On the Equilibrium Bed Slope in a Steady Nonuniform Flow

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Synopsis. This paper is an approach to an estimate of the equilibrium bed slope of the stream in which the sediment is mainly transported as bed-load, on the basis of the sediment transport theory under some assumptions. By using this procedure, the influences of the flow discharge and of the grain size of bed material upon the equilibrium bed slope are discussed in the case of a steady nonuniform flow with gradually varied width and rectangular cross sections.

1 Introduction

When the river improvement works such as widening the river width, converting the stream course and constructing dams across rivers are designed, one of the problems which used to afflict hydraulic engineers is how the configuration of the river bed will change after the construction works are performed. Supplied with much sediment load from the upperstream, the river bed profile changes its configuration, repeating deposition and scour. In a limited reach of the stream where deposition is nearly balanced with scour, however, the bed slope may be commonly called "the equilibrium slope".

Up to this time, theoretical studies on the equilibrium bed slope were performed by Dr. Mononobe and Dr. Aki, respectively. Combining the Sternberg's theory with the relation between the grain size of bed material and the tractive force which acts on the bed, Dr. Mononobe proposed[1]

$$i = I_0 e^{-\frac{cx}{2}} + \frac{c}{6} H_0 e^{\frac{cx}{6}}, \quad \cdots\cdots(1)$$

where x is the distance measured towards the down-stream from the referring point, c Sternberg's coefficient, I_0 and H_0 bed slope and water depth in the referring point. This formula treated the streams which were assumed to have rectangular cross sections of the same width and the uniform flow in every reach. Though he proposed thereafter another formula similar to Eq. (1) for the streams with gradually varied width, its method was essentially induced from the theory of the uniform flow in every reach. The relationship between the flow distance and the grain size of bed material in natural rivers found by Dr. Aki is[2]

$$x = a' - b' \log_{10} \lambda' D, \quad \cdots\cdots(2)$$

where a' and b' are constants peculiar to each river, and D the mean grain diameter calculated from the equation

$$D=\sum_{P=0}^{P=100\%} D\cdot\Delta p\Big/\sum_{P=0}^{P=100\%}\Delta p,$$

using the sieve analysis curve. Moreover, λ' is a coefficient expressed as $(100-P_m)/P_m$, where P_m is the percentage of finer material by weight corresponding to D.

Referring to Eq. (2), Dr. Aki gave the following formula;

$$i=I_0\cdot 10^{\frac{5(x_0-x)}{3.5b}}+\frac{3.45}{3.5b'}H_0\cdot 10^{\frac{1.5(x-x_0)}{3.5b'}}\qquad\cdots\cdots(3)$$

This formula, which has been proved its validity by the several field examples, has been often utilized in Japan.

With the development of the studies on the flow transporting sediment, the efforts(3)(4) have been made by the author and Dr. Iwagaki in recent years to explain analytically the mechanism of river-bed variation, though the scope of the studies were restricted within the streams with constant width. This paper presents a procedure to obtain the equilibrium bed slope in a steady nonuniform flow on the basis of the above described author's paper. It indicates also the questionable points among the various sediment problems found in natural rivers.

2 Derivation of Equations to Obtain Equilibrium Bed Slope

Suppose the case in which the bed of a stream with rectangular cross section has achieved a state of the practical equilibrium for the given flow discharge Q, and let x be the abscissa axis along the bed surface towards the downstream, and h the water depth vertical to x-axis as shown in Fig. 1. In any cross section, let i be the bed slope, v the mean velocity, and B the stream width. Then, the equation of motion for a steady nonuniform flow which has a gradually varied width is

