Local thermalization in the d + Au system

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Abstract

The extent of a locally equilibrated parton plasma in d + Au collisions at $\sqrt{s_{NN}} = 200$ GeV is investigated as a function of collision centrality in a nonequilibrium-statistical framework. Based on a three-sources model, analytical solutions of a relativistic diffusion equation are in precise agreement with recent data for charged-particle pseudorapidity distributions. The moving midrapidity source indicates the size of the local thermal equilibrium region after hadronization. In central d + Au collisions it contains about 19% of the produced particles, and its relative importance rises with decreasing centrality.

The production and identification of a transient quark-gluon plasma in local thermal equilibrium is of basic importance in relativistic heavy-ion physics. In this Letter we propose nonequilibrium-statistical methods to investigate analytically the gradual thermalization occuring in the course of particle production at the highest available energies. The approach is tailored to identify the fraction of produced particles in local thermal equilibrium from their distribution functions in pseudorapidity. It yields indirect evidence for the extent of a locally equilibrated parton plasma.

Recently pseudorapidity distributions of primary charged particles have become available [1] as functions of centrality in d + Au collisions at a nucleon-nucleon center-of-mass energy of 200 GeV. They are investigated within a nonequilibrium-statistical framework that is based on analytical solutions of a Relativistic Diffusion Model (RDM).

Our investigation is based on a linear Fokker-Planck equation (FPE) for three components $R_k(y, t)$ of the distribution function in rapidity space [2, 3, 4]

$$\frac{\partial}{\partial t} R_k(y,t) = \frac{1}{\tau_y} \frac{\partial}{\partial y} \Big[(y - y_{eq}) \cdot R_k(y,t) \Big] + \frac{\partial^2}{\partial^2 y} \Big[D_y^k \cdot R_k(y,t) \Big] \tag{1}$$

with the rapidity $y = 0.5 \cdot ln((E + p)/(E - p))$. The diagonal components D_y^k of the diffusion tensor contain the microscopic physics in the respective Au-like (k=1), d-like (k=2) and central (k=3) regions. They account for the broadening of the distribution functions through interactions and particle creations. In the present investigation the off-diagonal terms of the diffusion tensor are assumed to be zero. The rapidity relaxation time τ_y determines the speed of the statistical equilibration in y-space.

As time goes to infinity, the mean values of the solutions of Eqs. (1) approach the equilibrium value y_{eq}. We determine it from energy- and momentum conservation [5, 6] in the system of Au- and d-participants and hence, it depends on impact parameter. This dependence is decisive for a detailed description of the measured charged-particle distributions in asymmetric systems:

$$y_{eq}(b) = 1/2 \cdot ln \frac{< m_1^T(b) > exp(y_{max}) + < m_2^T(b) > exp(-y_{max})}{< m_2^T(b) > exp(y_{max}) + < m_1^T(b) > exp(-y_{max})} \tag{2}$$

with the beam rapidities $y_b = \pm y_{max}$, the transverse masses $< m_{1,2}^T(b) > = \sqrt{(m_{1,2}^2(b) + < p_T >^2)}$, and masses $m_{1,2}(b)$ of the Au- and d-like participants that depend

on the impact parameter b. The average numbers of participants $N_{1,2}(b)$ in the incident gold and deuteron nuclei are calculated from the geometrical overlap. The results are consistent with the Glauber calculations reported in [1] which we use in the further analysis. The corresponding equilibrium values of the rapidity vary from y_{eq} = - 0.169 for peripheral (80-100%) to y_{eq} = - 0.944 for central (0-20%) collisions. They are negative due to the net longitudinal momentum of the participants in the laboratory frame, and their absolute magnitudes decrease with impact parameter since the number of participants decreases for more peripheral collisions.

The RDM describes the drift towards y_{eq} in a statistical sense. Whether the mean values of the distribution functions R_1 and R_2 actually attain y_{eq} depends on the interaction time τ_{int} (the time the system interacts strongly, or the integration time of (1)). It can be determined from dynamical models or from parametrizations of two-particle correlation measurements. For central Au + Au at 200 A GeV, this yields about $\tau_{int} \simeq 10 fm/c$ [7], which is too short for R_1 and R_2 to reach equilibrium. Note, however, that this does not apply to R_{eq} which is born near local equilibrium at short times (in the present calculation, at t = 0 due to the $\delta-$function intitial conditions), and then spreads in time through diffusive interactions with other particles at nearly the same rapidity.

