# 17-*Figure Table for Sine and Cosine*

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(Received November 5, 1956)

**Synopsis.** The present table gives accurate values of sines and cosines to eighteen places of decimals for every one degree. Subsidiary tables are also given for $1'$ to $30'$ and for $5''$ to $30''$. These tables will afford an efficient means to compute accurate values of circular functions of any argument, when a 20-figure hand-driven calculating machine is available. Examples are given for illustration. Some techniques for treating large places of numbers by using the calculating machine are also described.

1. Every engineer or physicist keeps with him seven-figure tables, such as the Chamber's, for his numerical computation. Though they would be sufficient in former days, the modern engineering and science require more accurate tables than those of seven figures.

A nine-figure table for circular functions has been published[1] for such need, and special students keep it with them. But they often complain of its misprints. Also, though not so often, they have to face with urgent necessity for more accurate values.

The most complete table for circular functions would seem to be of fifteen places of decimals for every $10''$.[2] If one keeps such a complete table, the interpolation method would usually be preferable for obtaining accurate values of these functions so long as fifteen places of decimals are concerned. But, as far as I am aware, no student in this country keeps such a complete table with him.

In such a case, the present table will afford an efficient means, in virtue of which eighteen places of decimals of the wanted entry can be obtained by simple multiplication, followed by addition or subtraction, with a 20-figure hand-driven calculating machine which has now been used by most engineers or physicists.

2. We shall obtain sines and cosines for every one degree up to 45°. For this purpose it will be convenient to use the addition theorems

$$\left.\begin{aligned} \sin(x+h) &= \sin x \cos h + \cos x \sin h, \\ \cos(x+h) &= \cos x \cos h - \sin x \sin h, \end{aligned}\right\} \qquad (1)$$

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where we take

$$h = \frac{\pi}{180} = 0.017\,453\,292\,519\,943\,295\,8.$$

By Taylor's theorem we have

$$\cos h = 1 - \frac{h^2}{2!} + \frac{h^4}{4!} - \cdots, \qquad \sin h = h - \frac{h^3}{3!} + \frac{h^5}{5!} - \cdots, \tag{2}$$

from which Table 1 follows.

**Table 1.** Computation of sin 1° and cos 1°

$h = 1^\circ = 0.017\,453\,292\,519\,943\,295\,8$ rad.

$h^2 = 0.000\,304\,617\,419\,786\,708\,6$

| cos 1° | | sin 1° | |
|---|---|---|---|
| 1 | 1.000 | $h$ | 0.017 453 292 519 943 295 8 |
| $\times \frac{h^2}{2}$ | −0.000 152 308 709 893 354 3 | $\times \frac{h^2}{6}$ | −886 096 155 701 3 |
| $\times \frac{h^2}{12}$ | 3 866 323 851 6 | $\times h^2\,050$ | 13 496 016 2 |
| $\times h^2\,03\dot{3}$ | −39 258 3 | $\times h^2\,023\,81$ | −97 9 |
| $\times h^2\,017\,8$ | 0 2 | | |
| Sum | 0.999 847 695 156 391 239 2 | Sum | 0.017 452 406 437 283 512 8 |

In forming Table 1, a term in the power series is evaluated by multiplying the preceding term by certain factor; that is, for example

$$0.017\,453\,292\,519\,943\,295\,8 \times \frac{h^2}{6} = 886\,096\,155\,701\,3,$$

$$886\,096\,155\,701\,3 \times h^2\,050 = 13\,496\,016\,2,$$

$$13\,496\,016\,2 \times h^2\,023\,81 = 97\,9,$$

We thus obtain

$$\cos 1^\circ = 0.999\,847\,695\,156\,391\,239, = A \text{ say},$$

$$\sin 1^\circ = 0.017\,452\,406\,437\,283\,513, = B \text{ say}.$$

Equations (1) are then written

$$\left.\begin{aligned}\sin(x^\circ+1^\circ)&=A\sin x^\circ+B\cos x^\circ,\\ \cos(x^\circ+1^\circ)&=A\cos x^\circ-B\sin x^\circ.\end{aligned}\right\}\tag{3}$$

These are the "recurrence formulas" for the subsequent computation.