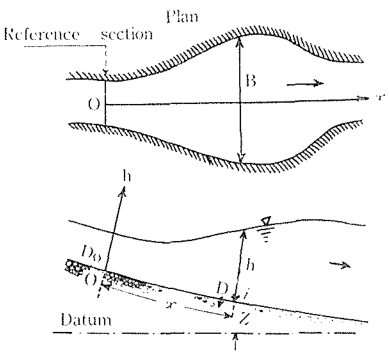

Fig. 1. Schematic plan and profile of a steady nonuniform flow

$$-i+\frac{dh}{dx}+I_f+\frac{\alpha\, d}{dx}\left(\frac{v^2}{2g}\right)=0,\quad\cdots\cdots(4)$$

where the variation of the width along the stream is so gentle that head losses are neglected except the frictional loss. Denoting the frictional slope as I_f, the shear velocity v^* is assumed to be

$$v^*=\sqrt{gRI_f},\qquad\cdots\cdots(5)$$

where R is the hydraulic mean depth, and g the gravity acceleration. Combining Eq. (4) with Eq. (5), the bed slope becomes

$$i = \frac{v^{*2}}{gR} + \frac{dh}{dx}\left(1 - \frac{h_c{}^3}{h^3}\right) - \frac{h_c{}^3}{h^3} \cdot \frac{h}{B} \cdot \frac{dB}{dx}, \quad \cdots\cdots(6)$$

where h_c is the critical depth expressed as $(\alpha Q^2 / gB^2)^{\frac{1}{3}}$. Using Eq. (6), the bed slope i in any section can be determined if each of h, R, dh/dx, h_c, v^* and dB/dx is given. Since dB/dx is known in general, and Q is given previously, then the equilibrium bed slope i can be determined if each of h, dh/dx and v^* is selected so as to satisfy the equilibrium conditions.

Now, the equation of continuity in respect to the sediment transport is

$$\frac{1}{(1-\lambda)} \cdot \frac{\partial Q_s}{\partial x} + \frac{\partial z}{\partial t} = 0, \quad \cdots\cdots(7)$$

where Q_s is the rate of transport in volume of material per unit time across the section, z the elevation of bed surface, and λ the porosity of bed-load.

If the river bed is in equilibrium state, $\partial z/\partial t$ is equal to zero; therefore, Eq. (7) gives

$$Q_s = B \cdot q_s = constant, \quad \cdots\cdots(8)$$

where q_s is the rate of transport in volume of material per unit time per unit width of section, and is regarded as a function of the mean diameter of sand grains, shear velocity, and specific weight of the sediment. Up to this time a number of experimental and theoretical formulas have been presented as to q_s. It seems, however, that there are few formulas which agree quite well with the field data in natural streams carrying so large and graded bed material. In this paper the following bed-load formula proposed by A. A. Kalinske and C. B. Brown[5] is adopted, because it is not only demensionless and has a simple form, but also is said to agree well with observed data in many experimental flumes.

$$\frac{q_s}{v^* D} = 10\left[\frac{v^{*2}}{g(S_s - 1) D}\right]^2, \quad \cdots\cdots(9)$$

where D is the mean diameter of the sand grains, and S_s specific weight of the sediment.

Although this equation is the one presented for the uniform flow, it is now assumed to be applicable even to the nonuniform flow. But in the small flume experiments, it was ascertained[6] that Eq. (9) was approximately applicable to the gradually varied nonuniform flow if v^* was denoted by $\sqrt{gRI_f}$.

By transforming Eq. (9), it becomes

$$q_s = \frac{10\, Bv^{*5}}{g^2 (S_s - 1)^2 D}. \quad \cdots\cdots(10)$$

It is well known that the vertical velocity distribution in the flows in open channel with definite rough bed follows the logarithmic law proposed by

Kármán and Prandtl. Though in the flows transporting sediment the determination of velocity distribution is a difficult matter because of the experimental troubles and the complexity of the mechanism in the sediment transportation, it is supposed that the following logarithmic formula will be also applicable to that case.

$$\frac{V}{v^*} = A' + B' \log_{10}\frac{y}{k_s}, \qquad (11)$$

where k_s is the equivalent roughness of the bottom bed, and V the velocity at a distance y from the bottom surface.

