
\end{aligned}$$

$$+ [-\beta^2 A_3 + \beta\gamma B_8 + \alpha\beta B_2 + \{(1-2\nu)\alpha\beta^2 + 2(1-\nu)\alpha^3\}E_1 + \beta^3 E_4 - 2\gamma R_2] \sin\cos\cosh$$

$$+ [-\beta^2 A_4 - \beta\gamma B_7 - \alpha\beta B_1 + \{(1-2\nu)\alpha\beta^2 + 2(1-\nu)\alpha^3\}E_2 - \beta^3 E_3 - 2\gamma U_1] \sin\sin\cosh$$

$$+ [-\beta^2 A_5 + \beta\gamma B_2 - \alpha\beta B_8 - \{(1-2\nu)\alpha\beta^2 + 2(1-\nu)\alpha^3\}E_7 + \beta^3 E_6 - 2\gamma R_4] \cos\cos\sinh$$

$$+ [-\beta^2 A_6 - \beta\gamma B_1 + \alpha\beta B_7 - \{(1-2\nu)\alpha\beta^2 + 2(1-\nu)\alpha^3\}E_8 - \beta^3 E_5 - 2\gamma U_3] \cos\sin\sinh$$

$$+ [-\beta^2 A_7 + \beta\gamma B_4 + \alpha\beta B_6 + \{(1-2\nu)\alpha\beta^2 + 2(1-\nu)\alpha^3\}E_5 + \beta^3 E_8 - 2\gamma U_2] \sin\cos\sinh$$

$$+ [-\beta^2 A_8 - \beta\gamma B_3 - \alpha\beta B_5 + \{(1-2\nu)\alpha\beta^2 + 2(1-\nu)\alpha^3\}E_6 + \beta^3 E_7 - 2\gamma R_1] \sin\sin\sinh$$

$$- \beta^2 z[(R_4 \cos\cos + U_3 \cos\sin + U_2 \sin\cos + R_1 \sin\sin)\cosh + (U_4 \cos\cos + R_3 \cos\sin + R_2 \sin\cos + U_1 \sin\sin)\sinh], \quad (7)$$

$$\widehat{zz} = [\gamma^2 A_1 + \alpha\gamma B_7 - \beta\gamma B_6 - \gamma^2(\beta E_2 + \alpha E_3) - 2(1-\nu)\alpha\beta\gamma E_8] \cos\cos\cosh$$
$$+ [\gamma^2 A_2 + \alpha\gamma B_8 + \beta\gamma B_5 + \gamma^2(\beta E_1 - \alpha E_4) + 2(1-\nu)\alpha\beta\gamma E_7] \cos\sin\cosh$$
$$+ [\gamma^2 A_3 - \alpha\gamma B_5 - \beta\gamma B_8 + \gamma^2(\alpha E_1 - \beta E_4) + 2(1-\nu)\alpha\beta\gamma E_6] \sin\cos\cosh$$
$$+ [\gamma^2 A_4 - \alpha\gamma B_6 + \beta\gamma B_7 + \gamma^2(\alpha E_2 + \beta E_3) - 2(1-\nu)\alpha\beta\gamma E_5] \sin\sin\cosh$$
$$+ [\gamma^2 A_5 + \alpha\gamma B_3 - \beta\gamma B_2 - \gamma^2(\beta E_6 + \alpha E_7) - 2(1-\nu)\alpha\beta\gamma E_4] \cos\cos\sinh$$
$$+ [\gamma^2 A_6 + \alpha\gamma B_4 + \beta\gamma B_1 + \gamma^2(\beta E_5 - \alpha E_8) + 2(1-\nu)\alpha\beta\gamma E_3] \cos\sin\sinh$$
$$+ [\gamma^2 A_7 - \alpha\gamma B_1 - \beta\gamma B_4 + \gamma^2(\alpha E_5 - \beta E_8) + 2(1-\nu)\alpha\beta\gamma E_2] \sin\cos\sinh$$
$$+ [\gamma^2 A_8 - \alpha\gamma B_2 + \beta\gamma B_3 + \gamma^2(\alpha E_6 + \beta E_7) - 2(1-\nu)\alpha\beta\gamma E_1] \sin\sin\sinh$$
$$+ \gamma^2 z[(R_4 \cos\cos + U_3 \cos\sin + U_2 \sin\cos + R_1 \sin\sin)\cosh + (U_4 \cos\cos + R_3 \cos\sin + R_2 \sin\cos + U_1 \sin\sin)\sinh], \quad (8)$$

$$\widehat{yz} = [\beta\gamma A_6 + \tfrac{1}{2}\{\alpha\beta B_4 + (\alpha^2 + 2\beta^2)B_1 - \alpha\gamma B_7\} - \nu\alpha\beta\gamma E_8 + (1-\nu)\alpha\gamma^2 E_3 + \{(1-\nu)\alpha^2\gamma + \beta^2\gamma\}E_5 + \beta U_3] \cos\cos\cosh$$
$$+ [-\beta\gamma A_5 + \tfrac{1}{2}\{-\alpha\beta B_3 + (\alpha^2 + 2\beta^2)B_2 - \alpha\gamma B_8\}$$

$$+ \nu\alpha\beta\gamma E_7 + (1-\nu)\alpha\gamma^2 E_4 + \{(1-\nu)\alpha^2\gamma + \beta^2\gamma\}E_6 - \beta R_4] \cos\sin\cosh$$

$$+ [\beta\gamma A_8 + \tfrac{1}{2}\{-\alpha\beta B_2 + (\alpha^2 + 2\beta^2)B_3 + \alpha\gamma B_5\}$$

$$+ \nu\alpha\beta\gamma E_6 - (1-\nu)\alpha\gamma^2 E_1 + \{(1-\nu)\alpha^2\gamma + \beta^2\gamma\}E_7 + \beta R_1] \sin\cos\cosh$$

$$+ [-\beta\gamma A_7 + \tfrac{1}{2}\{\alpha\beta B_1 + (\alpha^2 + 2\beta^2)B_4 + \alpha\gamma B_6\}$$

$$- \nu\alpha\beta\gamma E_5 - (1-\nu)\alpha\gamma^2 E_2 + \{(1-\nu)\alpha^2\gamma + \beta^2\gamma\}E_8 - \beta U_2] \sin\sin\cosh$$

$$+ [\beta\gamma A_2 + \tfrac{1}{2}\{\alpha\beta B_8 + (\alpha^2 + 2\beta^2)B_5 - \alpha\gamma B_3\}$$

$$- \nu\alpha\beta\gamma E_4 + (1-\nu)\alpha\gamma^2 E_7 + \{(1-\nu)\alpha^2\gamma + \beta^2\gamma\}E_1 + \beta R_3] \cos\cos\sinh$$

$$+ [-\beta\gamma A_1 + \tfrac{1}{2}\{-\alpha\beta B_7 + (\alpha^2 + 2\beta^2)B_6 - \alpha\gamma B_4\}$$

$$+ \nu\alpha\beta\gamma E_3 + (1-\nu)\alpha\gamma^2 E_8 + \{(1-\nu)\alpha^2\gamma + \beta^2\gamma\}E_2 - \beta U_4] \cos\sin\sinh$$

$$+ [\beta\gamma A_4 + \tfrac{1}{2}\{-\alpha\beta B_6 + (\alpha^2 + 2\beta^2)B_7 + \alpha\gamma B_1\}$$

$$+ \nu\alpha\beta\gamma E_2 - (1-\nu)\alpha\gamma^2 E_5 + \{(1-\nu)\alpha^2\gamma + \beta^2\gamma\}E_3 + \beta U_1] \sin\cos\sinh$$

$$+ [-\beta\gamma A_3 + \tfrac{1}{2}\{\alpha\beta B_5 + (\alpha^2 + 2\beta^2)B_8 + \alpha\gamma B_2\}$$

$$- \nu\alpha\beta\gamma E_1 - (1-\nu)\alpha\gamma^2 E_6 + \{(1-\nu)\alpha^2\gamma + \beta^2\gamma\}E_4 - \beta R_2] \sin\sin\sinh$$

$$+ \beta\gamma z[(R_3 \cos\cos - U_4 \cos\sin + U_1 \sin\cos - R_2 \sin\sin)\cosh$$

$$+ (U_3 \cos\cos - R_4 \cos\sin + R_1 \sin\cos - U_2 \sin\sin)\sinh], \qquad (9)$$

$$\widehat{zx} = [\alpha\gamma A_7 + \tfrac{1}{2}\{\beta\gamma B_6 - (2\alpha^2 + \beta^2)B_1 - \alpha\beta B_4\}$$

$$- \nu\alpha\beta\gamma E_8 + (1-\nu)\beta\gamma^2 E_2 + \{(1-\nu)\beta^2\gamma + \alpha^2\gamma\}E_5 + \alpha U_2] \cos\cos\cosh$$

$$+ [\alpha\gamma A_8 + \tfrac{1}{2}\{-\beta\gamma B_5 - (2\alpha^2 + \beta^2)B_2 + \alpha\beta B_3\}$$

$$+ \nu\alpha\beta\gamma E_7 - (1-\nu)\beta\gamma^2 E_1 + \{(1-\nu)\beta^2\gamma + \alpha^2\gamma\}E_6 + \alpha R_1] \cos\sin\cosh$$

$$+ [-\alpha\gamma A_5 + \tfrac{1}{2}\{\beta\gamma B_8 - (2\alpha^2 + \beta^2)B_3 + \alpha\beta B_2\}$$

$$+ \nu\alpha\beta\gamma E_6 + (1-\nu)\beta\gamma^2 E_4 + \{(1-\nu)\beta^2\gamma + \alpha^2\gamma\}E_7 - \alpha R_4] \sin\cos\cosh$$

$$+ [-\alpha\gamma A_6 + \tfrac{1}{2}\{-\beta\gamma B_7 - (2\alpha^2 + \beta^2)B_4 - \alpha\beta B_1\}$$

