Trial Method of Calculations in Chemical Engineering

by

Shinichiro IWASAKI

Faculty of Textile Science and Technology, Shinshu University, Ueda

(Recived September 10, 1964)

§ 1. General Description

The calculations in chemical Engineering require trial methods in many cases, eg. in the determination of equilibrium of chemical reactions,
in the determination of constants of viscosity of complicated behaviors,
in the determination of fluid transport problems,
in the calculations of heat transfer problems,
in the calculations of distillation, absorption, adsortion and extraction problems.
in the calculations of evaporation and drying problems and so on.

The solution of those problems are, at pressnt, done by trial method of repetition system or of "cut and try" system, which is often very primitive and tedious. Such a primitive method is used because there is no general efficient methodes to solve such a complicated equation—an equation which is more complicated than a second order algebraic equation in general.

The author of this paper has deviced some years ago a new general convenient method to solve such a complicated equation. It was found that by this new method, an equation of these complicated types may be solved within 15~20 minutes with ease. The auhor wishes that this method may become more popular, and make the chemical engineering calculations more rapid and easy.

§ 2. The Principle of the New Method to Solve a Complicated Equation $y=f(x)=o$

Suppose that the function $y=f(x)$ to be a complicated function, and it is not easy to find the value of x from the value of y, though the balue of y may be calculated from the value of x.

The function $y=f(x)$ is expressed in general by a curve in x–y– coordinate. Now, however complicated may the curve be, a very small part of the curve may be substitued by a straight line. This is a general geometric and experimental law. Now here we expand this law as follows: A very small part of the curve may be more accurately substitued by a regular parabola $y=a+bx+cx^2$ than by a straight line. This is the starting principle of the author's prosess. If we substitue the original curve in the neighborhood of x_0 (x_0

being very near the correct root of the equation) with this parabola, and braw this parabola, and cut this substitute curve by the st. line $y=0$, so we shall get the approximate root x_1, which is practically nearly equall to correct x. If we again substute the curve using x_1 instead of x_0, then the second apporoximate value x_2 will be much nearer to correct x.

We have two methods to draw the substituting parabola.

Method I

Calculate $y_0=f(x_0)$, $y_1=f(x_0-h)$, $y_2=f(x_0+k)$, where $(h\fallingdotseq k)$, and plot the points $A[y_1, (x_0-h)$, , $B[y_0, (x_0)]$ and $C[y_2, (x_0+k).]$ We have then to draw the regular parabola passing through ABC. We have here a theorem concernning parabola. "The locus of the middle points of parallel cords of a regular parabola is a straight line parallel to the axis of the parabola." By using this theorlem, we are able to find the fourth point D on the curve from any three points A, B, C lying on the curve. The procedures are as follows. (cf. fig. 1)

Draw cord $BB'//AC$, draw a straight line $M_1 M_2$ pararell to the y axis through the middle pt, M_1 of AC. Take $\overline{BM_2}=\overline{B'M_2}$, then B' is the fourth point lying on the regular parabola ABC. By this method, we can easily find out the fifth, sixth, points of the curve, and join these points by free hand. This curve is the substituting parabola in the neighborhood of x_0. Draw the straight line $y=0$, then the intersection with the parabola will give the nearly cornect value of x of the root.

Methode II

Compute $y_0=f(x_0)$, $y_1=f(x_0-h)$, $y_2=f(x_0+h)$ and plot $A(y_1, x_0-h)$ $B(y_0, x_0)$ and $C(y_2, x_0+h)$. Draw PQ//to Y axis, and CC'//to Y axis. Connect CB, and intersect PQ at R, draw RS//to AC intersecting CC' at S. Connect BS and intersect PQ at T, then T is the fourth point on the regular parabola ABC.

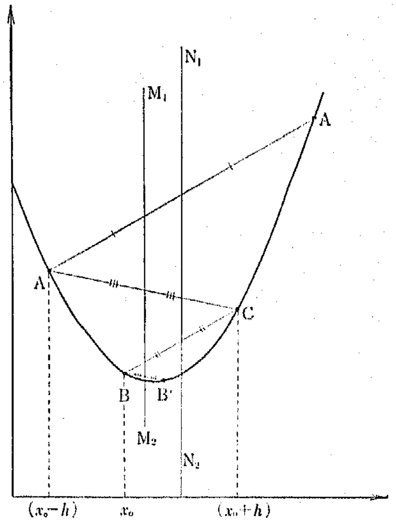

Fig. 1

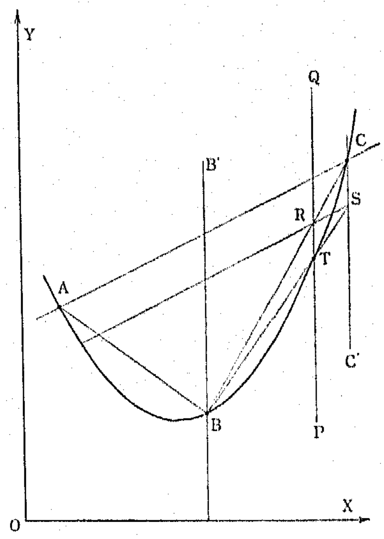

Fig. 2

Shift the PQ line parallel to y-axis along x-axis, make by the similar process points T_1, T_2, ⋯⋯⋯⋯, then all these points T_1, T_2, ⋯⋯⋯ lie on the same parabola. By joinning these points, we can easily draw the substitute curve for $y=f(x)$ by free hand. Draw st. line $y=o$, then its intersection with the parabola gives nearly correct value of reqiured x. The geometrical proof is simple, so that it is omited here.

The analytical meaning of the new process is as follows: If the function $y=f(x)$ may be expanded according to Taylor's theorem,

$$y=f(x)=y_0+f'(x_0)\Delta x+\frac{1}{2}f''(x_0)\Delta x^2+\cdots\cdots$$

and if we omit further terms higher than Δx^3, the above equation is practically the substitued curve (regular parabola) $y=a+bx+cx^2$ which we have just now corsidered. This parabola is tangential to the original curve at $x=x_0$. Thus the analytical meaning of the new process will be clear.

§ 3. Calculation of Equilibrium of Chemical Reactions.

Determinations of chemical equilibrium conditions require often solution of ligher algebraic equations. In such cases the new process is very convenient. By the new process, the probrems are solved within 15–20 minites in most cases.

Example I

Equilibrium constant K_p of the following reaction are given as (1).

$$\log K_p=\frac{4930}{T}-4.66 \tag{1}$$

It is required to find the extent of reaction progress of the reaclion at 600°C.

$$SO_2\,(\text{gas})+\frac{1}{2}O_2\,(\text{air})=SO_3\,(\text{gas}) \tag{2}$$

$$\log K_p \text{ at } 600°C=1.0 \qquad \therefore\ K_p=10.0 \tag{3}$$

Solution:

In the equilibrium state, we have

$$K_p=\frac{p_{SO_3}}{p_{SO_2}(p_{O_2})^{\frac{1}{2}}}=10.0 \tag{4}$$

$$\left.\begin{array}{llll}\text{We have, Initial concentraltion of} & SO_2 = & 6.0 & \text{vol\%} \\ \quad // \quad\quad // \quad\quad // & O_2 = & 8.0 & // \\ \quad // \quad\quad // \quad\quad // & N_2 = & 86.0 & // \\ \hline & \text{Total}= & 100.0 & // \end{array}\right\} \tag{5}$$

Reaction degree of conversion ⋯⋯⋯x%

$$\left.\begin{array}{ll}\therefore\ \text{Final concentration of} & SO_2=6.0-6x \text{ vol} \\ \quad // \quad\quad // \quad\quad // & O_2\ =8.0-3x\ // \\ \quad // \quad\quad // \quad\quad // & N_2\ =86 \quad\quad // \\ & SO_3=6x \quad\quad // \\ \hline & \text{Total}=(100-3x) \end{array}\right\} \tag{6}$$

$\therefore$ partial press. of components at equilibrium are:

$$p(\mathrm{so_3})=\frac{6x}{(100-3x)},\quad p(\mathrm{so_2})=\frac{(6-6x)}{(100-3x)},\quad p(\mathrm{o_2})=\frac{(8-3x)}{(100-3x)} \tag{7}$$

$$\therefore\ K_p=10=\frac{\dfrac{6x}{100-3x}}{\dfrac{(6-6x)}{100-3x}\times\left(\dfrac{8-3x}{100-3x}\right)^{\frac{1}{2}}} \tag{8}$$

Re-writing, wehave to solve

$$x^3-4.38x^2+6.40x-2.70=0 \tag{9}$$

Mathematically, x may take any value form $-\infty$ to $+\infty$, but for engineering purpose, x can not exeed 1.0 and can not become negative, and may perhaps take values of 0.60 or near-by value.

