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Characterization of wave propagation in thin laminated plates

Chao Du[1], Qing-Qing Ni*[2] and Toshiaki Natsuki [3]

[1]Interdisciplinary Graduate School of Science and Technology, Shinshu University, Ueda City, Nagano, Japan

*[2(a)] Key Laboratory of Ministry of Education, Zhejiang Sci-Tech University, Xiasha Higher Education Zone, Hangzhou 310018, China

[(b)] Dept. of Functional Machinery and Mechanics, Shinshu University, 3-15-1 Tokida, Ueda 386-8576, Japan

[3]Dept. of Functional Machinery and Mechanics, Shinshu University, 3-15-1 Tokida, Ueda 386-8576, Japan

* *Corresponding author*

E-mail: niqq@shinshu-u.ac.jp

Fax: +81-268-215438

Tel: +81-268-215438

Abstract

The formulae including the in-plane stiffness and bending stiffness coefficients, and coupling effect in out-plane were presented for analyzing characterization of wave propagation. Using the present formulae, the wave velocities can be calculated in any case of laminated plates including non-symmetric lamination. In this study, the characteristics of wave propagation in thin laminated plates were investigated in detail. Five modes including symmetric modes and anti-symmetric modes were obtained from the proposed formulae. For each mode, the wave velocities of laminated plates were investigated in different conditions such as plate thickness, stacking sequence and vibrational frequency.

Keywords

Wave propagation; Laminated plates; Lamb wave; Symmetric mode; Anti-symmetric mode

1. Introduction

In recent years, thin plate structures have been widely applied in many fields including aircraft and spacecraft external skins and pressure vessels due to weight and energy-saving. However, some damages like matrix cracking, delamination, transverse cracking and fiber breakage are required to be detected in real-time. The energy released from these damages may give rise to small surface displacements and cause transient elastic waves in materials. Thus, the wave propagation is necessary to detect damage mechanism and to predict damage life in materials or structures. Wave propagation of thin plates usually takes the form of lamb waves [1], and can be divided into three types: One type is called symmetric mode because the deformations are symmetric about the mid-plane of the plates; the second type is called anti-symmetric mode due to the deformations with anti-symmetric about the mid-plane; the third type is called shear horizontal (SH) mode, in which the transverse particle vibrations are horizontal to the plane. Lame et al. and Rayleigh et al. [2] have reported the propagation characteristics of the waves in a thin plate, which are dependent on the plate thickness and boundary conditions in addition to the elastic properties and density of the materials. For symmetric laminated plates, the formulae of wave velocity were presented by Tang et al. [3]. Yamada et al. have studied the source location of impact [4]. And also, by exciting waves and measuring the characterization of these waves propagating in materials, the mechanical properties of the materials could be determined [5, 6].

In this study, the valuation formulae of wave velocity for arbitrarily-laminated plates were derived based on first order shear deformation theory. Using the proposed formulae, the wave velocities can be calculated in any case of laminated plates including non-symmetric lamination. Moreover, the influences of the propagating direction, the plate thickness and the stacking sequence on the wave velocity were investigated in detail.

2. Theoretical approach

In this study, coupling stiffness and rotary inertia coefficients [7] are considered to develop the evaluation formula of the wave velocity in arbitrarily-laminated plates. Coordinate system of a laminated plate is shown in Fig. 1. The *x* and *y* axes are in the mid-plane, the *z* axis is normal to the lamina with its origin at the mid-plane, *f* indicates the direction of the fiber orientation, and *h* is the thickness of the plate. *u*, *v* and *w* are displacement components along the *x*, *y* and *z* directions, respectively. According to the first order shear deformation theory of laminated plates, the displacements of the laminated plates is defined as

$$\begin{aligned} u &= u_0(x,y,t) + z\psi_x(x,y,t) \\ v &= v_0(x,y,t) + z\psi_y(x,y,t) \\ w &= w(x,y,t) \end{aligned} \tag{1}$$

where u_0 and v_0 are the mid-plane displacement components, and ψ_x and ψ_y are the rotation components along *x* and *y* directions, respectively.

The constitutive relations related to the force and moment resultants can be given by

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \\ Q_x \\ Q_y \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} & 0 & 0 \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} & 0 & 0 \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} & 0 & 0 \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} & 0 & 0 \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} & 0 & 0 \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & A_{44} & A_{45} \\ 0 & 0 & 0 & 0 & 0 & 0 & A_{45} & A_{55} \end{bmatrix} \begin{bmatrix} \partial u_0/\partial x \\ \partial v_0/\partial y \\ \partial w/\partial y + \psi_y \\ \partial w/\partial x + \psi_x \\ \partial u_0/\partial y + \partial v_0/\partial x \\ \partial \psi_x/\partial x \\ \partial \psi_y/\partial y \\ \partial \psi_x/\partial y + \partial \psi_y/\partial x \end{bmatrix} \tag{2}$$

where Q_x and Q_y are the force and moment resultants per unit length along x and y directions, respectively. The A_{ij} are the in-plane stiffness coefficients, the D_{ij} are the bending stiffness coefficients, and the B_{ij} are the coupling stiffness in-plane considering the bending effects. $A_{ij,}$ B_{ij} and D_{ij} are defined as

$$\begin{aligned} (A_{ij}, B_{ij}, D_{ij}) &= \int_{-h/2}^{h/2} Q_{ij}^{(l)}(1, z, z^2)dz \\ A_{ij} &= k_i k_j \int_{-h/2}^{h/2} Q_{ij}^{(l)} dz \qquad i, j = 4,5 \end{aligned} \tag{3}$$

where the Q_{ij} are the transverse shear stiffness [8]. The superscript l refers to the layer number of the laminated plates. The k_i and k_j are shear correction factors with $k_4^2 = k_5^2 = k_4 k_5 = 5/6$.

