Mechanics on the Crease of Fabrics

(I) Stresses on Singles Yarn

by

Megumu SUZUKI

(Department of Mechanical Engineering, Faculty of Textile and Sericulture, Shinshu University)

INTRODUCTION

The crease recovery forces that show in bending fabrics depend upon the deformation recovery forces of the yarns forming fabrics and the single fibers forming yarn. Since yarns are twisted more or less, the relations between the stress and the deformation of singles yarn should be studied viscoelastically. In bending fabrics made complicatedly of yarns, the study of the mutual friction forces between fibers as well as the viscoelastical study is important. The fiction forces in bending fabrics actually change along the longitudinal direction of the yarn, and the analysis in Kirchhoff's[1] treatment is very complicated and so the friction forces are not discussed in this paper.

THEOREM

Let us consider the case of the plain fabrics with a view to simplifying the problem, of deformation and stress on the crease of fabrics.[2] A_1 and A_2 are the cross sections so cut by a plane containing a_1-a_2 and b_1-b_2 respectively as to be right angles to the fold that is bent with the 2-2 line of the weave sturctural diagram as shown in Fig. 1. The shapes of A_1 and A_2 are made

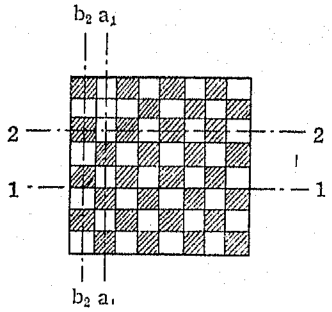


Fig 1. A plain fabric structural diagram

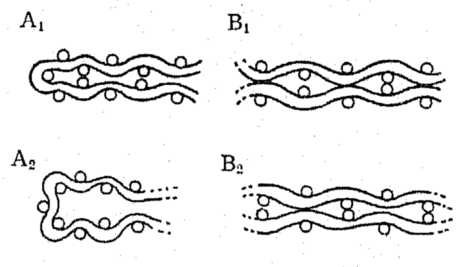


Fig 2-A. The cross sections cut by a plane containing $a_1—a_1$ or $b_2—b_2$ to be right angles at the hold line 2-2 (in Fig 1), A_1 and A_2

2-B. Schematic diagram that the concave (convex) is faced to the concave (convex)

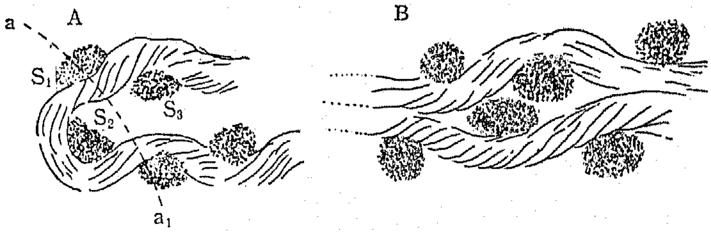


Fig. 3-A. The shape of the hold.
3-B. Diagram that fabrics are bent at random and are stable.

alternatively on the 2–2 fold. But whenever the fabrics are folded at random they are bent on the 1–1 line, because in Fig 2–B_1 and 2–B_2 the convex (concave) is faced to the convex (concave), in Fig 3-B the convex is stable facing to the concave and as the results the folded line seems not to be the 2–2 line but to be the 1–1 line.

The shape of the fold is shown in Fig 3–A, and in fact the maximum curvature of bent yarn is on the left side of aa_1 line. The cross sections of the yarn, S_1, S_2 and S_3……, are compressed by side pressure on the fold and thus become approximately elliptic.

When the fabrics are folded at the right angles to the warp or fill, the fabrics are bent to the radius of curvature equal to the yarn diameter. The actual curvature of single fibers forming yarn is much smaller than that of yarn. Its value is determined by the radius of the singles fiber and the yarn (or the number of fibers forming the yarn). As the yarn is twisted more or less, the single yarn itself is deformed by bending and torsion, and is moved from inside to outside as shown by $\widehat{aa'}$ in Fig 4, so that the bending deformation relaxes.

Then, the bending and torsional stresses on the twisted yarn are analyzed in this paper. It is assumed that the cross section of bent yarn is circular and not elliptic.