We compute $y_0=f(1.0)$, $y_1=f(0.80)$ and $y_2=f(0.60)$,

So we have
$$f(1.0)=+0.3200$$
$$f(0.80)=+0.1288$$
$$f(0.60)=-0.2208$$

On applying author's 2nd method, we have $x \fallingdotseq 0.714$ (10) (cf. fig 3)

And we have
$$\left.\begin{aligned} f(0.714)&=+0.000688\\ f(0.713)&=-0.000989\\ f(0.7135)&=-0.000149\end{aligned}\right\} \tag{11}$$

Therefore it is clear that x lies between 0.714 and 0.713, i.e. our first result x_1 is sufficiently accurate for ordinary purposes. Now we repeat the process again and we get as $x_2=0.713585$. We think this is a very exact value for many purposes. We see that the operation needs very little labor and time.

The new methode requires far less time and labor, when compared with the famous Newton's Methode. In the Newton's Method, the *tangential straight lines* are used as the substitute curves. In the new methode, the *tangeutial parabolas* are used as the substitute curves, so that much more accurate values are obtained by much less operations when compared with the former method.

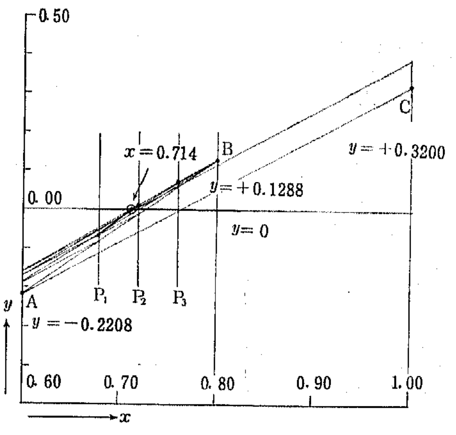

Fig. 3

§ 4. Problems in Fluid Transportations

Trial methodes are very often used in the problems of fluid transpotations. They are used e.g. in the determination ot required pipe diameters d for a given fluid flow rate Q for a given fluid head H_f and pipe length l. In this case we have

$$Q=u\times\frac{1}{4}\pi d^2 \tag{1}$$

$$H=\frac{u^2}{2g}\left(1+\lambda\frac{l}{d}+Zk\right) \tag{2}$$

$$\lambda=\frac{1}{\left\{2\log\left(\frac{r}{\varepsilon}\right)+1.74\right\}^2} \tag{3}$$

where we give

Q: fluid flow rate (cc/sec)
u: fluid velocity (cm/sec)
d: pipe diameter (cm)
H: fouid head (cm)
λ: friction coefficient of the pipe
l: pipe length (cm)
k: Coeff of resistances of pipe fittings

We have to find d. But since λ and k' s are generally functions of d, u and Reynold value $\left(\frac{du\rho}{\eta}\right)$, it is impossssible to find d by an ordinary process.

In the *clasical methode* of solution of the above proplems, we at first assūme λ_0, and calculate d_0 and therefore u_0. We then, check back λ_1 using d_0 and u_0. In generall, λ_0 and λ_1 do not coinside. We therefore, again calculate d_1 and u_1 using λ_1 and check back resultant λ_2. We repeat this procedure till λ_n and λ_{n+1}, coincide and d_n and d_{n+1} and u_n and u_{n+1} coincide to each other.

In the *author's method*:- H, Q, u, λ and k's are after all complicated function of d, $\therefore$ $Q=f(d)$. Mathemathically, d may tak eany value from $-\infty$ to $+\infty$, but in practicall problem, the provable range of d are generally known. Assume a nearby value of d_0, then u_0, λ_0 and k_0's are all calculated using d_0, and consequently Q_0 is determined. Asume, again, $d_1=d_0+\Delta d$, $d_2=d_0-\Delta d$. So we can calculate u_1, u_2, λ_1, λ_2, k_1's, and k_2's, and consequently Q_1, and Q_2. Now, however complicated may the function $Q=f(d)$ be, we can substitute the curve with the regular parabola in the range $Q_1=f(d_1)\sim Q_0=f(d_0)\sim Q_2=f(d_2)$. We draw the curve according to the methode II, and cut the curve with the st. line $Q=Q$, then the intersection will give the value d_3, which is very nearly correct. If nescessary, we repeat the procedure in the neighborhood of d_3.

Example I.

The level of the water head tank is kept at +20.0m constant. It is required to draw 12m³/h of water to a tank at level ±0.0m. The distance is 300m, find the vescesary diameter of the pipe. The roughness of pipe inside is assumed to be 0.1cm. The resistance of the pipe fittings are neglected.

Solution:

$$H_f=\frac{v^2}{2g}\left(0.5+1.0+\lambda\frac{l}{d}\right) \tag{1}$$

$$\lambda=\frac{1}{\left\{2\log\left(\frac{r}{\varepsilon}\right)+1.74\right\}^2} \tag{2}$$

$$Q=\frac{1}{4}\pi d^2\cdot v\times 3600\times 10^{-6}(m^3/h) \tag{3}$$

where we give

H_f =total frictional head loss (cm)
v =mean water velocity (cm/sec)
λ =coefficient of friction of pipe
L =leugth of pipe (cm)
d =$2r$ pipe diameter (cm)
ε =roughness of pipe inside (cm)

As λ is a function of d, this problem can not be solved by an ordinary calculation.

Assume $d_0=6.0\,\text{cm}\,\phi$, then $\lambda=\frac{1}{(4.6942)^2}=0.0455$, $\frac{l}{d}=\frac{30000}{6}=5000$

$$\therefore\ 2000=\frac{v_0^2}{2\times 980}(1.50=0.0455\times 5000)$$

$$\therefore\ v_0=131\,\text{cm/sec}$$

$$\therefore\ Q=\frac{1}{4}\times\pi\times(6.0)^2\times 131\times 3600^{\text{sec}}=13.35(\text{m}^3/\text{h})$$

By similar calculations, we have

d_0 $=6.0\,\text{cm}\,\phi$, then $Q_0=13.35\,\text{m}^3/\text{h}$
$d_1=d+1.0\,\text{cm}=7.0\,\text{cm}\,\phi$, then $Q_1=20.20\,\text{m}^3/\text{h}$
$d_2=d-1.0\,\text{cm}=5.0\,\text{cm}\,\phi$, then $Q_2=8.20\,\text{m}^3/\text{h}$

Then, by the authors 2nd method, we get for $Q=12.0\text{m}^3/\text{h}$, the pipe diameter $d=57.7\,\text{m/m}\,\phi$. (cf. fig 4)

Example 2.

Frictional coefficient λ of fluid transport is given in the theoritical formula as a function of Reynold value (Re) as follows.

$$\lambda=\frac{1}{[2\log(Re\cdot\sqrt{\lambda})-0.80]^2} \tag{1}$$

Find the actual value of λ when (Re) is 2500

Solution:-

This example is impossible to solve it in the classical method. In the author's method, we proceed as follows:- First, we put by rewiting,

$$\lambda\times\{\overline{2\log(Re)-0.80}+\log\lambda\}^2=f(\lambda)=1.0 \tag{2}$$

In most cases, λ is between $0.02\sim0.04$, therefore $\therefore$ we take $\lambda_0=0.04$