Our analytical diffusion model is consistent with, and complementary to parton cascade models where stopping involves large sudden jumps in rapidity from hard scatterings (eg. [8]), because even hard partons can participate significantly in equilibration processes, as is evidenced by the high-p_T suppression found in Au + Au at RHIC.

Nonlinear effects are not considered here. Their possible role in the context of relativistic heavy-ion collisions has been discussed in [9, 10, 4]. These account to some extent for the collective expansion of the system in $y-$space, which is not included a priori in a statistical treatment. In the linear model, the expansion is treated through effective diffusion coefficients D_y^{eff} that are larger than the theoretical values calculated from the dissipation-fluctuation theorem that normally relates D_y and τ_y to each other [11]. One can then deduce the collective expansion velocities from a comparison between data and theoretical result.

The FPE can be solved analytically in the linear case with constant D_y^k. For net-baryon rapidity distributions, the initial conditions are δ-functions at the beam rapidities $y_b = \pm y_{max}$. However, it has been shown that in addition there exists a central (k=3,

equilibrium) source at RHIC energies which accounts for about 14% of the net-proton yield in Au + Au collisions at 200 AGeV [4], and is most likely related to deconfinement. For d + Au, net-proton rapidity distributions are not yet available.

For produced particles, the initial conditions are not uniquely defined. Our previous experience with the Au + Au system regarding both net baryons [4], and produced hadrons [12] favors a three-sources approach, with δ-function initial conditions at the beam rapidities, supplemented by a source centered at the equilibrium value y_{eq}. This value is equal to zero for symmetric systems, but for the asymmetric d + Au case its deviation from zero according to (2) is decisive in the description of particle production.

Physically, the particles in this source are expected to be generated mostly from gluon-gluon collisions since only few valence quarks are present in the midrapidity region at $\sqrt{s_{NN}} = 200$ GeV [4]. Particle creation from a gluon-dominated source, in addition to the sources related to the valence part of the nucleons, has also been proposed by Bialas and Czyz [13]. The final width of this source corresponds to the local equilibrium temperature of the system which may approximately be obtained from analyses of particle abundance ratios, plus the broadening due to the collective expansion of the system. Formally, the local equilibrium distribution is a solution of (1) with diffusion coefficient $D_y^3 = D_y^{eq}$, and δ-function initial condition at the equilibrium value.

The PHOBOS-collaboration has analyzed their minimum-bias data successfully using a triple Gaussian fit [14]. This is consistent with our analytical three-sources approach, although additional contributions to particle production have been proposed [15]. Beyond the precise representation of the data, however, the Relativistic Diffusion Model offers an analytical description of the statistical equilibration during the collision and in particular, of the extent of the moving midrapidity source which is indicative of a locally equilibrated parton plasma prior to hadronization.

With $\delta-$function initial conditions for the Au-like source (1), the d-like source (2) and the equilibrium source (eq), we obtain exact analytical diffusion-model solutions as an incoherent superposition of the distribution functions $R_k(y, t)$ because the differential equation is linear. The three individual distributions are Gaussians with mean values

$$< y_{1,2}(t) >= y_{eq}[1 - exp(-t/\tau_y)] \mp y_{max} \exp{(-t/\tau_y)}. \qquad (3)$$

for the sources (1) and (2), and y_{eq} for the moving equilibrium source. Hence, all three mean values attain $y_{eq}(b)$ as determined from (2) for t$\to \infty$, whereas for short times the

mean rapidities are smaller than, but close to the Au- and d-like values in the sources 1 and 2. The variances are

$$\sigma^2_{1,2,eq}(t) = D_y^{1,2,eq}\tau_y[1 - \exp(-2t/\tau_y)].\tag{4}$$

The charged-particle distribution in rapidity space is then obtained as incoherent superposition of nonequilibrium and local equilibrium solutions of (1)

$$\frac{dN_{ch}(y, t = \tau_{int})}{dy} = N_{ch}^1 R_1(y, \tau_{int}) + N_{ch}^2 R_2(y, \tau_{int}) + N_{ch}^{eq} R_{eq}^{loc}(y, \tau_{int})\tag{5}$$

with the interaction time τ_{int} (total integration time of the differential equation). In the present work, the integration is stopped at the value of τ_{int}/τ_y that produces the minimum χ^2 with respect to the data and hence, the explicit value of τ_{int} is not needed as an input. The result for central collisions is $\tau_{int}/\tau_y \simeq 0.4$.