From the derivation of the classical governing equation for in-plane motion and the derivation of the equation, the equations of motion are given by

$$\begin{aligned} &\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = \rho^* \frac{\partial^2 u_0}{\partial t^2} + R\frac{\partial^2 \psi_x}{\partial t^2} \\ &\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = \rho^* \frac{\partial^2 v_0}{\partial t^2} + R\frac{\partial^2 \psi_y}{\partial t^2} \\ &\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x = R\frac{\partial^2 u_0}{\partial t^2} + I\frac{\partial^2 \psi_x}{\partial t^2} \\ &\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} - Q_y = R\frac{\partial^2 v_0}{\partial t^2} + I\frac{\partial^2 \psi_y}{\partial t^2} \\ &\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} = \rho^* \frac{\partial^2 w}{\partial t^2} \end{aligned} \tag{4}$$

and

$$(\rho^*, R, I) = \int_{-h/2}^{h/2} \rho(1, z, z^2)dz \tag{5}$$

where ρ is the mass density.

Considering the directional dependence, the waves propagating in the plane of laminated plates have the forms

$$
\begin{aligned}
u_0 &= A\alpha_x e^{i[k(mx+ny)-\omega t]} \\
v_0 &= A\alpha_y e^{i[k(mx+ny)-\omega t]} \\
\psi_x &= \Psi_x e^{i[k(mx+ny)-\omega t]} \\
\psi_y &= \Psi_y e^{i[k(mx+ny)-\omega t]} \\
w &= W e^{i[k(mx+ny)-\omega t]}
\end{aligned} \tag{6}
$$

where k is the wave number. m is the direction cosine between the wave propagation and the x axis, and n is the direction cosine between the wave propagation and the y axis. ω is the circular frequency, and $A\alpha_x$, $A\alpha_y$, Ψ_x, Ψ_y and W are the amplitudes of the waves.

Substituting Eq. (6) into Eq. (4), the matrix of coefficients for symmetric and anti-symmetric modes is expressed as

$$
\begin{vmatrix}
\overbrace{M_{11} & M_{12}}^{\text{S}} & M_{13} & M_{14} & 0 \\
M_{12} & M_{22} & M_{14} & M_{24} & 0 \\
M_{13} & M_{14} & \overbrace{M_{33} & M_{34} & M_{35}}^{\text{A}} \\
M_{14} & M_{24} & M_{34} & M_{44} & M_{45} \\
0 & 0 & -M_{35} & -M_{45} & M_{55}
\end{vmatrix} = 0 \tag{7}
$$

where

$$
\begin{aligned}
M_{11} &= (A_{11}m^2 + 2A_{16}mn + A_{66}n^2)k^2 - \rho^*\omega^2 \\
M_{12} &= [A_{16}m^2 + (A_{12} + A_{66})mn + A_{26}n^2]k^2 \\
M_{13} &= (B_{11}m^2 + 2B_{16}mn + B_{66}n^2)k^2 - R\omega^2 \\
M_{14} &= [B_{16}m^2 + (B_{12} + B_{66})mn + B_{26}n^2]k^2 \\
M_{15} &= 0 \\
M_{21} &= M_{12} \\
M_{22} &= (A_{66}m^2 + 2A_{26}mn + A_{22}n^2)k^2 - \rho^*\omega^2 \\
M_{23} &= M_{14} \\
M_{24} &= (B_{66}m^2 + 2B_{26}mn + B_{22}n^2)k^2 - R\omega^2 \\
M_{25} &= 0 \\
M_{31} &= M_{13} \\
M_{32} &= M_{14} \\
M_{33} &= (D_{11}m^2 + 2D_{16}mn + D_{66}n^2)k^2 + A_{55} - I\omega^2 \\
M_{34} &= [D_{16}m^2 + (D_{12} + D_{66})mn + D_{26}n^2]k^2 + A_{45} \\
M_{35} &= i(A_{55}m + A_{45}n)k \\
M_{41} &= M_{14} \\
M_{42} &= M_{24} \\
M_{43} &= M_{34} \\
M_{44} &= (D_{66}m^2 + 2D_{26}mn + D_{22}n^2)k^2 + A_{44} - I\omega^2 \\
M_{45} &= i(A_{45}m + A_{44}n)k \\
M_{51} &= 0 \\
M_{52} &= 0 \\
M_{53} &= -M_{35} \\
M_{54} &= -M_{45} \\
M_{55} &= (A_{55}m^2 + 2A_{45}mn + A_{44}n^2)k^2 - \rho^*\omega
\end{aligned}
\tag{8}
$$

According to previous research [3, 9], it is known that *S*-matrix is regarded as the symmetric mode, while *A*-matrix is regarded as the anti-symmetric mode. In this study, these modes are involved in one matrix including other elements. Thus, the wave velocity can be calculated more exactly in any case of laminates, including non-symmetric laminates. The phase velocity (ω/k) can be obtained when the determinant is set equal to zero.