Multiplication for large places of numbers can be effected by the operations

$$\left.\begin{aligned}(A+B\times10^{-9})^2&=2AB\times10^{-9}+A^2,\\ (A+B\times10^{-9})(A'+B'\times10^{-9})&=(AB'+A'B)\times10^{-9}+AA',\end{aligned}\right\}\tag{4}$$

provided that a 20-figure hand-driven calculating machine is available; the right half in $2AB\times10^{-9}$ and $(AB'+A'B)\times10^{-9}$ being curtailled without affecting significant figures of the result.

Multiplication for larger places of numbers, which occurs in certain cases in the present work, is effected by the operation

$$\begin{aligned}&(A+B\times10^{-9}+C\times10^{-18}+\cdots)(A'+B'\times10^{-9}+C'\times10^{-18}+\cdots)\\ &\quad=AA'+(AB'+A'B)\times10^{-9}+(BB'+AC'+A'C)\times10^{-18}+\cdots.\end{aligned}$$

It is noted that in performing the multiplication preference will be given to the order indicated in the above right-hand side; that is to say, the left principal part in $AA'$ is first written down on the computation sheet, and then the remaining part in $AA'$ is superposed by the subsequent constituent product $(AB'+A'B)\times10^{-9}$, and so on. In this way we can obtain the wanted product. Thus we have for example

$$\begin{aligned}\pi^2&=(3.141\,592\,653\,589\,793\,238\,462\,643\,383)^2\\ &=9.869\,604\,401\,089\,358\,618\,834\,490\,998.\end{aligned}$$

3. Thus Table I attached will be obtained. It is noted here that the following correction should be made to avoid unfavourable accumulation of errors caused by rounding-off the nineteenth place of decimals in the course of computation.

By the recurrence formulas (3), computation proceeds on up to sin 15°and cos 15°, whose true values are known otherwise, and hence the computed entries can be compared with their true entries. The last digit in the computed sin 15° was 5, while its true value is 2, so that all the preceding entries, as well as sin 15°, were adjusted suitably, by assuming a uniform accumulation of errors. For cosines also, the same adjustment has been done. Starting from the true values of sin 15° and cos 15°, computation proceeds on up to sin 30° and cos 30° whose true values are also known. Similar correction was here made, and then computation proceeds on up to sin 45° and cos 45°. Correction was again made. In this way Table I has been finished.

Table 2 shows the comparison for the cited correction, in which (T) denotes true entries and (R) those computed by repeated application of the recurrence formulas (3).

**Table 2.** Comparison for correction (for every one degree)

| $x^\circ$ | $\sin x^\circ$ | | $\cos x^\circ$ | |
|---|---|---|---|---|
| 15° | 0.258 819 045 102 520 762 | (T) | 0.965 925 826 289 068 287 | (T) |
| | 0.258 819 045 102 520 765 | (R) | 0.965 925 826 289 068 286 | (R) |
| 30° | 0.500 000 000 000 000 000 | (T) | 0.866 025 403 784 438 647 | (T) |
| | 0.499 999 999 999 999 999 | (R) | 0.866 025 403 784 438 648 | (R) |
| 45° | 0.707 106 781 186 547 524 | (T) | 0.707 106 781 186 547 524 | (T) |
| | 0.707 106 781 186 547 527 | (R) | 0.707 106 781 186 547 521 | (R) |

It is added, in constructing Table I, that, after each step of the use of the recurrence formulas (3) has finished, the entries obtained were checked every time by the relation

$$\sin^2 x^\circ + \cos^2 x^\circ = 1, \qquad \text{or} \qquad \sin^2 x^\circ = 1 - \cos^2 x^\circ;$$

the latter form being by far suited for the checking. For example, with the first equation in (4),

$$\sin^2 12^\circ = 0.043\,227\,271\,178\,699\,552,$$

and on the other hand*)

$$1 - \cos^2 12^\circ = 0.043\,227\,271\,178\,699\,552.$$

*) Sequence of numbers appearing on the right dial of a calculating machine will be as follows (with the notation of (4)):

