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Analyses of k_t distributions at RHIC by means of some selected statistical and stochastic models

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Abstract. The new data on k_t distributions obtained at RHIC are analyzed by means of selected models of statistical and stochastic origin in order to estimate their importance in providing new information on hadronization process, in particular on the value of the temperature at freeze-out to hadronic phase.

PACS. 25.75.-q Relativistic heavy ion collisions – 12.40.Ee Statistical (extensive and non-extensive) models – 02.50.Ey Stochastic models

1 Introduction

Very recently high k_t distributions at RHIC have been reported in Refs. [1–3]. These data are of potentially high interest as a possible source of information on the conditions existing at the freeze-out in heavy-ion collisions. This resulted in a number of works, mostly of statistical or thermal origin [4], stressing different possible dynamical aspects, like the role of resonances or the flow phenomenon. In our work we would like to show that one can account summarily for such (and others) effects considered in the literature by using simple minimal extensions of the known statistical or stochastic models, which were already successfully applied in other analysis of experimental data. They are:

(i) The modified statistical model inspired by Tsallis statistics [5], which generalizes the usual Boltzmann-Gibbs statistics to nonextensive systems parametrized by a nonextensivity parameter q (for $q \to 1$ one returns to the usual Boltzmann-Gibbs extensive scenario); it has been already successfully used in this context [6–8]. Parameter q summarizes in such approach all deviations from the Boltzmann-Gibbs statistics including those caused by flow phenomena and resonances [4].

(ii) A suitable adaptation of the recently proposed model derived from the Fokker-Planck equation for the Orstein-Uhlenbeck (O-U) process [9–11] but this time used in the transverse rapidity space, i.e., for $y_t = \frac{1}{2}\ln[(m_t+k_t)/(m_t-k_t)]$ (where $m_t = \sqrt{m^2+\langle k_t\rangle^2}$), in which one allows for the mass m to be treated as free parameter in order to account for some specific features of data (like flow phenomenon) which cannot be explained in a usual way.

As a kind of historical reference point we shall use classical statistical model developed long time ago by Hagedorn [12] in which transverse momentum distribution of produced secondaries is given by the following formula [13] (with T_0 being parameter identified with temperature, T_h denoting the so called Hagedorn temperature [12,13] and m_π being pion mass),

$$\frac{d^2\sigma}{2\pi k_t dk_t} = C\int_{m_\pi}^{\infty} dm\rho(m)\sqrt{m^2+k_t^2}K_1\left(\frac{\sqrt{m^2+k_t^2}}{T_0}\right); \quad (1)$$

$$\rho(m) = \frac{e^{m/T_h}}{(m^2+m_0^2)^{5/4}}. \quad (2)$$

As one can see in Fig. 1 although fits to k_t distributions at $\sqrt{s_{NN}} = 200$ GeV obtained by BRAHMS Collaboration [1] are quite good, they start to deviate from data at highest values of k_t and became very bad there, what is very clearly seen in Fig. 2 where we show our fits to STAR data [2] covering larger span of transverse momenta. Although one can argue that for such large values of k_t statistical approach must give way to some more detailed dynamical calculations [4], there are examples that suitable modifications of statistical approach can lead to quite reasonable results in leptonic, hadronic and nuclear collisions. What we have in mind here are some non-extensive generalizations of statistical model as discussed in [7,6,8]) and some specific realization of stochastic approach as proposed by [9,10,14]. In what follows we shall therefore apply these two methods to nuclear data of Refs. [1–3].

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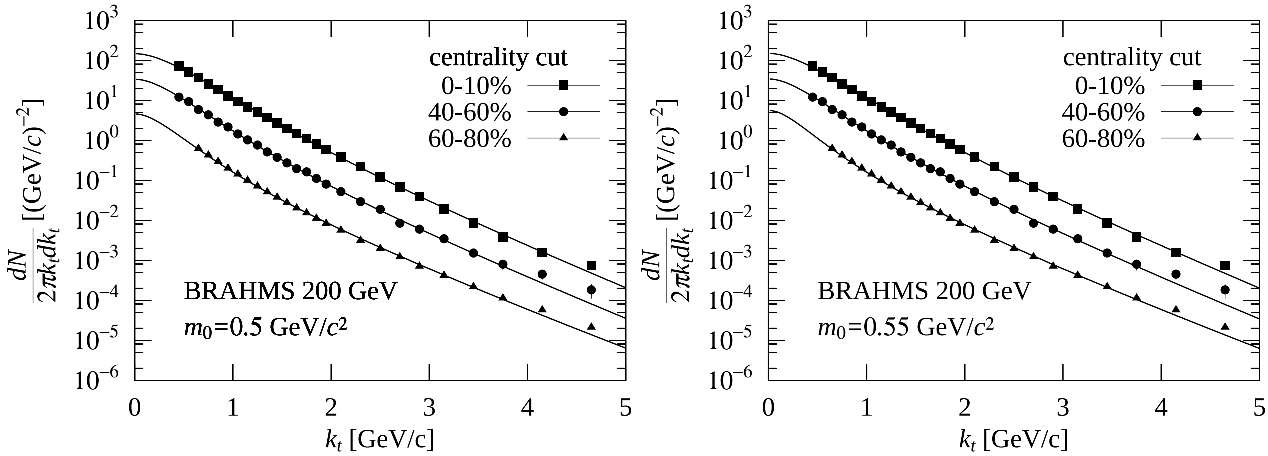

Fig. 1. Results of application of simple statistical model, cf. Eq. (1), to data for k_t-distributions at $\sqrt{s_{NN}} = 200$ GeV measured for different centralities by BRAHMS Collaboration [1].