T. Tsubaki[7] stated in respect to the values of A' and B' from his experimental observations in large flumes, that they might be taken approximately as 8.5 and 5.75, respectively. Strictly speaking, however, it seems that there remains some doubt as to whether A' and B' are constant or not. It was pointed out by Dr. Iwagaki[8] that the plots of the velocity distribution in the position extremely near the bottom tended to deviate a little from the logarithmic law, and that A' was not constant but was a function of the Froude number. As the more rigorous discussions are not permitted in the present state of knowledge, it is assumed that the mean velocity formula in the movable beds is

$$\frac{v}{v^*} = 6.0 + 5.75 \log_{10}\frac{R}{k_s}, \qquad (12)$$

where v is the mean velocity.

Now, the efforts have been made by many investigators to obtain the relations between the equivalent roughness k_s and the characteristics of flow transporting sediment. In the present day, however, it may be the most widely accepted theory that k_s/D, which is the rate of the equivalent roughness to the mean diameter of sand grains, depends exceedingly upon Ψ which is a function of the tractive force and the sediment characteristics. From the field data in the several natural streams, T. Tsubaki and A. Furuya proposed[9]

$$\log_{10}\frac{k_s}{D} = 3.48\,(1 - 0.225\,\Psi^{-\frac{1}{2}}), \qquad (13)$$

where Ψ is expressed as $v^{*2} / g\,(s_s - 1)\,D$.

Since these data were collected from streams which had fairly gentle bed slopes and bed materials of fine size, it seems that Eq. (13) may be applicable to streams, the beds of which are covered with dunes. On the other hand, the studies have been scarcely presented on the values of k_s / D in natural streams, in which the bed slope and the sediment size are comparatively large. As to the experimental studies, Dr. Iwagaki[10] gave the following formula from his experiments performed under the condition of smooth bed and fairly steep bed slopes. It is written

$$\log_{10}\frac{k_s}{D} = 1 + 0.769 \log_{10}\Psi. \qquad (14)$$

Although the application of this formula is, needless to say, to be limited in the experimental flumes, it is assumed that Eq. (14) may be applicable to natural streams which have fairly steep bed slopes and are supposed to have smooth beds. At any rate, the investigations of k_s / D in natural streams will remain hereafter as one of the most important themes among the sediment problems.

Now, if the distribution of grain size along the stream and the flow discharge Q are given, the equilibrium bed slope may be estimated according to the following procedure.

The value of Q_s in every section is always constant from Eq. (8) when the flow is in the equilibrium conditions as stated previously. To obtain Q_s, a reference section, in which the flow condition is regarded approximately uniform and I_f is nearly equal to i_0, must be selected in the reach. i_0 means the bed slope in the reference section. Therefore, the following equations will be written down

$$\frac{Q_0}{B_0 v_0{}^*} = h_0 (6.0 + 5.75 \log_{10} \frac{R_0}{k_{s0}}), \quad \cdots\cdots (15)$$

$$v_0{}^* = \sqrt{g R_0 i_0}, \quad \cdots\cdots (16)$$

$$\log_{10} \frac{k_{s0}}{D_0} = 1 + 0.769 \log_{10} \frac{v_0{}^{*2}}{g (S_s - 1) D_0}, \quad \cdots\cdots (17)$$

$$R_0 = B_0 h_0 / (B_0 + 2h_0), \quad \cdots\cdots (18)$$

where R_0, h_0, k_{s0}, $v_0{}^*$, and D_0 are the corresponding values in the reference section. Of course, Eq. (17) should be replaced by Eq. (13) when the river bed is supposed to be covered with dunes. From Eq. (15), (16), (17), and (18), four unknowns such as $v_0{}^*$, R_0, h_0, and k_{s0} may be determined by the trial and error method, respectively. Consequently, Q_s becomes

$$Q_s = B_0 \cdot q_{s0} = \frac{10 B_0 v_0{}^{*5}}{g^2 (S_s - 1)^2 D_0}. \quad \cdots\cdots (19)$$

Combining Eq. (10) with Eq. (19), the shear velocity becomes

$$v^* = \left(\frac{Q_s \cdot D}{m \cdot B}\right)^{0.20}, \quad \cdots\cdots (20)$$

where m is $10/g^2 (S_s - 1)^2$.