I. Bending Rigidity of Yarn

The bending equation for apparent central fiber of bent yarn as shown in Fig 5. is

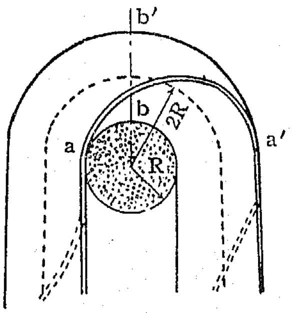


Fig 4. Diagram of the relaxation of fibers on the bending deformation of yarn.

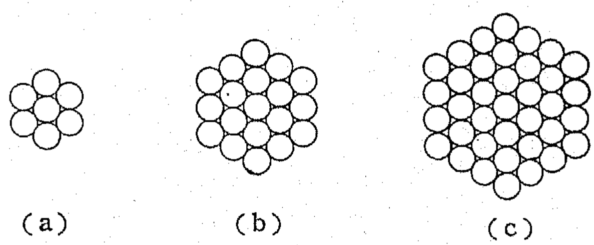


Fig 5. Ideal cross section of yarn.

$$1/\rho = M_c/EI \tag{1}$$

where ρ is the radius of curvature of apparent centeral fiber of bent yarn, E young's modulus, I moment of inertia and M_c the bending moment of apparent centnal fiber.

$$1/\rho = y''/(1+y'^2)^{3/2}$$

and

$$M_c = Py$$

where P is the load and the Cartesian coordinate axes are taken in the tangent and normal directions on the bending plane, and y' and y'' are the first and second derivatives on x.

In the general case that the radius of curvature of bent yarn and the bending moment are shown by the above equations, the calculation is elliptic integral, "famous Lamb's[3] calculation". The bending moment on side fibers can be represented in Fig. 6,[4] where, M is the bending moment on singles fiber, r the radius of coil formed by singles fiber in the yarn, h_o and β pitch and pitch angle of coil formed by singles filament and ϕ the cylindrical coordinate from the bending plane to one point on singles fiber. Then, bending moment M_b and torsional moment M_d are shown as follows:

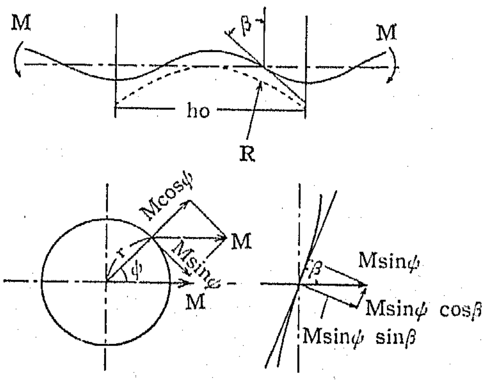


Fig 6. Moment diagram of the side fiber in bending yarn.

$$M_b = M(\cos^2\phi + \sin^2\phi \sin^2\beta)^{1/2}$$

$$M_d = M \sin\phi \cos\beta$$

The elastic strain energy dA stored in the differential element of length dS on singles fiber is

$$dA = M_b^2/2EI \cdot dS + M_d^2/2GJ \cdot dS$$

where G and J are the modulus of rigidity and the polar moment of inertia of the fiber cross section.

$$dS = r/\cos\beta \cdot d\phi$$

Introducing M_b and M_d into the strain energy dA stored in one pitch of coil,

$$A = M^2 r/\cos\beta \int_0^{2\pi} \left\{ \cos^2\phi + \sin^2\phi \sin^2\beta/2EI + \sin^2\phi \cos^2\beta/2GJ \right\} dS$$

$$= \pi r M^2/\cos\beta \left\{ 1 + \sin^2\beta/2EI + \cos^2\beta/2GJ \right\} \tag{2}$$

Here $$\tan\beta = h_o/2\pi r$$

therefore,

$$A = h_o M^2/4\sin\beta\left\{1 + \sin^2\beta/EI + \cos^2\beta/2GI\right\} \tag{3}$$