$$- \nu\alpha\beta\gamma E_5 - (1-\nu)\beta\gamma^2 E_3 + \{(1-\nu)\beta^2\gamma + \alpha^2\gamma\}E_8 - \alpha U_3] \sin\sin\cosh$$

$$+ [\alpha\gamma A_3 + \tfrac{1}{2}\{\beta\gamma B_2 - (2\alpha^2 + \beta^2)B_5 - \alpha\beta B_8\}$$

$$- \nu\beta\gamma E_4 + (1-\nu)\beta\gamma^2 E_6 + \{(1-\nu)\beta^2\gamma + \alpha^2\gamma\}E_1 + \alpha R_2] \cos\cos\sinh$$

$$+ [\alpha\gamma A_4 + \tfrac{1}{2}\{-\beta\gamma B_1 - (2\alpha^2 + \beta^2)B_6 + \alpha\beta B_7\}$$

$$+ \nu\alpha\beta\gamma E_3 - (1-\nu)\beta\gamma^2 E_5 + \{(1-\nu)\beta^2\gamma + \alpha^2\gamma\}E_2 + \alpha U_1] \cos\sin\sinh$$

$$+ [-\alpha\gamma A_1 + \tfrac{1}{2}\{\beta\gamma B_4 - (2\alpha^2 + \beta^2)B_7 + \alpha\beta B_6\}$$
$$+ \nu\alpha\beta\gamma E_2 + (1-\nu)\beta\gamma^2 E_8 + \{(1-\nu)\beta^2\gamma + \alpha^2\gamma\}E_3 - \alpha U_4] \sin\cos\sinh$$
$$+ [-\alpha\gamma A_2 + \tfrac{1}{2}\{-\beta\gamma B_3 - (2\alpha^2 + \beta^2)B_8 - \alpha\beta B_5\}$$
$$- \nu\alpha\beta\gamma E_1 - (1-\nu)\beta\gamma^2 E_7 + \{(1-\nu)\beta^2\gamma + \alpha^2\gamma\}E_4 - \alpha R_3] \sin\sin\sinh$$
$$+ \alpha\gamma z [(R_2 \cos\cos + U_1 \cos\sin - U_4 \sin\cos - R_3 \sin\sin) \cosh$$
$$+ (U_2 \cos\cos + R_1 \cos\sin - R_4 \sin\cos - U_3 \sin\sin) \sinh], \quad (10)$$

$$\widehat{xy} = [\alpha\beta A_4 + \tfrac{1}{2}\{\alpha\gamma B_7 + (\alpha^2 - \beta^2)B_1 - \beta\gamma B_6\} - (1-\nu)\gamma^3 E_5$$
$$+ \{(1-\nu)\alpha^3 + (2-\nu)\alpha\beta^2\}E_3 + \{(2-\nu)\alpha^2\beta + (1-\nu)\beta^3\}E_2] \cos\cos\cosh$$
$$+ [-\alpha\beta A_3 + \tfrac{1}{2}\{\alpha\gamma B_8 + (\alpha^2 - \beta^2)B_2 + \beta\gamma B_5\} - (1-\nu)\gamma^3 E_6$$
$$+ \{(1-\nu)\alpha^3 + (2-\nu)\alpha\beta^2\}E_4 - \{(2-\nu)\alpha^2\beta + (1-\nu)\beta^3\}E_1] \cos\sin\cosh$$
$$+ [-\alpha\beta A_2 + \tfrac{1}{2}\{-\alpha\gamma B_5 + (\alpha^2 - \beta^2)B_3 - \beta\gamma B_8\} - (1-\nu)\gamma^3 E_7$$
$$- \{(1-\nu)\alpha^3 + (2-\nu)\alpha\beta^2\}E_1 + \{(2-\nu)\alpha^2\beta + (1-\nu)\beta^3\}E_4] \sin\cos\cosh$$
$$+ [\alpha\beta A_1 + \tfrac{1}{2}\{-\alpha\gamma B_6 + (\alpha^2 - \beta^2)B_4 + \beta\gamma B_7\} - (1-\nu)\gamma^3 E_8$$
$$- \{(1-\nu)\alpha^3 + (2-\nu)\alpha\beta^2\}E_2 - \{(2-\nu)\alpha^2\beta + (1-\nu)\beta^3\}E_3] \sin\sin\cosh$$
$$+ [\alpha\beta A_8 + \tfrac{1}{2}\{\alpha\gamma B_3 + (\alpha^2 - \beta^2)B_5 - \beta\gamma B_2\} - (1-\nu)\gamma^3 E_1$$
$$+ \{(1-\nu)\alpha^3 + (2-\nu)\alpha\beta^2\}E_7 + \{(2-\nu)\alpha^2\beta + (1-\nu)\beta^3\}E_6] \cos\cos\sinh$$
$$+ [-\alpha\beta A_7 + \tfrac{1}{2}\{\alpha\gamma B_4 + (\alpha^2 - \beta^2)B_6 + \beta\gamma B_1\} - (1-\nu)\gamma^3 E_2$$
$$+ \{(1-\nu)\alpha^3 + (2-\nu)\alpha\beta^2\}E_8 - \{(2-\nu)\alpha^2\beta + (1-\nu)\beta^3\}E_5] \cos\sin\sinh$$
$$+ [-\alpha\beta A_6 + \tfrac{1}{2}\{-\alpha\gamma B_1 + (\alpha^2 - \beta^2)B_7 - \beta\gamma B_4\} - (1-\nu)\gamma^3 E_3$$
$$- \{(1-\nu)\alpha^3 + (2-\nu)\alpha\beta^2\}E_5 + \{(2-\nu)\alpha^2\beta + (1-\nu)\beta^3\}E_8] \sin\cos\sinh$$
$$+ [\alpha\beta A_5 + \tfrac{1}{2}\{-\alpha\gamma B_2 + (\alpha^2 - \beta^2)B_8 + \beta\gamma B_3\} - (1-\nu)\gamma^3 E_4$$
$$- \{(1-\nu)\alpha^3 + (2-\nu)\alpha\beta^2\}E_6 - \{(2-\nu)\alpha^2\beta + (1-\nu)\beta^3\}E_7] \sin\sin\sinh$$
$$+ \alpha\beta z [(R_1 \cos\cos - U_2 \cos\sin - U_3 \sin\cos + R_4 \sin\sin) \cosh$$
$$+ (U_1 \cos\cos - R_2 \cos\sin - R_3 \sin\cos + U_4 \sin\sin) \sinh], \quad (11)$$

where, for shortness,

$$R_1 = -\alpha\beta E_1 + \alpha\gamma E_6 + \beta\gamma E_7,$$
$$R_2 = \alpha\gamma E_1 - \beta\gamma E_4 + \alpha\beta E_6,$$

$$R_3 = \beta\gamma E_1 - \alpha\gamma E_4 + \alpha\beta E_7,$$

$$R_4 = -\alpha\beta E_4 - \beta\gamma E_6 - \alpha\gamma E_7;$$

$$U_1 = \alpha\gamma E_2 + \beta\gamma E_3 - \alpha\beta E_5,$$

$$U_2 = \alpha\beta E_2 + \alpha\gamma E_5 - \beta\gamma E_8,$$

$$U_3 = \alpha\beta E_3 + \beta\gamma E_5 - \alpha\gamma E_8,$$

$$U_4 = -\beta\gamma E_2 - \alpha\gamma E_3 - \alpha\beta E_8.$$

For securing the correctness of the above calculus, I have checked that the above stress-components, equations (6)~(11), do satisfy the three stress-equations

$$\frac{\partial\widehat{xx}}{\partial x} + \frac{\partial\widehat{xy}}{\partial y} + \frac{\partial\widehat{zx}}{\partial z} = 0,$$

$$\frac{\partial\widehat{xy}}{\partial x} + \frac{\partial\widehat{yy}}{\partial y} + \frac{\partial\widehat{yz}}{\partial z} = 0,$$

$$\frac{\partial\widehat{zx}}{\partial x} + \frac{\partial\widehat{yz}}{\partial y} + \frac{\partial\widehat{zz}}{\partial z} = 0.$$

5. Simultaneous Equations for Constants

Now Fourier's integral theorem gives

$$F_i(x, y) = \frac{1}{\pi^2}\int_0^\infty d\alpha\int_0^\infty d\beta\int_{-\infty}^\infty\int_{-\infty}^\infty F_i(\xi, \eta)\cos\alpha(x-\xi)\cos\beta(y-\eta)\,d\xi\,d\eta$$

$$= \frac{1}{\pi^2}\int_0^\infty d\alpha\int_0^\infty d\beta\int_{-\infty}^\infty\int_{-\infty}^\infty F_i(\xi, \eta)[\cos\alpha x\cos\beta y\cos\alpha\xi\cos\beta\eta$$

$$+ \cos\alpha x\sin\beta y\cos\alpha\xi\sin\beta\eta + \sin\alpha x\cos\beta y\sin\alpha\xi\cos\beta\eta$$

$$+ \sin\alpha x\sin\beta y\sin\alpha\xi\sin\beta\eta]\,d\xi\,d\eta. \tag{12}$$

In connexion with the Fourier's integral representation, equation (12), let us introduce the following quantities:

$$Z_1 = \frac{1}{\pi^2}\int_{-\infty}^\infty\int_{-\infty}^\infty F_1(\xi, \eta)\cos\alpha\xi\cos\beta\eta\,d\xi\,d\eta,$$

$$Z_2 = \frac{1}{\pi^2}\int_{-\infty}^\infty\int_{-\infty}^\infty F_1(\xi, \eta)\cos\alpha\xi\sin\beta\eta\,d\xi\,d\eta,$$

$$Z_3 = \frac{1}{\pi^2}\int_{-\infty}^\infty\int_{-\infty}^\infty F_1(\xi, \eta)\sin\alpha\xi\cos\beta\eta\,d\xi\,d\eta,$$