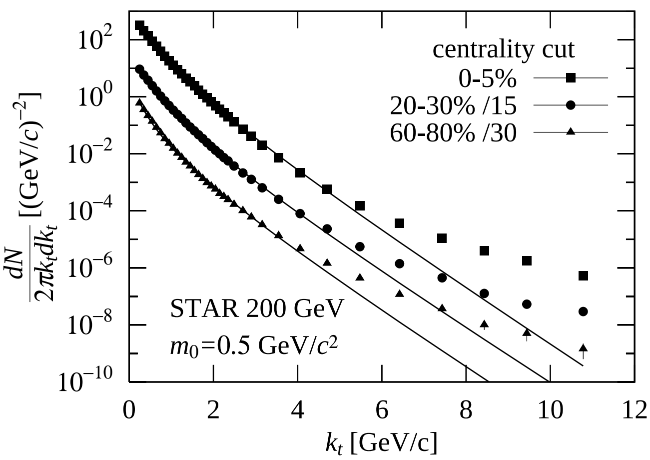

Fig. 2. Results of application of simple statistical model, cf. Eq. (1), to data for k_t-distributions at $\sqrt{s_{NN}} = 200$ GeV measured for different centralities by STAR Collaboration [2].

Table 1. Values of parameters C, T_h and T_0 in eq. (1) used to obtain results presented in Figs 1 and 2. The values of χ^2/n.d.f. for BRAHMS data are the same for all centralities and equal to 19.3/23 and 18.3/23 for $m_0 = 0.5$ and 0.55 GeV, respectively. For STAR data they are equal 532/32 for C.C = $0-5\%$, 249/32 for C.C= $20-30\%$ and 308/32 for C.C= $60-80\%$.

BRAHMS Coll. [1] $m_0 = 0.5$ GeV (fixed)			
C.C. (%)	C	T_h (GeV)	T_0 (GeV)
0-10	177±11	0.180±0.007	0.169±0.006
10-20	127±9	0.172±0.008	0.162±0.006
20-40	83±7	0.156±0.008	0.149±0.007
40-60	44±5	0.133±0.010	0.128±0.009
60-80	177±11	0.095±0.0001	0.093±0.0001
BRAHMS Coll. [1] $m_0 = 0.55$ GeV (fixed)			
0-10	204±13	0.172±0.008	0.162±0.006
10-20	146±10	0.163±0.008	0.155±0.006
20-40	96±8	0.148±0.008	0.142±0.007
40-60	51±6	0.124±0.010	0.121±0.009
60-80	204±13	0.075±0.00007	0.075±0.0001
STAR Coll. [2] $m_0 = 0.5$ GeV (fixed)			
0-5	816±15	0.086±0.0001	0.085±0.0001
20-30	382±7	0.077±0.0001	0.076±0.0001
60-80	106±2	0.037±0.00001	0.037±0.00001

In next Section we shall analyze data using nonextensive generalization of statistical model by means of Tsallis statistics. In Section 3 we shall analyze data using stochastic approach in transverse rapidity space. Our conclusions are presented in Section 4.

2 Analysis of k_t distributions by generalized statistical model based on Tsallis statistics

In many fields in physics, which use statistical and stochastic approaches as their tools, it was recognized since some time already that the usual Boltzmann-Gibbs approach encounters serious problems when applied to systems possessing memory effects, correlations (especially long-range correlations but also those caused by the production of resonances in multiparticle production processes or by the flow effects present there) or which phase space has some (multi) fractal structure [5]. Such systems are all, in a sense, *small*, by what we means that the effective range of correlations they experience is of the order of dimension of the system itself. Therefore they will not show property of extensivity leading to Boltzmann-Gibbs form of entropy, which is the basis of any statistical or stochastical model. One can therefore argue that in such cases one has to resort to some dynamical approach in which effects mentioned above would be properly accounted for. The problem is, however, that there is no unique model of this type and usually several approaches are competing among themselves in describing experimental data. The other possibility is to realize that most probably our system is not extensive (in the abovementioned sense) and that this fact should be accounted for by using the non-extensive form of entropy, for example the so called Tsallis [5]. It turns out that such situations are encountered also in domain of multiparticle production processes at high energy collisions (cf., [6], to which we refer for all details). In fact, there already exists a number of detailed analysis using a non-extensive approach ranging from k_t distributions in e^+e^- annihilations [7] and in $p+\bar{p}$ collisions [8] to rapidity distributions in some selected reactions [6]. In [7, 8] a kind of non-extensive q-version of Hagedorn approach has been used whereas [6] were exploring information theoretical approach to statistical models including as option also its non-extensive version [1].

