

Diffusion or bounce back in relativistic heavy-ion collisions?

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Abstract. The time evolution of pseudorapidity distributions of produced charged hadrons in d+Au collisions at $\sqrt{s_{NN}} = 200$ GeV is investigated. Results of a nonequilibrium-statistical Relativistic Diffusion Model with three sources are compared with a macroscopic "bounce back" model that does not allow for statistical equilibration at large times, but instead leads to motion reversal. When compared to the data, the results of the diffusion approach are more precise, thus emphasizing that the system is observed to be on its way to thermal equilibrium.

PACS. 25.75.-q Relativistic heavy-ion collisions – 24.60.Ky Fluctuation phenomena

1 Introduction

It has recently been shown that rapidity distributions of net protons [1, 2] and pseudorapidity distributions of produced charged hadrons [3, 4, 7] in relativistic heavy-ion collisions can be described very precisely in a nonequilibrium-statistical Relativistic Diffusion Model (RDM) with three sources. There are two sources with initial rapidities close to the beam values that arise mainly from the valence quarks, and a third midrapidity source that is mostly due to gluon-gluon collisions, which reaches statistical equilibrium with respect to the variable pseudorapidity during the time evolution in the diffusion model. The particles in this equilibrium source move collectively with very large velocities [2] close to light velocity similar to a blast wave [8]. They may have hadronized from an equilibrated quark-gluon plasma.

In case of the asymmetric system d+Au at the highest RHIC energy $\sqrt{s_{NN}} = 200$ GeV, we could demonstrate that this analytical model of a many-particle system on its way to statistical equilibrium reproduces the complicated features of the experimental distribution functions to a high degree of accuracy. Signatures such as the steeper slope of the distributions in the deuteron direction as compared to the gold direction, the gradual change of the pseudorapidity distributions with centrality, and the dominance of particle production in the Au-like rapidity region towards more central collisions arises in a natural way from the nonequilibrium-statistical description. The measured pseudorapidity distributions remain rather asymmetric because strong interaction stops before the system reaches statistical equilibrium.

In this work we compare the time evolution of the pseudorapidity distribution in the nonequilibrium-statistical RDM [1–4] with a schematic macroscopic model that does *not* incorporate statistical equilibrium as a limit for large times (except for the midrapidity source), but instead leads to a separation of the beam-like distribution functions in pseudorapidity space. In the course of their time evolution, the mean values of the beam-like distribution functions change their signs. This corresponds to motion reversal, or a "bounce back" of the produced particles in the beam-like sources with respect to the beam directions. Such a model does not lead to statistical equilibrium in the system for large times since the partial distributions separate in pseudorapidity space.

In case of an asymmetric system such as d+Au investigated here, it is possible to decide whether agreement with the data occurs before or after motion reversal. Although a corresponding distinction is not possible for symmetric systems such as Au+Au, the results for d+Au will indicate that symmetric systems behave in an analogous way.

The Relativistic Diffusion Model is reconsidered briefly in the next section. Afterwards, the bounce back approach is outlined, and both models with their corresponding time evolution are then compared to the available PHOBOS minimum-bias [9] and centrality dependent [10] d+Au data at $\sqrt{s_{NN}} = 200$ GeV, and to each other. Such a comparison is conceptually quite important because a preference for the diffusion case would emphasize that the relativistic systems are indeed observed to be on their way to thermal equilibrium.

2 Nonequilibrium-Statistical Approach

The nonequilibrium-statistical approach is based on diffusion equations which eventually lead to statistical equilibrium for large times. In the three-sources model, we have successfully used linear Fokker-Planck equations (FPEs) [1,3,4] for the components $R_k(y, t)$ of the distribution function for produced charged hadrons in rapidity space

$$\frac{\partial}{\partial t} R_k(y, t) = \frac{1}{\tau_y} \frac{\partial}{\partial y} \left[(y - y_{eq}) \cdot R_k(y, t) \right] + \frac{\partial^2}{\partial y^2} \left[D_y^k \cdot R_k(y, t) \right] \quad (1)$$

with the rapidity $y = 0.5 \cdot \ln((E+p)/(E-p))$. The diagonal components D_y^k of the rapidity diffusion tensor contain the microscopic physics in the respective beam-like ($k = 1, 2$) and central ($k = 3$) regions. They account for the broadening of the distribution functions through interactions and particle creations. The off-diagonal terms of the diffusion tensor account for correlations between the three sources. They are expected to be small, and we neglect them in this work. The rapidity relaxation time τ_y determines the speed of the statistical equilibration in y -space.