$$
\left.\begin{aligned}
Z_4 &= \frac{1}{\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_1(\xi,\ \eta)\sin\alpha\xi\sin\beta\eta\, d\xi\, d\eta,\\
X_1 &= \frac{1}{\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_2(\xi,\ \eta)\cos\alpha\xi\cos\beta\eta\, d\xi\, d\eta,\\
X_2 &= \frac{1}{\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_2(\xi,\ \eta)\cos\alpha\xi\sin\beta\eta\, d\xi\, d\eta,\\
X_3 &= \frac{1}{\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_2(\xi,\ \eta)\sin\alpha\xi\cos\beta\eta\, d\xi\, d\eta,\\
X_4 &= \frac{1}{\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_2(\xi,\ \eta)\sin\alpha\xi\sin\beta\eta\, d\xi\, d\eta,\\
Y_1 &= \frac{1}{\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_3(\xi,\ \eta)\cos\alpha\xi\cos\beta\eta\, d\xi\, d\eta,\\
Y_2 &= \frac{1}{\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_3(\xi,\ \eta)\cos\alpha\xi\sin\beta\eta\, d\xi\, d\eta,\\
Y_3 &= \frac{1}{\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_3(\xi,\ \eta)\sin\alpha\xi\cos\beta\eta\, d\xi\, d\eta,\\
Y_4 &= \frac{1}{\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_3(\xi,\ \eta)\sin\alpha\xi\sin\beta\eta\, d\xi\, d\eta.
\end{aligned}\right\} \tag{13}
$$

Then equation (12) are written out in the forms

$$
\left.\begin{aligned}
F_1(x,\ y) &= \int_0^{\infty}\int_0^{\infty}(Z_1\cos\alpha x\cos\beta y + Z_2\cos\alpha x\sin\beta y\\
&\qquad + Z_3\sin\alpha x\cos\beta y + Z_4\sin\alpha x\sin\beta y)\, d\alpha\, d\beta,\\
F_2(x,\ y) &= \int_0^{\infty}\int_0^{\infty}(X_1\cos\alpha x\cos\beta y + X_2\cos\alpha x\sin\beta y\\
&\qquad + X_3\sin\alpha x\cos\beta y + X_4\sin\alpha x\sin\beta y)\, d\alpha\, d\beta,\\
F_3(x,\ y) &= \int_0^{\infty}\int_0^{\infty}(Y_1\cos\alpha x\cos\beta y + Y_2\cos\alpha x\sin\beta y\\
&\qquad + Y_3\sin\alpha x\cos\beta y + Y_4\sin\alpha x\sin\beta y)\, d\alpha\, d\beta.
\end{aligned}\right\} \tag{14}
$$

With equations (8), (10), and (9), the boundary conditions (1) are satisfied by putting

$$
\begin{aligned}
&[\gamma^2 A_1 - \beta\gamma B_6 + \alpha\gamma B_7 - \gamma^2(\beta E_2 + \alpha E_3)\\
&\qquad - 2(1-\nu)\alpha\beta\gamma E_8]\cosh\gamma c + \gamma^2 c\sinh\gamma c\cdot U_4 = Z_1,
\end{aligned} \tag{15}
$$

$$
\begin{aligned}
&[\gamma^2 A_2 + \beta\gamma B_5 + \alpha\gamma B_8 + \gamma^2(\beta E_1 - \alpha E_4)\\
&\qquad + 2(1-\nu)\alpha\beta\gamma E_7]\cosh\gamma c + \gamma^2 c\sinh\gamma c\cdot R_3 = Z_2,
\end{aligned} \tag{16}
$$

$$[\gamma^2 A_3 - \alpha\gamma B_5 - \beta\gamma B_8 + \gamma^2(\alpha E_1 - \beta E_4)$$
$$+ 2(1-\nu)\alpha\beta\gamma E_6] \cosh\gamma c + \gamma^2 c \sinh\gamma c \cdot R_2 = Z_3, \tag{17}$$

$$[\gamma^2 A_4 - \alpha\gamma B_6 + \beta\gamma B_7 + \gamma^2(\alpha E_2 + \beta E_3)$$
$$- 2(1-\nu)\alpha\beta\gamma E_5] \cosh\gamma c + \gamma^2 c \sinh\gamma c \cdot U_1 = Z_4, \tag{18}$$

$$[\alpha\gamma A_7 + \tfrac{1}{2}\{-(\alpha^2+\gamma^2)B_1 - \alpha\beta B_4 + \beta\gamma B_6\} + \alpha\gamma(\alpha E_5 - \beta E_8) + \alpha U_2$$
$$+ (1-\nu)\beta\gamma(\gamma E_2 + \beta E_5 + \alpha E_8)] \cosh\gamma c + \alpha\gamma c \sinh\gamma c \cdot U_2 = X_1, \tag{19}$$

$$[\alpha\gamma A_8 + \tfrac{1}{2}\{-(\alpha^2+\gamma^2)B_2 + \alpha\beta B_3 - \beta\gamma B_5\} + \alpha\gamma(\alpha E_6 + \beta E_7) + \alpha R_1$$
$$+ (1-\nu)\beta\gamma(-\gamma E_1 + \beta E_6 - \alpha E_7)] \cosh\gamma c + \alpha\gamma c \sinh\gamma c \cdot R_1 = X_2, \tag{20}$$

$$[-\alpha\gamma A_5 + \tfrac{1}{2}\{\alpha\beta B_2 - (\alpha^2+\gamma^2)B_3 + \beta\gamma B_8\} + \alpha\gamma(\beta E_6 + \alpha E_7) - \alpha R_4$$
$$+ (1-\nu)\beta\gamma(\gamma E_4 - \alpha E_6 + \beta E_7)] \cosh\gamma c - \alpha\gamma c \sinh\gamma c \cdot R_4 = X_3, \tag{21}$$

$$[-\alpha\gamma A_6 + \tfrac{1}{2}\{-\alpha\beta E_1 - (\alpha^2+\gamma^2)B_4 - \beta\gamma B_7\} + \alpha\gamma(-\beta E_5 + \alpha E_8) - \alpha U_3$$
$$+ (1-\nu)\beta\gamma(-\gamma E_3 + \alpha E_5 + \beta E_8)] \cosh\gamma c - \alpha\gamma c \sinh\gamma c \cdot U_3 = X_4, \tag{22}$$

$$[\beta\gamma A_6 + \tfrac{1}{2}\{(\beta^2+\gamma^2)B_1 + \alpha\beta B_4 - \alpha\gamma B_7\} + \beta\gamma(\beta E_5 - \alpha E_8) + \beta U_3$$
$$+ (1-\nu)\alpha\gamma(\gamma E_3 + \alpha E_5 + \beta E_8)] \cosh\gamma c + \beta\gamma c \sinh\gamma c \cdot U_3 = Y_1, \tag{23}$$

$$[-\beta\gamma A_5 + \tfrac{1}{2}\{(\beta^2+\gamma^2)B_2 - \alpha\beta B_3 - \alpha\gamma B_8\} + \beta\gamma(\beta E_6 + \alpha E_7) - \beta R_4$$
$$+ (1-\nu)\alpha\gamma(\gamma E_4 + \alpha E_6 - \beta E_7)] \cosh\gamma c - \beta\gamma c \sinh\gamma c \cdot R_4 = Y_2, \tag{24}$$

$$[\beta\gamma A_8 + \tfrac{1}{2}\{-\alpha\beta B_2 + (\beta^2+\gamma^2)B_3 + \alpha\gamma B_5\} + \beta\gamma(\alpha E_6 + \beta E_7) + \beta R_1$$
$$+ (1-\nu)\alpha\gamma(-\gamma E_1 - \beta E_6 + \alpha E_7)] \cosh\gamma c + \beta\gamma c \sinh\gamma c \cdot R_1 = Y_3, \tag{25}$$

$$[-\beta\gamma A_7 + \tfrac{1}{2}\{\alpha\beta B_1 + (\beta^2+\gamma^2)B_4 + \alpha\gamma B_6\} + \beta\gamma(-\alpha E_5 + \beta E_8) - \beta U_2$$
$$+ (1-\nu)\alpha\gamma(-\gamma E_2 + \beta E_5 + \alpha E_8)] \cosh\gamma c - \beta\gamma c \sinh\gamma c \cdot U_2 = Y_4; \tag{26}$$

$$[\gamma^2 A_5 - \beta\gamma B_2 + \alpha\gamma B_3 - \gamma^2(\beta E_6 + \alpha E_7)$$
$$- 2(1-\nu)\alpha\beta\gamma E_4] \sinh\gamma c + \gamma^2 c \cosh\gamma c \cdot R_4 = 0, \tag{27}$$

$$[\gamma^2 A_6 + \beta\gamma B_1 + \alpha\gamma B_4 + \gamma^2(\beta E_5 - \alpha E_8)$$
$$+ 2(1-\nu)\alpha\beta\gamma E_3] \sinh\gamma c + \gamma^2 c \cosh\gamma c \cdot U_3 = 0, \tag{28}$$

$$[\gamma^2 A_7 - \alpha\gamma B_1 - \beta\gamma B_4 + \gamma^2(\alpha E_5 - \beta E_8)$$
$$+ 2(1-\nu)\alpha\beta\gamma E_2] \sinh\gamma c + \gamma^2 c \cosh\gamma c \cdot U_2 = 0, \tag{29}$$

$$[\gamma^2 A_8 - \alpha\gamma B_2 + \beta\gamma B_3 + \gamma^2(\alpha E_6 + \beta E_7)$$
$$- 2(1-\nu)\alpha\beta\gamma E_1] \sinh\gamma c + \gamma^2 c \cosh\gamma c \cdot R_1 = 0, \tag{30}$$

