

LIFETIME OF COMPONENT AND ITS ECONOMIC SIDE

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Lifetime means the lasting for the duration of a life irrespective of animal, machine, or other systems. It may be said that the problem of lifetime is an important measure not only for animals and machines, but also for all sorts of systems, as well as the patterns of many forms of life are analogous to other systems' transformations.

This paper aims at a simple description about lifetime of component and its related matters.

Animal death in nature often comes of ending faster than physiological longevity, moreover, death rate of each period in life varies according to each animal. We use the life tables for ecological comparison such every kind of lifetime by a good reason that the rules of lifetime are deterministic. Life tables of animals are based upon the case of human being, and consist of each term such as living number (number of survivors at age x), mortality number (number of deaths in $(x, x+1)$), death rate (proportion of deaths in $(x, x+1)$) and expected lifetime (observed expectation of life at age x). These also give how constant young decreases as time goes by after birth.

But in general lifetime is stochastic.

SYSTEM LIFE AS A FUNCTION OF COMPONENT LIVES

The environmental condition in which the apparatus is placed has considerable importance. It would readily be understood that equipment used in a calm laboratory and by people accustomed to handling delicate apparatus is not at all subject to the same constraints. What is more, mechanical vibrations, thermal shocks, humidity, shortcomings of the operators and so on may considerably shorten the lifetime of the equipment.

We shall suppose that the equipment considered functions in well-defined conditions and that one has to choose a parameter that well represents the amount of use. This parameter will be called the age of the equipment. The lifetime of equipment will be the age that it has attained at its death, that is, when it has fallen in failure.

The lifetime of equipment may not be described in a precise fashion without the language of the theory of probability. We thus suppose that the lifetime of a component may be represented by a random variable.

Now we consider a monotone system of independent components in which the i th component functions for a random length of time having a distribution R_i , $i=1, \dots, n$. In terms of the distributions R_i and the reliability function $r(P)$, how we express the distribution of the amount of time that the system functions? We first note that the system will function for an amount of time t or greater if and only if it is still functioning at time t . Since $1-R_i(t)$ is the probability that the i th component is functioning at time t , we have that R , the distribution of the amount of time that the system functions is given by

$$\begin{aligned} 1-R(t) &= P \{ \text{system life} \geq t \} \\ &= r[\mathbf{I}-\mathbf{R}(t)] \end{aligned}$$

where $\mathbf{I}-\mathbf{R}(t) = (1-R_1(t), \dots, 1-R_n(t))$.

Case 1

In a series system,

$$1-R(t) = \prod_{i=1}^n [1-R_i(t)]$$

Case 2

In a parallel system,

$$1-R(t) = 1 - \prod_{i=1}^n R_i(t)$$

Case 3

The following two examples are the special cases in a series system.

System with spare circuits: This is a method of exchanging circuit with damaged component for new ones. Now let m be all numbers of circuits, and then the numbers of spare circuits are $(m-1)$. If we let R_j denote the probability of reliable work with a circuit, the system reliability is given by

$$1-R(t) = 1 - \prod_{j=1}^m R_j(t) = 1 - \prod_{j=1}^m [1 - \prod_{i=1}^n (1-R_i(t))].$$

When the system has the same components, that is, $R_i = R_0$,

$$1-R(t) = 1 - [1 - (1-R_0)^n]^m$$

System with spare components: This is a method of exchanging damaged

components for new spare ones. In this system,

$$1-R(t) = \prod_{j=1}^n [1 - \prod_{i=1}^m R_i(t)]$$

When the system has the same components,

$$1-R(t) = [1 - R_0^m]^n$$

The latter method has clearly high reliability, but all the system will be complicated. Practically we had better use the combination of both methods. In any case the big problem with these methods is a lack of information related to the lifetime distribution and probability of reliable work in each component.