If the specific weight of the sediment is constant in every section, the elimination of m gives

$$v^* = v_0{}^* \left(\frac{B_0 D}{B D_0}\right)^{0.20}. \quad \cdots\cdots (21)$$

Moreover, combining the law of continuity of the flow with Eq. (12) gives

$$\frac{Q}{B v^*} = h (6.0 + 5.75 \log_{10} \frac{R}{k_s}). \quad \cdots\cdots (22)$$

Since v^* is calculated from Eq. (21), h and k_s in any section may be determined from Eq. (14) and Eq. (22) by the trial and error method. The values of dh/dx in every section are evaluated from the relations between h and x, while as the number of sections increase, more accurate will be the values of dh/dx. Then, the bed slope i can be calculated from Eq. (6). Likewise the elevation of the stream bed is computed from

$$z=\int_o^L i\,dL+z_L, \qquad \cdots\cdots(23)$$

where z_L is the elevation of stream bed at the position where the standard of elevation is given, as at the position of dams, sluice gates or overfalls. L is the distance measured towards the upperstream from the position described above.

3 Equilibrium Bed Slope over Upperstream of a Dam

(a) Procedure of Calculation

In this article, the application of the foregoing procedure to the problems of sedimentation upstream a dam is described, and the factors upon which the equilibrium bed slope depends are discussed from the results calculated on the several examples. If the sediment transported from the upperstream of a dam is so numerous, the reservoir will be soon filled up with sediment as high as the elevation of the dam crest, as seen in the debris barriers; while these examples are often found in Japan, especially at the low dams. When the rate of the sediment transport through any cross section is balanced with that of the sediment transport discharged to the downstream over the dam, it may be considered that there yields the equilibrium bed slope. In order to simplify the computation, a mathematically regular shape of stream channel was adopted which had a narrow reach of the same width by 30m and had a reservoir of 8 km long as shown in Fig. 2.

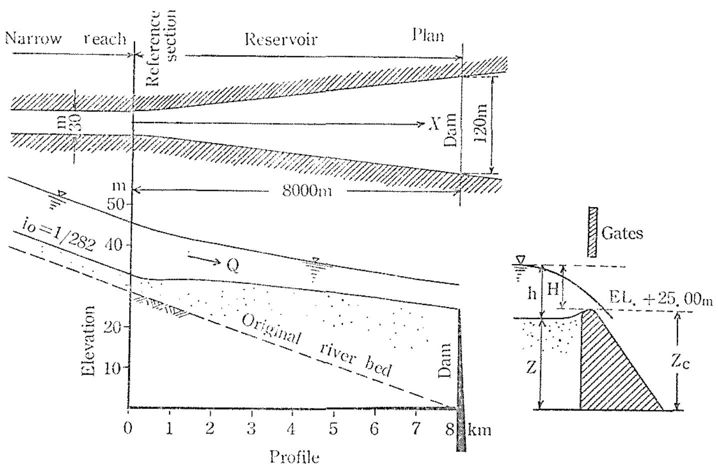

Fig. 2. Form of reservoir used as calculation examples

The width of the channel increased linearly towards the dam from 30 m to 120 m, except the reach between the sections 0 and 1 where the width increased parabolically. Every cross section of the channel was regarded rectangular. The narrow reach was so long that the flow was assumed to be uniform, the bed slope being 1/282. The effective overflow width of the dam was given by 105.6 m under the flow discharge of 3000m^3/s and 2000 m^3/s, gates of the dam being opened fully.

These data were given by modifying the values of an existing reservoir in Japan, though the small irregularities and curved parts of the reservoir were eliminated to simplify the calculation. Seven runs of the computation examples were performed by employing the various combinations of flow discharge and grain size distribution as shown in Table 1.

Now, the evaluation of grain size distribution in a reservoir is of great difficulty especially in the case when the bed material is moving; it is supposed that an intimate relation may exist between the variations of channel width and the grain size of bed material. Since it is impossible to know the relation between them exactly in the present state of knowledge, adequately assumed data on the grain size distribution are tried to be used with reference to the field data obtained at the low water discharge as shown in Table 1. Of course, the grain sizes obtained at the low water discharge may not be equal to those at the high water discharge; the former seems to show smaller values than the latter especially in the vicinity of the dam.