$$[\alpha\gamma A_3 + \tfrac{1}{2}\{\beta\gamma B_2 - (\alpha^2 + \gamma^2)B_5 - \alpha\beta B_8\} + \alpha\gamma(\alpha E_1 - \beta E_4) + \alpha R_2$$
$$+ (1-\nu)\beta\gamma(\beta E_1 + \alpha E_4 + \gamma E_6)] \sinh \gamma c + \alpha\gamma c \cosh \gamma c \cdot R_2 = 0, \tag{31}$$

$$[\alpha\gamma A_4 + \tfrac{1}{2}\{- \beta\gamma B_1 - (\alpha^2 + \gamma^2)B_6 + \alpha\beta B_7\} + \alpha\gamma(\alpha E_2 + \beta E_3) + \alpha U_1$$
$$+ (1-\nu)\beta\gamma(\beta E_2 - \alpha E_3 - \gamma E_5)] \sinh \gamma c + \alpha\gamma c \cosh \gamma c \cdot U_1 = 0, \tag{32}$$

$$[- \alpha\gamma A_1 + \tfrac{1}{2}\{\beta\gamma B_4 + \alpha\beta B_6 - (\alpha^2 + \gamma^2)B_7\} + \alpha\gamma(\beta E_2 + \alpha E_3) - \alpha U_4$$
$$+ (1-\nu)\beta\gamma(- \alpha E_2 + \beta E_3 + \gamma E_8)] \sinh \gamma c - \alpha\gamma c \cosh \gamma c \cdot U_4 = 0, \tag{33}$$

$$[- \alpha\gamma A_2 + \tfrac{1}{2}\{- \beta\gamma B_3 - \alpha\beta B_5 - (\alpha^2 + \gamma^2)B_8\} + \alpha\gamma(- \beta E_1 + \alpha E_4) - \alpha R_3$$
$$+ (1-\nu)\beta\gamma(\alpha E_1 + \beta E_4 - \gamma E_7)] \sinh \gamma c - \alpha\gamma c \cosh \gamma c \cdot R_3 = 0, \tag{34}$$

$$[\beta\gamma A_2 + \tfrac{1}{2}\{- \alpha\gamma B_3 + (\beta^2 + \gamma^2)B_5 + \alpha\beta B_8\} + \beta\gamma(\beta E_1 - \alpha E_4) + \beta R_3$$
$$+ (1-\nu)\alpha\gamma(\alpha E_1 + \beta E_4 + \gamma E_7)] \sinh \gamma c + \beta\gamma c \cosh \gamma c \cdot R_3 = 0, \tag{35}$$

$$[-\beta\gamma A_1 + \tfrac{1}{2}\{- \alpha\gamma B_4 + (\beta^2 + \gamma^2)B_6 - \alpha\beta B_7\} + \beta\gamma(\beta E_2 + \alpha E_3) - \beta U_4$$
$$+ (1-\nu)\alpha\gamma(\alpha E_2 - \beta E_3 + \gamma E_8)] \sinh \gamma c - \beta\gamma c \cosh \gamma c \cdot U_4 = 0, \tag{36}$$

$$[\beta\gamma A_4 + \tfrac{1}{2}\{\alpha\gamma B_1 - \alpha\beta B_6 + (\beta^2 + \gamma^2)B_7\} + \beta\gamma(\alpha E_2 + \beta E_3) + \beta U_1$$
$$+ (1-\nu)\alpha\gamma(- \beta E_2 + \alpha E_3 - \gamma E_5)] \sinh \gamma c + \beta\gamma c \cosh \gamma c \cdot U_1 = 0, \tag{37}$$

$$[- \beta\gamma A_3 + \tfrac{1}{2}\{\alpha\gamma B_2 + \alpha\beta B_5 + (\beta^2 + \gamma^2)B_8\} + \beta\gamma(- \alpha E_1 + \beta E_4) - \beta R_2$$
$$+ (1-\nu)\alpha\gamma(\beta E_1 + \alpha E_4 - \gamma E_6)] \sinh \gamma c - \beta\gamma c \cosh \gamma c \cdot R_2 = 0. \tag{38}$$

These conditional equations (15)~(38) will enable us to determine the 24 unknowns. To solve these equations simultaneously, it is to be noticed here that these 24 equations are separated into two groups; that is to say,

1) The first group contains as unknowns

$$A_4,\ A_7,\ A_6,\ A_1;\ B_4,\ B_7,\ B_6,\ B_1;\ E_8,\ E_3,\ E_2,\ E_5.$$

2) The second group contains as unknowns

$$A_8,\ A_3,\ A_2,\ A_5;\ B_8,\ B_3,\ B_2,\ B_5;\ E_4,\ E_7,\ E_6,\ E_1.$$

Thus the seeming difficulties in solving the enormous simultaneous equations can greatly be reduced.

Furthermore, the conditional equations (15)~(38) are rearranged in the following forms:

$$\gamma A_1 - \beta B_6 + \alpha B_7 - \gamma(\beta E_2 + \alpha E_3) - 2(1-\nu)\alpha\beta E_8$$
$$- \gamma c \tanh \gamma c(\beta\gamma E_2 + \alpha\gamma E_3 + \alpha\beta E_8) = \frac{Z_1}{\gamma \cosh \gamma c}, \tag{39}$$

$$\gamma A_2 + \beta B_5 + \alpha B_8 + \gamma(\beta E_1 - \alpha E_4) + 2(1-\nu)\alpha\beta E_7$$
$$+ \gamma c \tanh \gamma c(\beta\gamma E_1 - \alpha\gamma E_4 + \alpha\beta E_7) = \frac{Z_2}{\gamma \cosh \gamma c}, \tag{40}$$

$$\gamma A_3 - \alpha B_5 - \beta B_8 + \gamma(\alpha E_1 - \beta E_4) + 2(1-\nu)\alpha\beta E_6$$
$$+ \gamma c \tanh \gamma c(\alpha\gamma E_1 - \beta\gamma E_4 + \alpha\beta E_6) = \frac{Z_3}{\gamma \cosh \gamma c}, \tag{41}$$

$$\gamma A_4 - \alpha B_6 + \beta B_7 + \gamma(\alpha E_2 + \beta E_3) - 2(1-\nu)\alpha\beta E_5$$
$$+ \gamma c \tanh \gamma c(\alpha\gamma E_2 + \beta\gamma E_3 - \alpha\beta E_5) = \frac{Z_4}{\gamma \cosh \gamma c}, \tag{42}$$

$$\alpha\gamma A_7 + \tfrac{1}{2}\{-(\alpha^2+\gamma^2)B_1 - \alpha\beta B_4 + \beta\gamma B_6\} + \alpha\gamma(\alpha E_5 - \beta E_8)$$
$$+ (1-\nu)\beta\gamma(\gamma E_2 + \beta E_5 + \alpha E_8)$$
$$+ (1 + \gamma c \tanh \gamma c)\alpha(\alpha\beta E_2 + \alpha\gamma E_5 - \beta\gamma E_8) = \frac{X_1}{\cosh \gamma c}, \tag{43}$$

$$\alpha\gamma A_8 + \tfrac{1}{2}\{-(\alpha^2+\gamma^2)B_2 + \alpha\beta B_3 - \beta\gamma B_5\} + \alpha\gamma(\alpha E_6 + \beta E_7)$$
$$+ (1-\nu)\beta\gamma(-\gamma E_1 + \beta E_6 - \alpha E_7)$$
$$+ (1 + \gamma c \tanh \gamma c)\alpha(-\alpha\beta E_1 + \alpha\gamma E_6 + \beta\gamma E_7) = \frac{X_2}{\cosh \gamma c}, \tag{44}$$

$$-\alpha\gamma A_5 + \tfrac{1}{2}\{\alpha\beta B_2 - (\alpha^2+\gamma^2)B_3 + \beta\gamma B_8\} + \alpha\gamma(\beta E_6 + \alpha E_7)$$
$$+ (1-\nu)\beta\gamma(\gamma E_4 - \alpha E_6 + \beta E_7)$$
$$+ (1 + \gamma c \tanh \gamma c)\alpha(\alpha\beta E_4 + \beta\gamma E_6 + \alpha\gamma E_7) = \frac{X_3}{\cosh \gamma c}, \tag{45}$$

$$-\alpha\gamma A_6 + \tfrac{1}{2}\{-\alpha\beta B_1 - (\alpha^2+\gamma^2)B_4 - \beta\gamma B_7\} + \alpha\gamma(-\beta E_5 + \alpha E_8)$$
$$+ (1-\nu)\beta\gamma(-\gamma E_3 + \alpha E_5 + \beta E_8)$$
$$+ (1 + \gamma c \tanh \gamma c)\alpha(-\alpha\beta E_3 - \beta\gamma E_5 + \alpha\gamma E_8) = \frac{X_4}{\cosh \gamma c}, \tag{46}$$

$$\beta\gamma A_6 + \tfrac{1}{2}\{(\beta^2+\gamma^2)B_1 + \alpha\beta B_4 - \alpha\gamma B_7\} + \beta\gamma(\beta E_5 - \alpha E_8)$$
$$+ (1-\nu)\alpha\gamma(\gamma E_3 + \alpha E_5 + \beta E_8)$$
$$+ (1 + \gamma c \tanh \gamma c)\beta(\alpha\beta E_3 + \beta\gamma E_5 - \alpha\gamma E_8) = \frac{Y_1}{\cosh \gamma c}, \tag{47}$$

$$-\beta\gamma A_5 + \tfrac{1}{2}\{(\beta^2+\gamma^2)B_2 - \alpha\beta B_3 - \alpha\gamma B_8\} + \beta\gamma(\beta E_6 + \alpha E_7)$$
$$+ (1-\nu)\alpha\gamma(\gamma E_4 + \alpha E_6 - \beta E_7)$$
$$+ (1 + \gamma c \tanh \gamma c)\beta(\alpha\beta E_4 + \beta\gamma E_6 + \alpha\gamma E_7) = \frac{Y_2}{\cosh \gamma c}, \tag{48}$$