3. Valuation of wave propagation

In this study, the characterization of the wave propagation in symmetric laminated plates was valuated based on the proposed formula. The wave velocities were calculated with the properties shown in Table 1.

Figure 2 shows the velocity dispersion curves of a 16-ply unidirectional laminated plate in different direction of wave propagation. We assume that the fiber orientation is in the direction of 0^o. It is well known that the velocity of symmetric mode is a constant, while the velocity of anti-symmetric mode has different values for changing frequency. S_0' and S_0 as shown in Fig.2 are symmetric modes, while A_0, A_1 and A_2 were anti-symmetric modes.

Comparing Fig. 2-a), 2-b) and 2-c), the influence of the propagating direction on the wave velocity is investigated. The velocities of S_0' mode are 1453 m/s, 1887 m/s and 1453 m/s along the

fiber direction, 45^o and 90^o directions, respectively. The velocities of S_0 mode are 4660 m/s, 3555 m/s and 2541 m/s, respectively. In these modes, the waves excite particle displacement components along not only propagating direction but also perpendicular to propagating direction. Generally, the wave of S_0' mode is called the quasi-in-plane shear wave, and the wave of S_0 mode called quasi-extensional wave. The quasi-extensional wave has larger component of its particle displacement in the direction of wave propagation [10]. The velocities of A_1 and A_2 modes occurred in the frequencies of over 453 kHz and 457 kHz, respectively. It can be found that the velocities decreased sharply with increasing frequency. The A_0 mode has the smallest velocity in all of modes. Figure 3 shows the velocity curves of A_0 mode of a 16-ply unidirectional laminated plate in different propagating directions. At the same frequency, the wave velocity is the largest value when the wave propagates along the fiber orientation, and it has the smallest value in the direction perpendicular to the fiber orientation. However, the velocities of A_0 mode increase with increasing frequency and then reach a plateau.

Figure 4 shows the velocity dispersion curves of an 8-ply unidirectional laminated plate in the direction perpendicular to fiber. The velocities of S_0' and S_0 modes are 1453 m/s and 2541 m/s, respectively, and are the same as that in a 16-ply unidirectional laminated plate. The velocities of A_1 and A_2 modes appear when the frequencies (called the critical frequency) are over 905 kHz and 913 kHz, respectively. Comparing with the results shown in Fig. 2-c), the critical frequencies in the 8-ply unidirectional laminated plate are about twice as large as those in the 16-ply unidirectional laminated plate. For A_0 mode, the velocity curves of the 8-ply plate with different thicknesses are shown in Fig. 5. It is clearly observed that the velocity of A_0 mode in a thick laminated plate is larger than that in the thin one when the frequency is low. However, all of the velocities increase with increasing frequency and hold the same value.

Figure 6 shows the velocity dispersion curves of a quasi-isotropic laminated plate ($[45/-45/0/90]_{2s}$) and a cross-ply laminated plate ($[0/90]_{4s}$). The velocities of S_0' and S_0 mode are 1969 m/s and 3510 m/s for $[45/-45/0/90]_{2s}$, and 1453 m/s and 3753 m/s for $[0/90]_{4s}$, respectively. It is found that the stacking sequence affects the wave propagation of symmetric modes. For the frequency larger than 455 kHz, the velocities of A_1 and A_2 modes for $[45/-45/0/90]_{2s}$ and $[0/90]_{4s}$ are close to the same value. The velocity dispersion curves of A_0 mode in plates with different stacking sequences are shown in Fig. 7. The velocities in the plates of $[45/-45/0/90]_{2s}$ and $[0/90]_{4s}$ are just between 0^o and 90^o directions of unidirectional laminated plates.

4. Conclusions

According to the first order shear deformation theory, the formulae including the effects of the shear deformation and rotary inertia were derived to evaluate the velocity of wave propagation in the arbitrarily-laminated plates. Based on the present theoretical analysis, the influences of the plate thicknesses and the frequency on the velocities of wave propagation in the symmetric laminated plates were investigated.

Five modes including symmetric modes (S_0' and S_0) and anti-symmetric modes (A_0, A_1 and A_2) were derived with the present formulae. The dispersion of symmetric modes is independent on the frequency of vibration, and the velocities for each mode were hardly affected by the thickness of laminated plates. For anti-symmetric modes, the velocities of A_0 mode increased with increasing frequency, while the velocities of A_1 and A_2 modes decreased. The frequencies, at which A_1 and A_2 modes occurred, were affected by the thickness of laminated plates. Furthermore, the results of the laminated plates with different stacking sequences were obtained and the velocity dependence of stacking angles was made clear.

The further study will be done to verify the theoretical results with experiments and to investigate the wide range of application to a delamination problem.

Acknowledgements

The work was supported partly by Grant-in-Aid for Global COE Program and the project for "Creation of Innovation Centers for Advanced Interdisciplinary Research Areas (Shinshu University)" in promoting science and technology from the Ministry of Education, Culture, Sports, Science and Technology of Japan.