In our work we shall apply Tsallis formalism, treated as simplest possible extension of the usual statistical approach with parameter q (the so called nonextensivity parameter or entropic index) summarizing deviations from the usual statistical approach (without, however, specifying their dynamical origin). It leads to (T_0 denotes temperature):

$$\frac{d^2\sigma}{2\pi k_t dk_t} = C\int_0^\infty \left[1-(1-q)\frac{\sqrt{k_t^2+k_l^2+m^2}}{T_0}\right]^Q dk_l. \quad (3)$$

There exist two different formulations leading to slightly different forms of parameter Q:

(a) In first one uses the so-called escort probability distributions [19], $P_i = p_i^q/\sum_i p_i^q$ (cf., for example, analysis of k_t distributions in e^+e^- annihilations [7] or in $p\bar{p}$ collisions[8]), in this case $Q = q/(1-q)$.

(b) In second approach one uses normal definition of probabilities resulting in $Q = 1/(1-q)$. In this case, as shown in [15,17], parameter q is given by the normalized variance of all intrinsic fluctuations present in the hadronizing system under consideration:

$$q = 1+\omega = 1+\left(\langle\beta^2\rangle - \langle\beta\rangle^2\right)/\langle\beta\rangle^2. \quad (4)$$

This conjecture originates from the observation that:

$$\left[1-(1-q)\beta_0 H_0\right]^{\frac{1}{1-q}} = \int_0^\infty e^{-\beta H_0} f(\beta) d\beta \quad (5)$$

where $f(\beta)$ describes fluctuation of parameter β and has form of Gamma distribution [15,17] (in our case $H_0 = \sqrt{k_l^2+k_t^2+m^2}$ and fluctuations are in temperature, i.e., $\beta = 1/T$ and $\beta_0 = \langle\beta\rangle$ with respect to $f(\beta)$) [2].

[1] It should be mentioned at this point that proper formulation of Hagedorn model using Tsallis q-statistics has been proposed in [16]. We shall not pursue this problem here.

[2] It must be mentioned at this point that this suggestion, which in [15] has been derived only for $q > 1$ case, has been shown to be valid also for $q < 1$ case [17] and extended to general form of fluctuations leading then to the new concept of *superstatistics* proposed in [18]. The most recent discussion of physical meaning of q parameter when applied to multiparticle production processes (and in this context also of the possible origin of statistical formulas as well) with relevant references can be found in [6].

We have analyzed BRAHMS [1], STAR [2] and PHENIX [3] data and our results are shown in Fig. 3 and in Table 2. It turns out that both form of parameter Q in Eq. (3) result in practically identical curves, therefore here we are showing only results obtained for $Q = q/(1-q)$. The values of parameters are also very close to each other with tendency of C, T_0 and q estimated by using $Q = 1/(1-q)$ being slightly bigger then those obtained for $Q = q/(1-q)$. It is worth to stress at this point that such comparison of these two approaches has been made for the first time here and, as one can see from the presented results, it confirms previous expectation (made in [6]) that in case of only limited phenomenological applications, as is the case of our work, the results from using Eq. (3) with $Q = q/(1-q)$ (i.e., parameters: $C^{(a)} = c$, $T_0^{(a)} = l$ and $q^{(a)} = q$) are simply connected to those using $Q = 1/(1-q)$ (i.e., to parameters: $C^{(b)} = C$, $T_0^{(b)} = L$ and $q^{(b)} = \hat{Q}$), namely:

$$\hat{Q} \simeq 1 - \frac{1-q}{q}, \qquad L \simeq \frac{l}{q}, \qquad C \simeq cq. \quad (6)$$

As one can see from Table 2 these relations are indeed satisfied (some small differences present could be attributed to the fact that both sets of results represent results of separate and independent fitting procedures, without making use of Eq. (6)). It means therefore that in all phenomenological applications one can use either of the two form of parameter Q in Eqs. (3), and, if necessary, to use Eq. (6) to translate results from one scheme to another. In both cases the pion mass value has been used, $m = 0.14$ GeV (and we have checked that additional changes in mass m of the type introduced recently in [14], would not affect the final results as long as m is limited to, say, $m < 0.2$ GeV). The estimated fluctuations of temperature are of the order of $30-45$ MeV. It is interesting to observe that these fluctuations are weaker at small centralities and grow for more peripheral collisions matching very nicely similar fluctuations seen in $p+p$ data [2] shown here for comparison. One should add here also result from similar analysis of e^+e^- data [7] reporting even higher values of nonextensivity parameter q (reaching value of $q \simeq 1.2$), i.e., much stronger fluctuations. These results confirm therefore, for the first time, another expectation made in [6] saying that precisely such trend should be observed. This is because Eq. (4) can be also interpreted as being a measure of the *total heat capacity* C_h of the hadronizing system (cf. [6]):

$$\frac{1}{C_h} = \frac{\sigma^2(\beta)}{\langle\beta\rangle^2} = \omega = q-1. \quad (7)$$