$$\beta\gamma A_8 + \tfrac{1}{2}\{-\alpha\beta B_2 + (\beta^2+\gamma^2)B_3 + \alpha\gamma B_5\} + \beta\gamma(\alpha E_6 + \beta E_7)$$
$$+ (1-\nu)\alpha\gamma(-\gamma E_1 - \beta E_6 + \alpha\gamma E_7)$$
$$+ (1+\gamma c \tanh \gamma c)\beta(-\alpha\beta E_1 + \alpha\gamma E_6 + \beta\gamma E_7) = \frac{Y_3}{\cosh \gamma c}, \tag{49}$$

$$-\beta\gamma A_7 + \tfrac{1}{2}\{\alpha\beta B_1 + (\beta^2+\gamma^2)B_4 + \alpha\gamma B_6\} + \beta\gamma(-\alpha E_5 + \beta E_8)$$
$$+ (1-\nu)\alpha\gamma(-\gamma E_2 + \beta E_5 + \alpha E_8)$$
$$+ (1+\gamma c \tanh \gamma c)\beta(-\alpha\beta E_2 - \alpha\gamma E_5 + \beta\gamma E_8) = \frac{Y_4}{\cosh \gamma c}; \tag{50}$$

$$\gamma A_5 - \beta B_2 + \alpha B_3 - \gamma(\beta E_6 + \alpha E_7) - 2(1-\nu)\alpha\beta E_4$$
$$- \gamma c \coth \gamma c(\alpha\beta E_4 + \beta\gamma E_6 + \alpha\gamma E_7) = 0, \tag{51}$$

$$\gamma A_6 + \beta B_1 + \alpha B_4 + \gamma(\beta E_5 - \alpha E_8) + 2(1-\nu)\alpha\beta E_3$$
$$+ \gamma c \coth \gamma c(\alpha\beta E_3 + \beta\gamma E_5 - \alpha\gamma E_8) = 0, \tag{52}$$

$$\gamma A_7 - \alpha B_1 - \beta B_4 + \gamma(\alpha E_5 - \beta E_8) + 2(1-\nu)\alpha\beta E_2$$
$$+ \gamma c \coth \gamma c(\alpha\beta E_2 + \alpha\gamma E_5 - \beta\gamma E_8) = 0, \tag{53}$$

$$\gamma A_8 - \alpha B_2 + \beta B_3 + \gamma(\alpha E_6 + \beta E_7) - 2(1-\nu)\alpha\beta E_1$$
$$+ \gamma c \coth \gamma c(-\alpha\beta E_1 + \alpha\gamma E_6 + \beta\gamma E_7) = 0, \tag{54}$$

$$\alpha\gamma A_3 + \tfrac{1}{2}\{\beta\gamma B_2 - (\alpha^2+\gamma^2)B_5 - \alpha\beta B_8\} + \alpha\gamma(\alpha E_1 - \beta E_4)$$
$$+ (1-\nu)\beta\gamma(\beta E_1 + \alpha E_4 + \gamma E_6)$$
$$+ (1+\gamma c \coth \gamma c)\alpha(\alpha\gamma E_1 - \beta\gamma E_4 + \alpha\beta E_6) = 0, \tag{55}$$

$$\alpha\gamma A_4 + \tfrac{1}{2}\{-\beta\gamma B_1 - (\alpha^2+\gamma^2)B_6 + \alpha\beta B_7\} + \alpha\gamma(\alpha E_2 + \beta E_3)$$
$$+ (1-\nu)\beta\gamma(\beta E_2 - \alpha E_3 - \gamma E_5)$$
$$+ (1+\gamma c \coth \gamma c)\alpha(\alpha\gamma E_2 + \beta\gamma E_3 - \alpha\beta E_5) = 0, \tag{56}$$

$$-\alpha\gamma A_1 + \tfrac{1}{2}\{\beta\gamma B_4 + \alpha\beta B_6 - (\alpha^2+\gamma^2)B_7\} + \alpha\gamma(\beta E_2 + \alpha E_3)$$
$$+ (1-\nu)\beta\gamma(-\alpha E_2 + \beta E_3 + \gamma E_8)$$
$$+ (1+\gamma c \coth \gamma c)\alpha(\beta\gamma E_2 + \alpha\gamma E_3 + \alpha\beta E_8) = 0, \tag{57}$$

$$-\alpha\gamma A_2 + \tfrac{1}{2}\{-\beta\gamma B_3 - \alpha\beta B_5 - (\alpha^2+\gamma^2)B_8\} + \alpha\gamma(-\beta E_1 + \alpha E_4)$$
$$+ (1-\nu)\beta\gamma(\alpha E_1 + \beta E_4 - \gamma E_7)$$
$$+ (1+\gamma c \coth \gamma c)\alpha(-\beta\gamma E_1 + \alpha\gamma E_4 - \alpha\beta E_7) = 0, \tag{58}$$

$$\beta\gamma A_2 + \tfrac{1}{2}\{-\alpha\gamma B_3 + (\beta^2+\gamma^2)B_5 + \alpha\beta B_8\} + \beta\gamma(\beta E_1 - \alpha E_4)$$

$$+ (1-\nu)\alpha\gamma(\alpha E_1 + \beta E_4 + \gamma E_7)$$
$$+ (1+\gamma c \coth \gamma c)\beta(\beta\gamma E_1 - \alpha\gamma E_4 + \alpha\beta E_7) = 0, \tag{59}$$
$$-\beta\gamma A_1 + \tfrac{1}{2}\{-\alpha\gamma B_4 + (\beta^2+\gamma^2)B_6 - \alpha\beta B_7\} + \beta\gamma(\beta E_2 + \alpha E_3)$$
$$+ (1-\nu)\alpha\gamma(\alpha E_2 - \beta E_3 + \gamma E_8)$$
$$+ (1+\gamma c \coth \gamma c)\beta(\beta\gamma E_2 + \alpha\gamma E_3 + \alpha\beta E_8) = 0, \tag{60}$$
$$\beta\gamma A_4 + \tfrac{1}{2}\{\alpha\gamma B_1 - \alpha\beta B_6 + (\beta^2+\gamma^2)B_7\} + \beta\gamma(\alpha E_2 + \beta E_3)$$
$$+ (1-\nu)\alpha\gamma(-\beta E_2 + \alpha E_3 - \gamma E_5)$$
$$+ (1+\gamma c \coth \gamma c)\beta(\alpha\gamma E_2 + \beta\gamma E_3 - \alpha\beta E_5) = 0, \tag{61}$$
$$-\beta\gamma A_3 + \tfrac{1}{2}\{\alpha\gamma B_2 + \alpha\beta B_5 + (\beta^2+\gamma^2)B_8\} + \beta\gamma(-\alpha E_1 + \beta E_4)$$
$$+ (1-\nu)\alpha\gamma(\beta E_1 + \alpha E_4 - \gamma E_6)$$
$$+ (1+\gamma c \coth \gamma c)\beta(-\alpha\gamma E_1 + \beta\gamma E_4 - \alpha\beta E_6) = 0. \tag{62}$$

The above conditional equations (39)~(62) may be arranged in schematic forms as given in pp. 27~28, in which, for brevity, κ and κ' are represented by

$$\gamma c \tanh \gamma c = \kappa, \qquad \gamma c \coth \gamma c = \kappa'. \tag{63}$$

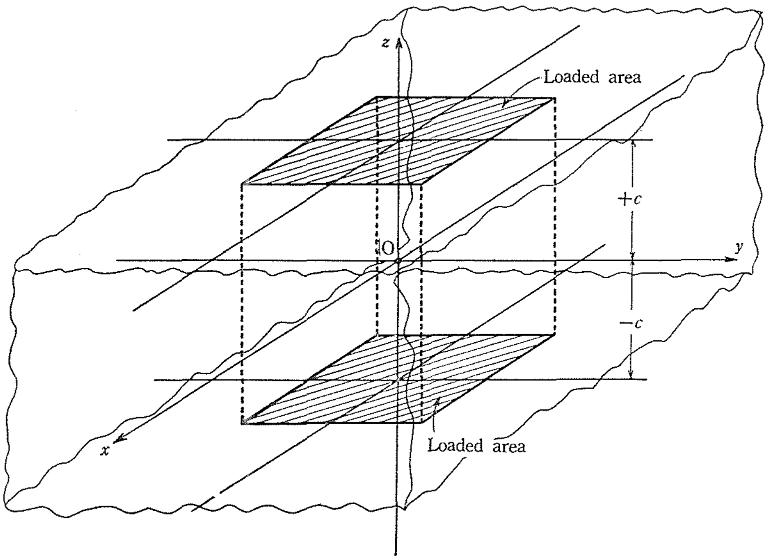

Fig. 3

(I) First set of equations for A_4, A_7, A_6, A_1, B_4, B_7, B_6, B_1, E_8, E_3, E_2, E_5.