The average numbers of charged particles in the Au- and d-like regions $N_{ch}^{1,2}$ are proportional to the respective numbers of participants $N_{1,2}$,

$$N_{ch}^{1,2} = N_{1,2}\frac{(N_{ch}^{tot} - N_{ch}^{eq})}{(N_1 + N_2)}\tag{6}$$

with the constraint $N_{ch}^{tot} = N_{ch}^1 + N_{ch}^2 + N_{ch}^{eq}$. Here the total number of charged particles in each centrality bin N_{ch}^{tot} is determined from the data. The average number of charged particles in the equilibrium source N_{ch}^{eq} is a free parameter that is optimized together with the variances and τ_{int}/τ_y in a χ^2-fit of the data using the CERN minuit-code [16]. With known τ_{int}, including its dependence on centrality, one could then determine τ_y and D_y, but this is beyond the scope of the present work.

The result of the RDM calculation is shown in Fig. 1 for central collisions (0-20%) of d + Au. The charged-particle yield is dominated by hadrons produced from the Au-like source, but there is a sizeable equilibrium source that is more important than the d-like contribution. This thermalized source is moving since y_{eq} has a negative value for d + Au, whereas it is zero for symmetric systems.

The total yield is compared to PHOBOS data [1] which refer to the pseudorapidity $\eta = -ln[tan(\theta/2)]$ since particle identification was not available. As a consequence, there is a small difference to the model result in y-space ($y \approx \eta$) which is most pronounced in the midrapidity region. It is removed when the theoretical result is converted to η-space

through the Jacobian

$$J(\eta, \langle m \rangle / \langle p_T \rangle) = \cosh(\eta) \cdot [1 + (\langle m \rangle / \langle p_T \rangle)^2 + \sinh^2(\eta)]^{-1/2}. \qquad (7)$$

Here we approximate the average mass $< m >$ of produced charged hadrons in the central region by the pion mass m_π, and use a mean transverse momentum $< p_T > = 0.4$ GeV/c. In the Au-like region, the average mass is larger due to the participant protons, but since their number $Z_1 < 5.41$ is small compared to the number of produced charged hadrons in the d + Au system, the increase above the pion mass remains small: $< m > \approx m_p \cdot Z_1 / N_{ch}^1 + m_\pi \cdot (N_{ch}^1 - Z_1) / N_{ch}^1 \approx 0.17 GeV$. This increase turns out to have a negligeable effect on the results of the numerical optimization, where we use $< m > / < p_T > = 0.45$ for the Jacobian transformations in the three regions. For reasonable deviations of the mean transverse momentum from 0.4 GeV/c, the results remain consistent with the data within the experimental error bars.

The equilibrium source in the light and asymmetric d + Au system is found to contain only 19% of the produced charged hadrons in central collisions. A previous result [12] for Au + Au in the three-sources-RDM shows that the equilibrium source for particle production tends to be larger in the heavy system [12] at the same energy. Note, however, that the results of the χ^2-minimization are not unique for Au + Au due to the symmetry of the system.

The results for the mean values and variances of produced charged hadrons in d + Au collisions as functions of impact parameter are shown in Fig. 2. Here the average impact parameters for the five centrality cuts k are determined according to $< b_k > = \int b\sigma_k(b)db / \int \sigma_k(b)db$ with the geometrical cross sections $\sigma_k(b)$ in each bin. Whereas the total particle number and the particles created from the Au-like source decrease almost linearly with increasing impact parameter, the magnitude of the equilibrium source is roughly independent of centrality. As a consequence, particle production in the equilibrium source is relatively more important in peripheral collisions. The variance of the central source lies for sufficiently small impact parameters between the values for the Au- and d-like sources.