Whereas an equilibrium-statistical description of relativistic heavy-ion collisions works well for certain observables such as abundance ratios of produced hadrons (thermal model), a nonequilibrium-statistical formulation as outlined here is required for distribution functions and other more sophisticated observables. A description of the dynamics should be part of such a formulation. In the present work, the dynamics enters through the time variable in the FPE that describes the evolution of the three sources for particle production in rapidity space.

For time to infinity, both the mean values and the variances of the three sources reach the equilibrium values with respect to the variable rapidity. Then the incoherent sum of the three distributions represents the overall statistical equilibrium distribution in rapidity space and after Jacobi-transformation, in pseudorapidity space. Comparison with the data for d+Au at the highest RHIC energy shows, however, that strong interaction stops before this equilibrium distribution is reached.

In the nonequilibrium-statistical model as well as in the bounce back approach, the initial particle production in the three sources is assumed to occur with full strength at very short times. In the diffusion case, we use in this work the initial conditions $R_{1,2}(y, t = 0) = \delta(y \pm y_{max})$ with the maximum rapidity $y_{max} = 5.36$ at the highest RHIC energy of $\sqrt{s_{NN}} = 200$ GeV (beam rapidities are $y_{1,2} = \mp y_{max}$), and $R_3(y, t = 0) = \delta(y)$. A midrapidity gluon-dominated symmetric source had also been proposed by Bialas and Czyz [6]. It may originate from a thermally equilibrated quark-gluon plasma.

These initial conditions are slightly modified for the central source as compared to our previous investigation [4], where we used $R_3(y, t = 0) = \delta(y - y_{eq})$. The motivation is twofold: We want to use identical initial conditions when comparing diffusion and bounce back approach, and we want to investigate whether the modified initial condi-

tion allows to maintain the quality of the agreement with the data in the diffusion approach.

The new initial condition for the midrapidity source corresponds to initial ($t=0$) particle production at rest, independently of the mass of the collision partners: the third source is created at $y=0$. Hence, this gluon-dominated source is initially insensitive to the mass distribution in the system, and it is only at larger times - when the particles in this source have already been created - that the drift towards the equilibrium value sets in. Since there are very few valence quarks in the midrapidity region at the highest RHIC energy, this initial condition is probably more realistic than the one we used in [4].

The mean values in the three sources have the time dependence

$$\langle y_{d1,2}(t) \rangle = y_{eq} [1 - \exp(-t/\tau_y)] \mp y_{max} \exp(-t/\tau_y) \quad (2)$$

for the sources (1) and (2), and

$$\langle y_{d3}(t) \rangle = y_{eq} [1 - \exp(-t/\tau_y)] \quad (3)$$

for the moving central source. The three mean values reach the equilibrium value for time to infinity. In our previous RDM-calculation [4] with slightly different initial condition, the mean value of the central source was at the equilibrium value $\langle y_3(t) \rangle = y_{eq}$ independently of time, thus assuming instant equilibration in this source regarding the mean values. The variances are obtained as in [5]. It will turn out that for d+Au at the highest RHIC energy, we can not determine from a comparison with the data which of the two possibilities for the initial conditions of the central source is more realistic because the χ^2 is nearly identical in both cases.

We follow the subsequent diffusion-model time evolution in pseudorapidity space up to the interaction time τ_{int} , when the produced charged hadrons cease to interact strongly. The quotient τ_{int}/τ_y is determined from the minimum χ^2 with respect to the data, simultaneously with the minimization of the other free parameters - in particular, the variances of the three partial distribution functions, and the number of particles produced in the central source. In the nonequilibrium-statistical approach, the equilibrium value of the rapidity and its dependence on centrality is calculated from energy and momentum conservation in the system of participants as

$$y_{eq}(b) = \frac{1}{2} \ln \frac{\langle m_1^T(b) \rangle \exp(-y_{max}) + \langle m_2^T(b) \rangle \exp(y_{max})}{\langle m_2^T(b) \rangle \exp(-y_{max}) + \langle m_1^T(b) \rangle \exp(y_{max})} \quad (4)$$

with the transverse masses $\langle m_{1,2}^T(b) \rangle = \sqrt{(m_{1,2}^2(b) + \langle p_T \rangle^2)}$, and masses $m_{1,2}(b)$ of the "target" (Au)- and "projectile" (d)-participants that depend on the impact parameter b . The average numbers of participants $\langle N_{1,2}(b) \rangle$ from the Glauber calculations reported in [10] for minimum bias d + Au at the highest RHIC energy are $\langle N_1 \rangle = 6.6$, $\langle N_2 \rangle = 1.7$, which we had also used in [4, 5]. With $\langle p_T \rangle = 0.4$ GeV/c the result is $y_{eq} = -0.664$, with $\langle p_T \rangle = 1$ GeV/c we obtain $y_{eq} = -0.60$.