γA_4	γA_7	γA_6	γA_1	$\frac{B_4}{2}$	$\frac{B_7}{2}$	$\frac{B_6}{2}$	$\frac{B_1}{2}$	$-E_8$	$-E_3$	$-E_2$	$-E_5$	= const.
1	0	0	0	0	2β	-2α	0	0	$-(1+\kappa)\beta\gamma$	$-(1+\kappa)\alpha\gamma$	$[2(1-\nu)+\kappa]\alpha\beta$	$\frac{Z_4}{\gamma\cosh\gamma c}$
0	1	0	0	-2β	0	0	-2α	$(1+\kappa')\beta\gamma$	0	$-[2(1-\nu)+\kappa']\alpha\beta$	$-(1+\kappa')\alpha\gamma$	0
0	0	1	0	2α	0	0	2β	$(1+\kappa')\alpha\gamma$	$-[2(1-\nu)+\kappa']\alpha\beta$	0	$-(1+\kappa')\beta\gamma$	0
0	0	0	1	0	2α	-2β	0	$[2(1-\nu)+\kappa]\alpha\beta$	$(1+\kappa)\alpha\gamma$	$(1+\kappa)\beta\gamma$	0	$\frac{Z_1}{\gamma\cosh\gamma c}$
β	0	0	0	0	$\beta^2+\gamma^2$	$-\alpha\beta$	$\alpha\gamma$	0	$-[(1-\nu)\alpha^2+\beta^2(2+\kappa')]\gamma$	$-[(1+\nu)+\kappa']\alpha\beta\gamma$	$[\beta^2(1+\kappa')+(1-\nu)\gamma^2]\alpha$	0
0	β	0	0	$-(\beta^2+\gamma^2)$	0	$-\alpha\gamma$	$-\alpha\beta$	$[(1-\nu)\alpha^2+\beta^2(2+\kappa)]\gamma$	0	$-[\beta^2(1+\kappa)+(1-\nu)\gamma^2]\alpha$	$-[(1+\nu)+\kappa]\alpha\beta\gamma$	$-\frac{Y_4}{\cosh\gamma c}$
0	0	β	0	$\alpha\beta$	$-\alpha\gamma$	0	$\beta^2+\gamma^2$	$[(1+\nu)+\kappa]\alpha\beta\gamma$	$-[\beta^2(1+\kappa)+(1-\nu)\gamma^2]\alpha$	0	$-[(1-\nu)\alpha^2+\beta^2(2+\kappa)]\gamma$	$\frac{Y_1}{\cosh\gamma c}$
0	0	0	β	$\alpha\gamma$	$\alpha\beta$	$-(\beta^2+\gamma^2)$	0	$[\beta^2(1+\kappa')+(1-\nu)\gamma^2]\alpha$	$[(1+\nu)+\kappa']\alpha\beta\gamma$	$[(1-\nu)\alpha^2+\beta^2(2+\kappa')]\gamma$	0	0
α	0	0	0	0	$\alpha\beta$	$-(\alpha^2+\gamma^2)$	$-\beta\gamma$	0	$-[(1+\nu)+\kappa']\alpha\beta\gamma$	$-[\alpha^2(2+\kappa')+(1-\nu)\beta^2]\gamma$	$[\alpha^2(1+\kappa')+(1-\nu)\gamma^2]\beta$	0
0	α	0	0	$-\alpha\beta$	0	$\beta\gamma$	$-(\alpha^2+\gamma^2)$	$[(1+\nu)+\kappa]\alpha\beta\gamma$	0	$-[\alpha^2(1+\kappa)+(1-\nu)\gamma^2]\beta$	$-[\alpha^2(2+\kappa)+(1-\nu)\beta^2]\gamma$	$\frac{X_1}{\cosh\gamma c}$
0	0	α	0	$\alpha^2+\gamma^2$	$\beta\gamma$	0	$\alpha\beta$	$[\alpha^2(2+\kappa)+(1-\nu)\beta^2]\gamma$	$-[\alpha^2(1+\kappa)+(1-\nu)\gamma^2]\beta$	0	$-[(1+\nu)+\kappa]\alpha\beta\gamma$	$-\frac{X_4}{\cosh\gamma c}$
0	0	0	α	$-\beta\gamma$	$\alpha^2+\gamma^2$	$-\alpha\beta$	0	$[\alpha^2(1+\kappa')+(1-\nu)\gamma^2]\beta$	$[\alpha^2(2+\kappa')+(1-\nu)\beta^2]\gamma$	$[(1+\nu)+\kappa']\alpha\beta\gamma$	0	0

(64)

(II) Second set of equations for A_8, A_3, A_2, A_5, B_8, B_3, B_2, B_5, E_4, E_7, E_6, E_1.

γA_8	γA_3	γA_2	γA_5	$\frac{B_8}{2}$	$\frac{B_3}{2}$	$\frac{B_2}{2}$	$\frac{B_5}{2}$	$-E_4$	$-E_7$	$-E_6$	$-E_1$	= const.
1	0	0	0	0	2β	-2α	0	0	$-(1+\kappa')\beta\gamma$	$-(1+\kappa')\alpha\gamma$	$[2(1-\nu)+\kappa']\alpha\beta$	0
0	1	0	0	-2β	0	0	-2α	$(1+\kappa)\beta\gamma$	0	$-[2(1-\nu)+\kappa]\alpha\beta$	$-(1+\kappa)\alpha\gamma$	$\frac{Z_3}{\gamma \cosh \gamma c}$
0	0	1	0	2α	0	0	2β	$(1+\kappa)\alpha\gamma$	$-[2(1-\nu)+\kappa]\alpha\beta$	0	$-(1+\kappa)\beta\gamma$	$\frac{Z_2}{\gamma \cosh \gamma c}$
0	0	0	1	0	2α	-2β	0	$[2(1-\nu)+\kappa']\alpha\beta$	$(1+\kappa')\alpha\gamma$	$(1+\kappa')\beta\gamma$	0	0
β	0	0	0	0	$\beta^2+\gamma^2$	$-\alpha\beta$	$\alpha\gamma$	0	$-[(1-\nu)\alpha^2+\beta^2(2+\kappa)]\gamma$	$-[(1+\nu)+\kappa]\alpha\beta\gamma$	$[\beta^2(1+\kappa)+(1-\nu)\gamma^2]\alpha$	$\frac{Y_3}{\cosh \gamma c}$
0	β	0	0	$-(\beta^2+\gamma^2)$	0	$-\alpha\gamma$	$-\alpha\beta$	$[(1-\nu)\alpha^2+\beta^2(2+\kappa')]\gamma$	0	$-[\beta^2(1+\kappa')+(1-\nu)\gamma^2]\alpha$	$-[(1+\nu)+\kappa']\alpha\beta\gamma$	0
0	0	β	0	$\alpha\beta$	$-\alpha\gamma$	0	$\beta^2+\gamma^2$	$[(1+\nu)+\kappa']\alpha\beta\gamma$	$-[\beta^2(1+\kappa')+(1-\nu)\gamma^2]\alpha$	0	$-[(1-\nu)\alpha^2+\beta^2(2+\kappa)]\gamma$	0
0	0	0	β	$\alpha\gamma$	$\alpha\beta$	$-(\beta^2+\gamma^2)$	0	$[\beta^2(1+\kappa)+(1-\nu)\gamma^2]\alpha$	$[(1+\nu)+\kappa]\alpha\beta\gamma$	$[(1-\nu)\alpha^2+\beta^2(2+\kappa)]\gamma$	0	$-\frac{Y_2}{\cosh \gamma c}$
α	0	0	0	0	$\alpha\beta$	$-(\alpha^2+\gamma^2)$	$-\beta\gamma$	0	$-[(1+\nu)+\kappa]\alpha\beta\gamma$	$-[\alpha^2(2+\kappa)+(1-\nu)\beta^2]\gamma$	$[\alpha^2(1+\kappa)+(1-\nu)\gamma^2]\beta$	$\frac{X_2}{\cosh \gamma c}$
0	α	0	0	$-\alpha\beta$	0	$\beta\gamma$	$-(\alpha^2+\gamma^2)$	$[(1+\nu)+\kappa']\alpha\beta\gamma$	0	$-[\alpha^2(1+\kappa')+(1-\nu)\gamma^2]\beta$	$-[\alpha^2(2+\kappa')+(1-\nu)\beta^2]\gamma$	
0	0	α	0	$\alpha^2+\gamma^2$	$\beta\gamma$	0	$\alpha\beta$	$[\alpha^2(2+\kappa')+(1-\nu)\beta^2]\gamma$	$-[\alpha^2(1+\kappa')+(1-\nu)\gamma^2]\beta$	0	$-[(1+\nu)+\kappa']\alpha\beta\gamma$	
0	0	0	α	$-\beta\gamma$	$\alpha^2+\gamma^2$	$-\alpha\beta$	0	$[\alpha^2(1+\kappa)+(1-\nu)\gamma^2]\beta$	$[\alpha^2(2+\kappa)+(1-\nu)\beta^2]\gamma$	$[(1+\nu)+\kappa]\alpha\beta\gamma$	0	$-\frac{X_3}{\cosh \gamma c}$

(65)

6. The Solution —— Stress-components

The process of solving two sets of equations (64) and (65), and that involved in substituting the values of constants into equations (6)~(11) need a sort of technique; but they are not written here for avoiding diffuseness.

The resulting solution can be rearranged into the following forms:

$$\widehat{xx}=\frac{1}{\pi^2}\int_0^\infty d\alpha\int_0^\infty d\beta\int_{-\infty}^\infty\int_{-\infty}^\infty\frac{1}{\cosh\gamma c}\Big(\frac{1}{1-\kappa+\kappa'}\Big[\Big(\frac{\alpha^2}{\gamma^2}+2\nu\frac{\beta^2}{\gamma^2}-\frac{\alpha^2}{\gamma^2}\kappa'\Big)\cosh\gamma z$$

$$+\frac{\alpha^2}{\gamma^2}\gamma z\sinh\gamma z\Big]F_1(\xi,\eta)\cos\alpha(x-\xi)\cos\beta(y-\eta)$$

$$-\frac{1}{1+\kappa-\kappa'}\frac{2\alpha}{\gamma}\Big[\Big\{1+\frac{\beta^2}{\gamma^2}(\nu+\kappa-\kappa')-\frac{\alpha^2}{2\gamma^2}\kappa'\Big\}\sinh\gamma z+\frac{\alpha^2}{2\gamma^2}\gamma z\cosh\gamma z\Big]$$

$$\times F_2(\xi,\eta)\sin\alpha(x-\xi)\cos\beta(y-\eta)$$

$$-\frac{1}{1+\kappa-\kappa'}\frac{2\beta}{\gamma}\Big[\Big\{\nu\frac{\beta^2}{\gamma^2}-\frac{\alpha^2}{\gamma^2}(\kappa-\kappa')-\frac{\alpha^2}{2\gamma^2}\kappa'\Big\}\sinh\gamma z+\frac{\alpha^2}{2\gamma^2}\gamma z\cosh\gamma z\Big]$$