To simplify the computation in this paper, the mean diameters of sand grains were assumed to decrease exponentially with time towards the downstream according to Eq.(2), the rates of decrease of D being selected independently of the field data. In Run 1 and 2, the grain diameters were tentatively assumed constant along the reach only to investigate the influence of the grain size

Table 1. Grain size distribution and flow discharge (in mm)

Section	Run 1	Run 2	Run 3	Run 4	Run 5	Run 6	Run 7	*Observed data
0	50.00	20.00	50.00	50.00	50.00	50.00	50.00	35.6
1	50.00	20.00	44.59	42.92	39.76	39.76	44.59	20.4
2	50.00	20.00	39.77	36.85	31.62	31.62	39.77	40.0 31.7
3	50.00	20.00	35.47	31.62	25.15	25.15	35.47	—
4	50.00	20.00	31.62	27.15	20.00	20.00	31.62	—
5	50.00	20.00	28.20	23.30	15.91	(15.19 5.00)	28.20	0.19
6	50.00	20.00	25.15	20.01	12.65	5.00	25.15	—
7	50.00	20.00	22.43	17.17	10.06	5.00	22.43	0.10
8	50.00	20.00	20.00	14.74	8.00	5.00	20.00	0.08
Flow discharge (m^3/s)	3000						2000	Low water discharge

(*These field data were observed in 1950 by the engineers of Res. Inst. of Civ. Eng. in the Ministry of Construction.)

itself on the equilibrium bed slope, though they were fairly different from the field data. Although this assumption on the exponential decrease of grain size has not been proved as true in the reservoir where the variations in stream width along the reach are fairly large, further investigations on many field data will make it possible to eliminate this assumption in the future.

The reference section was taken at the downstream end of the narrow reach. It should be noted that even if the stream width along the reach is constant the uniform flow could not exist when the grain size is not constant along the reach. Then, the frictional slope I_f in the reference section was taken approximately as equal to i_0, errors being supposed to be negligiblly small. The reference elevation of river bed was determined at the dam site. As shown in Fig. 2, the reference elevation may be

$$z = z_c + H - h, \qquad (24)$$

where z is the reference elevation, z_c the elevation of the dam crest, h water depth, and H overflow depth, which is determined from the well-known formula,

$$Q = CbH^{3/2}. \qquad (25)$$

As to the values of the coefficient of discharge in Eq. (25), T. kusama indicated from his experiments that C increased suddenly as soon as the dam pool was filled up with sediment as high as the dam crest. In this paper, C was taken 2.20, referring to the above suggestion.