On the other hand, the work W done outward by bending moment M on one pitch is

$$W = Mh_o/2\rho \tag{4}$$

From the equations (2) and (4) or (3) and (4)

$$1/\rho = M\pi r/h_o\cdot\cos\beta\left\{1 + \sin^2\beta/EI + \cos^2\beta/2GI\right\} \tag{5a}$$

$$= M/2\cdot\cos\beta\left\{1 + \sin^2\beta/EI + \cos^2\beta/2GI\right\} \tag{5b}$$

Therefore, a reciprocal of bending rigidity of one coil in fiber, $1/B_s$, is

$$1/B_s = \pi r/h_o\cdot\cos\beta\left\{1 + \sin^2\beta/EI + \cos^2\beta/2GI\right\} \tag{6a}$$

$$= 1/2\sin\beta\left\{1 + \sin^2\beta/EI + \cos^2\beta/2GI\right\} \tag{6b}$$

Acording to the equation (6b), B_s is given by pitch angle, bending rigidity EI, and torsional rigidity GI. The bending rigidity, B, of yarn having n fibers around the apparent central fiber is given by the equations (1), (6a) and the following equation (7)

$$B = nB_s + EI \tag{7}$$

Generally the bending rigidity of yarn having side filament-layers, is represented : ——

$$B = z_1B_1 + z_2B_2 + z_3B_3 + \cdots\cdots + z_nB_n + EI \tag{8}$$

where z_1, z_2, $\cdots\cdots z_n$ are the number of fibers and B_1, B_2, $\cdots\cdots B_n$ are the bending rigidity of each layer

II. Stresses on Bent Yarn.

The bending moment M on the yarn bent at the radius of curvature R_p is

$$M = pM_s + M_c \tag{9}$$

where M_c and M_s are the bending moment on apparent centeral fiber and side fiber, and p is the number of side fibers. Then,

$$M_c = EI/R_p \tag{10}$$

$$M_s = B_s/P_p \tag{11}$$

where EI and B_s are the bending rigidity of apparent centeral fiber and side fiber, and B_s is given by the equation (6b)

(1) *bending stress on apparent centeral fiber.*

The bending stress on apparent central fiber σ_c is shown:——

$$\sigma_c = M_c/Z = 32/\pi d^3 \cdot M_c \tag{12}$$

where Z is the coefficient of cross section, and since the cross section of singles fiber is not always circular, the diameter d is calculated by changing the cross section area of singles yarn to that of circle.[5] Then, the cross section area of singles fiber $= m/\rho$, where m is the mass per unit length and measured by the cantilever type microbalance, and ρ is the apparent density of the fiber.

(2) *Maximum and minimum of the bending and the torsional moments.*

If bending moment on singles fiber with vector M is M_s, the bending moment M_b and the torsional moment M_d are represented respectively: ——

$$M_b = M_s\sqrt{\cos^2\phi + \sin^2\phi \sin^2\beta} \tag{13}$$

$$M_d = M_s \sin\phi \cdot \cos\beta \tag{14}$$

Table 1. Maximum and Minimum of the bendig and the torsional moments.

	座標の位置	最大・最小値
max of M_b	$\phi=0,\ \pi$	$(M_b)\max=M_s$
min of M_b	$\phi=\pi/2,\ 3\pi/2$	$(M_b)\min=M_s\sin\beta$
max of M_d	$\phi=\pi/2$	$(M_d)\max=M_s\cos\beta$
min of M_d	$\phi=3\pi/2$	$(M_d)\min=M_s\cos\beta$

The maximum and minimum of M_b and M_d as to ϕ are shown in Table 1.

(3) *bending and torsional stresses on side fiber.*

The bending and the torsional stresses for the equations (13) and (14) are shown as follows:

$$\sigma_s = M_b/Z = 32/\pi d^3 \cdot M_s\sqrt{\cos^2\phi + \sin^2\phi \sin^2\beta} \tag{15}$$

$$\tau_s = M_d/Z_o = 16/\pi d^3 \cdot M_s \sin\phi \cos\beta \tag{16}$$

where Z and Z_o are the coefficient of cross section and the polar coefficient of cross section.

NUMERICAL EXAMPLES

The bending stress σ_s, the torsional stress τ_s on the side fiber and bending stress σ_c on the apparent central fiber are determined by the equations (15), (16) and (12) respectively.