$-AB = 99\,282\,229\,776\,23\ldots,$

$-2\,AB \times 10^{-9} = 18\,564\,459\,552 = 99\,999\,999\,998\,564\,459\,552,$

$-2\,AB \times 10^{-9} - A^2 = 99\,043\,227\,271\,178\,699\,552.$

It may therefore be assumed that most of the entries in Table 1 would be correct to the last digit, and in the worst case this might be affected by one unit or so compared with its true value. The user of this table should note this uncertainty in the last digit. This is the reason why the present article is entitled "17–Figure..." and not "18–Figure..."

When tan $x°$ is desired, the division with a calculating machine is effected by the operation

$$\frac{C + D \times 10^{-9}}{A + B \times 10^{-9}} = \left[\left(-\frac{B}{A}C + D\right) \times 10^{-9} + C\right] \frac{1}{A}; \tag{5}$$

and thus for example

$$\tan 15° = \frac{0.258\,819\,045\,102\,520\,762}{0.965\,925\,826\,289\,068\,287} = 0.267\,949\,192\,431\,122\,706.$$

4. A first subsidiary table is given in Table II attached, which gives entries corresponding to $1'$, $2'$, $3'$,... $30'$. In virtue of this, when the argument terminates in minute, sines and cosines for intermediate argument can be computed by simple multiplication, and then addition or subtraction.

To construct Table II, the following recurrence formulas were used.

$$\left.\begin{aligned} \sin(x' + 1') &= A' \sin x' + B' \cos x', \\ \cos(x' + 1') &= A' \cos x' - B' \sin x', \end{aligned}\right\} \tag{6}$$

in which

$$A' = \cos 1' = 0.999\,999\,957\,692\,025\,328,$$

$$B' = \sin 1' = 0.000\,290\,888\,204\,563\,425.$$

Values of cos $1'$ and sin $1'$ are computed in Table 3, in which very

**Table 3.** Computation of cos $1'$ and sin $1'$

| | | | |
|---|---|---|---|
| $h = 1' = 0.000\,290\,888\,208\,665\,721\,6$ rad. | | | |
| $h^2 = 0.000\,000\,084\,615\,949\,940\,7$ | | | |
| cos $1'$ | | sin $1'$ | |
| 1 | 1.000 | $h$ | 0.000 290 888 208 665 721 6 |
| $\times \frac{h^2}{2}$ | −0.000 000 042 307 974 970 4 | $\times \frac{h^2}{6}$ | −4 102 297 0 |
| $\times h^2\, 08\dot{3}$ | 298 3 | $\times h^2\, 050$ | 0 0 |
| Sum | 0.999 999 957 692 025 327 9 | Sum | 0.000 290 888 204 563 424 6 |

rapid convergences can be seen. For the fractional part of argument in minute, preference will therefore be given to the computation by power series, since its convergence is more rapid than that of Table 3; and hence the correction for the fractional part is an easy work.

5. As examples, let us find $\sin 28° 40' 00''$ and $\cos 28° 40' 00''$. We then have

$$\begin{aligned}\sin 28° 40' &= \sin(29° - 20') = \sin 29° \cos 20' - \cos 29° \sin 20' \\ &= 0.484\,809\,620\,246\,337\,029 \times 0.999\,983\,076\,857\,744\,189 \\ &\quad - 0.874\,619\,707\,139\,395\,800 \times 0.005\,817\,731\,354\,993\,834 \\ &= 0.479\,713\,113\,250\,246\,227,\end{aligned}$$

which is correct to the last place of decimals, since the direct way of obtaining this value by power series gives

$$\sin 28° 40' = 0.479\,713\,113\,250\,246\,227\,21.$$

Also we have

$$\begin{aligned}\cos 28° 40' &= \cos(29° - 20') = \cos 29° \cos 20' + \sin 29° \sin 20' \\ &= 0.877\,425\,397\,954\,581\,912.\end{aligned}$$