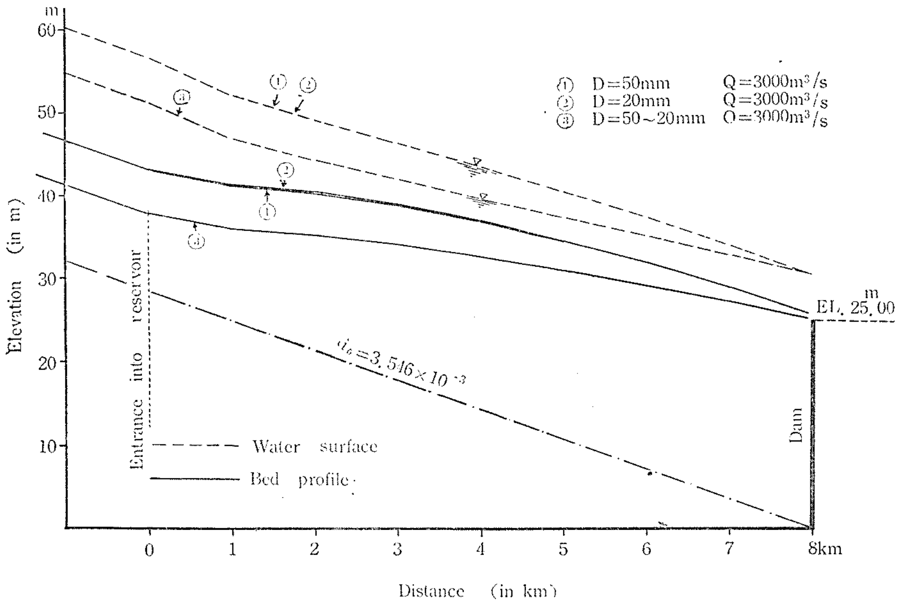

Fig. 3. Calculated results in Run 1, 2, and 3

(b) Discussion of the Results Obtained

(i) Run 1 and Run 2 were calculated under the condition in which the grain size of bed material was assumed constant over the reach, being 50mm for Run 1 and 20 mm for Run 2, respectively, as shown in Fig. 3. The sand grain was regarded to have a specific gravity of 2.65. The computed water surface curve and the bed profile curve in these two cases are similar to each other, in spite of the difference of grain size. To compare with them, Run 3 was calculated under the condition in which the grain size was assumed to decrease towards the downstream from 50 mm to 20 mm exponentially as shown in Fig. 3. It is easily noticed that the obtained bed slope and water surface slope are obviously far gentler than those of the above two. These results seem to indicate that the equilibrium bed slope depends much more upon the rate of the grain size decrease along the reach than upon the grain size itself.

(ii) Fig. 4 shows the three cases, in which the flow discharge is all the same and the rates of the grain size decrease are different respectively

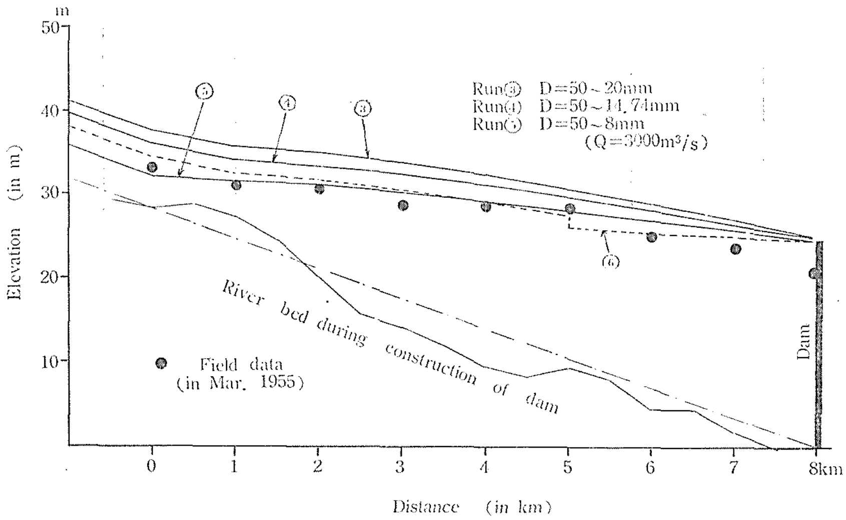

Fig. 4. Calculated results for different rates of grain size decrease

(Run 3, 4, and 5). The grain size at the reference section was previously given as 50 mm in each run. Run 5, in which the rate of the grain size decrease was the greatest of the three, gave the gentlest bed slope and water surface slope. The grain size of bed material in natural rivers, especially in the reach of the upperstream of dam, is considered to change with the lapse of time, being subjected to some laws, the solution of which will remain as an important problem in the future. Accordingly, it seems that the distribution of grain size in natural rivers cannot help being presumed, in the present day. In Fig. 4, the plots of the field data, which show the deepest elevations of river bed at the low water obtained in March

Table 2. Calculated results in Run 5

Section	B (m)	D (mm)	v (m/s)	k_s (m)	h (m)	h_c (m)
0	30.00	50.00	0.4969	0.2008	13.50	10.39
1	38.71	39.76	0.4511	0.1641	11.16	8.769
2	50.32	31.62	0.4090	0.1339	9.231	7.361
3	61.93	25.15	0.3747	0.1110	8.023	6.410
4	73.55	20.00	0.3458	0.09305	7.189	5.716
5	85.16	15.91	0.3208	0.07862	6.578	5.184
6	96.77	12.65	0.2987	0.06682	6.113	4.760
7	108.39	10.06	0.2789	0.05704	5.748	4.414
8	120.00	8.00	0.2610	0.04886	5.455	4.124