Rewriting we have

$$f(\lambda_0)=0.04\times(4.602)^2=0.845 \tag{3}$$

For $\lambda_1=0.05$, $\log(\lambda_1)=\bar{2}.6990=-1.3001$

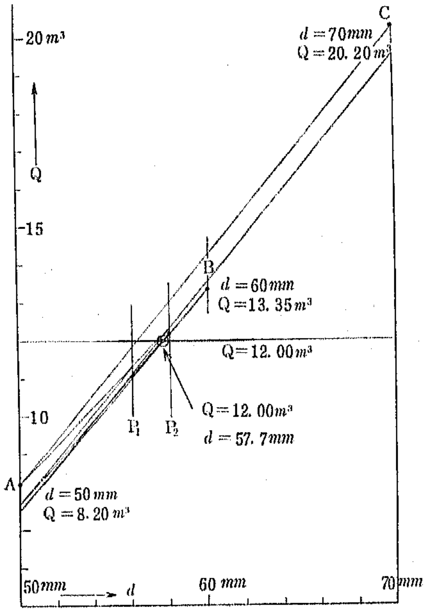

Fig. 4

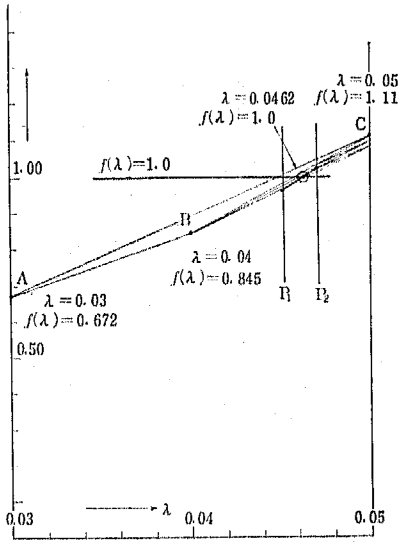

Fig. 5

$\therefore\ f(\lambda_1)=0.05\times[6.00-1.300]^2=0.05\times4.7=1.11$ (4)

For $\lambda_2=0.03,\ \log\lambda_2=2.4771=-1.5229=-1.523$

$\therefore\ f(\lambda_2)=0.03\times[6.00-1.523]^2=0.03\times4.477=0.672$ (5)

$\therefore$ By the awhors 2nd graplincal method, we have

$\lambda_3=0.0462$ (6) (cf. Fig 5)

§ 5. Problems in Distillation

In the calculations of problems of ditillations considering two compnent system, we have often to solve a very complicated symultenous equations i e.

$$\left.\begin{aligned}\ln\left(\frac{L_0}{L}\right)&=\int_x^{x_0}\frac{1}{(y-x)}dx \\ f(x,y)&=0\end{aligned}\right\}\qquad\begin{aligned}&(1)\\&(2)\end{aligned}$$

where we give by

L_0······initial amount of kettle liquid (mol)
L ······final // (mol)
y ······composition of vapor phase in mol fraction
x ······ // // liquid phase in mol fraction
x ······final composition of liqwid phase in mol froclion
$f(x, y)=0$······equilirium condition of vapor-and liquid phase.

We will explain by an example.

Example: 100 mol of liquid is charged into the kettle. Its composition is

80 mol% of x. Simple distillation is carried out till the kettle-liquid is 40 mol. It is required to find the final composition of the kettle-liquid, amount of total distillate and its composition. Vapor-liquid phase relation of the mixture is given as follows by experiments.

x (mol%) (lauid phase)	y (mol%) (vapor phase)	x	y	x	y
0.00	0.000	0.35	0.618	0.70	0.875
0.05	0.136	0.40	0.667	0.75	0.900
0.10	0.250	0.45	0.711	0.80	0.924
0.15	0.346	0.50	0.750	0.85	0.944
0.20	0.429	0.55	0.787	0.90	0.964
0.30	0.562	0.65	0.848	1.00	1.000

In the clasical method, only "cut and try" system has been possible. In the authors method, we assume an appropriate value of $x=x_1$ and carry out the integration of formula (1) by Simpson formtula, and find out cornesponding value of $L=L_1$. We further assume $x_2=x_1+\Delta x$, $x_3=x_1-\Delta x$, and calculate cornesponding values of $L=L_2$, $L=L_3$, From thes values, we find the correct value $L=L_4$ by our seconde methode. In this case we assume at first $x_1=0.60$.

From the table, we calcalate $y-x$, and $\frac{1}{y-x}$ as follows

x	y	$y-x$	$\frac{1}{y-x}$
0.50	0.750	0.250	4.00
0.55	0.787	0.237	4.22
0.60	0.818	0.218	4.58
0.65	0.848	0.198	5.03
0.70	0.875	0.175	5.72
0.75	0.900	0.150	6.67
0.80	0.924	0.124	8.06

$$\therefore \int_{0.60}^{0.80}\frac{1}{(y-x)}dx=\frac{1}{3}\times 0.05\times[8.06+4\times 6.67+2\times 5.72+4\times 5.03+4.58]$$

$$=\frac{1}{3}\times 0.05\times[70.88]=1.1815=J_1,$$

$$\therefore \ln\left(\frac{100}{L_1}\right)=1.1815,\quad \log\left(\frac{100}{L}\right)=1.815\times\frac{1}{2.30}=0.5137$$

$$\log 100-\log L_1=0.5137$$

$$\log L_1=2-0.5137=1.4863$$

$$\therefore L_1=30.64 \tag{3}$$

For $x_2=70\%$

$$\int_{0.70}^{0.80}\frac{1}{(y-x)}dx=\frac{1}{3}\times 0.05\times[8.06+4\times 6.67+5.72]$$

$$=\frac{1}{3}\times\frac{1}{20}\times 40.46=0.6743=J_2$$

$$\therefore\ \ln\left(\frac{100}{L_2}\right)=0.6743$$

$$\therefore\ \log\left(\frac{100}{L_2}\right)=\frac{0.6743}{2.30}=0.2931$$

$$\therefore\ \log 100-\log L_2=0.2931$$

$$\therefore\ \log L_2=2-0.2931=1.7069$$

$$L_2=50.82=50.93$$

$$L_2=50.93\,\text{mol} \tag{4}$$

For $x_3=50\%$

$$\int_{0.50}^{0.80}\frac{1}{(y-x)}dx=\int_{0.60}^{0.80}\frac{1}{(y-x)}dx+\int_{0.50}^{0.60}\frac{1}{(u-x)}dx=$$

$$J_1+\frac{1}{3}\times 0.05\times[4.58+4\times 4.22+4.00]=1.1813+\frac{1}{3}\times\frac{1}{20}25.46$$

$$=1.1813=0.4243=1.6058$$

$$\therefore\ \ln\left(\frac{100}{L}\right)=1.6056$$

$$\therefore\ \log\left(\frac{100}{L_4}\right)=\frac{1.6058}{2.30}=0.6982$$

$$\log 100-\log L_4=0.6982$$

$$\therefore\ \log L_3=2-0.6982=1.3018$$

$$\therefore\ L_3=20.04 \tag{5}$$

We have therefore

X	L	$D=(100-L)$
50	20.04	79.96
60	30.65	69.35
70	50.93	49.07

$$\therefore\ x_4=0.652 \tag{6}$$

by the author's 2nd method.
(cf. fig. 6)

Now on the other hand, we have

$$\therefore\ x_0L_0=x_4L_4+xd\,(100-L_4) \tag{7}$$

$$\therefore\ x_0=0.80,\ L_0=100,\ \text{and}\ x=0.652,$$

$$100-L_4=60=D \tag{8}$$

$$\therefore\ xd=\frac{0.80\times 100-0.652\times 40}{0.60\times 100}$$

$$=\frac{80-26.08}{60}=\frac{53.92}{60}=0.8987$$

$$\therefore\ xd=89.87\% \tag{9}$$

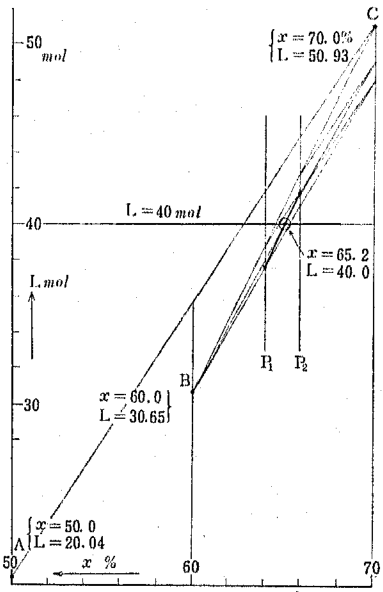

Fig. 6

§ 6. Problems in the Heat Transfer

In problems of heat transfer, we

have often to treat the counter-current heat exchangers. In heat exchangers, we have the following relation

$$Q=AU(\Delta\theta)_m=AU\frac{\Delta_1-\Delta_2}{\ln\left(\frac{\Delta_1}{\Delta_2}\right)} \tag{1}$$

where we denote by

Q: heat exchanged per hour (kcal/hour)
A: heat exchanging area (m^2)
$(\Delta\theta)_m$: mean temperature difference (C°)
Δ_1, Δ_2: temperature difference at the entrance and the exit of the exchanger
U: overall coefficient of heat trausfer (kcal/m²h. C°).