The check calculation for these values is thus:

$$\begin{aligned}(\sin 28° 40')^2 &= 0.230\,124\,671\,024\,243\,562, \\ 1 - (\cos 28° 40')^2 &= 0.230\,124\,671\,024\,243\,565.\end{aligned}$$

6. For the correction for Table II, sines and cosines for 10′, 20′and 30′ were computed beforehand by power series which should give true

**Table 4.** Comparison for correction (for every one minute)

| $x'$ | $\sin x'$ | | $\cos x'$ | |
|---|---|---|---|---|
| 10′ | 0.002 908 877 984 361 934 | (T) | 0.999 995 769 205 486 236 | (T) |
| | 0.002 908 877 984 361 934 | (R) | 0.999 995 769 205 486 236 | (R) |
| 20′ | 0.005 817 731 354 993 834 | (T) | 0.999 983 076 857 744 189 | (T) |
| | 0.005 817 731 354 993 832 | (R) | 0.999 983 076 857 744 187 | (R) |
| 30′ | 0.008 726 535 498 373 935 | (T) | 0.999 961 923 064 171 289 | (T) |
| | 0.008 726 535 498 373 935 | (R) | 0.999 961 923 064 171 289 | (R) |

entries. Corrections similar to those in constructing Table I are also necessary. For reference sake, a comparison of computed entries with those by power series is given in Table 4, in which (T) denotes true entries by power series and (R) those by the recurrence formulas (6).

In Table 4, perfect coincidence can be seen in the ranges 1′-10′ and 21′-30′, so that there is no need to correct entries in these ranges in Table II. As for the remaining range, 11′-20′, there are discrepancies of two units in the last digit between (T) and (R), so that, supposing a uniform accumulation of these errors, suitable correction has been made in these digits in Table II.

7. When the argument has a fractional part in minute, the computation for this part may easily be performed by power series. Thus, to find for example sin 28° 40′. 433 615 398, the fractional part is computed as indicated in Table 5.

**Table 5.** Computation for fractional part in minute

$h = 0'.433\,615\,398 = 0.000\,126\,133\,606\,374\,094$ rad.

$h^2 = 0.000\,000\,015\,909\,686\,657$

| cos $h$ | | sin $h$ | |
|---|---|---|---|
| 1 | 1.000 | $h$ | 0.000 126 133 606 374 094 |
| $\times\ h^2\ 500$ | − 0.000 000 007 954 843 328 | $\times\ h^2\ 1\dot{6}6$ | − 334 458 |
| $\times\ h^2\ 08\dot{3}$ | 11 | | 0 |
| Sum | 0.999 999 992 045 156 683 | Sum | 0.000 126 133 606 039 636 |

We then have, with the results of Table 5,

$$\begin{aligned}\sin\,(28^\circ\,40' + h) &= \sin\,28^\circ\,40'\,\cos h + \cos 28^\circ\,40'\,\sin\,h \\ &= 0.479\,713\,113\,250\,246\,227 \times 0.999\,999\,992\,045\,156\,683 \\ &\quad + 0.877\,425\,397\,954\,581\,912 \times 0.000\,126\,133\,606\,039\,636,\end{aligned}$$

that is we obtain

$$\sin\,28^\circ\,40'.433\,615\,398 = 0.479\,823\,782\,263\,678\,347.$$

8. A second subsidiary table is given in Table III, in which sines and cosines for every five seconds are given. Reference, however, may seldom

be made to this table, because of the very rapid convergence of the fractional part.

This table was constructed by means of the "leading-difference method,"[3] which is suited especially for this case where fourth-order differences can be neglected. Attention was of course directed to the perfect elimination from the accumulation of errors due to higher differences, by taking three additional places of decimals into account. Entries in Table III therefore are rigorously correct to the last place of decimals.