Table 2. (Continued)

Section	I_f	I_h	I_b	i	z	**Manning's n
0	3.546	0	0	3.546	32.23*	0.029
1	2.934	(−1.290* / −1.099)	−1.625	0.019*	31.63*	0.0287
2	2.527	−0.772	−1.080	0.675	31.28	0.0278
3	2.248	−0.500	−0.767	0.981	30.45	0.0270
4	2.029	−0.359	−0.571	1.099	29.41	0.0263
5	1.843	−0.275	−0.439	1.129	28.30	0.0256
6	1.677	−0.219	−0.346	1.112	27.18	0.0249
7	1.527	−0.180	−0.279	1.068	26.09	0.0244
8	1.390	−0.151	−0.228	1.011	25.05	0.0238

* The values obtained by taking Δx as 200 m for the reach between the sections 0 and 1

** Manning's coefficient of roughness

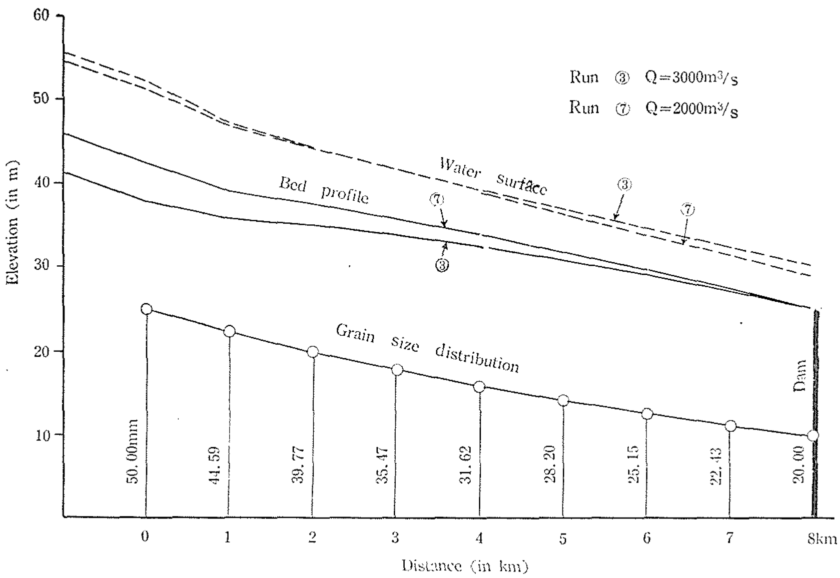

Fig. 5. Calculated results for different flow discharges

1955 by sounding in the foregoing existing reservoir are shown. Since the water level was usually varied by regulating the dam gates in this reservoir, the distribution of grain size did not seem to be approximately represented with one exponential curve; the grain size in the reach between the sections 5 and 8 was found to be extremely smaller than that of the upperstream reach. Considering such circumstances, Run 6 was calculated in a case in which two kinds of decrease curve of grain size were assumed to exist as shown in Table 1. In such a case, it was noticed that a sudden change of the bed elevation was supposed to yield at the discontinuous part of grain size as shown in Fig. 4.

(iii) To investigate the influence of the variation in flow discharge on the equilibrium bed slope for the same grain size distribution, the water surface and bed profile curves were drawn in Fig. 5 for the flow discharges 3000 m^3/s and 2000 m^3/s. Fig. 5 indicates that the larger discharge tends to make the bed slope and water surface slope gentler than the smaller one does. Although the water surface curves in Run 3 and Run 7 look almost alike except in the vicinity of dam, a fairly remarkable difference is found between them as to the bed profile curves; that is, the bed profile curve for the flow discharge of 3000 m^3/s is far lower than that of 2000 m^3/s, especially in the narrow reach. These facts suggest that the scouring power in the narrow reach may be enormous at the high stage. It should be noted that in the natural rivers the grain size of bed material does not be supposed to remain constant with the decrease of flow discharge. This is also an important problem to remain in the future.