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Table 1 Material properties of GFRP

E_1(GPa)	E_2(GPa)	G_{33}(GPa)	υ_{12}	ρ (g/cm^3)
38.05	10.45	4.03	0.32	1.91

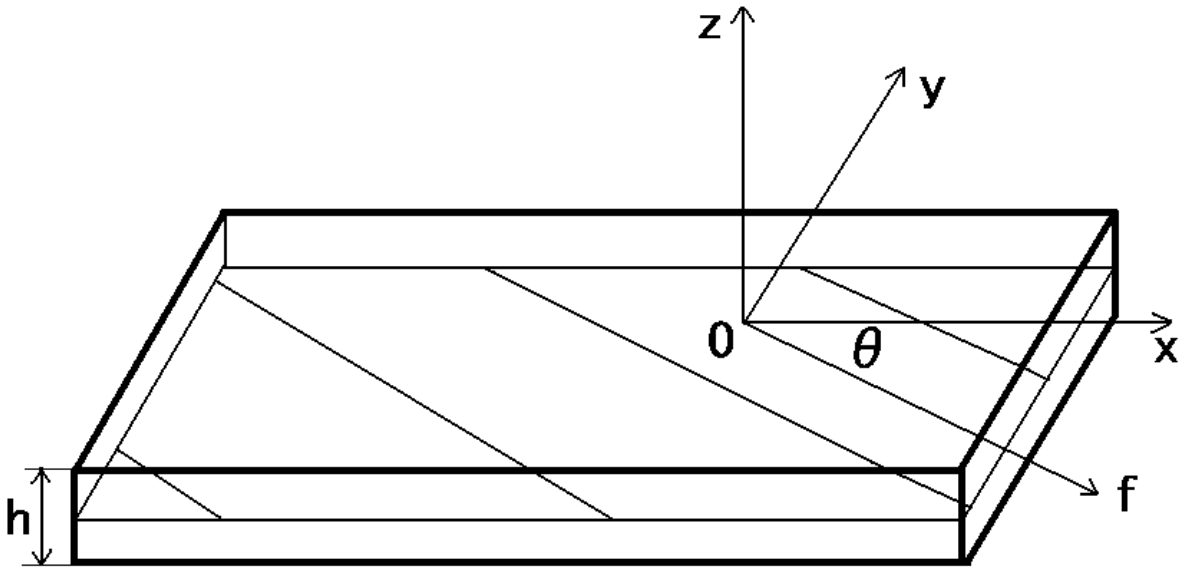
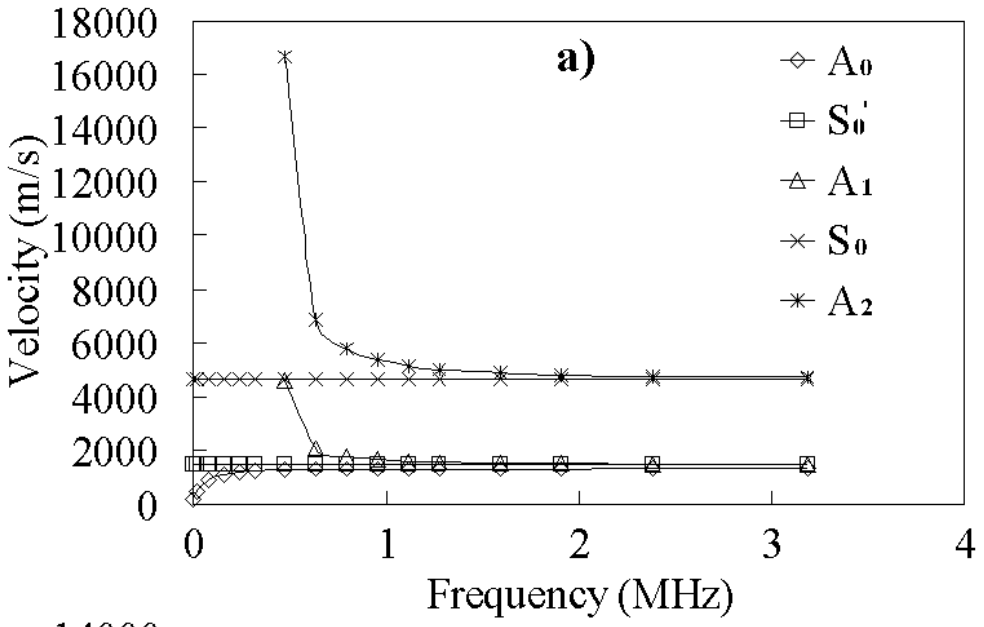
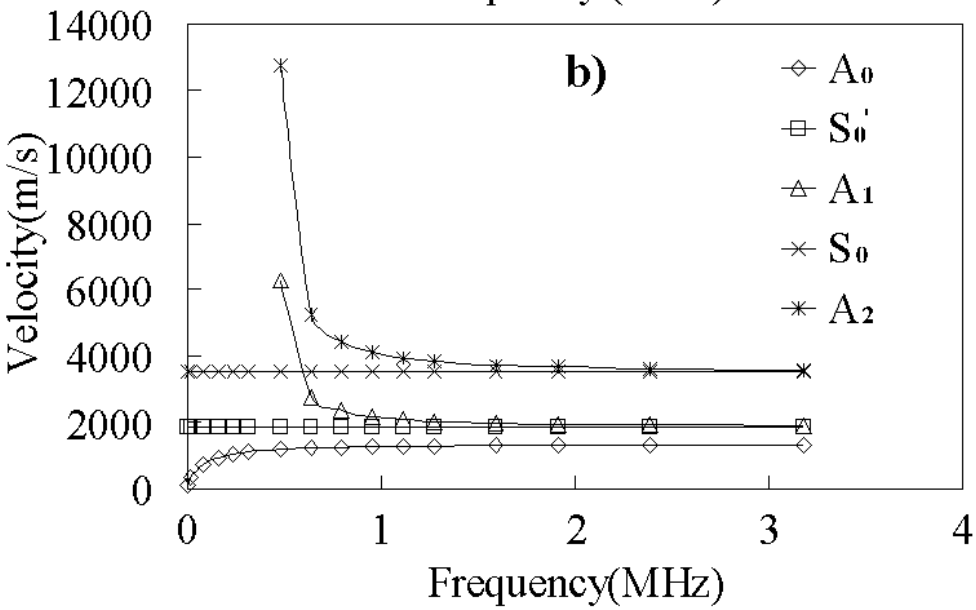