As the heat capacity C_h is proportional to the volume, $C_h \sim V$, in our case V would be the volume of interaction (or hadronization), it is expected to grow with volume and, respectively, q is expected to decrease with V, which

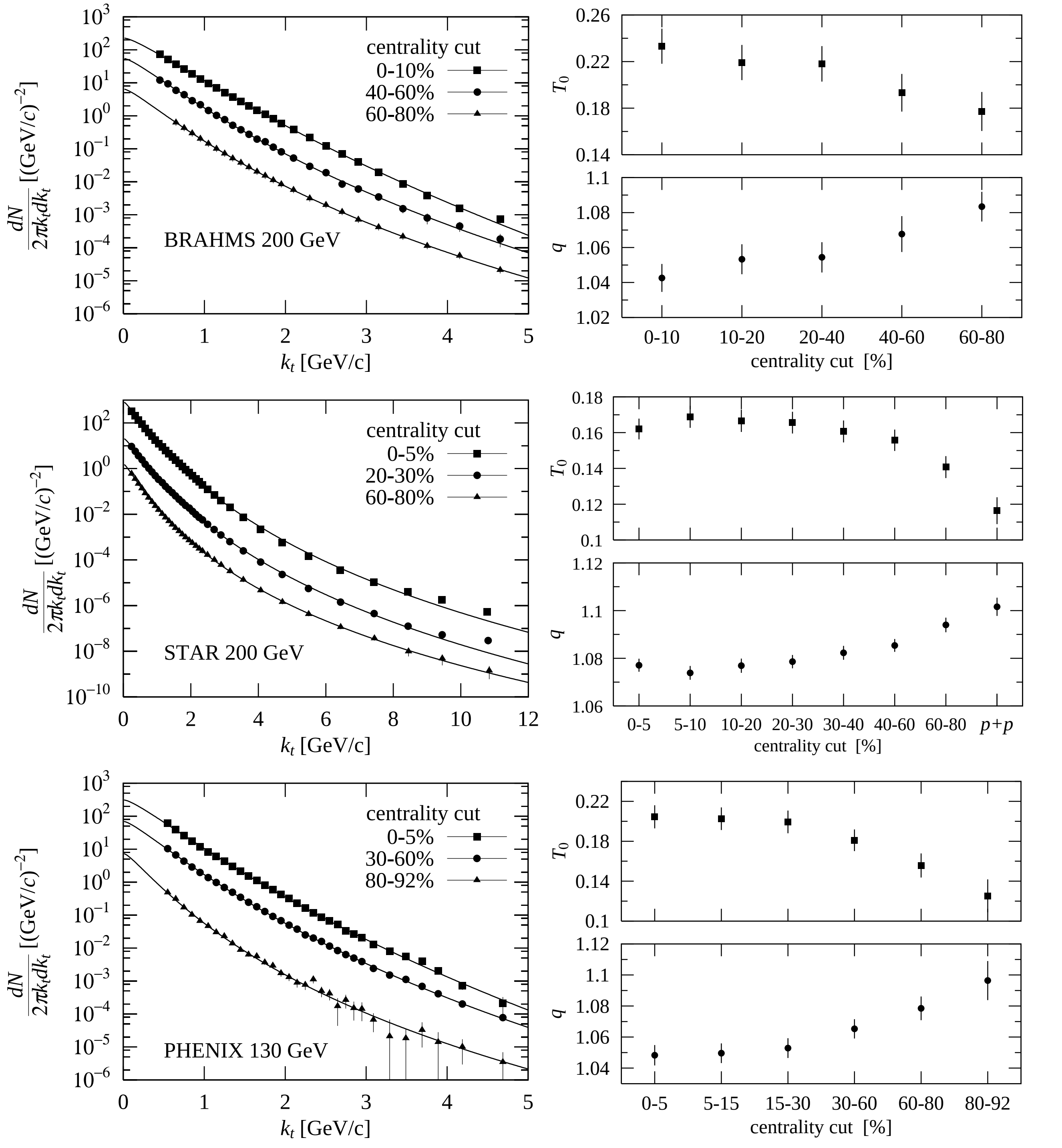

Fig. 3. Results of application of non-extensive approach given by Eq. (3) with $Q = q/(1-q)$ to data for k_t-distributions at $\sqrt{s_{NN}} = 200$ GeV measured for different centralities by BRAHMS [1] and STAR [2] Collaborations and to data at $\sqrt{s_{NN}} = 130$ GeV as measured by PHENIX Collaboration [3]. The results obtained using $Q = 1/(1-q)$ instead look essentially the same, therefore they are not shown separately. For differences in values of obtained parameters see Table 2.