$$\times F_3(\xi,\eta)\cos\alpha(x-\xi)\sin\beta(y-\eta)\Big)d\xi\,d\eta, \tag{66}$$

$$\widehat{yy}=\frac{1}{\pi^2}\int_0^\infty d\alpha\int_0^\infty d\beta\int_{-\infty}^\infty\int_{-\infty}^\infty\frac{1}{\cosh\gamma c}\Big(\frac{1}{1-\kappa+\kappa'}\Big[\Big(\frac{\beta^2}{\gamma^2}+2\nu\frac{\alpha^2}{\gamma^2}-\frac{\beta^2}{\gamma^2}\kappa'\Big)\cosh\gamma z$$

$$+\frac{\beta^2}{\gamma^2}\gamma z\sinh\gamma z\Big]F_1(\xi,\eta)\cos\alpha(x-\xi)\cos\beta(y-\eta)$$

$$-\frac{1}{1+\kappa-\kappa'}\frac{2\alpha}{\gamma}\Big[\Big\{\nu\frac{\alpha^2}{\gamma^2}-\frac{\beta^2}{\gamma^2}(\kappa-\kappa')-\frac{\beta^2}{2\gamma^2}\kappa'\Big\}\sinh\gamma z+\frac{\beta^2}{2\gamma^2}\gamma z\cosh\gamma z\Big]$$

$$\times F_2(\xi,\eta)\sin\alpha(x-\xi)\cos\beta(y-\eta)$$

$$-\frac{1}{1+\kappa-\kappa'}\frac{2\beta}{\gamma}\Big[\Big\{1+\frac{\alpha^2}{\gamma^2}(\nu+\kappa-\kappa')-\frac{\beta^2}{2\gamma^2}\kappa'\Big\}\sinh\gamma z+\frac{\beta^2}{2\gamma^2}\gamma z\cosh\gamma z\Big]$$

$$\times F_3(\xi,\eta)\cos\alpha(x-\xi)\sin\beta(y-\eta)\Big)d\xi\,d\eta, \tag{67}$$

$$\widehat{zz}=\frac{1}{\pi^2}\int_0^\infty d\alpha\int_0^\infty d\beta\int_{-\infty}^\infty\int_{-\infty}^\infty\frac{1}{\cosh\gamma c}\Big(\frac{1}{1-\kappa+\kappa'}\Big[(1+\kappa')\cosh\gamma z-\gamma z\sinh\gamma z\Big]$$

$$\times F_1(\xi,\eta)\cos\alpha(x-\xi)\cos\beta(y-\eta)$$

$$-\frac{1}{1+\kappa-\kappa'}\frac{\alpha}{\gamma}\Big[\kappa'\sinh\gamma z-\gamma z\cosh\gamma z\Big]F_2(\xi,\eta)\sin\alpha(x-\xi)\cos\beta(y-\eta)$$

$$-\frac{1}{1+\kappa-\kappa'}\frac{\beta}{\gamma}\Big[\kappa'\sinh\gamma z-\gamma z\cosh\gamma z\Big]F_3(\xi,\eta)\cos\alpha(x-\xi)\sin\beta(y-\eta)\Big)d\xi\,d\eta, \tag{68}$$

$$\widehat{xz} = \frac{1}{\pi^2}\int_0^\infty d\alpha\int_0^\infty d\beta\int_{-\infty}^\infty\int_{-\infty}^\infty \frac{1}{\cosh\gamma c}\Big(-\frac{1}{1-\kappa+\kappa'}\frac{\alpha}{\gamma}\Big[\kappa'\sinh\gamma z - \gamma z\cosh\gamma z\Big]$$
$$\times F_1(\xi,\eta)\sin\alpha(x-\xi)\cos\beta(y-\eta)$$
$$+\frac{1}{1+\kappa-\kappa'}\Big[\Big\{1+\frac{\beta^2}{\gamma^2}(\kappa-\kappa')-\frac{\alpha^2}{\gamma^2}\kappa'\Big\}\cosh\gamma z+\frac{\alpha^2}{\gamma^2}\gamma z\sinh\gamma z\Big]$$
$$\times F_2(\xi,\eta)\cos\alpha(x-\xi)\cos\beta(y-\eta)$$
$$+\frac{1}{1+\kappa-\kappa'}\frac{\alpha\beta}{\gamma^2}\Big[\kappa\cosh\gamma z-\gamma z\sinh\gamma z\Big]F_3(\xi,\eta)\sin\alpha(x-\xi)\sin\beta(y-\eta)\Big)d\xi\, d\eta, \tag{69}$$

$$\widehat{yz} = \frac{1}{\pi^2}\int_0^\infty d\alpha\int_0^\infty d\beta\int_{-\infty}^\infty\int_{-\infty}^\infty \frac{1}{\cosh\gamma c}\Big(-\frac{1}{1-\kappa+\kappa'}\frac{\beta}{\gamma}\Big[\kappa'\sinh\gamma z-\gamma z\cosh\gamma z\Big]$$
$$\times F_1(\xi,\eta)\cos\alpha(x-\xi)\sin\beta(y-\eta)$$
$$+\frac{1}{1+\kappa-\kappa'}\frac{\alpha\beta}{\gamma^2}\Big[\kappa\cosh\gamma z-\gamma z\sinh\gamma z\Big]F_2(\xi,\eta)\sin\alpha(x-\xi)\sin\beta(y-\eta)$$
$$+\frac{1}{1+\kappa-\kappa'}\Big[\Big\{1+\frac{\alpha^2}{\gamma^2}(\kappa-\kappa')-\frac{\beta^2}{\gamma^2}\kappa'\Big\}\cosh\gamma z+\frac{\beta^2}{\gamma^2}\gamma z\sinh\gamma z\Big]$$
$$\times F_3(\xi,\eta)\cos\alpha(x-\xi)\cos\beta(y-\eta)\Big)d\xi\, d\eta, \tag{70}$$

$$\widehat{xy} = \frac{1}{\pi^2}\int_0^\infty d\alpha\int_0^\infty d\beta\int_{-\infty}^\infty\int_{-\infty}^\infty \frac{1}{\cosh\gamma c}\Big(-\frac{1}{1-\kappa+\kappa'}\frac{\alpha\beta}{\gamma^2}\Big[(1-2\nu-\kappa')\cosh\gamma z$$
$$+\gamma z\sinh\gamma z\Big]F_1(\xi,\eta)\sin\alpha(x-\xi)\sin\beta(y-\eta)$$
$$-\frac{1}{1+\kappa-\kappa'}\frac{\beta}{\gamma}\Big[\Big\{1-2\nu\frac{\alpha^2}{\gamma^2}-\frac{\alpha^2-\beta^2}{\gamma^2}(\kappa-\kappa')-\frac{\alpha^2}{\gamma^2}\kappa'\Big\}\sinh\gamma z$$
$$+\frac{\alpha^2}{\gamma^2}\gamma z\cosh\gamma z\Big]F_2(\xi,\eta)\cos\alpha(x-\xi)\sin\beta(y-\eta)$$
$$-\frac{1}{1+\kappa-\kappa'}\frac{\alpha}{\gamma}\Big[\Big\{1-2\nu\frac{\beta^2}{\gamma^2}+\frac{\alpha^2-\beta^2}{\gamma^2}(\kappa-\kappa')-\frac{\beta^2}{\gamma^2}\kappa'\Big\}\sinh\gamma z$$
$$+\frac{\beta^2}{\gamma^2}\gamma z\cosh\gamma z\Big]F_3(\xi,\eta)\sin\alpha(x-\xi)\cos\beta(y-\eta)\Big)d\xi\, d\eta. \tag{71}$$

It can be verified easily that, when $z=\pm c$, the three stress-components $\widehat{zz}$, $\widehat{xz}$, and $\widehat{yz}$, given in equations (68), (69), and (70) respectively, reduce to

$$\left.\begin{aligned}(\widehat{zz})_{z=\pm c} &= \frac{1}{\pi^2}\int_0^\infty d\alpha\int_0^\infty d\beta\int_{-\infty}^\infty\int_{-\infty}^\infty F_1(\xi,\eta)\cos\alpha(x-\xi)\cos\beta(y-\eta)d\xi\, d\eta,\\ (\widehat{xz})_{z=\pm c} &= \frac{1}{\pi^2}\int_0^\infty d\alpha\int_0^\infty d\beta\int_{-\infty}^\infty\int_{-\infty}^\infty F_2(\xi,\eta)\cos\alpha(x-\xi)\cos\beta(y-\eta)d\xi\, d\eta,\end{aligned}\right\} \tag{72}$$

$$(\widehat{yz})_{z=\pm c} = \frac{1}{\pi^2}\int_0^\infty d\alpha\int_0^\infty d\beta\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_3(\xi,\eta)\cos\alpha(x-\xi)\cos\beta(y-\eta)d\xi\, d\eta,$$

which are the given boundary conditions (14).