But since U is not a constant, but a function of h (film coefficient of heat trausfer kcal/m². h. C°), thickness of wall t and its thermal conductivity λ(kcal/m. h. C°), and as the film coefficients of heat transfer h are again a function of fluid velocity, fluid viscoscty, pipe diameter and other factors, the solution become a little complicated.

$$\left.\begin{aligned} \left(\frac{hD}{\lambda_f}\right)&=0.023\left(\frac{Du\rho}{\eta_f}\right)^{0.8}\left(\frac{C_p\eta_f}{\lambda_f}\right)^{\frac{1}{3}} && (2)\\ \frac{1}{U}&=\frac{1}{h_1}+\frac{t}{\lambda_w}+\frac{1}{h_2} && (3)\end{aligned}\right\}$$

where we denote by

h_1, h_2: film coefficients of hest transfer of inside and out side fluids (kcal/m². h. C°)
t: thickness of the wall (m) or (cm)
λ: Thermal conductivity of wall material (kcal/m. h. C°)
D: pipe diameter (cm) or (m)
u: fluid velocity (cm/sec)
ρ: deusity of the fluid (g/cm³)
η_f: viscosity of the fluid at wall temperature (pois)
C_p: specific heat capacity of the fluid (kcal/kg mol) or (cal/g. C°)
λ_f: thermal conductivity of the fluid (kcal/m. h. C°) or (cal/cm. sec. C°)

The solution is not so simple, and requires a solution of a very complicated equation, which has been unable to solve by a classical methode.

Example:- A counter-current heat-exchanger of heating area 10.0m², is charged with hot water of 100C° and 6.0m³/h. The hot water is cooled to 40C° with cold water of 20C°. Find the quantity of cold water per hour and its final temperature.

Solution:- We first assume the amount of cold water to be q_0 (m³/h). Then, the final temperature of cooling water, its viscosity, its velocity, its Reynold value, mean temperature difference $(\Delta\theta)_m$ and U are determind, and consequently possible heat-transfer Q_0 will be given. If our assumed value of q_0 be just correct, then Q_0 should be equall to the required heat quantity $Q=6000^l\times(100°-40°)=360000$ kcal/h. If they do not agree, we assume

$q_1=q_0+\Delta q$, and $q_2=q_0-\Delta q$, and proceed as before according the authors seconde graphical method, and can find the value q_3 when Q is just 360000 (kcal/h).

§ 7. Problems in the Gas Absorption of Packing Tower.

In the counter current gas absorption of a liquid in a packing tower, we have

$$\int_{y_2}^{y_1}\frac{dy}{(y-y_i)}=\frac{1}{G}k_G\cdot a\cdot s\cdot H \tag{1}$$

$$\int_{x_1}^{x_2}\frac{dx}{(x-x_i)}=\frac{1}{L}k_L\cdot a\cdot s\cdot H \tag{2}$$

wher we denota by

G=velocity of carrier gas (mol/h)
L=velocity of solvent liquid (mol/h)
x=concentration of solution (mol/mol)
y=concentration of gas (mol/mol)
H=height of tower (m)
S=sectional area of tower (m^2)
A=total absorbing area (m^2)
a=absorbing area per vol. (m^2/m^3)
x_1, y_1=concentrations in the bottom of the tower (mol%)
x_2, y_2=concentrations in the top of the tower (mol%)
k_G=gas absorption constant
k_L=liquid absorption constant
$y_i=f(x_i)$=equilibrium relation of $x\sim y$

We have

$$\int_{y_2}^{y_1}\frac{dy}{(y-y_i)}=\frac{1}{G}\cdot k_G\cdot a\cdot s\cdot H \tag{1}$$

and

$$\int_{x_1}^{x_2}\frac{dx}{(x-x_i)}=\frac{1}{L}\cdot k_L\cdot a\cdot s\cdot H \tag{2}$$

We can calculate H from k_G, k_L, a, G, L, x_1, y_1, x_2, y_2, but its is not easy to calculate k_G and k_L form H, a, G, L, x_1, y_1, x_2, y_2. In the latter case, "cut and try" was the only methode.

In the author'ts method, we at first assume the ratio ($\gamma=k_G/k_L$), then the left hand side of eqn, (1) may be calculated by graqhical method and Simpson formula. In the right hand side of eqn. (1), all value except k_G is known, so that we can find the value of k_G, therefore $k_L=(k_G/\gamma_0)$. The left hand side of the eqn. (2) may be calculated since γ_0 is assumed. In the right hand side of eqn. (2), all values of L. k_L, a, and S are allready known, therefore we can chech back H to be H'_0. If H and H'_0 do coinside, our assumption was correct-i. e. all γ_0, k_{G-O}, k_{L-O}, are correct. It not, we further assume $\gamma_1=\gamma_0+\Delta\gamma$ and $\gamma_2=\gamma_0-\Delta\gamma$ and proceed to check back H'_1 and H'_2. We plott H'_0, H'_1 and H'_2 against γ_0, γ_1 and γ_2, and by using the author's 2nd Method, we can find out the correct velue of γ and so find out k_G and k_L.

§ 8. Problems of Drying

In the problems of the falling rate drying, the solution of a complicated equation becomes often nescary. For example in the falling rate drying of a cloth in a batch dryer, we have

$$A_0 \cdot k \cdot f_v \cdot dZ = M \cdot \frac{1}{f_w} \cdot \frac{1}{(H_w - H)} \cdot dW \tag{1}$$

and

$$\int_0^Z A_0 \cdot k \cdot f_v \cdot dZ = M \int_W^{W_0} \left(\frac{1}{f_w}\right) \cdot \frac{1}{(H_w - H)} dW \tag{2}$$

Where we denote by

A_0	total drying area (m^2)
K	drying constant [63.0 kg water/m^2·h. (kg water/kg dry air)]
f	wind factor
Z	time (h)
M	wt. of dry goods (kg)
W, W_0	water content of good at $Z=Z$, and $Z=Z_0$ (dry basis %)
fw	falling rate factor, which is a function of W, and given in form of experimental data
H	humidity of drying air (kg water/kg dry air)
H_w	humidity of saturated air (kg water/kg dry air)
$(H_w-H)_m$	log. meanof $(H_w-H.)$

If for example, we wish to find the drying time wanted to dry the goods from $W_0=100\%$ to $W=13\%$, we first calculate the time to dry to $W_1=20\%$, $W_2=15\%$ and $W_3=10\%$, which is resp. Z_1, Z_2 and Z_3 hours by using eqn (2), and by applying the authors second graphical mithod, find the uescesary time Z.

Example:-

A material is dried in a batch system dryer. The dry weight is 90 kg, and the drying surface is 90 m^2. The water content W_0 is 100% (dry basis). In the dryer a hot air currint of 150(m^3/min) is circulated with the wind velocety of (2m/sec). The temperature of the current T_1 is 80°C and its humidity H_1 is 0.007 (kg aq/kg dry air). The falling rate factor fw is a function of W and is giben as follows

$W\%$	100~30	25%	20	15	10
fw	1.00	0.60	0.20	0.10	0.05

It is requred to find the time nescesary to dry the goods from $W=100\%$ to 13%.

Solution:

We have by reduction,

$$\frac{1}{G} \cdot A_0 \cdot k \cdot f_v \cdot f_w \cdot = \ln \frac{(H_w - H_1)}{(H_w - H)} \tag{3}$$

and in this case we have

G (wind quantity kg/h) $= 1.25^{(kg/m^3)} \times 150^{(m^3/min)} \times 60^{(min)}$
$= (1.25 \times 9000)$(kg/h)

A_0 (drying area m^2)$=90$(m^2)
M (goods dry weight kg)$=90$(kg)
k (Evaporation constant)$=63$kg aq/m^2. h. (kg aq/kg dry air)
f_v (windfactor)$=1.30$
f_w (falling rate factor) as given in the table above.
H_1 humidity of blown-on air$=0.007$ (kg aq/kg dry air)
H humidity of blown-off air (kg ap/kg dry air)
H_w humidity of drying air at saturated temp. (kg aq/kg dry air)
$(H_w-H_1)=0.0155$ (kg aq/kg dry air)

we have

$W\%$	100~30	25	20	15	10%
f_w	0.655		0.1310	0.0655	0.0328

From ln $\left(\frac{H_w-H_1}{H_w-H}\right)$, we find log $\left(\frac{H_w-H_1}{H_w-H}\right)$, and then $\frac{H_w-H_1}{H_w-H}$, and thin H_w-H, and then $(H_w-H_1)-(H_w-H)$, and then $(H_w-H)_m$ which is logalithmic mean of (H_w-H_1) and (H_w-H).