The average numbers of charged particles in the target- and projectile-like regions $N_{ch}^{1,2}$ are proportional to the respective numbers of participants $N_{1,2}$,

$$N_{ch}^{1,2} = N_{1,2} \frac{(N_{ch}^{tot} - N_{ch}^{eq})}{(N_1 + N_2)} \quad (5)$$

with the constraint $N_{ch}^{tot} = N_{ch}^1 + N_{ch}^2 + N_{ch}^{eq}$. Here the total number of charged particles N_{ch}^{tot} is determined from the data. The average number of charged particles in the equilibrium source N_{ch}^{eq} is a free parameter that is optimized together with the variances and τ_{int}/τ_y in a χ^2 -fit of the data using the CERN minuit-code. Due to the accuracy of the data, these five free parameters (for both scenarios, diffusion and bounce back) are determined with great precision.

The FPE is solved analytically as outlined in [4, 5], the result is converted to pseudorapidity space, and compared to data, fig.1, right-hand column. Here the time evolution parameter p in the numerical calculation is defined as ¹

$$p = 1 - \exp(-t/\tau_y). \quad (6)$$

We have shown in [4, 5] that with this RDM approach, the centrality dependence of the measured pseudorapidity distributions [10] from central to very peripheral collisions can be modeled in considerable detail. For peripheral collisions, the asymmetry of the overall distribution is not yet pronounced because here the d- and the Au-like partial distributions are similar in size due to the small number of participants. Towards more central collisions, the number of gold participants rises, and the corresponding partial distribution of produced particles becomes more important. In addition, the distributions drift towards the equilibrium value. Both effects produce the asymmetric shape, which is also seen in minimum-bias.

The minimum-bias result that we present here in more detail also shows the asymmetric shape, which is very well reproduced in the diffusion calculation. At larger values of the time evolution parameter p , all three subdistributions tend to become symmetric in y with respect to the equilibrium value y_{eq} , indicating the approach to thermal equilibrium. At $p=0.999$, the equilibrium state is already closely approached. The slight asymmetry is due to the conversion from rapidity- to pseudorapidity space which tends to produce a dip at $\eta = 0$. For time to infinity, statistical equilibrium in pseudorapidity space would be reached.

It is interesting to determine actual values for the diagonal components of the diffusion tensor from the expression for the variances, eq.(11), provided the interaction time is known. As an example, for an interaction time of 7fm/c in d+Au and $\tau_{int}/\tau_y=0.78$, the resulting values of the three diffusion coefficients in minimum-bias are $D_y^{1,2,3} = 1.55, 0.46$ and 0.98 c/fm. These are effective values as explained in ref.[2] because they include the influence of collective expansion in addition to the statistical fluctuations.

¹ There is a difference of a factor of two in the exponent as compared to the definition of p used in [5], which causes different t/τ_y values for given p .

Table 1. Produced charged hadrons in minimum-bias d + Au collisions at $\sqrt{s_{NN}} = 200$ GeV, $y_{1,2} = \mp 5.36$ in the "bounce back" model (1st line), and in the Relativistic Diffusion Model (2nd line). The equilibrium value of the rapidity in the RDM is y_{eq} , the time parameter (see text) is p , the corresponding value of interaction time over relaxation time is τ_{int}/τ_y , the variance of the central source in y -space is σ_3^2 . The number of produced charged particles is $N_{ch}^{1,2}$ for the sources 1 and 2 and N_{ch}^3 for the central source, the percentage of charged particles produced in the midrapidity source is n_{ch}^3 .

y_{eq}	p	τ_{int}/τ_y	σ_3^2	N_{ch}^1	N_{ch}^2	N_{ch}^3	n_{ch}^3 (%)
–	0.30	0.36	1.32	66	17	3	3
-0.664	0.54	0.78	4.19	55	14	22	24