9. Log sin $\theta, e^{\sin\theta}$, etc. may also be computed by the aid of my alternative article.[4] Let us for example find log sin 28°40′. We first write

$$\log \sin 28^\circ 40' = \log 0.479\,713\,113\,250\,246\,227$$
$$= -\frac{1}{M} + \log 4.797\,131\,132\,502\,462\,27,$$

where

$$\frac{1}{M} = \log_e 10 = 2.302\,585\,092\,994\,045\,602.$$

We now have, with the "$e^n$ Table,"[5]

$$4.797\,131\,132\,502\,462\,27 = \exp\left(\frac{3}{2} + \frac{1}{16} + \frac{1}{192} + \frac{1}{3\,584} + \frac{1}{32\,768}\right) \times e^{\varepsilon};$$

but we have, by successive multiplication,

$$\exp\left(\frac{3}{2} + \frac{1}{16} + \frac{1}{192} + \frac{1}{3\,584} + \frac{1}{32\,768}\right) = 4.797\,130\,222\,322\,406\,706,$$

and then, with (5),

$$\frac{4.797\,131\,132\,502\,462\,27}{4.797\,130\,222\,322\,406\,706} = 1.000\,000\,189\,734\,281\,410.$$

Hence $\varepsilon$ becomes, by the procedure of iteration,

$$\varepsilon = 0.0^6\,189\,734\,281\,410 - \frac{\varepsilon^2}{2} = 0.0^6\,189\,734\,281\,410 - 18\,000$$
$$= 0.0^6\,189\,734\,263\,410,$$

so that we have

$$\log 4.797\,131\,132\,502\,462\,27 = 1.568\,018\,058\,502\,864\,600.$$

Thus the wanted logarithm becomes

$$\log\ \sin 28^\circ\ 40' = -0.734\,567\,034\,491\,181\,00;$$

the corresponding value by the Chamber's tables being

$$\log\ \sin 28^\circ\ 40' = (9.680\,981\,6 - 10) \times \frac{1}{M} = -0.734\,567\,0.$$

10. In handling a calculating machine, emphasis should be laid on the adoption of a special device that all of its figure-dials are marked by white-paint dots every three figures, thus making the order of numbers treated clear and definite.

In the course of printing of the present article, I was informed by Messrs. Y. Ishii and T. Ikeda, the Fuji Communication Apparatus Mfg. Co. Ltd., Kawasaki, Kanagawa Prefecture, that a very accurate trigonometric table has appeared, and they were kind in lending me a photo-copy from the numerical table in its original paper. This gives sines and cosines for every one degree to 30 places of decimals; but, in spite of its extreme accuracy, it gives nothing more than the cited entries. By virtue of this table, the principal Table I attached was corrected, so that our Table I is correct to the last place of decimals, these two being in perfect coincidence so far as 17 places of decimals are concerned.

Recently, in reply to my sending of a private copy of the present work, Professor W. Shibagaki, University of Kyushu, Fukuoka, wrote me and commented that the following tables have been published: C. E. van Orstrand: "Tables of the Exponential Function and of the Circular Sine and Cosine to Radian Argument," in which sin $x$ and cos $x$ are given to 23 places of decimals for $x = 0$ to 100 for every 1.0 radian, for $x = 0.0$ to 10 for every 0.1 radian, and for $x = 0.000$ to 1.600 for every 0.001 radian, together with $e^x$ and $e^{-x}$ to 33 figures for $x = 0.0$ to 50.0 for every 0.1.

## References

1) K. Hayashi: "Nine-Figure Trigonometric Functions," Iwanami, Tokyo.

2) E. T. Whittaker and G. Robinson: "The Calculus of Observations," 2nd ed., London and Glasgow, 1937.

3) Loc. cit. 2), Chap. IV.

4) B. Tanimoto: "17-Figure $e^n$ Table and its Applications," Journal of the Faculty of Engineering, Shinshu University, vol. 6 (1956)—a preceding article of this Volume.

5) Loc. cit. 4), "$e^n$ Table" attached.