$(H_w-H)_m$	0.01075	0.01285	0.0145	0.0155	0.0153
$\frac{1}{(H_w-H)_m}$	93.0	78.0	69.0	65.5	65.7
$\frac{1}{(H_w-H)_m}\cdot\frac{1}{f_w}$	93.0	130.0	344.0	655.0	1317

$$\int_{0.30}^{1.00}\frac{1}{(H_w-H)_m}\cdot\frac{1}{f_w}\cdot dW=93.0\times(1.00-0.30)=65.1 \qquad J_0$$

$$\int_{0.20}^{0.30}\frac{1}{(H_w-H)_m}\cdot\frac{1}{f_w}\cdot dW=\frac{1}{3}\times0.05\times[93.0+4\times130.0+344.0]$$
$$=\frac{957}{60}=15.95 \qquad J_1$$

$$\int_{0.15}^{0.30}\frac{1}{(H_w-H)}\cdot\frac{1}{f_w}\cdot dW=\frac{3}{8}\times0.05\times[93.0+3\times130.0+3\times344+655.0]$$
$$=40.6 \qquad J_2$$

$$\int_{0.10}^{0.20}\frac{1}{(H_w-H)}\cdot\frac{1}{f_w}\cdot dW=\frac{1}{3}\times0.05\times[344.0+4\times655.0+1317.0]$$
$$=\frac{1}{60}\times4281=71.35 \qquad J_3$$

Time wanted to dry from $W=100\%$ to $W=30\%$ is

$$Z_0=\frac{M\times J_0}{A_0k.f_v}=\frac{90\times65.1}{90\times63\times1.30}=0.795 \text{ hours}$$

similarly

$$Z_1=W_{100}\rightarrow W_{20}=\frac{90\times(65.1+15.95)}{90\times63\times1.30}=0.991 \text{ hours}$$

$$Z_2=W_{100}\rightarrow W_{15}=\frac{90\times(65.1+40.6)}{90\times63\times1.30}=1.291 \text{ hours}$$

$$Z_3=W_{100}\rightarrow W_{10}=\frac{90\times(65.1+15.95+71.25)}{90\times63\times1.30}=1.880 \text{ hours}$$

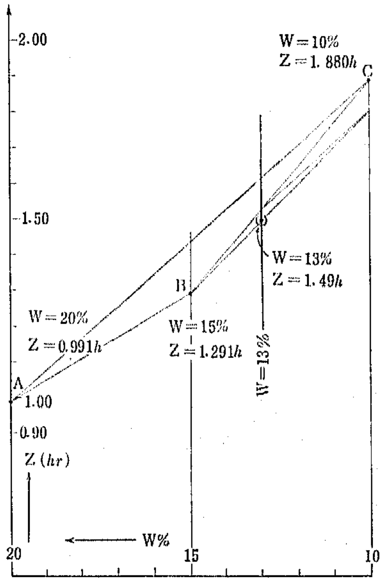

Fig. 7

By the authors second graphic method, we has

Z_4=time needed for $W_{100} \rightarrow W_{13}$
$=1.490$ hours (cf. fig. 7)

§9. A novel method to estimate the activation energy bassed on the cumulative reaction (method A).

Classical method of estimating activation energy consists, as is well known, in finding velocity constants k_1 and k_2 at two constant temperatures T_1 and T_2 respectively, and solving the simultaneous equation,

$$\left.\begin{aligned}\ln k_1 &= \ln A + \frac{-E}{RT_2} \\ \ln k_1 &= \ln A + \frac{-E}{RT_1}\end{aligned}\right\} \qquad (9\text{-}1)$$

We have found quite a new method of estimating activation energy based on cumulative reaction. This method may be very useful in case when the the experimental procedures (for velocity constants at constant temperatures) are difficult to carry out.

A) Principle of the new method.

Authors' new method consists in carrying out at least two experiments in which two known kinds of temperature-changes were taken. (Consider, for example, a first order reaction).

$$\ln\frac{a}{a-x_1} = \int_{\mathrm{I}} k dZ = \int Ae^{-\frac{E}{RT}} dZ = A\int_{\mathrm{I}} e^{-\frac{E}{RT}} dZ \qquad (9\text{-}2)$$

$$\ln\frac{a}{a-x_2} = \int_{\mathrm{II}} k dZ = \int_{\mathrm{II}} Ae^{-\frac{E}{RT}} dZ = A\int_{\mathrm{II}} e^{-\frac{E}{RT}} dZ \qquad (9\text{-}3)$$

Dividing each side of (9-2) by the same side of (9-3) respectively, we obtain:—

$$\frac{\ln\frac{a}{a-x_1}}{\ln\frac{a}{a-x_2}} = \frac{\int_{\mathrm{I}} e^{-\frac{E}{RT}} dZ}{\int_{\mathrm{II}} e^{-\frac{E}{RT}} dZ} = \phi(E) = C, \text{ say} \qquad (9\text{-}4)$$

The left side of equation (9-4) is a known value. The middle side of the equation (9-4) is also a known value if E and temperature-time change are known. Thus equation (9-4) is a function of E. It is, however, rather difficult to solve the equation $\phi(E)=C$. for E by ordinary mathematical methods. The author devised a method to simplify such a solution. This method was particularly dercribed in §2. By this method the complicated equation can be easily solved within ten to thirty minutes. By solving this equation, a noval method to find the acitivation evergy was established.

B) Experimental method.

Hydrolysis of ethylacetate was tested. The temperature was not kept constant *i. e.* it changed with the time, and was measured every five minutes and samples were taken out at 20–30 minutes interval and titrated, and reaction rates ln [$a/(a-x)$] were calculated. One the other hand, the value of E was assumed as E=14,000 cal., E=15,500 cal. and E=18,500 cal. and the equation was solved according to the aforementioned principle and method.

C) Experimental results.

Experimental results obtained are given in Table (9–1).

We repeated similar experiments several times, and their result are given in Table (9–2).

Table. (9—1) (The example of experiment D)

Combination	$\phi(E)=\dfrac{\int_0^{Z_1} e^{-E/RT}dZ}{\int_0^{Z_1} e^{-E/RT}dZ}$ $E_1=14000$	$E_2=15500$	$E_3=18500$	$C=\dfrac{\ln\frac{V_\infty-V_0}{V_\infty-V_{Z_1}}}{\ln\frac{V_\infty-V_0}{V_\infty-V_{Z_2}}}$	E (cal/mol) solved by $\phi(E)=C$
II/I	1.583	1.593	1.613	1.578	13200
III/I	2.269	2.304	2.371	2.273	14300
IV/I	3.076	3.150	3.313	3.255	17500
III/II	1.432	1.445	1.470	1.440	15000
IV/II	1.942	1.975	2.053	2.062	18800
IV/III	1.355	1.367	1.391	1.431	21600

mean : E=16700

∴ frequency factor : $A=10^{9.0060}$

Note : I, II, III and IV denote reacrtions of 40min., 60 min., 80 min., and 100 min. respectively, for example, III/I means

$$\phi(E)=\frac{\int_0^{80} \mathrm{k}dZ}{\int_0^{40} \mathrm{k}dZ}$$

Table (9—2)

Experimental number		E (cal/mol)	A (min^{-1})	Relative difference of experimental & calculated values: By using values of new method A	By using values of classical method
Values by new method A	exp. 1	16700	$10^{9.0060}$	0.8%	2.2%
	exp. 2	16800	$10^{9.0816}$	1.6%	1.0%
	exp. 3	18100	$10^{10.0286}$	0.9%	1.5%
	exp. 4	15600	$10^{8.2057}$	3.7%	4.6%
	exp. 5	17000	$10^{9.2164}$	1.2%	2.3%
Mean of 1∼5		16800	$10^{9.1076}$		
Values by classical method		16720	$10^{9.0313}$		

These results prove that our novel method gives better results as a whole than the ordinary classical method.