Table 2. Values of characteristic parameters used to fit data on k_t-distributions at different centralities by using non-extensive approach as given by Eq. (3) with $Q = q/(1-q)$ and $Q = 1/(1-q)$ for data at $\sqrt{s_{NN}} = 200$ GeV obtained by BRAHMS [1] and STAR [2] Collaborations and at $\sqrt{s_{NN}} = 130$ GeV obtained by PHENIX Collaboration [3]. For results obtained using $Q = 1/(1-q)$ we provide also explicit values of the corresponding fluctuations of temperature as given by $\Delta T_0 = \sqrt{q-1} \cdot T_0$. The order of magnitude of the corresponding errors for T_0, q and ΔT_0: δT_0, δq and $\delta \Delta T_0$, , respectively, are listed below as well. In analysis we have used errors either as provided by experiments (for STAR and PHENIX) or assuming systematic error on the level of 5% (for BRAHMS).

BRAHMS Coll. [1]									
	Eq. (3) with $Q = q/(1-q)$ δT_0 =0.005-0.007, δq =0.003-0.005.				Eq. (3) with $Q = 1/(1-q)$ δT_0 =0.005-0.007, δq =0.002-0.004, $\delta \Delta T_0$ =0.001-0.002.				
C.C. (%)	χ^2/n.d.f.	C	T_0 (GeV)	q	χ^2/n.d.f.	C	T_0 (GeV)	q	ΔT_0 (GeV)
0-10	11.2/23	1033±78	0.232	1.043	11.2/23	1033±78	0.223	1.041	0.045
10-20	12.9/23	797±66	0.224	1.049	12.9/23	797±65	0.213	1.047	0.046
20-40	12.7/23	525±49	0.215	1.055	12.7/23	525±49	0.204	1.052	0.047
40-60	10.5/23	304±38	0.193	1.067	10.5/23	304±38	0.181	1.063	0.045
60-80	2.85/22	41±5	0.175	1.084	2.85/22	41±5	0.161	1.077	0.045
STAR Coll. [2]									
	Eq. (3) with $Q = q/(1-q)$ δT_0 =0.002-0.003, δq = 0.001-0.002.				Eq. (3) with $Q = 1/(1-q)$ δT_0 =0.002-0.003, δq ≅0.001, $\delta \Delta T_0$ ≅0.001.				
C.C. (%)	χ^2/n.d.f.	C	T_0 (GeV)	q	χ^2/n.d.f.	C	T_0 (GeV)	q	ΔT_0 (GeV)
0-5	170/32	4684±231	0.171	1.071	170/32	4686±231	0.159	1.066	0.041
5-10	68/32	3393±184	0.176	1.068	67.8/32	3393±185	0.165	1.064	0.041
10-20	69/32	2767±144	0.171	1.073	69.2/32	2768±144	0.160	1.068	0.042
20-30	45/32	1928±102	0.169	1.075	44.7/32	1928±102	0.157	1.070	0.042
30-40	44/32	1391±78	0.165	1.078	43.9/32	1391±78	0.153	1.072	0.041
40-60	19/32	896±50	0.153	1.085	19.2/32	896±50	0.141	1.079	0.040
60-80	14/32	414±25	0.137	1.095	14.2/32	413±25	0.125	1.087	0.037
$p+p$	9.7/29	62±7	0.117	1.099	9.62/29	61.9±7.1	0.107	1.090	0.032
PHENIX Coll. [3]									
	Eq. (3) with $Q = q/(1-q)$ δT_0 =0.011-0.016, δq =0.008-0.010.				Eq. (3) with $Q = 1/(1-q)$ δT_0 =0.011-0.016, δq =0.005-0.011, $\delta \Delta T_0$ =0.003-0.005.				
C.C. (%)	χ^2/n.d.f.	C	T_0 (GeV)	q	χ^2/n.d.f.	C	T_0 (GeV)	q	ΔT_0 (GeV)
0-5	5.13/29	1694±409	0.201	1.049	5.1/29	1694±411	0.192	1.047	0.042
5-15	3.62/29	1330±316	0.199	1.051	3.6/29	1330±316	0.190	1.048	0.042
15-30	5.55/29	846±206	0.196	1.054	5.6/29	846±206	0.186	1.051	0.042
30-60	2.63/29	433±113	0.178	1.066	2.6/29	433±113	0.167	1.074	0.045
60-80	10.6/29	139±48	0.152	1.080	10.6/29	139±48	0.141	1.062	0.035
80-92	9.10/29	74±45	0.121	1.098	9.1/29	74±42	0.110	1.089	0.033

is indeed the case if one puts together results for e^+e^-, $p\bar{p}$ and AA collisions.