3 Bounce Back Approach

In this schematic model of motion reversal in pseudorapidity space, the beam-like partial distribution functions change their signs in the course of the time evolution of the collision. The behaviour of the system is approximated through a linear partial differential equation which is similar to a Fokker-Planck equation, but differs in that it does not lead to statistical equilibrium in the system for large times. Instead it causes a separation of the beam-like sources in rapidity space:

$$\frac{\partial}{\partial t} R_k(y, t) = \frac{1}{\tau_y} \frac{\partial}{\partial y} \left[(y - y_k) \cdot R_k(y, t) \right] + \frac{\partial^2}{\partial y^2} \left[D_y^k \cdot R_k(y, t) \right]. \quad (7)$$

Here the drift term does not account for the equilibration towards $y_{eq}(b)$ as in the FPE, but rather describes the motion of both peripheral distributions in rapidity space towards smaller absolute values of the rapidity, and eventually into the other hemisphere of rapidity space - which means motion reversal, or bounce back. The drift term that induces this behaviour of the beam-like partial distributions is $(y - y_k) = (y \mp y_{max})$ for $k=1,2$. The drift term for the midrapidity source is $(y - y_3) = y$.

We use the same τ_y as in the RDM case in order to have a direct comparison of the two scenarios. In the bounce-back scenario, there is no relaxation towards statistical equilibrium and hence, τ_y should not be considered to be a relaxation time in this case, it is just a time parameter that controls the speed of the movement in y - (or η -)space.

Because the three-component system does not approach statistical equilibrium with respect to the variable rapidity for time to infinity, the underlying system of partial differential equations for the bounce back case differs in an important aspect from the ordinary Fokker-Planck, or Uhlenbeck-Ornstein [11] framework, although it looks formally quite similar ².

The initial conditions for the beam-like partial distribution functions are taken to be the same as in the nonequilibrium-statistical diffusion approach. Since the

² A preliminary version of the bounce back approach had been proposed in [12]