Figure 1 Coordinate axes of a laminated plate.


a)
Velocity (m/s)
Frequency (MHz)
A0
S0'
A1
S0
A2


b)
Velocity(m/s)
Frequency(MHz)
A0
S0'
A1
S0
A2

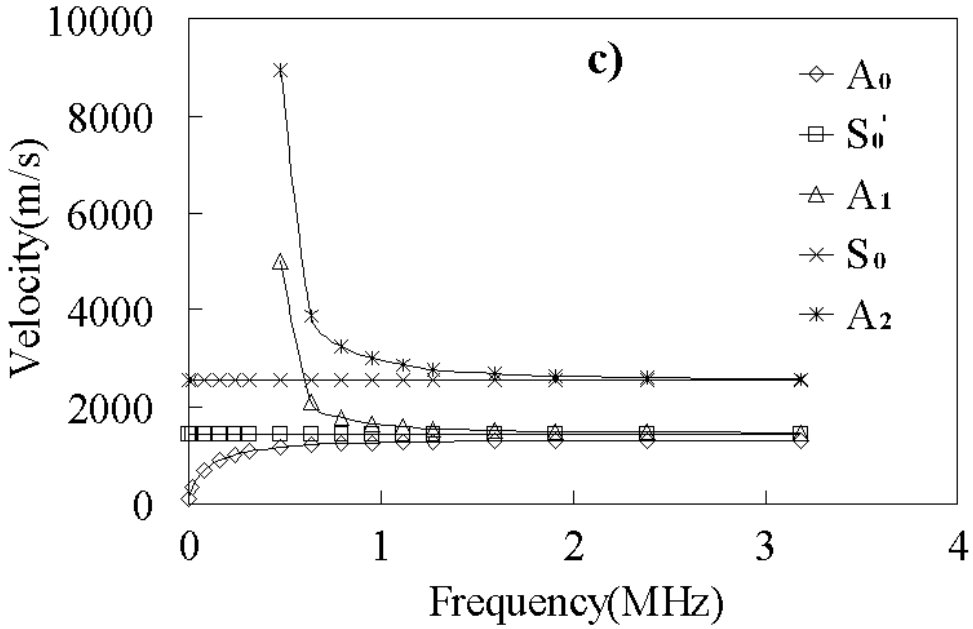

c)
Velocity(m/s)
Frequency(MHz)
A0
S0'
A1
S0
A2

Figure 2 Dispersion curves of 16-ply unidirectional laminated plate.

(a) Wave propagating along fiber orientation. (b) Wave propagating along the direction of 45 degree with fiber orientation. (c) Wave propagating perpendicular to fiber orientation.