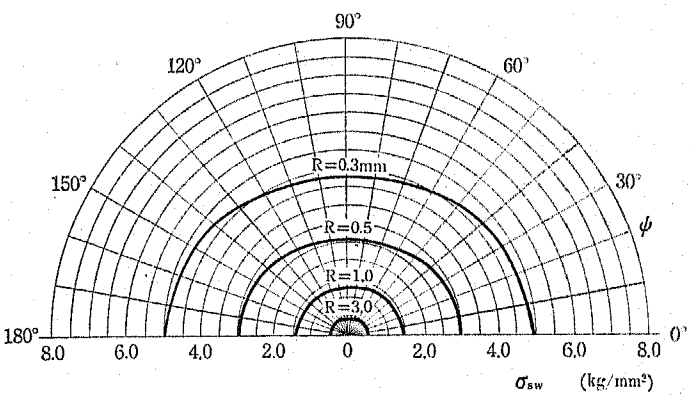
Introducing the equations (11), (12), (6b) and $\tan\beta = h_o/2\pi r$ into the equations (15) and (16),

$$\sigma_c = EI/RZ \tag{17}$$

$$\sigma_s = 1/RZ \cdot h_0/2\pi r \cdot 1/\sqrt{1+(h_0/2\pi r)^2}\left[\frac{\sqrt{\cos^2\psi+\sin^2\psi(h_0/2\pi r)^2/1+(h_0/2\pi r)^2}}{\left\{1+\frac{(h_0/2\pi r)^2}{1+(h_0/2\pi r)^2}\right\}/EI+\left\{\frac{1}{1+(h_0/2\pi r)^2}\right\}/2GI}\right] \tag{18}$$

$$\tau_s = 1/RZ_0 \cdot h_0/2\pi r \cdot 1/\sqrt{1+(h_0/2\pi r)^2}\left[\frac{\sin\psi \cdot 1/\sqrt{1+(h_0/2\pi r)^2}}{\left\{1+\frac{(h_0/2\pi r)^2}{1+(h_0/2\pi r)^2}\right\}/EI+\left\{\frac{1}{1+(h_0/2\pi r)^2}\right\}/2GI}\right] \tag{19}$$

Therefore, the bending stress σ_s, the torsional stress τ_s on the side fiber in the warp and fill and the bending stress σ_c on apparent centeral fiber are shown in Fig 7, 8 and 9, where the young's modulus E is about 400 kg/mm^2,



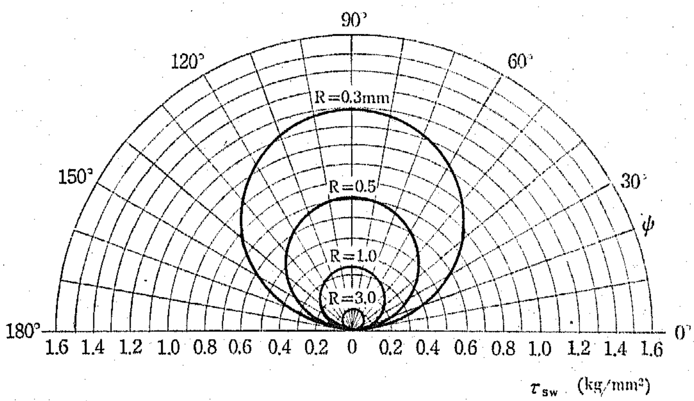
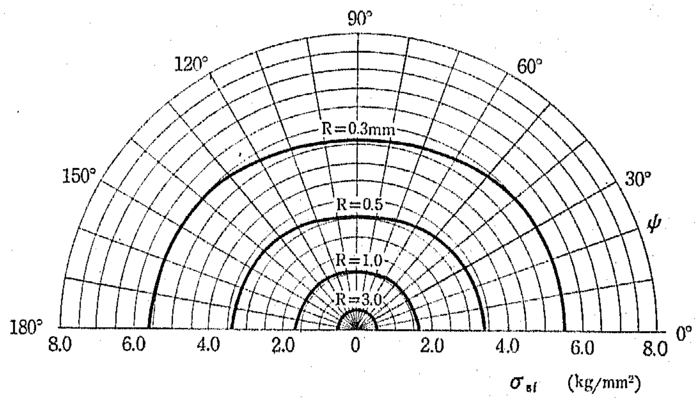


Fig 7. Diagram of the bending stress σs_w and the torsional stress τ_{sw} on the warp side fiber.