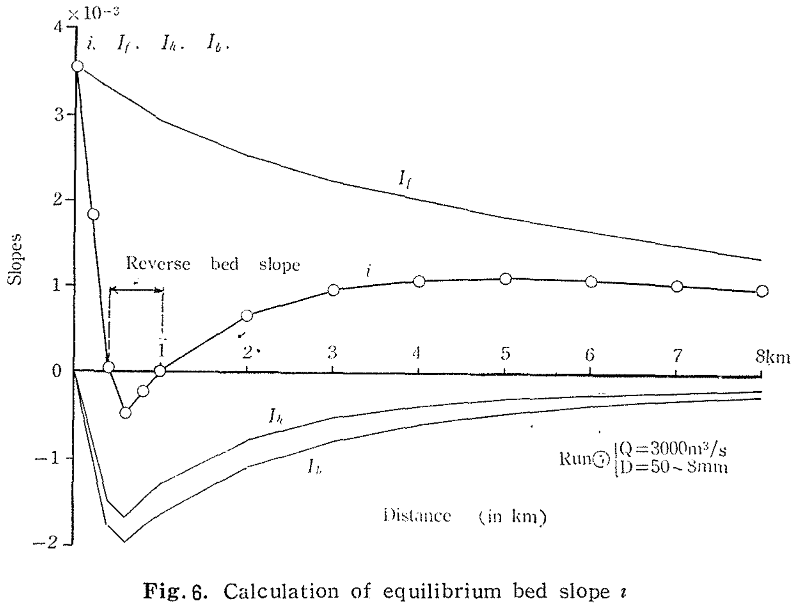

Fig. 6. Calculation of equilibrium bed slope i

(iv) Eq. (6) may be rewritten as

$$i = I_f + I_h + I_b, \quad \cdots\cdots (26)$$

where I_h and I_b are the corrective slopes for the variations in the water depth and the stream width along the reach, respectively. When dB/dx is fairly large, I_h tends also to be comparatively large. Consequently, at the entrance into the reservoir where the stream width begins gradually to increase, I_h plus I_b becomes negatively so large that sometimes the reverse bed slope may yield. Fig. 6 shows the calculated values of I_f, I_h, and I_b in Run 5. This phenomenon to yield the reverse bed slope is often experienced when a stream flow with movable bed pours out from a narrow reach to a wide area.

5 Conclusions

The conclusions may be summarized as follows:

(1) The equilibrium bed slope for a steady nonuniform flow depends upon the flow discharge and the grain size distribution of bed material, and it may be determined according to the procedure described in article 2.

(2) For the same flow discharge, the greater the rate of the decrease of grain size becomes, the gentler may be the equilibrium bed slope.

(3) For the same grain size distribution, the larger the flow discharge becomes, the gentler may be the slope.

(4) In the reach where the stream width is gradually increasing, the bed slope is very gentle and sometimes even the reverse bed slope happens.

It should be noted that the following assumptions or restrictions were used in this computation.

(1) The scope of this study is resricted within the bed-load movement.

(2) In this paper the streams which have regular shapes and rectangular cross sections are treated.

(3) The shear velocity along the stream bed in a steady nonuniform flow is computed from Eq. (5).

(4) Eq. (10), (12), and (13) or (14) are adopted as the equation of the mean velocity, the bed-load transport, and the equivalent sand roughness, respectively.

Moreover, to obtain the equilibrium bed slope behind a dam the mean diameters of sand grains in the reservoir were assumed to decrease exponentially towards downstream as shown in Eq. (2). It is supposed that there are many problems to be discussed in respect to the relation between the grain size distribution and the channel width.

Above all, the fourth assumption should be noticed. Though these existing formulas are proved to be applicable to the experiments in the laboratory, they seem to be insufficient in the point of accuracy to apply to natural rivers yet. Accordingly, the reader will realize that the foregoing development is applicable so long as the assumed conditions are actually fulfilled. In spite of the uncertainty in these three formulas, further investigations on the many field data will make it possible to apply this procedure to the practical use.

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