§ 10. Another novel method to estimate the activation energy based on the cumulative reaction (method B).

Another novel method to estimate activation energy based on the cumulative reaction was deviced by the author. This method (method B) may be simpler in some cases in calculations than that given in § 9.

A) Principle of the method B.

Let us consider, for example, an unimolecular reaction. We have,

$$\ln\frac{a}{a-x}=\int_0^Z kdZ=\int_0^Z Ae^{-\frac{E}{RT}}\,dZ=A\int_0^Z\left(e^{-\frac{E}{RT}}\right)dZ \qquad (10\text{-}1)$$

therefore

$$\left.\begin{aligned}\log\left[\ln\frac{a}{a-x_1}\right]&=\log A+\log\left[\int_0^{Z_1}\left(e^{-\frac{E}{RT}}\right)dZ\right.\\ \log\left[\ln\frac{a}{a-x_2}\right]&=\log A+\log\left[\int_0^{Z_2}\left(e^{-\frac{E}{RT}}\right)dZ\right.\end{aligned}\right\} \qquad (10\text{-}2)$$

If our experiments have no experimental error, and also if the assumed value of E be correct, the plots of $\log\left[\ln\frac{a}{a-x}\right]$ aginst $\log\left[\int_0^Z\left[e^{-\frac{E}{RT}}\right)dZ\right]$ should be a straight line and the inclination of the straight line should be just 45° ($\tan\phi=1.00$) paticularly. Adopt E_1, E_2, and E_3, as apporoximate value of E, in eqn. (10–2), and plot the straight lines, and calculate, by least square mean method, their inclinations ϕ_1, ϕ_2 and ϕ_3 respectively as in Fig. 8.

Then plot $\tan\phi$ against E as in Fig. 9. and draw curve P_1-P_2-P_3. This curve is very nearly a straight line. Draw a horizontal line of $\tan\phi=1.00$, and let it intersect P_1-P_2-P_3 at P_4, then P_4 gives the correct value of E. Substituting value of E in eqn. (10–2), we get several values of log A. By taking their mean, we get corresponding value of A.

B) Experiments:

Cumulative hydrolysis of sugar was taken as a sample. Thus the activation energy and frequency factor were found to be 20.15 Kcal/mol and $10^{12.8481}$ min^{-1}. respectively. Experimental and calculated values by these data are shown in table (10–1). They agree very good to each other. The agreements are even better than in Table of § 9.

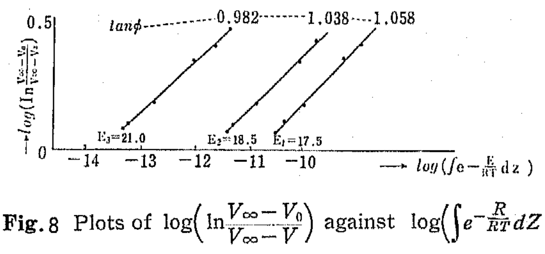

Fig. 8 Plots of $\log\left(\ln\frac{V_\infty-V_0}{V_\infty-V}\right)$ against $\log\left(\int e^{-\frac{R}{RT}}dZ\right)$

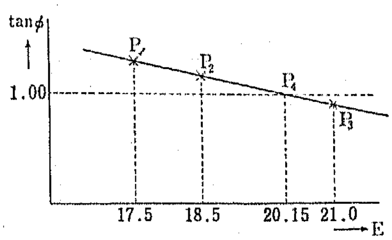

Fig. 9 Plots of tan ϕ against E.

Table (10—1) Experimental & Calculated values of Expt. of § 10 by adopting values of E and A obtained by Method B.

time [min]	titration value [ml]	$\ln\frac{V_\infty - V_0}{V_\infty - V_Z}\times 10^2$ (experimt.)	$\int kdx \times 10^2$ (calculated)	relative difference in %
0	0.71	—	—	—
40	3.04	73.69	72.93	−1.0
50	3.65	107.25	101.93	−4.9
60	4.02	134.91	134.95	0
80	4.65	213.26	213.43	+0.8
90	4.84	257.66	259.72	+0.8
∞	5.18	—	—	—

$E=20.15$ [Kcal/mol] $A=10^{12.8481}$ [min^{-1}]

§ 11. Solution of symultenous equations of complicated natures

$$\left.\begin{array}{ll} f(x, y)=0 & (1) \\ \phi(x, y)=0 & (2) \end{array}\right\}$$

The solution of the above symultaneovs equations is reduced to the solution of a equation of 4th order when $f(x, y)=0$ and $\phi(x, y)=0$ are functions of 2nd order with respect to x and y. It will be reduced to a solutution of still higher order when they are equations of higher than 2nd orders concerning x and y. The solution will again become more complicated if they are equations of more complicated natures: *e.g.* equations containing exponential, loglithmic or trigonometric functions etc. In all these cases, the solution of the symultenous equations are very difficult, or impossible by ordinary frocedure, whereas by the authors method, we can solve them within 15～30 minutes in most cases. We can solve them, in several cases, even within 5～10 minutes using slide rules.

The principle of authors process of solution are conveniently divided into several groups according to the properties of the equations.

A) Group A:–When either or both of $f(x_0, y)=0$ and $f(x, y_0)=0$ are soluble, and also either or both of $\phi(x_0, y)=0$ and $\phi(x, y_0)=0$ are soluble equations. Here a soluble equation means an equation of 2nd or lower order with respect to x and y, and an insoluble equation means an equation of 3rd or higher order with respect to x and y, including equations containing exponential, logailithmic, trigonometric or data-functions (functions of experimental data). Such an equation is generally difficult or impossible to solve by an ordinary method. Now concidering the case when $f(x_0, y)=0$ and $\phi(x_0, y)=0$ are soluble equations, we proceed in this case as follows.

A–a:–

We compute

$$f(x_0-h, y_{f1})=0,\quad f(x_0, y_{f2})=0,\quad f(x_0+h, y_{f3})=0 \tag{1}$$

and

$$\phi(x_0-h, y_{\phi1})=0,\quad \phi(x_0, y_{\phi2})=0,\quad \phi(x_0+h, y_{\phi3})=0 \tag{2}$$

These values of y'_{f_s} and $y_{\phi}'_s$ are computable without difficulty since the form of $f(x, y)$ and $\phi(x, y)$ are given, and $f(x_0, y)=0$ and $\phi(x_0, y)=0$ are soluble equations. We then plot the points (x_0-h, y_{f1}), (x_0, y_{f2}) and (x_0+h, y_{f3}), on the co-cordinate. These three points sustisfy the condition of $f(x, y)=0$. We join these three points by substitue parabola by one of the methods described before. We get thus a substitube curve for $f(x, y)=0$ in the range

$$(x_0-h) \text{ to } (x_0+h) \tag{3}$$

Similarty we plot the points $(x_0-h, y_{\phi 1})$, $(x_0, y_{\phi 2})$ and $(x_0+h, y_{\phi 3})$ on the co-ordinate, and join them by a parabola. These three points sutisfy the condition of $\phi(x, y)=0$. We get thus a substitute curve for $\phi(x, y)=0$ in the range

$$(x_0-h) \text{ to } (x_0+h) \tag{4}$$

The intersection of the 2 curves (3) and (4) will give the values of roots of the original symultenous equations.

A–b):–

There is an other procedure of solution.

If our assumed value x_0 was just the cornect value of the root x, then y_{f2} must be the cornect value of y, and also $y_{\phi 2}$ must also be equall to y, and concequently eq*all* to y_{f2}. Therefore $(y_{f2}-y_{\phi 2})$ must be equall to zero. If x_0 is not equall to correct x, then $(y_{f2}-y_{\phi 2})$ would not be equall to zero. $(y_{f2}-y_{\phi 2})$ is a function of (x_0-x) which varies from negative to positive or from positive to negative just at the point when $x_0=x$; and $(y_f-y_\phi) \sim (x_0-x)$ relation can be expressed by a parabola when the range $(x_0-h) \sim (x_0+h)$ is not so large. Therefore, we plot (y_f-y_ϕ) against x on the co-cordinates, and draw a parabola through the three points $[(y_{f1}-y_{\phi 1}), (x_0-h)]$, $[(y_{f2}-y_{\phi 2}), x_0]$ and $[(y_{f3}-y_{\phi 3}), (x_0+h)]$. The point where this substitute parabola cuts the $y=0$ axis, will give the value of correct x. If x is known, we can comput y_f and y_ϕ without difficulty, and comfirm that $y_f=y_\phi=y$.