It has been seen also that equations (66)~(71) for the required stress-components satisfy three stress-equations of the type

$$\frac{\partial\widehat{xx}}{\partial x} + \frac{\partial\widehat{xy}}{\partial y} + \frac{\partial\widehat{xz}}{\partial z} = 0.$$

7. Corresponding Displacement-components

We shall find expressions for displacement corresponding to the above stress-components. For this purpose we first calculate the three normal strain-components as follows:

$$\begin{aligned} e_{xx} &= \frac{1}{E}\{\widehat{xx} - \nu(\widehat{yy} + \widehat{zz})\} \\ &= \frac{1}{2\pi^2\mu}\int_0^\infty d\alpha\int_0^\infty d\beta\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{1}{\cosh\gamma c}\Big(\frac{1}{1-\kappa+\kappa'}\,\frac{\alpha^2}{\gamma^2}\Big[(1-2\nu-\kappa')\cosh\gamma z \\ &\qquad + \gamma z\sinh\gamma z\Big]F_1(\xi,\eta)\cos\alpha(x-\xi)\cos\beta(y-\eta) \\ &\quad - \frac{1}{1+\kappa-\kappa'}\,\frac{2\alpha}{\gamma}\Big[\Big\{1-\nu\frac{\alpha^2}{\gamma^2}+\frac{\beta^2}{\gamma^2}(\kappa-\kappa')-\frac{\alpha^2}{2\gamma^2}\kappa'\Big\}\sinh\gamma z \\ &\qquad + \frac{\alpha^2}{2\gamma^2}\gamma z\cosh\gamma z\Big]F_2(\xi,\eta)\sin\alpha(x-\xi)\cos\beta(y-\eta) \\ &\quad + \frac{1}{1+\kappa-\kappa'}\,\frac{\alpha^2\beta}{\gamma^3}\Big[(2\nu+2\kappa-\kappa')\sinh\gamma z-\gamma z\cosh\gamma z\Big] \\ &\qquad \times F_3(\xi,\eta)\cos\alpha(x-\xi)\sin\beta(y-\eta)\Big)d\xi\, d\eta, \end{aligned} \tag{73}$$

$$\begin{aligned} e_{yy} &= \frac{1}{E}\{\widehat{yy} - \nu(\widehat{xx} + \widehat{zz})\} \\ &= \frac{1}{2\pi^2\mu}\int_0^\infty d\alpha\int_0^\infty d\beta\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{1}{\cosh\gamma c}\Big(\frac{1}{1-\kappa+\kappa'}\,\frac{\beta^2}{\gamma^2}\Big[(1-2\nu-\kappa')\cosh\gamma z \\ &\qquad + \gamma z\sinh\gamma z\Big]F_1(\xi,\eta)\cos\alpha(x-\xi)\cos\beta(y-\eta) \\ &\quad + \frac{1}{1+\kappa-\kappa'}\,\frac{\alpha\beta^2}{\gamma^3}\Big[(2\nu+2\kappa-\kappa')\sinh\gamma z-\gamma z\cosh\gamma z\Big] \\ &\qquad \times F_2(\xi,\eta)\sin\alpha(x-\xi)\cos\beta(y-\eta) \end{aligned}$$

$$- \frac{1}{1+\kappa-\kappa'}\frac{2\beta}{\gamma}\Big[\Big\{1-\nu\frac{\beta^2}{\gamma^2}+\frac{\alpha^2}{\gamma^2}(\kappa-\kappa')-\frac{\beta^2}{2\gamma^2}\kappa'\Big\}\sinh\gamma z+\frac{\beta^2}{2\gamma^2}\gamma z\cosh\gamma z\Big]$$

$$\times F_3(\xi,\eta)\cos\alpha(x-\xi)\sin\beta(y-\eta)\Big)d\xi\,d\eta, \qquad (74)$$

$$e_{zz}=\frac{1}{E}\{\widehat{zz}-\nu(\widehat{xx}+\widehat{yy})\}$$

$$=\frac{1}{2\pi^2\mu}\int_0^\infty d\alpha\int_0^\infty d\beta\int_{-\infty}^\infty\int_{-\infty}^\infty\frac{1}{\cosh\gamma c}\Big(\frac{1}{1-\kappa+\kappa'}\Big[(1-2\nu+\kappa')\cosh\gamma z-\gamma z\sinh\gamma z\Big]$$

$$\times F_1(\xi,\eta)\cos\alpha(x-\xi)\cos\beta(y-\eta)$$

$$+\frac{1}{1+\kappa-\kappa'}\frac{\alpha}{\gamma}\Big[(2\nu-\kappa')\sinh\gamma z$$

$$+\gamma z\cosh\gamma z\Big]F_2(\xi,\eta)\sin\alpha(x-\xi)\cos\beta(y-\eta)$$

$$+\frac{1}{1+\kappa-\kappa'}\frac{\beta}{\gamma}\Big[(2\nu-\kappa')\sinh\gamma z+\gamma z\cosh\gamma z\Big]$$

$$\times F_3(\xi,\eta)\cos\alpha(x-\xi)\sin\beta(y-\eta)\Big)d\xi\,d\eta. \qquad (75)$$

Since we have

$$e_{xx}=\frac{\partial u}{\partial x},\qquad e_{yy}=\frac{\partial v}{\partial y},\qquad e_{zz}=\frac{\partial w}{\partial z},$$

the displacement-components become thus:

$$u=\frac{1}{2\pi^2\mu}\int_0^\infty d\alpha\int_0^\infty d\beta\int_{-\infty}^\infty\int_{-\infty}^\infty\frac{1}{\cosh\gamma c}\Big(\frac{1}{1-\kappa+\kappa'}\frac{\alpha}{\gamma^2}\Big[(1-2\nu-\kappa')\cosh\gamma z$$

$$+\gamma z\sinh\gamma z\Big]F_1(\xi,\eta)\sin\alpha(x-\xi)\cos\beta(y-\eta)$$

$$+\frac{1}{1+\kappa-\kappa'}\frac{2}{\gamma}\Big[\Big\{1-\nu\frac{\alpha^2}{\gamma^2}+\frac{\beta^2}{\gamma^2}(\kappa-\kappa')-\frac{\alpha^2}{2\gamma^2}\kappa'\Big\}\sinh\gamma z+\frac{\alpha^2}{2\gamma^2}\gamma z\cosh\gamma z\Big]$$

$$\times F_2(\xi,\eta)\cos\alpha(x-\xi)\cos\beta(y-\eta)$$

$$+\frac{1}{1+\kappa-\kappa'}\frac{\alpha\beta}{\gamma^3}\Big[(2\nu+2\kappa-\kappa')\sinh\gamma z-\gamma z\cosh\gamma z\Big]$$

$$\times F_3(\xi,\eta)\sin\alpha(x-\xi)\sin\beta(y-\eta)\Big)d\xi\,d\eta, \qquad (76)$$

$$v=\frac{1}{2\pi^2\mu}\int_0^\infty d\alpha\int_0^\infty d\beta\int_{-\infty}^\infty\int_{-\infty}^\infty\frac{1}{\cosh\gamma c}\Big(\frac{1}{1-\kappa+\kappa'}\frac{\beta}{\gamma^2}\Big[(1-2\nu-\kappa')\cosh\gamma z+\gamma z\sinh\gamma z\Big]$$

$$\times F_1(\xi,\eta)\cos\alpha(x-\xi)\sin\beta(y-\eta)$$

$$+\frac{1}{1+\kappa-\kappa'}\frac{\alpha\beta}{\gamma^3}\Big[(2\nu+2\kappa-\kappa')\sinh\gamma z-\gamma z\cosh\gamma z\Big]$$

$$\times F_2(\xi, \eta) \sin \alpha(x-\xi) \sin \beta(y-\eta)$$

$$+ \frac{1}{1+\kappa-\kappa'} \frac{2}{\gamma}\left[\left\{1 - \nu\frac{\beta^2}{\gamma^2} + \frac{\alpha^2}{\gamma^2}(\kappa-\kappa') - \frac{\beta^2}{2\gamma^2}\kappa'\right\} \sinh \gamma z + \frac{\beta^2}{2\gamma^2}\gamma z \cosh \gamma z\right]$$

$$\times F_3(\xi, \eta) \cos \alpha(x-\xi) \cos \beta(y-\eta)\Big) d\xi\, d\eta, \qquad (77)$$

$$w = \frac{1}{2\pi^2\mu}\int_0^\infty d\alpha \int_0^\infty d\beta \int_{-\infty}^\infty \int_{-\infty}^\infty \frac{1}{\cosh \gamma c}\left(\frac{1}{1-\kappa+\kappa'} \frac{1}{\gamma}\left[(2-2\nu+\kappa') \sinh \gamma z - \gamma z \cosh \gamma z\right]\right.$$

$$\times F_1(\xi, \eta) \cos \alpha(x-\xi) \cos \beta(y-\eta)$$

$$+ \frac{1}{1+\kappa-\kappa'} \frac{\alpha}{\gamma^2}\left[-(1-2\nu+\kappa') \cosh \gamma z\right.$$

$$\left. + \gamma z \sinh \gamma z\right] F_2(\xi, \eta) \sin \alpha(x-\xi) \cos \beta(y-\eta)$$

$$+ \frac{1}{1+\kappa-\kappa'} \frac{\beta}{\gamma^2}\left[-(1-2\nu+\kappa') \cosh \gamma z + \gamma z \sinh \gamma z\right]$$

$$\times F_3(\xi, \eta) \cos \alpha(x-\xi) \sin \beta(y-\eta)\Big) d\xi\, d\eta, \qquad (78)$$

in which, as before,

$$\kappa = \gamma c \tanh \gamma c, \qquad \kappa' = \gamma c \coth \gamma c, \qquad \alpha^2 + \beta^2 = \gamma^2,$$

and

$$F_1(x, y) = (\widehat{zz})_{z=\pm c}, \quad F_2(x, y) = (\widehat{xz})_{z=\pm c}, \quad F_3(x, y) = (\widehat{yz})_{z=\pm c};$$

μ being the modulus of rigidity and ν Poisson's ratio for the material.

It seems difficult to integrate the above solution by means of analytical method, and then we have to rely on methods of numerical integration, which need a considerable amount of time and labour. Numerical evaluations of the above solution would therefore be given, when an automatic computer is available.

As a succeeding work of the present one, the case in which a plate of infinite extension is subjected to a normal pressure distribution and a shearing-force distribution are applied on one bounding plane of the plate will appear in the succeeding issue of this Journal.

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Note. It is added that every effort was exerted to secure the correctness of the printing:

1) The type-setting was made by the original manuscript, to avoid slips which might occur in the course of copying.

2) Every step of proof-readings was assisted by two men, who heard my utterance in reading the manuscript.

3) The last proof-reading was finished by the stereotype, in which I could find no errors.