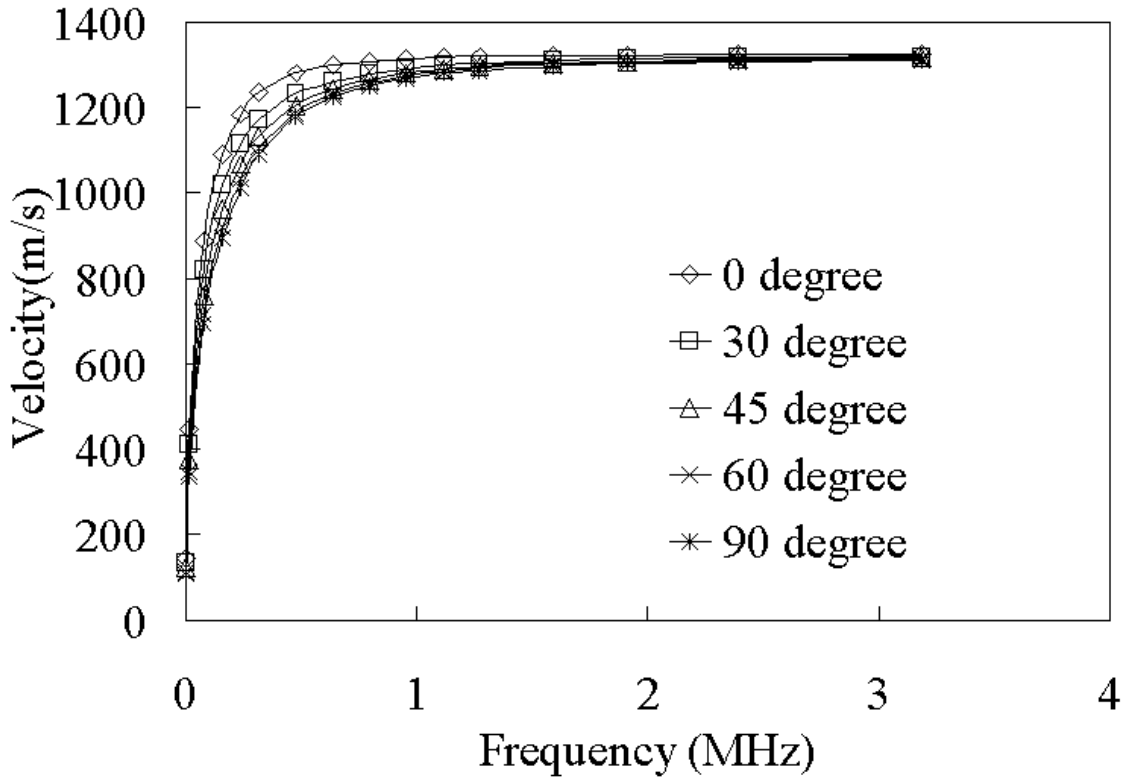


Figure 3 Velocity curves of A_0 mode of 16-ply unidirectional laminated plate in different propagating directions.

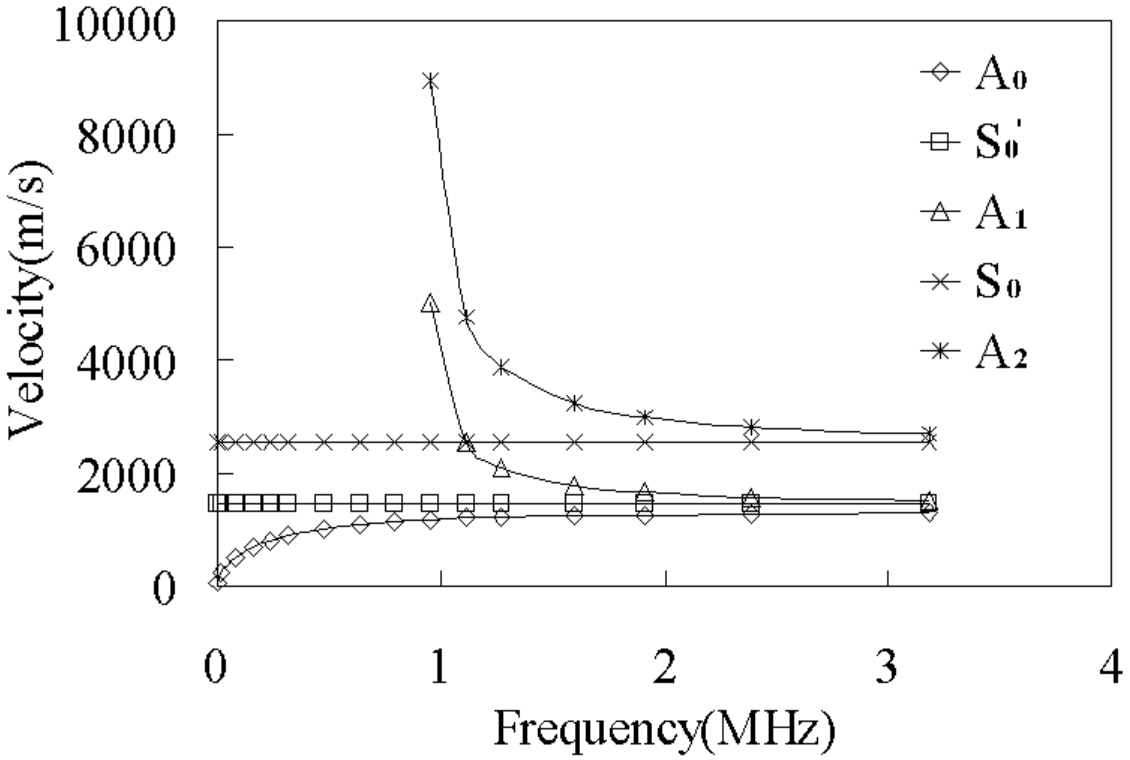


Figure 4 Dispersion curves of 8-ply unidirectional laminated plate in the direction perpendicular to fiber.