B) Group B: When the original equations may be reduced to

$$y=F(x) \tag{1$'$}$$

$$x=\Phi(y) \tag{2$'$}$$

In this case, we compute $y_1=F(x_0-h)$, $y_2=F(x_0)$, $y_3=F(x_0+h)$ and $x_1=\Phi(y_0-k)$, $x_2=\Phi(y_0)$, $x_3=\Phi(y_0+k)$, and draw the substitute parabolas for $y=F(x)$ and $x=\Phi(y)$ through the points (y_1, x_0-h), (y_2, x_0) (y_3, x_0+h) and the points (y_0-k, x_1), (y_0, x_2) and (y_0+k, x_3) according to the methods described in the former chapters. The intersection of the two parabolas gives sufficiently exact values of required x and y.

C) Group *C*: Either of the functions $f(x_0, y)=0$ or $f(x, y_0)=0$ (1)

or $\Phi(x_0, y)=0$ or $\phi(x, y_0)=0$ (2)

is unsoluble with respect to y or x respect. Suppose at first, $f(x_0, y)=0$ to be the soluble function and $\phi(x, y)$ is insluble. In this case, we compute three points $f(y_1, x_0-h)=0$, $f(y_2, x_0)=0$ and $f(y_3, x_0+h)=0$ (C–1)

In this case, all values of y_1, y_2, y_3 are computable. We draw the substitute parabola through these three points. This parabola shows the curve of points which satisfies the relation $f(x, y)=0$ (C–2)

We then compute on the unsolved equation $\phi(x, y=)0$, the follourng set of points

$$\left.\begin{array}{l}\Phi(x_0-h, y_0-k)=\phi_1,\ \ \Phi(x_0-h, y_0)=\phi_2,\ \ \Phi(x_0-h, y_0+k)=\phi_3\\ \Phi(x_0, y_0-k)=\phi_4,\ \ \Phi(x_0, y_0)=\phi_5,\ \ \Phi(x_0, y_0+k)=\phi_6\\ \Phi(x_0+h, y_0-k)=\phi_7,\ \ \Phi(x_0+y_0)=\phi_8,\ \ \Phi(x_0+h, y_0+k)=\phi_9\end{array}\right\} \quad \text{(C-3)}$$

respctively. Of cause all these computation of ϕ's are possible since the form of $\phi(x, y)$ is given. By using the process describld before, we can find out

$$\left.\begin{array}{llll}\text{From points} & \phi_1,\ \phi_2,\ \phi_3, & \text{the point} & \Phi(x_0-h, \bar{y}_1)=0\\ \text{〃}\ \ \text{〃} & \phi_4,\ \phi_5,\ \phi_6, & \text{〃}\ \ \text{〃} & \Phi(x_0, \bar{y}_2)=0\\ \text{〃}\ \ \text{〃} & \phi_7,\ \phi_8,\ \phi_9, & \text{〃}\ \ \text{〃} & \Phi(x_0, \bar{y}_3)=0\\ \text{〃}\ \ \text{〃} & \phi_1,\ \phi_4,\ \phi_7, & \text{〃}\ \ \text{〃} & \Phi(\bar{x}_1, y_0-h)=0\\ \text{〃}\ \ \text{〃} & \phi_2,\ \phi_5,\ \phi_8, & \text{〃}\ \ \text{〃} & \Phi(\bar{x}_2, y_0)=0\\ \text{〃}\ \ \text{〃} & \phi_3,\ \phi_6,\ \phi_9, & \text{〃}\ \ \text{〃} & \Phi(\bar{x}_3, y_0+k)=0\end{array}\right\} \quad \text{(C-4)}$$

All these points of (C-4) satisfy the relation $\Phi(x, y)=0$, therefore they are points on the curve $\Phi(x, y)=0$. We plot them on the co-ordinates. The intersection of the curve of (C-1) and (C-4) will give sufficiently exact values of required roots of the symultenous equations

$$\left.\begin{array}{l}f(x, y)=0\\ \phi(x, y)=0\end{array}\right\}$$

If on the contrary, when $f(x_0, y)=0$ is the innoluble equation, and
$\phi(x_0, y)=0$ is the soluble equation,

the procedures are reverse:-
We compute at first $\phi(x_0-h, y_1)=0$ $\phi(x_0, y_2)=0$ and $\phi(x_0+h, y_3)=0$ (C-5)
all these values of y_1, y_2, and y_3 are computable, and all three points (x_0-h, y_1), (x_0, y_2) and (x_0+h, y_3) satisfy the relation $\phi(x, y)=0$, and therefore lie on the curve of $\phi(x, y)=0$, and we draw the substitute parabola through these three points.

We then draw the substitute parabola of insoluble function $f(x, y)=0$. For this purpose we compute the follouring set of points. Of cause all these computations of f' s are possible since the form of $f(x, y)$ is given.

$$\left.\begin{array}{l}f(x_0-h, y_0-k)=f_1,\ \ f(x_0-h, y_0)=f_2,\ \ f(x_0-h, y_0+k)=f_3\\ f(x_0, y_0-k)=f_4,\ \ f(x_0, y_0)=f_5,\ \ f(x_0, y_0+k)=f_6\\ f(x_0+h, y_0-k)=f_7,\ \ f(x_0+h, y_0)=f_8,\ \ f(x_0+h, y_0+k)=f_9\end{array}\right\} \quad \text{(C-6)}$$

By using the process described before, we can find out

$$\left.\begin{array}{llll}\text{from points} & f_1,\ f_2,\ f_3, & \text{the point} & f(x_0-h, \bar{y}_1)=0\\ \text{〃}\ \ \text{〃} & f_4,\ f_5,\ f_6, & \text{〃}\ \ \text{〃} & f(x_0, \bar{y}_2)=0\\ \text{〃}\ \ \text{〃} & f_7,\ f_8,\ f_9, & \text{〃}\ \ \text{〃} & f(x_0+h, \bar{y}_3)=0\\ \text{〃}\ \ \text{〃} & f_1,\ f_4,\ f_7, & \text{〃}\ \ \text{〃} & f(\bar{x}_1, y_0-k)=0\\ \text{〃}\ \ \text{〃} & f_2,\ f_5,\ f_8, & \text{〃}\ \ \text{〃} & f(\bar{x}_2, y_0)=0\\ \text{〃}\ \ \text{〃} & f_3,\ f_6,\ f_9, & \text{〃}\ \ \text{〃} & f(\bar{x}_3, y_0+k)=0\end{array}\right\} \quad \text{(C-7)}$$

All these points of (C-7), satisfy the relation of $f(x, y)=0$, are therefore the points on the curve $f(x, y)=0$. We plot them on the co-ordinates. The intersection of the curve of (C-5) and (C-7) will give sufficiently exact values of required roots of the symultenous equations.

$$\left.\begin{array}{l} f(x, y)=0 \\ \phi(x, y)=0 \end{array}\right\}$$

D) Group D: Neither of $f(x_0, y)=0$, $f(x, y_0)=0$, $\phi(x_0, y)=0$ nor $\phi(x, y_0)=0$ is soluble.
In this case, we compute

$$\left.\begin{array}{l} f(x_0-h, y_0-k)=f_1,\ \ f(x_0-h, y_0)=f_2,\ \ f(x_0-h, y_0+k)=f_3 \\ f(x_0, y_0-k)=f_4,\ \ f(x_0, y_0)=f_5,\ \ f(x_0, y_0+\mathrm{k})=f_6 \\ f(x_0+h, y_0-k)=f_7,\ \ f(x_0+h, y_0)=f_8,\ \ f(x_0+h, y_0+k)=f_9 \end{array}\right\} \quad \text{(D-8)}$$