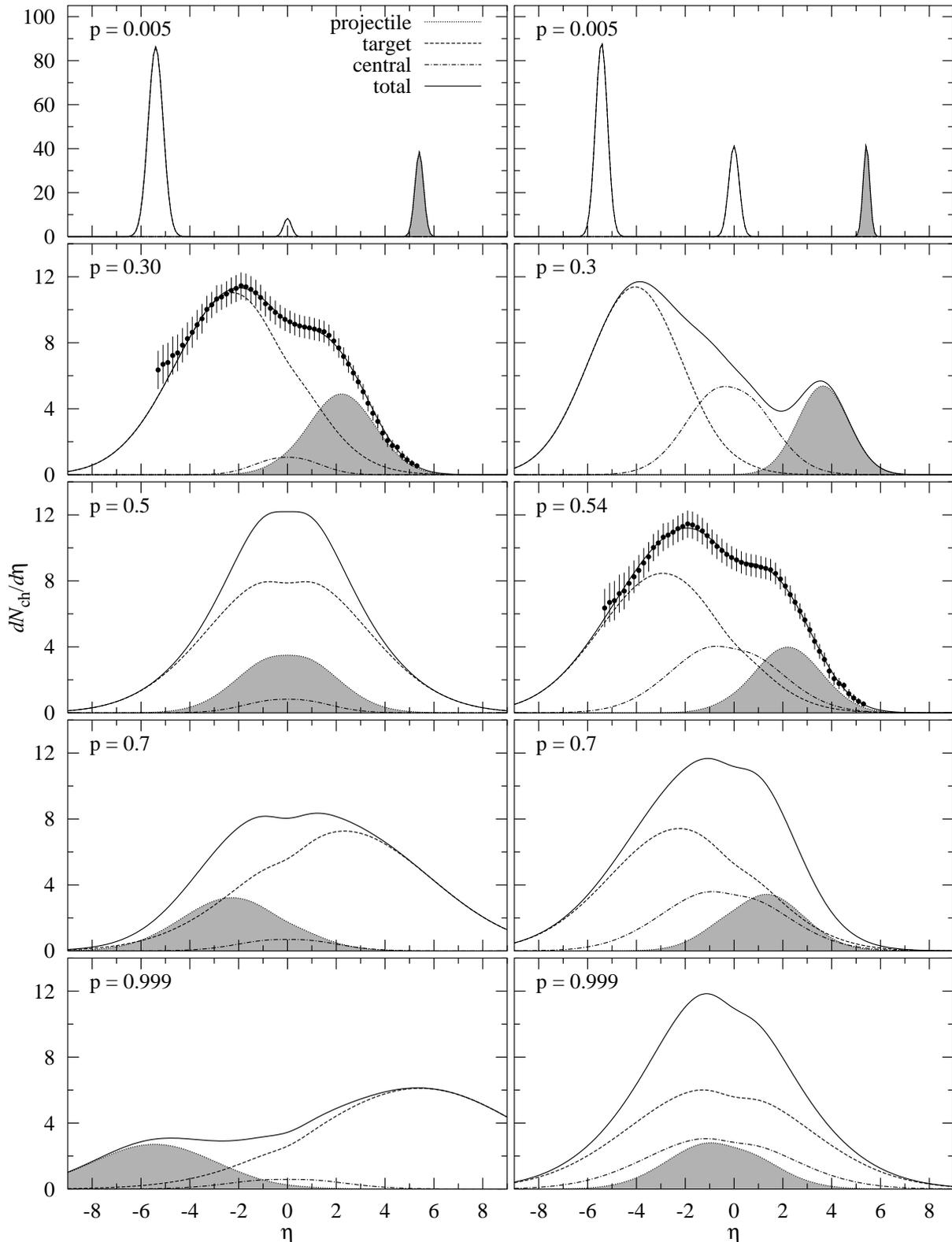


Fig. 1. Time evolution of pseudorapidity distributions for produced charged particles from minimum-bias d + Au collisions at $\sqrt{s_{NN}} = 200$ GeV. Results for five time-steps (p -values, cf. text) are shown. In the left-hand column, the bounce back calculation with three sources is displayed. Here the Au- and d-like (grey) distributions reverse their motion in the mean at large times, and then tend to separate. Agreement with the data can only be achieved "on the way in" at $p=0.30$, before bounce back occurs. Dash-dotted curves show the midrapidity sources for hadron production. The right-hand column shows the diffusion-model calculation where statistical equilibrium would be achieved for large times at the equilibrium value η_{eq} . The diffusion-model evolution reaches agreement with the data much slower, at $p=0.54$. The minimum $\chi^2/d.o.f.$ -values with respect to the PHOBOS data [9] are 2.4/48 for diffusion, and 7.4/48 for bounce back.

equation is linear, a superposition of the partial distribution functions using the initial conditions $R_{1,2}(y, t = 0) = \delta(y \pm y_{max})$ with the maximum rapidity y_{max} , and $R_3(y, t = 0) = \delta(y)$ yields the exact solution. In the solution, the mean values are obtained analytically from the moments equations as

$$\langle y_{bb1,2}(t) \rangle = \pm y_{max} [1 - \exp(-t/\tau_y)] \mp y_{max} \exp(-t/\tau_y) \quad (8)$$

for the sources (1) and (2), and

$$\langle y_{bb3}(t) \rangle = 0 \quad (9)$$

for the third source, which rests at 0 in the bounce back case during the whole time evolution. It is therefore obvious that the mean values of the beam-like distribution functions start their time evolution at $t=0$ with values $y_{1,2}(t=0) = \mp y_{max}$, and for time to infinity they reach $y_{1,2}(t \rightarrow \infty) = \pm y_{max}$. Hence, they change from one rapidity hemisphere into the other one at a value of the bounce back time

$$\frac{t_{bb}}{\tau_y} = -\ln(1/2) = 0.6932 \quad (10)$$

which corresponds to a value of the time evolution parameter $p=0.5$, as can be seen in the middle frame of the bounce back case in fig. 1. We use the same τ_y in sections 3,4 in order to have a direct comparison of the two scenarios. Of course in the bounce back scenario, there is no relaxation towards statistical equilibrium and hence, τ_y should not be considered to be a relaxation time in this case. Here it is a time parameter that controls the speed of the movement in y - (or η -)space.