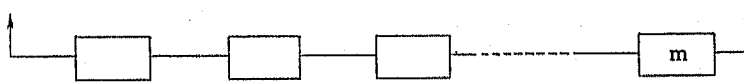


Figure 1 System with spare circuits

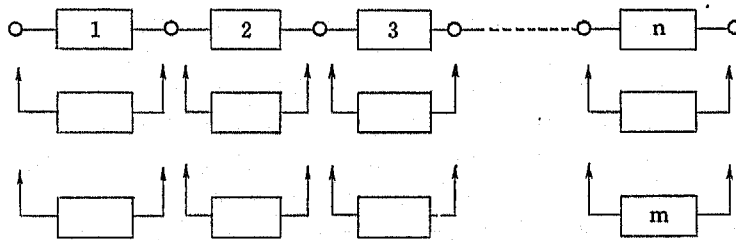


Figure 2 System with spare components

For a continuous distribution F , we define $\lambda(t)$, the failure rate function, by

$$\lambda(t) = \frac{f(t)}{1-F(t)}$$

where $f(t) = dF(t)/dt$. If F is the distribution of the lifetime of an item, then $\lambda(t)$ represents the probability density that a t year old item will fail. That is, $\lambda(t)dt$ is the conditional probability an item will fail between times t and $t+dt$ given that it has functioned to time t . We say that F is an increasing failure rate (IFR) distribution if $\lambda(t)$ is an increasing function of t . Similarly, we say that F is a decreasing failure rate (DFR) distribution if $\lambda(t)$ is a decreasing function of t .

When the lifetime distribution of each component in a monotone system is IFR, the system lifetime is not always IFR. For example, the lifetime distribution of a parallel system of two independent components, the i th component having an exponential distribution with mean $1/i$, $i=1, 2$, is given by

$$\begin{aligned} 1-R(t) &= 1 - (1-e^{-t})(1-e^{-2t}) \\ &= e^{-2t} + e^{-t} - e^{-3t}. \end{aligned}$$

Therefore,

$$\begin{aligned} \lambda(t) &= \frac{dR(t)/dt}{1-R(t)} \\ &= \frac{2e^{-2t} + e^{-t} - 3e^{-3t}}{e^{-2t} + e^{-t} - e^{-3t}}. \end{aligned}$$

By differentiation, the sign of $\lambda'(t)$ is determined by $e^{-5t} + 4e^{-4t} - e^{-3t}$, that is, $\lambda'(t)$ is positive for small values of t and negative for large values of t . Consequently, $\lambda(t)$ is initially strictly increasing and then strictly decreasing, so that R is not IFR.

LIFETIME FROM THE ECONOMIC VIEWPOINT

Decisions concerning production or purchase quantities in enterprises are made such that they will maximize profits. In government operations the profit motive and pure cost considerations may be replaced by considerations of general welfare. However, even in that environment the time value for money is very important. From the economic side there are three major methods for evaluating proposals, that is, present worth, equivalent annual cost and annual rate of return. In the process of comparing these methods the number of years life for each asset must be specified. There is some danger of confusing such concepts as physical life, accounting life, service life and economic life. Among these concepts, economic life is important. For our purposes, economic life is the period that an asset spends on its intended service prior to being liquidated or moved to a next task. An incorrect specification of the life period may change the decision. A poor estimate of economic life will give a serious damage to the company. Obviously the question of economic life should not be taken lightly.

The economic life of different types of equipment may be influenced by many factors. Particular major factors are based upon deterioration and obsolescence. Deterioration can manifest itself in the form of gradually increased cost of maintenance and operation or in the form of sudden catastrophic failure.

An automobile is an good example of a piece of equipment which may have its economic life determined by increased maintenance and repair costs. As it ages, its life can be extended only by the investment of increasingly large sums of money. It sometimes becomes more economical to replace it rather than repair it, and its economic life is defined by that reason. On the other hand, an example of light bulbs commonly show sudden failure in which deterioration is latent until it stops to function. In this case economic life and physical life coincide.

In the case of digital computers, even if they would have the remaining lifetime, they may be replaced by new generation of computers with a progression of technological improvements. This example is progressive deterioration. An alternative method of calculation to determine the economic life of progressive deterioration items is to calculate the present worth of an infinite stream of identical items, with periodic renewals every n years, using continuous discounting. The resulting equation can be differentiated to determine a minimum cost life. Other more complete discussion of this is achieved by Jorgenson et al (1).

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