All these computations are possible, since the form of $f(x, y)$ is given. By using the process described before, we can find out

$$\left.\begin{array}{llll} \text{from points} & f_1,\ f_2,\ f_3, & \text{the point} & f(x_0-h, \bar{y}_1)=0 \\ \text{〃} \quad \text{〃} & f_4,\ f_5,\ f_6, & \text{〃} \quad \text{〃} & f(x_0, \bar{y}_2)=0 \\ \text{〃} \quad \text{〃} & f_7,\ f_8,\ f_9, & \text{〃} \quad \text{〃} & f(x_0+h, \bar{y}_3)=0 \\ \text{〃} \quad \text{〃} & f_1,\ f_4,\ f_7, & \text{〃} \quad \text{〃} & f(\bar{x}_1, y_0-k)=0 \\ \text{〃} \quad \text{〃} & f_2,\ f_5,\ f_8, & \text{〃} \quad \text{〃} & f(\bar{x}_2, y_0)=0 \\ \text{〃} \quad \text{〃} & f_3,\ f_6,\ f_9, & \text{〃} \quad \text{〃} & f(\bar{x}_3, y_0+k)=0 \end{array}\right\} \quad \text{(D-9)}$$

All these points of (C-9), satisfy the relation of $f(x, y)=0$, are therefore the points on the curve $f(x, y)=0$. We plot them on the co-ordinates. The curve gives the curve of $f(x, y=0)$.
Quite similarly, we compute on $\phi(x, y)$, *i. e.*——
We compute

$$\left.\begin{array}{l} \phi(x_0-h, y_0-k)=\phi_1,\ \ \phi(x_0-h, y_0)=\phi_2,\ \ \phi(x_0-h, y_0+k)=\phi_3 \\ \phi(x_0, y_0-k)=\phi_4,\ \ \phi(x_0, y_0)=\phi_5,\ \ \phi(x_0, y_0+k)=\phi_5 \\ \phi(x_0+h, y_0-k)=\phi_7,\ \ \phi(x_0+h, y_0)=\phi_8,\ \ \phi(x_0+h, y_0+k)=\phi_9 \end{array}\right\} \quad \text{(D-10)}$$

All these computations of ϕ's are also possible, since the form of $\phi(x, y)$ is given. By using the process described before, we can find out

$$\left.\begin{array}{llll} \text{from points} & \phi_1,\ \phi_2,\ \phi_3, & \text{the point} & \phi(x_0-h, \bar{y}_1)=0 \\ \text{〃} \quad \text{〃} & \phi_4,\ \phi_5,\ \phi_6, & \text{〃} \quad \text{〃} & \phi(x_0, \bar{\bar{y}}_2)=0 \\ \text{〃} \quad \text{〃} & \phi_7,\ \phi_8,\ \phi_9, & \text{〃} \quad \text{〃} & \phi(x_0+h, \bar{\bar{y}}_3)=0 \\ \text{〃} \quad \text{〃} & \phi_1,\ \phi_4,\ \phi_7, & \text{〃} \quad \text{〃} & \phi(\bar{\bar{x}}_1, y_0-k)=0 \\ \text{〃} \quad \text{〃} & \phi_2,\ \phi_5,\ \phi_8, & \text{〃} \quad \text{〃} & \phi(\bar{\bar{x}}_2, y_0)=0 \\ \text{〃} \quad \text{〃} & \phi_3,\ \phi_6,\ \phi_9, & \text{〃} \quad \text{〃} & \phi(\bar{\bar{x}}_3, y_0+k)=0 \end{array}\right\} \quad \text{(D-11)}$$

All these points of (D-11) satisfy the relation of $\phi(x, y)=0$, are therefore the points on the curve $\phi(x, y)=0$. We plot them on the co-ordinates. The curve gives the curve of $\phi(x, y)=0$.

The intersection of the curves of (D-9) and (D-11) will give sufficiently exact values of the roots of the symultenous equationa.

$$\left.\begin{array}{l} f(x, y)=0 \\ \phi(x, y)=0 \end{array}\right\}$$

Example

Solve the following equations for $2<x<3$

$$\left.\begin{array}{ll} f(x, y)=x^2-y^2=1 & (1) \\ \phi(x, y)=(x-2)^2+4(y-1)^2=4 & (2) \end{array}\right\}$$

Each equation is a second order equation, so that the symultenous equation is reduced to a fourth order equation, and can not be solved by ordinary method. But $f(x_0, y)=0$, $f(x, y_0)=0$, $\phi(x_0, y)=0$ and $\phi(x, y_0)=0$ are all soluble equations, so that this problem may be solved by group *A–a*

Solution 1.

$$f(2, y)=2^2-y^2=1 \quad \therefore\ y^2=3.000 \quad \therefore\ y_{f1}=+1.732$$
$$f(2.5, y)=(2.5)^2-y^2=1 \quad \therefore\ y^2=5.250 \quad \therefore\ y_{f2}=2.291$$
$$f(3, y)=3^2-y^2=1 \quad \therefore\ y^2=8.000 \quad \therefore\ y_{f3}=2.828$$

We plot points (2, 1.732), (2.5, 2.291) and (3, 2.828) and draw the substitute parabola through these three points. This curve gives the curve of $f(x, y)=1$.

$$\phi(2, y)=4(y-1)^2=4 \quad \therefore\ (y-1)^2=1 \quad \therefore\ (y-1)=1 \quad \therefore\ y_{\phi1}=2.000$$
$$\phi(2.5, y)=(2.5-2)^2+4(y-1)^2=4 \quad \therefore\ 0.25+4(y-1)^2=4$$
$$\therefore\ 4(y-1)^2=4-0.25 \quad \therefore\ 2(y-1)=1.936 \quad \therefore\ (y-1)=0.968$$
$$\therefore\ y_{\phi2}=1.968$$
$$\phi(3, y)=(3-2)^2+4(y-1)^2=4 \quad \therefore\ 4(y-1)^2=3 \quad \therefore\ 2(y-1)=1.732$$
$$\therefore\ y-1=0.866 \quad \therefore\ y_{\phi3}=1.866$$

We plot points (2, 2.000), (2.5, 1.968) and (3, 1.866), and draw the substitute parabola through these three points. This curve gives the curve of $\phi(x, y)=4$
The intersection gives $x=2.22$, $y=1.990$

The above calculations can be done with a sliderule very rapidly, for only an approximate value of x is necessary.

By the above operations, we know that x lies in the neighborhood of 2.20. So we repeat the operation on $x=2.10$, 2.20, 2.30.
We get

for $f(2.10, y)=0$, $y_f=1.847$ and for $\phi(2.10, y)=0$, $y_\phi=1.999$ $\therefore\ y_f-y_\phi=-0.152$
$f(2.20, y)=0$, $y_f=-1.960$ $\phi(2.20, y)=0$, $y_\phi=1.995$ $y_f-\phi_y=-0.035$
$f(2.30, y)=0$, $y_f=2.070$ $\phi(2.30, y)=0$, $y_\phi=1.989$ $y_f-y_\phi=+0.081$

We plot $\delta=(y_f-y_\phi)$ againt x. The curve is nearly a straight line, and we find for $x=2.230$ the value of $\delta=(y_f-y_\phi)=0$. (cf. Fig. 10)

$$\text{Ans}\ \begin{cases} x=2.230 \\ y=1.993 \end{cases}$$

Otherwise, we plot $f(x, y)=0$ curve and $\phi(x, y)=0$ curve according the above values, and the intersection of the two substitute parabolas gives the required answers, *i.e.* $x=2.230$ and $y=1.993$.

Comfirmations:

for $f(x, y)=0$ curve:

$(2.230)^2-y^2=1 \quad \therefore\ y^2=4.973-1=3.973 \quad \therefore\ y_f=1.993$

for $\phi(x, y)=0$ curve:

$(2.230-2.0)^2+4(y-1)^2=4$

$\therefore\ 4(y-1)^2=4-(0.23)^2=3.9471$

$\therefore\ 2(y-1)=1.986 \quad \therefore\ (y-1)=0.993 \quad \therefore\ y_\phi=1.993$

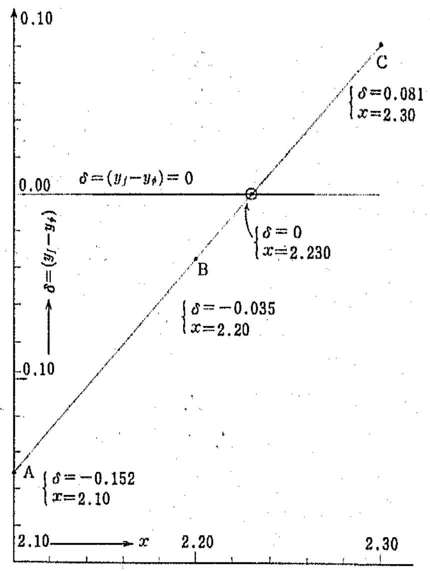

Fig. 10