3 Analysis of k_t distributions using stochastic approach in y_t space

Whereas previous approach was concerned with extension of the purely statistical approach the one presented now will go bit further by modelling hadronization process by a kind of diffusion mechanism Refs. [9,10,14] in which the original energy of projectiles is being dissipated in some well defined way into a number of produced secondaries occurring in different part of the phase space [3]. In the case considered here it is diffusion process taking place in the k_t space. Actually, it turns out that it is more suitable to consider such diffusion as taking place in the $y_t = \sinh^{-1}(k_t/m)$ space. In this case one obtains the following Fokker-Planck equation:

$$\frac{\partial P_t(y_t,t)}{\partial t} = \gamma \left[\frac{\partial y_t P_t(y_t,\, t)}{\partial y_t} + \frac{\sigma_t^2}{2\gamma} \frac{\partial^2 P_t(y_t,t)}{\partial y_t^2} \right], \quad (8)$$

[3] It should be mentioned here that there exist also non-extensive versions of such diffusion process applied to multiparticle production data [20] but we shall not pursue this possibility here.

Table 3. Values of characteristic parameters used to fit data on k_t-distributions at different centralities by using stochastic approach as given by Eq. (9) and presented in Fig. 4 for data at $\sqrt{s_{NN}} = 200$ GeV obtained by BRAHMS [1] and STAR [2] Collaborations and at $\sqrt{s_{NN}} = 130$ GeV obtained by PHENIX Collaboration [3]. The order of magnitude of the corresponding errors for T_0, δT_0, are listed below as well.

BRAHMS Coll.; $\delta T_0 = 0.008$-0.012; $\delta m = 0.024$-0.031				
C. C. (%)	χ^2/n.d.f.	C	T_0 (GeV)	m (GeV)
0-10	39.9/23	140±9	0.201	0.784
10-20	24.2/23	108±7	0.199	0.725
20-40	20.1/23	72±5	0.196	0.671
40-60	11.9/23	38±4	0.185	0.577
60-80	4.06/22	4.3±0.5	0.184	0.515

STAR Coll.; $\delta T_0 = 0.004$-0.006; $\delta m = 0.009$-0.014				
C. C. (%)	χ^2/n.d.f.	C	T_0 (GeV)	m (GeV)
0-5	221/32	484±22	0.169	0.533
5-10	124/32	370±18	0.170	0.547
10-20	121/32	310±14	0.168	0.513
20-30	92.9/32	217±10	0.168	0.498
30-40	89.9/32	158±8	0.165	0.473
40-60	43.9/32	99.6±5.0	0.157	0.419
60-80	22.3/32	43.2±2.3	0.143	0.349
$p+p$	17.4/29	5.29±0.63	0.126	0.298

PHENIX Coll.; $\delta T_0 = 0.020$-0.037; $\delta m = 0.058$-0.078				
C. C. (%)	χ^2/n.d.f.	C	T_0 (GeV)	m (GeV)
0-5	8.06/29	161±34	0.185	0.734
5-15	5.61/29	124±26	0.185	0.729
15-30	7.27/29	80±18	0.185	0.700
30-60	3.49/29	39±9	0.177	0.594
60-80	11.1/29	12±4	0.158	0.460
80-92	9.01/29	6.1±3.6	0.131	0.317

Its solution can be expressed by Gaussian distribution,

$$\frac{d^2\sigma}{2\pi k_t dk_t} = CP_t(y_t,t) = \frac{C}{\sqrt{2\pi V_t^2(t)}}\exp\left[-\frac{y_t^2}{2V_t^2(t)}\right], \quad (9)$$

with[4]

$$2V_t^2(t) = \frac{\sigma_t^2}{\gamma}(1-e^{-2\gamma t}). \quad (10)$$

In Fig. 4 we show our results of using Eq. (9). It should be noticed that now, following [14], we have regarded mass m to be a free parameter. Only then we can obtain good agreement with data. In a sense, variable mass m corresponds in this approach to the non-extensivity parameter q introduced in Section 2 in that it summarily accounts for some additional effects not accounted by simple diffusion process (like, for example, effect of resonances and flow).

In the stochastic approach considered here we do not have direct access to the temperature T_0. It is accessible only if we additionally assume validity of the Einstein's fluctuation-dissipation relation, which in our case means that measure of the size of diffusion (dissipation), $V_t^2(t)$, can be expressed by the temperature T_o and mass m:

$$V^2(t) = \frac{T_0}{m}. \quad (11)$$

Therefore our results for V^2 shown in Fig. 4 (see inlets), where $V(t)^2$ increases with increasing centrality, would indicate that temperature T_0 obtained by applying Einstein's relation with m kept constant would increases with centrality as well, contrary to what has been obtained above by applying q-statistical approach. We have allowed then (following [14]) the mass m to be a free parameter and the best fit is obtained when m decreases with centrality, see inlets in Fig. 4. The resulting temperature, $T_0 \simeq m \cdot V_t^2$, behaves then in essentially the same way as function of centrality as in the q-statistical approach, cf., Table 3 and Fig. 5[5].