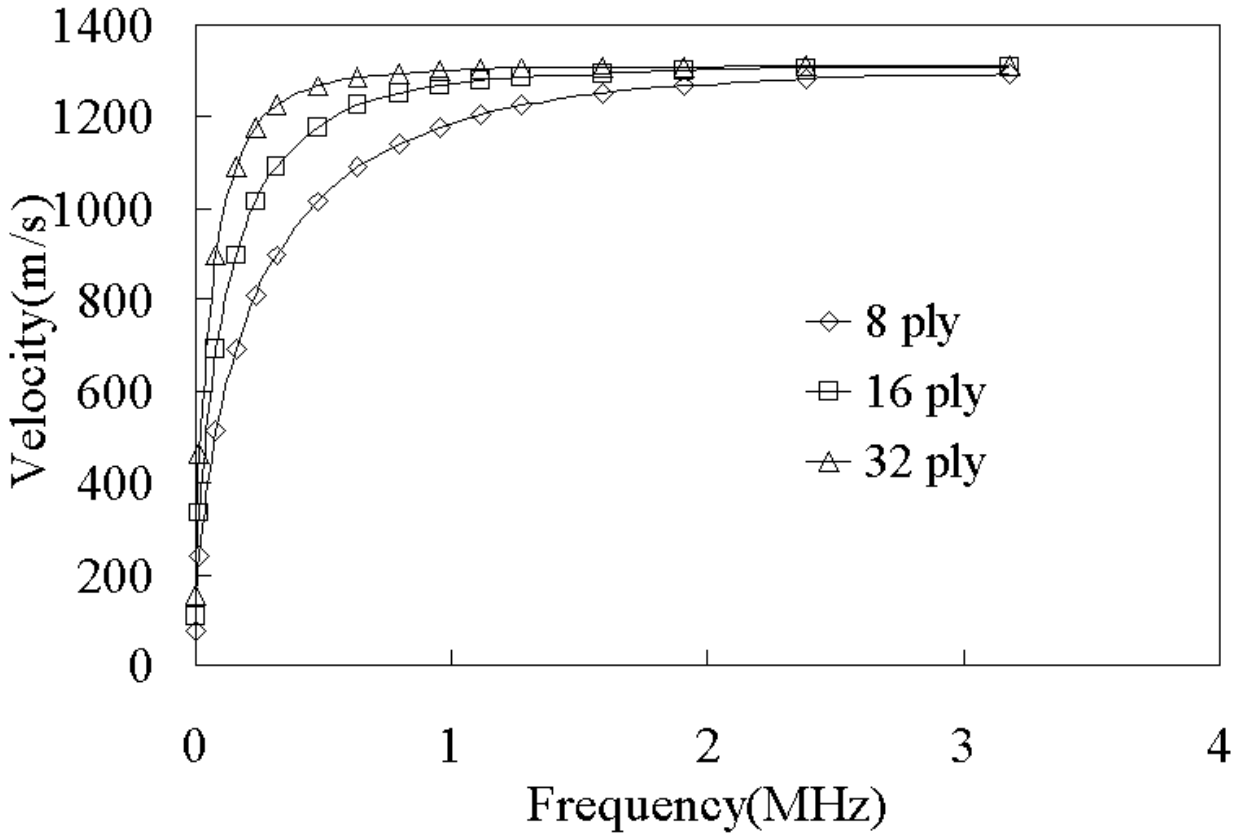


Figure 5 Velocity curves of A_0 mode of unidirectional laminated plates in the direction perpendicular to fiber with different thicknesses.