The variances are as in the diffusion case

$$\sigma_{1,2,3}^2(t) = D_y^{1,2,3} \tau_y [1 - \exp(-2t/\tau_y)]. \quad (11)$$

The conversion to pseudorapidity is required because particle identification is not available: $\eta = -\ln[\tan(\theta/2)]$ with the scattering angle θ measured relative to the direction of the deuteron beam. Hence, particles that move in the direction of the gold beam have negative, particles that move in the deuteron direction have positive pseudorapidities. The conversion from y - to η - space of the rapidity density

$$\frac{dN}{d\eta} = \frac{p}{E} \frac{dN}{dy} = J(\eta, \langle m \rangle / \langle p_T \rangle) \frac{dN}{dy} \quad (12)$$

is performed through the Jacobian

$$J(\eta, \langle m \rangle / \langle p_T \rangle) = \cosh(\eta) \cdot [1 + (\langle m \rangle / \langle p_T \rangle)^2 + \sinh^2(\eta)]^{-1/2}. \quad (13)$$

We approximate the average mass $\langle m \rangle$ of produced charged hadrons in the central region by a value somewhat higher than the pion mass m_π , and use a mean transverse momentum $\langle p_T \rangle = 0.4$ GeV/c such that the relevant parameter in the Jacobian becomes $\langle m \rangle / \langle p_T \rangle = 0.45/c$.

Due to the conversion from y - to η -space, the partial distribution functions are different from Gaussians. The charged-particle distribution in rapidity space is obtained in both the diffusion model, and the bounce back case as incoherent superposition of nonequilibrium and local equilibrium solutions of (1)

$$\frac{dN_{ch}(y, t = \tau_{int})}{dy} = N_{ch}^1 R_1(y, \tau_{int}) + N_{ch}^2 R_2(y, \tau_{int}) + N_{ch}^3 R_3(y, \tau_{int}) \quad (14)$$

with the interaction time τ_{int} (total integration time of the differential equation). In the present work, the integration is stopped at the value of τ_{int}/τ_y that produces the minimum χ^2 with respect to the data and hence, the explicit value of τ_{int} is not needed as an input. The resulting values for τ_{int}/τ_y are given in table 1 together with the widths of the central distributions, and the particle numbers in the three sources.

We show the time evolution in the bounce back case together with the fit to the PHOBOS data [9] in the left-hand column of fig.1. It is evident that the two beam-like distribution functions move towards smaller pseudorapidities as time increases, reach agreement with the data at $p=0.30$ before motion reversal in the mean is achieved, then move on in pseudorapidity space to reverse their motion in the mean at $p=0.5$, and finally proceed until they would separate according to the time evolution given by eq.(7). At $p = 0.999$, a very broad distribution in pseudorapidity has emerged, which deviates considerably from the near-thermal distribution that can be seen in the diffusion case at the same time step.

The RHIC data of the asymmetric d+Au system show very clearly that motion reversal in the mean is not observed, since the maximum occurs at negative pseudorapidities in the gold direction. Only a certain fraction of the produced particles moves opposite to the respective beam direction. Although in the bounce back case, the underlying system of differential equations shows a memory in the exit channel (P- and T-like distributions emerge separately at large times) the effect is not observed, because the measurement occurs before that would happen.

Comparing to the diffusion-model time evolution on the right-hand side of fig.1, it is obvious that agreement with the data is reached much faster in the bounce back case ($p=0.30$ as compared to $p=0.54$). The reason is found in the respective equations for the mean values (2), (8). To first order in t/τ_{int} , the diffusion result is

$$\langle y_{d1,2} \rangle \approx \mp y_{max} \pm y_{max} t/\tau_y \quad (15)$$

whereas in the bounce back case

$$\langle y_{bb1,2} \rangle \approx \mp y_{max} \pm 2y_{max} t/\tau_y \quad (16)$$

and hence, the drift in the bounce back case for fixed t/τ_y is much stronger than in the diffusion case.

Because the underlying partial differential equation conserves the norm, the particle number in each distribution function does not change in our schematic model once