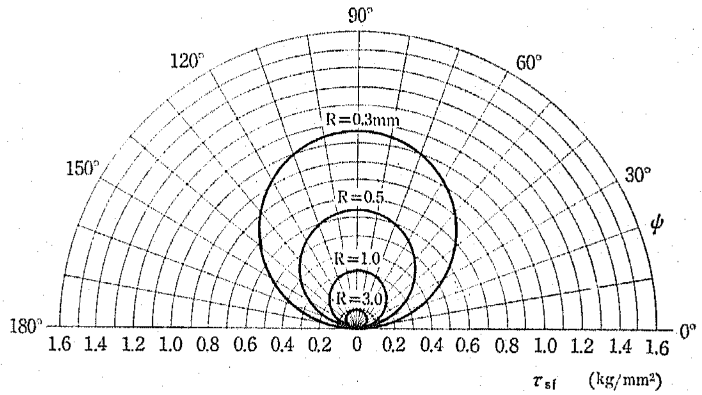


Fig 8. Diagram of the bending stress σ_{sf} and the torional stress τ_{sf} on the fill side fiber.

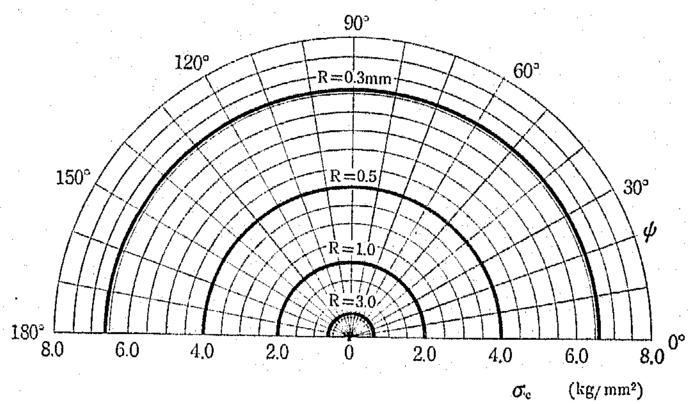


Fig 9. Diagram of the stress σ_c on the apparent central fiber

the modulus of rigidity G about 80 kg/mm^2 in acetate fiber, the length of one pitch h_o 0.74 mm (warp), 1.00 mm (fill), the diametir of singles fiber d 0.01 mm, the coil radius formed by the side fiber r 0.065 mm, and the radii of curvature in bending R 0.3, 0.5, 1.0 and 3.0 mm.

DISCUSSION

Acetate yarn, 50'^{s}, has about 110 fibers. As T. Nakajima[6] reported on cotton yarn that yarn count was not always proportional to the number of fibers and about 50 percent of the cross section area of the yarn was fibers and the rest was space it can be considered that acetate yarn has much spaces. Then it is assumed that the cross section is circular by bending and not elliptic. The young's modulus E and the modulus of rigidity G are Sakurada's[7] and Meredith's[5] results, The diameter of singles fiber is calculated by Posselt's[6] method or the relation between the yarn length (46080m), the yarn weight (453.6g) and the specific weight (1.32). Consequently it is about1.0×10^{-2}mm and the diameter of 50'^{s} yarn is about 1.3×10^{-1} mm. The twist in turns per inch T is shown by the next relation,

$$T=F\sqrt{N}$$

where N is the yarn count and F is constant determined by yarn kinds. Ordinarily, the constant F is taken as follows[8)9)]:——

$$F=4.85 \text{ (warp)}, \ 3.5 \text{ (fill)}$$

As the results, the coil length is shown in Table 2. In the case of the usual twist, Z twist or S twist, the coil length h_o of 50'^{s} yarn is from 0.5mm to 1.5mm in the warp or fill.

Table 2. Note on the warp and fill of acetate yarn.

	warp	fill
F	4.75	3.50
$T(^1/\text{mm})$	1.36	1.00
h_o(mm)	0.74	1.00
β	60°58′	67°45′

The bending stress on the warp is larger than that on the fill, and the torsional stress is reverse, because the constant F of the warp yarn is larger than that of the fill. The smaller the radius of curvature of bending, the larger their stresses are.