**Table** I. Values of sin $x°$ and cos $x°$

| $x°$ | sin $x°$ | cos $x°$ |
|---|---|---|
| 1 | 0.017 452 406 437 283 513 | 0.999 847 695 156 391 239 |
| 2 | 0.034 899 496 702 500 972 | 0.999 390 827 019 095 730 |
| 3 | 0.052 335 956 242 943 833 | 0.998 629 534 754 573 874 |
| 4 | 0.069 756 473 744 125 301 | 0.997 564 050 259 824 248 |
| 5 | 0.087 155 742 747 658 174 | 0.996 194 698 091 745 532 |
| 6 | 0.104 528 463 267 653 471 | 0.994 521 895 368 273 337 |
| 7 | 0.121 869 343 405 147 481 | 0.992 546 151 641 322 035 |
| 8 | 0.139 173 100 960 065 444 | 0.990 268 068 741 570 315 |
| 9 | 0.156 434 465 040 230 869 | 0.987 688 340 595 137 726 |
| 10 | 0.173 648 177 666 930 349 | 0.984 807 753 012 208 059 |
| 11 | 0.190 808 995 376 544 812 | 0.981 627 183 447 663 953 |
| 12 | 0.207 911 690 817 759 337 | 0.978 147 600 733 805 638 |
| 13 | 0.224 951 054 343 864 998 | 0.974 370 064 785 235 229 |
| 14 | 0.241 921 895 599 667 723 | 0.970 295 726 275 996 472 |
| 15 | 0.258 819 045 102 520 762 | 0.965 925 826 289 068 287 |
| 16 | 0.275 637 355 816 999 186 | 0.961 261 695 938 318 862 |
| 17 | 0.292 371 704 722 736 728 | 0.956 304 755 963 035 481 |
| 18 | 0.309 016 994 374 947 424 | 0.951 056 516 295 153 572 |
| 19 | 0.325 568 154 457 156 669 | 0.945 518 575 599 316 810 |
| 20 | 0.342 020 143 325 668 733 | 0.939 692 620 785 908 384 |
| 21 | 0.358 367 949 545 300 273 | 0.933 580 426 497 201 749 |
| 22 | 0.374 606 593 415 912 035 | 0.927 183 854 566 787 401 |
| 23 | 0.390 731 128 489 273 755 | 0.920 504 853 452 440 327 |
| 24 | 0.406 736 643 075 800 208 | 0.913 545 457 642 600 896 |
| 25 | 0.422 618 261 740 699 436 | 0.906 307 787 036 649 963 |
| 26 | 0.438 371 146 789 077 417 | 0.898 794 046 299 166 993 |
| 27 | 0.453 990 499 739 546 792 | 0.891 006 524 188 367 862 |
| 28 | 0.469 471 562 785 890 776 | 0.882 947 592 858 926 942 |
| 29 | 0.484 809 620 246 337 029 | 0.874 619 707 139 395 800 |
| 30 | 0.500 000 000 000 000 000 | 0.866 025 403 784 438 647 |
| 31 | 0.515 038 074 910 054 210 | 0.857 167 300 702 112 287 |

| $x^\circ$ | $\sin x^\circ$ | $\cos x^\circ$ |
|---|---|---|
| 32 | 0.529 919 264 233 204 954 | 0.848 048 096 156 425 970 |
| 33 | 0.544 639 035 015 027 082 | 0.838 670 567 945 424 030 |
| 34 | 0.559 192 903 470 746 830 | 0.829 037 572 555 041 692 |
| 35 | 0.573 576 436 351 046 096 | 0.819 152 044 288 991 790 |
| 36 | 0.587 785 252 292 473 129 | 0.809 016 994 374 947 424 |
| 37 | 0.601 815 023 152 048 280 | 0.798 635 510 047 292 846 |
| 38 | 0.615 661 475 325 658 280 | 0.788 010 753 606 721 957 |
| 39 | 0.629 320 391 049 837 453 | 0.777 145 961 456 970 880 |
| 40 | 0.642 787 609 686 539 326 | 0.766 044 443 118 978 035 |
| 41 | 0.656 059 028 990 507 285 | 0.754 709 580 222 771 998 |
| 42 | 0.669 130 606 358 858 214 | 0.743 144 825 477 394 235 |
| 43 | 0.681 998 360 062 498 500 | 0.731 353 701 619 170 483 |
| 44 | 0.694 658 370 458 997 287 | 0.719 339 800 338 651 139 |
| 45 | 0.707 106 781 186 547 524 | 0.707 106 781 186 547 524 |