4 Concluding remarks

We have provided here systematic analysis of recent RHIC data on k_t distributions [1–3] by using three different kinds of statistical approaches: Hagedorn model [12], two versions of the modified statistical based on Tsallis statistics [5] and a suitable adaptation of the stochastic model proposed in [9]. We have found that Hagedorn-type model [12] cannot fit data at large k_t (its widely used for quick estimations simplified version with $\rho(m) = 1$, which is then just a simple Boltzmann gas model with only one parameter, the temperature T_0, fails completely even for smaller k_t, cf. Table 4). However, these data can still be reasonably well fitted either by non-extensive extensions of statistical model [5] or by picture of some suitable diffusion process taking place in transverse rapidity space [9,10]. This is specially true if one limits itself to $k_t < 5$ GeV/c range as the case of BRAHMS [1] and PHENIX [3] data, the $k_t < 12$ GeV/c range considered in STAR experiment [2] seems to be already too big to be fitted properly even with these two approaches (the corresponding values of the χ^2 are considerably bigger in this case and the values of parameters obtained for STAR and BRAHMS data, which were taken at the same collision and at the same energy, are also different).

As is shown in Fig. 5, the temperatures T_0 obtained in modified statistical and stochastic approaches (with varying mass m) are essentially very similar to each other and follow the same dependence on the centrality, namely T_0

[4] See also [9]. Actually, Eq. (9) is the same as the formula used already long time ago in [11].

[5] It should be stressed here that for constant value of mass, $m = 0.14$ GeV as used for q-statistics case above, we would have obtained somewhat higher values of χ^2's. In addition, it is interesting to observe at this point that the fact that we can fit data within modified stochastic approach only by allowing for a kind of "quasiparticles" of mass m, different for different centralities, corresponds in a sense to introducing parameter q to the usual statistical model. The possible dynamical origin and meaning of such variable mass is, however, still lacking.