The bending stress σ_s on the side fiber represents compressive (tensile) stress from $\phi=0°$ to 180° and tensile (compressive) stress from 180° to 360°. The torsional stress is maximum at $\phi=90°$ and 270° and minimum at 0° and 180°, and on the contrary the bending stress is maximum at 0° and 180°, and minimum at 90° and 270°.

When singles yarn has specially obvious central axis for the example of 50'^{s}, the force P on the cross section of the yarn is :

$$P=109\pi(d/2)^2\binom{\sigma_s}{\tau_s}+\pi(d/2)^2\sigma_c \qquad (20)$$

where $\binom{\sigma_s}{\tau_s}$ is the resultant stress and d is the diameter of singles fiber.

As the yarn axis is not always obvious, however the above equation (20) is represented approximately as follows:

$$P' = 110\pi(d/2)^2\binom{\sigma_s}{\tau_s} \tag{21}$$

In many cases, the yarn axis does not exist in the center and shows a sine curve by the torsional force. This is explained by the X-ray photographs[10] of the twisted cord with steel wire in the center and polyvinylchrolide cord around it. The yarn axis, therefore, need not be considered and the bending force on the singles yarn is shown by the equation (21).

CONCLUSION

The crease recovery force of fabrics depends upon the deformation recovery forces of single fibers forming yarns in fabrics. Since yarns are twisted more or less, both deformations of "bending" and "torsion" should be studied viscoelastically to the elastic limit in the small deformation and the plastic region in the large deformation, but the construction of fabrics is very complicated and so the study of the mutual friction forces between fibers as well as the viscoelastical study of the crease is important. The calculation, therefore, is very complicated and so the frction forces are not discussed in this paper, because the friction forces change along the longitudinal-direction of the yarn. It is also assumed that yarn has the circular form in bending with a view to simplifying the calculations. When the warp (fill) is bent at the axis of the fill (warp), the bending stress on apparent central fiber, and the bending and the torsional stress on the side fiber in an example of acetate yarn $50'^s$ are discussed in this paper.

On the basis of this research several important conclusion can be reached:

(1) The bending stress on the warp is larger than that on the fill and the torsional stress is reverse, because the twist factor of the warp is larger than that of the fill. The smaller the radius of curvature in bending, the larger their stresses are.

(2) The bending stress σ_s on the side fiber represents compressive (tensile) stress from $\psi=0°$ to 180° and tensile (compressive) stress from $\psi=180°$ to 360°. The torsional stress is maximum at $\psi=90°$ and 270° and minimum at 0° and 180°, and on the contrary the bending stress is maxmum at 0° and 180° and minimum at 90° and 270°.

(3) As the yarn axis does not exist in the center by X-ray photographs, the bending loads on yarn with m fibers is:

$$P' = m\pi(d/2)^2\binom{\sigma_s}{\tau_s}$$

where P' is the bending loads, m the number of fibers, d the diameter of singles fiber and $\binom{\sigma_s}{\tau_s}$ the resultant stress of the bending stress and the torsional stress.

ACKNOWLEDGEMENT

My thanks are due to Dr. Yūkichi Go, Proffessor of Faculty of Textile and Sericulture, Shinshu University, for his helpful advices and trouble taken to recommend this paper.

LITERATURE CITED

1) S. Owada; Studies on Mechanical Properties of the Twisted Wire (1953)
2) Y. Go, and others; Reports of Japan Cotton Technical Institute 39, 127 (1955)
3) A. E. H. Love; The Mathematical Theory of Elasticity (1933)
4) S. Ikeda and I. Ueno; Proceeding of the 1st National Congress for Applied Mechanics (1951)
5) R. Meredith; J. Text. Inst., 45, 7 (1954)
6) T. Nakajima; Cotton Spinning (1937)
7) I. Sakurada; Science of Fibers (1956)
8) S. Backer; Text. Res. J. 22 668 (1952)
9) Handbook of Chemical Fibers in Japan (1953)
10) R. Miyashita; Graduation Thesis, Faculty of Textile and Sericulture, Shinshu University (1955)