**Table** II. Values of $\sin x'$ and $\cos x'$

| $x'$ | $\sin x'$ | $\cos x'$ |
|---|---|---|
| 1 | 0.000 290 888 204 563 425 | 0.999 999 957 692 025 328 |
| 2 | 0.000 581 776 384 513 068 | 0.999 999 830 768 104 892 |
| 3 | 0.000 872 664 515 235 150 | 0.999 999 619 228 249 431 |
| 4 | 0.001 163 552 572 115 895 | 0.999 999 323 072 476 846 |
| 5 | 0.001 454 440 530 541 535 | 0.999 998 942 300 812 196 |
| 6 | 0.001 745 328 365 898 309 | 0.999 998 476 913 287 700 |
| 7 | 0.002 036 216 053 572 466 | 0.999 997 926 909 942 736 |
| 8 | 0.002 327 103 568 950 269 | 0.999 997 292 290 823 845 |
| 9 | 0.002 617 990 887 417 994 | 0.999 996 573 055 984 726 |
| 10 | 0.002 908 877 984 361 934 | 0.999 995 769 205 486 236 |
| 11 | 0.003 199 764 835 168 402 | 0.999 994 880 739 396 396 |
| 12 | 0.003 490 651 415 223 731 | 0.999 993 907 657 790 382 |
| 13 | 0.003 781 537 699 914 277 | 0.999 992 849 960 750 534 |

| $x'$ | $\sin x'$ | $\cos x'$ |
|---|---|---|
| 14 | 0.004 072 423 664 626 421 | 0.999 991 707 648 366 348 |
| 15 | 0.004 363 309 284 746 570 | 0.999 990 480 720 734 483 |
| 16 | 0.004 654 194 535 661 162 | 0.999 989 169 177 958 758 |
| 17 | 0.004 945 079 392 756 666 | 0.999 987 773 020 150 149 |
| 18 | 0.005 235 963 831 419 581 | 0.999 986 292 247 426 793 |
| 19 | 0.005 526 847 827 036 445 | 0.999 984 726 859 913 988 |
| 20 | 0.005 817 731 354 993 834 | 0.999 983 076 857 744 189 |
| 21 | 0.006 108 614 390 678 361 | 0.999 981 342 241 057 014 |
| 22 | 0.006 399 496 909 476 683 | 0.999 979 523 009 999 240 |
| 23 | 0.006 690 378 886 775 498 | 0.999 977 619 164 724 801 |
| 24 | 0.006 981 260 297 961 552 | 0.999 975 630 705 394 794 |
| 25 | 0.007 272 141 118 421 639 | 0.999 973 557 632 177 473 |
| 26 | 0.007 563 021 323 542 602 | 0.999 971 399 945 248 254 |
| 27 | 0.007 853 900 888 711 335 | 0.999 969 157 644 789 713 |
| 28 | 0.008 144 779 789 314 788 | 0.999 966 830 730 991 582 |
| 29 | 0.008 435 658 000 739 966 | 0.999 964 419 204 050 756 |
| 30 | 0.008 726 535 498 373 935 | 0.999 961 923 064 171 289 |

**Table** III. Sin $x''$ and cos $x''$ for every $5''$

| $x''$ | $\sin x''$ | $\cos x''$ |
|---|---|---|
| 5 | 0.000 024 240 684 053 103 | 0.999 999 999 706 194 618 |
| 10 | 0.000 048 481 368 091 961 | 0.999 999 998 824 778 473 |
| 15 | 0.000 072 722 052 102 332 | 0.999 999 997 355 751 565 |
| 20 | 0.000 096 962 736 069 970 | 0.999 999 995 299 113 896 |
| 25 | 0.000 121 203 419 980 632 | 0.999 999 992 654 865 466 |
| 30 | 0.000 145 444 103 820 074 | 0.999 999 989 423 006 276 |