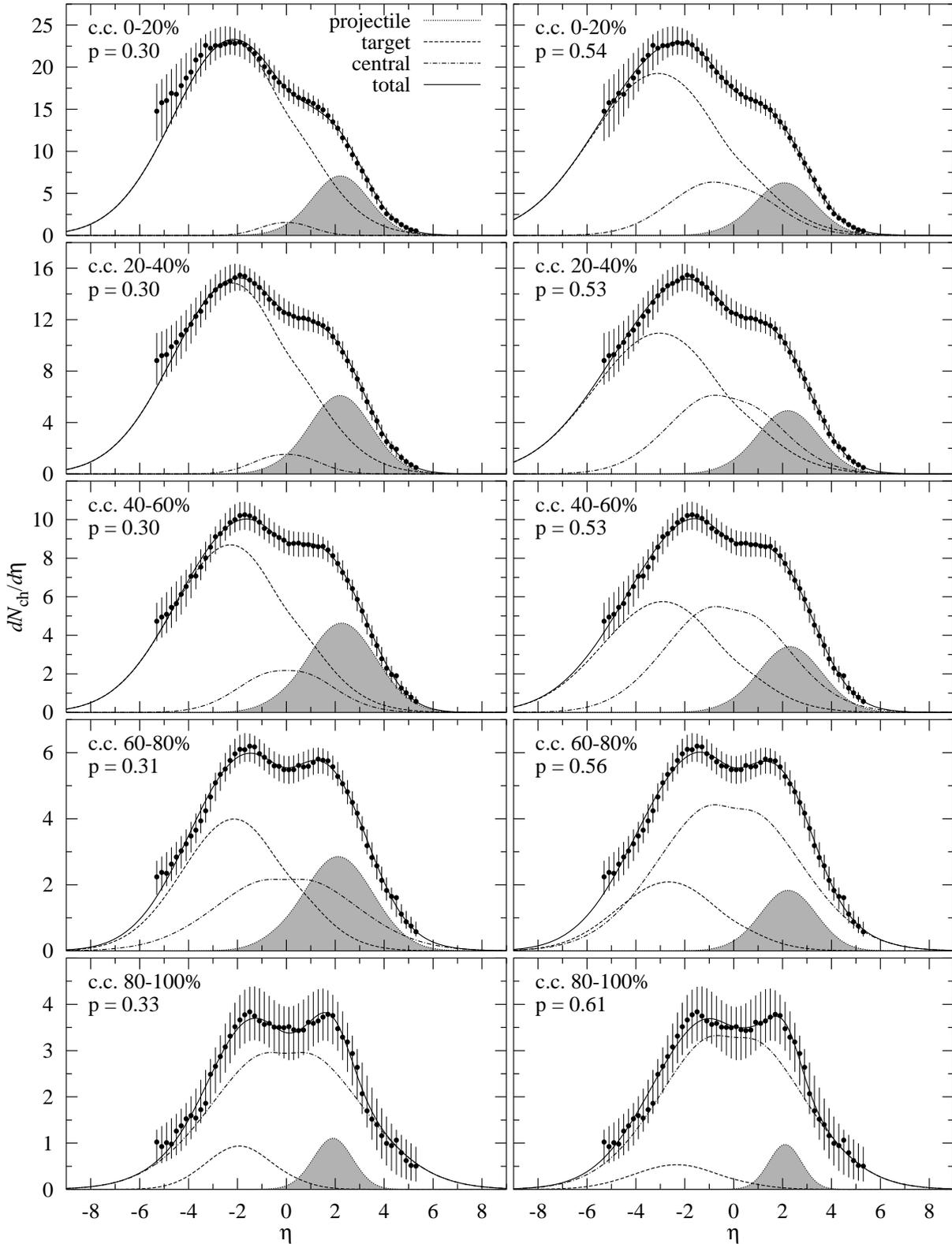


Fig. 2. Calculated pseudorapidity distributions of produced charged particles in $d + \text{Au}$ collisions at $\sqrt{s_{NN}} = 200$ GeV for five different centralities. Central collisions are shown in the top frames, peripheral at the bottom. In the left-hand column, results of the bounce back calculation with three sources are shown. The right-hand column gives results of the diffusion-model calculation where statistical equilibrium would be achieved for large times at the equilibrium value η_{eq} . The diffusion-model evolution provides a more detailed agreement with the PHOBOS data [10].

the particles are created. The χ^2 -minimization yields a different partition of the produced charged hadrons among the three sources as compared to the diffusion case, table 1. In particular, the percentage of particles in the midrapidity source is much smaller as in the diffusion case, only 3 % instead of 24 %. As is evident from fig.1 and table 1, the diffusion case yields better agreement with the data. However, both the bounce back and the diffusion evolution still appear to be acceptable models, and a clear distinction between the two becomes apparent only in the subsequent undetected time evolution. Hence, it seems not straightforward so far to conclude from this analysis that the system is observed to be on its way to statistical equilibrium. To prove that this is likely to be the correct physical interpretation, as outlined in [1–5], one indeed needs additional physical information.

A possibility to obtain this information is the investigation of the centrality dependence as performed in [4,5] for the RDM-case. Such an impact-parameter dependent analysis is very sensitive to the details of the model. We have performed this analysis for the bounce back case, and repeated it for the RDM-case with the same initial condition for the central source. We use the same Glauber values for the average number of participants in each centrality bin as in [10,4,5].