The model calculations are converted to η-space and compared with PHOBOS data for five centrality cuts [1] and minimum bias [14] in Fig. 3. The minimization procedure yields precise results so that reliable values for the relative importance of the three sources for particle production can be determined, Table 1. The rapidity relaxation times and

diffusion coefficients can also be obtained from (3),(4), but this requires an independent information about the interaction times. A small discrepancy in case of the most peripheral collisions (80-100%) is a consequence of the three straggling data points in the region $-4 < \eta < -3$. The observed shift of the distributions towards the Au-like region in more central collisions, and the steeper slope in the deuteron direction as compared to the gold direction appear in the Relativistic Diffusion Model as a consequence of the gradual approach to equilibrium.

Given the structure of the underlying differential equation that we use to model the equilibration, together with the initial conditions and the constraints imposed by Eqs. (2) and (6), there is no room for substantial modifications of this result. In particular, changes in the impact-parameter dependence of the mean values in (3) that are not in accordance with (2) vitiate the precise agreement with the data.

To conclude, we have investigated charged-particle production in d + Au collisions at $\sqrt{s_{NN}}$= 200 GeV as function of centrality within the framework of an analytically soluble three-sources model. Excellent agreement with recent PHOBOS pseudorapidity distributions has been obtained, and from a χ^2-minimization we have determined the diffusion-model parameters very accurately.

For central d + Au collisions, a fraction of only 19% of the produced particles arises from the locally equilibrated midrapidity source. Although this fraction increases towards more peripheral collisions, the formation of a thermalized parton plasma prior to hadronization can probably only be expected for more central collisions.

The d + Au results show clearly that only the midrapidity part of the distribution function reaches thermal equilibrium, whereas the interaction time is too short for the d- and Au-like parts to attain the thermal limit. The same is true for the heavy Au + Au system at the same energy, but there the precise fraction of particles produced in the equilibrium source is more difficult to determine due to the symmetry of the problem.

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Table 1. Produced charged hadrons as functions of centrality in d + Au collisions at $\sqrt{s_{NN}} = 200$ GeV, $y_b = \pm 5.36$ in the Relativistic Diffusion Model. The variance of the central source in $y-$space is σ_{eq}^2. The number of produced charged particles is $N_{ch}^{1,2}$ for the sources 1 and 2 and N_{ch}^{eq} for the equilibrium source, the percentage of charged particles produced in the thermalized source is n_{ch}^{eq}.

$Centrality(\%)$	σ_{eq}^2	N_{ch}^1	N_{ch}^2	N_{ch}^{eq}	$n_{ch}^{eq}(\%)$
0-20	3.99	131	19	35	19
20-40	3.95	78	17	31	25
40-60	5.70	33	11	38	46
60-80	7.44	9	5	35	71
80-100	6.89	2	2	24	86
min. bias	4.04	56	15	21	23

Figure captions

Fig. 1. Charged-particle rapidity spectrum dN_{ch}/dy in the Relativistic Diffusion Model (RDM) for central (0-20%) d + Au collisions at $\sqrt{s_{NN}} = 200$ GeV, upper curve. Dashed curves are contributions from the Au- and d-like sources, respectively. The dashed area is the yield from the moving thermalized central source of partonic origin, cf. text. PHOBOS pseudorapidity data $dN_{ch}/d\eta$ ($\eta \approx y$) [1] are shown to illustrate the difference in $y-$ and $\eta-$space. See Fig. 3 for a fit with the proper Jacobian transformation.

Fig. 2. Mean values (upper part) and variances (lower part) of the three sources for produced charged particles in rapidity space as functions of the mean impact parameter for five centrality bins, symbols. Lines are drawn to guide the eye. Dashed lines have Au- and d-like sources, solid lines correspond to the thermalized central source. The top solid line gives the mean total number of produced charged particles.

Fig. 3. Calculated pseudorapidity distributions of charged particles from d + Au collisions at $\sqrt{s_{NN}} = 200$ GeV for five different collision centralities, and minimum-bias in comparison with PHOBOS data [1, 14]. The analytical RDM-solutions are optimized in a fit to the data. The corresponding minimum χ^2-values (top left to bottom right) are 4.7, 5.9, 2.4, 1.7, 1.9, 2.1. See Fig. 2 and text for the resulting diffusion-model parameters.

PHOBOS d+Au at 200 GeV

centrality 80-100%

centrality 60-80%

centrality 40-60%

centrality 20-40%

centrality $\bar{0}$-20%

Minimum-Bias

$dN_{ch}/d\eta$

η

13