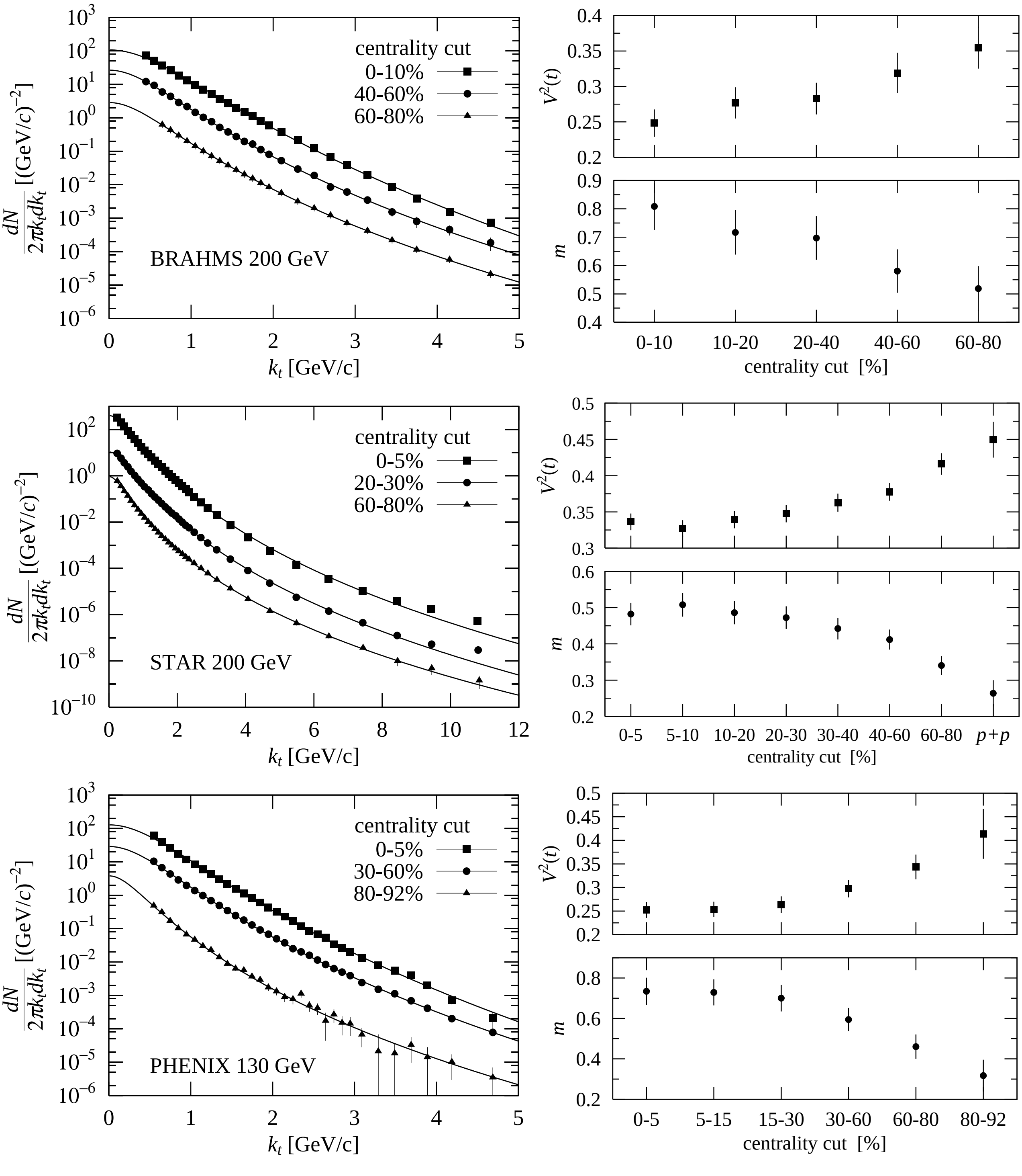

Fig. 4. Results of application of stochastic approach as given by Eq. (9) to data for k_t-distributions at $\sqrt{s_{NN}} = 200$ GeV measured for different centralities by BRAHMS [1] and STAR [2] Collaborations and at $\sqrt{s_{NN}} = 130$ GeV obtained by PHENIX Collaboration [3]. Notice that mass m is treated here as free parameter, in similar way as in [14].

Table 4. Comparison of investigated models: simple statistical model (i.e., Hagedorn model as given by eq. (1) but with $\rho(m) = 1$, in which case it is just a simple statistical Boltzmann gas model with only one parameter, namely temperature T_0), non-extensive Tsallis distribution (NETD) and Ornstein-Uhlenbeck process (O-U), using data on k_t distributions at $\sqrt{s_{NN}} = 200$ GeV obtained by BRAHMS Collaboration [1] for smallest and largest centralities.

C.C (%)		Simple statistical model, Eq. (1) with $\rho(m) = 1$			NETD Eq. (3) (with $Q = q/(1-q)$)			O-U Eq. (9)		
		T_0 (GeV)	q	m (GeV)	T_0 (GeV)	q	m (GeV)	T_0 (GeV)	q	m (GeV)
0-10	χ^2/n.d.f	177/23			10.2/23			39.9/23		
		0.302	—	—	0.232	1.043	—	0.201	—	0.784
60-80	χ^2/n.d.f	567/22			2.76/22			4.06/22		
		0.325	—	—	0.175	1.084	—	0.184	—	0.515

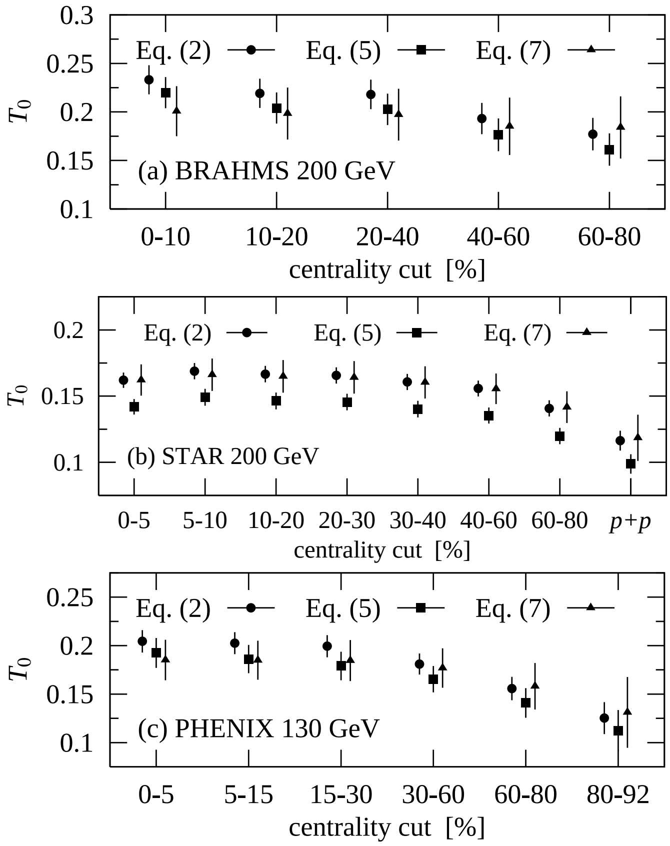

Fig. 5. Comparison of temperatures of hadronization obtained by using different approaches as given by: (a) - Eq. (3) with $Q = q/(1-q)$; (b) - Eq. (3) and $Q = 1/(1-q)$; (c) - Eq. (9). In the later case T_0 has been obtained from the values of V_t^2 and m obtained in Fig. 4 by using Einstein's relation: $T_0 = m \cdot V^2(t)$.

decreases when collision is more peripheral. However, because stochastic approach seems to be more dynamical than q-statistical one (where the true dynamical origin of the nonextensivity parameter is not yet firmly established, see [6, 15]), we regard as the most valuable our finding that stochastic approach [9, 10, 14] works so well and can serve to provide first simple estimations of any new data in the future. On the other hand we have also demonstrated that the two possible approaches using q-statistics are equivalent to each other, at least in the frame of limited phenomenological approach presented here. One should also stress at this point that q statistical approach offers unique information on fluctuations in the system, which can be translated into information on its volume. Our results for AA and pp collisions taken together with old results for e^+e^- annihilations indicate in this respect distinct growth of the expected volume of interactions from the most elementary annihilation processes to the nuclear collisions.

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