The results as presented in fig.2 show that the RDM gives consistently better agreement with the data for all centralities, in particular for more central collisions where the diffusion approach is expected to function best because of the larger number of participants. The χ^2 -values of the bounce back calculation vary from 12.3 for 0 – 20% (central) to 4.9 for 80 – 100% (peripheral), as compared to 1.5 (central) and 4.6 (peripheral) in the diffusion case. To obtain better results in the bounce back case, deviations from the Glauber values would be required which are not realistic. As an example, for $\langle N_1 \rangle = 7$ and $\langle N_2 \rangle = 1$ in minimum bias we obtain $\chi^2 = 2.1$, which is comparable to the diffusion result with the Glauber values $\langle N_1 \rangle = 6.6$ and $\langle N_2 \rangle = 1.7$, but the value for the deuteron participants is unrealistic.

The interpretation provided by the Relativistic Diffusion Model is also favoured by the experimental fact that the distributions of produced particles in transverse momentum space are very close to thermal distributions. Hence, it seems natural to infer that the system tends to approach thermal equilibrium, and that this tendency is seen in the longitudinal variables as well.

We have confined this investigation to a comparison with the PHOBOS data because these provide a large number of data points with high accuracy, giving better constraints of the free parameters than the corresponding BRAHMS [13], and STAR results [14].

4 Conclusion

To conclude, we have compared two schematic models for charged-hadron production in relativistic d+Au collisions at $\sqrt{s_{NN}} = 200$ GeV for minimum bias, and depending on centrality. The nonequilibrium-statistical Relativistic

Diffusion Model (RDM) describes the gradual approach of the system towards statistical equilibrium. The rapidity distribution functions become symmetric at large times with respect to the equilibrium value of the rapidity, which is obtained from energy- and momentum conservation in the system of participants.

Since strong interaction stops before the system reaches equilibrium, the pseudorapidity distribution for minimum-bias collisions remains rather asymmetric. In this approach, all the details of the experimental distribution function such as the different slopes in the respective gold and deuteron directions are precisely reproduced for all centralities with good χ^2 -values. The midrapidity source contains 24% of the produced charged hadrons.

In the bounce back approach, the underlying differential equation would lead to a re-separation of the partial beam-like distribution functions in η - space, rather than to equilibrium. The midrapidity source is very small in size (3%). It remains centered at rest, $\langle \eta_3(t) \rangle = 0$.

When comparing the bounce back time evolution to the available PHOBOS-data [9], it turns out that satisfactory agreement can only be achieved "on the way in" before motion reversal, because the experimental distribution peaks in the negative pseudorapidity hemisphere, fig.1. The data are reached faster during the time evolution as compared to the diffusion case because the drift is stronger. The system is quite transparent, and it is not possible to actually observe motion reversal (bounce back) in the mean of the produced-hadron distributions at RHIC energies. Most likely, this is true at LHC energies as well, since the transparency is expected to be even more pronounced at higher energies [7].

This result is particularly relevant for the interpretation of heavy symmetric systems like Au+Au or Pb+Pb, which are expected to behave similarly, but where the two possibilities - agreement with the data "on the way in" (partial transparency), or after motion reversal (bounce back) - can not be distinguished easily due to the symmetry of the system. From the analogy with our investigation of an asymmetric system, however, it appears certain that partial transparency is the correct physical description at RHIC energies and above. For symmetric systems with $y_{eq} = 0$, the two descriptions outlined in this work are difficult to distinguish when compared to data.

For asymmetric systems, however, the differences of the two models are quite pronounced, in particular when the centrality dependence is considered. The RDM approach is more precise, and we expect that it yields converging χ^2 -minimizations with respect to the experimental pseudorapidity distributions for every relativistic heavy-ion collision also at other energies, for example, Si+Al at AGS or S+Au at SPS energies. The distribution of produced particles among the three sources, however, will vary substantially depending on incident energy, size and asymmetry of the system, and centrality. The third source vanishes at sufficiently low energy, for example, at $\sqrt{s_{NN}} = 19.6$ GeV for Au+Au [7].

In summary, consistency with the data at RHIC energies can only be achieved based on partial transparency

without motion reversal in the mean, independently of the specific model. Comparing bounce back and nonequilibrium-statistical approach in detail, the agreement of the diffusion approach with the data is definitely better. This result supports the view that the relativistic many-body system is indeed observed to be on its way towards statistical equilibrium.

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