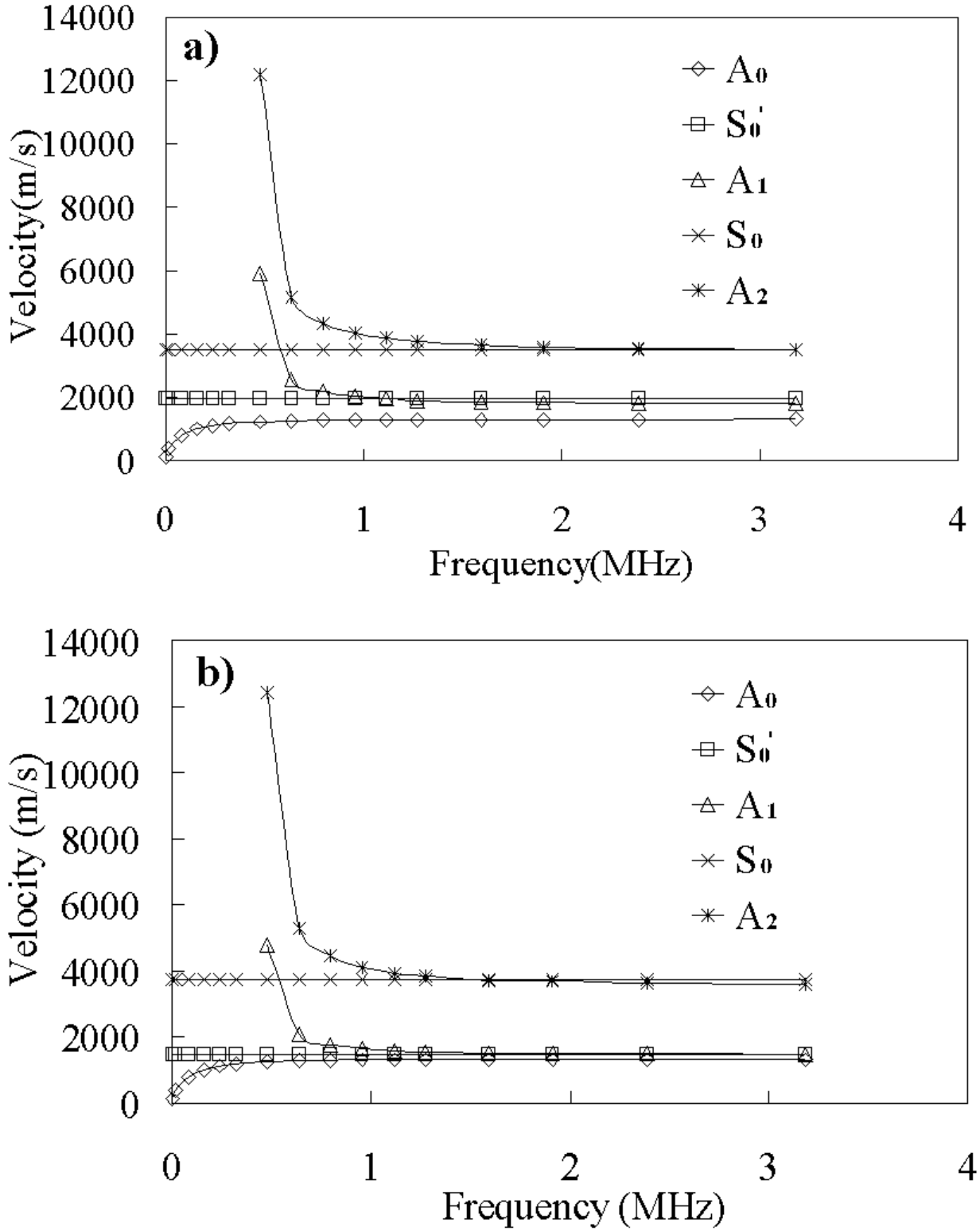


Figure 6 Influence of stacking sequence on dispersion curves along 0 degree direction of propagation.

(a) Dispersion curves of $[45/-45/0/90]_{2s}$ laminated plate. (b) Dispersion curves of $[0/90]_{4s}$ laminated plate.

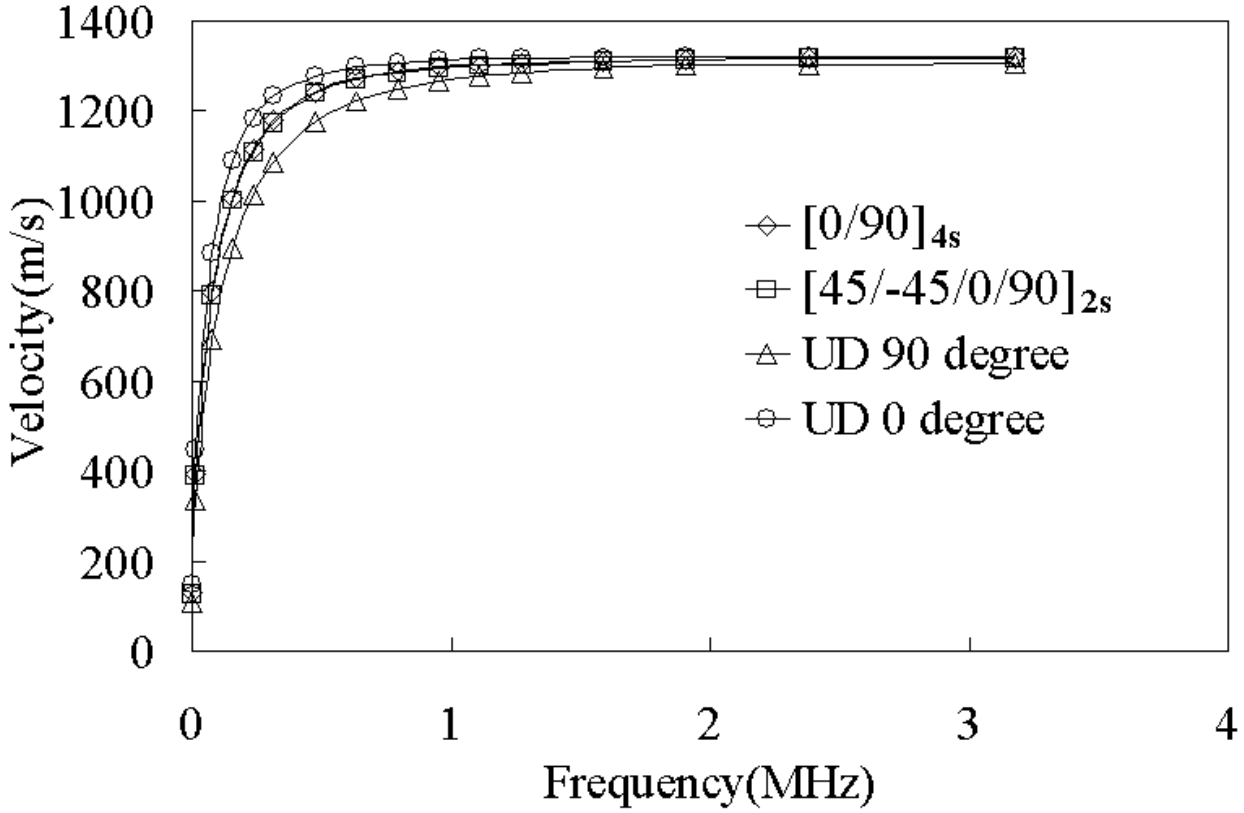


Figure 7 Velocity curves of A_0 mode of 16-ply laminated plate with different stacking sequences.