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Modified Hagedorn formula including temperature fluctuation

-Estimation of temperatures at RHIC experiments-

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Abstract. We have systematically estimated the possible temperatures obtained from an analysis of recent data on p_t distributions observed at RHIC experiments. Using the fact that observed p_t distributions cannot be described by the original Hagedorn formula in the whole range of transverse momenta (in particular above 6 GeV/c), we propose a modified Hagedorn formula including temperature fluctuation. We show that by using it we can fit p_t distributions in the whole range and can estimate consistently the relevant temperatures, including their fluctuations.

PACS. 25.75.-q Relativistic heavy ion collisions – 12.40.Ee Statistical (extensive and non-extensive) models – 02.50.Ey Stochastic models

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1 Introduction

One of the characteristic features in every high energy collision experiment is the production of large numbers of secondaries (mostly pions). From the very beginning of the history of the multiparticle production processes, it was realized that a possible way to treat them was to employ some sort of statistical approach [1]. This idea found its most mature formulation in the statistical bootstrap model proposed by Hagedorn [2], in which the exponential growth of the number of hadronic resonances with mass is one of the most fundamental issues [3]. The proposed formula is

$$\frac{d^3\sigma}{dp^3} = C \int dm\rho(m) \exp\left(-\sqrt{p_l^2 + p_t^2 + m^2}\beta_0\right) . \quad (1)$$

In Eq.(1), $\rho(m)$ denotes the density of resonances given by

$$\rho(m) = \frac{\exp(m\beta_H)}{(m^2 + m_0^2)^{5/4}}, \quad (2)$$

where $\beta_H = 1/(k_B T_H)$, the inverse of the so called Hagedorn temperature T_H, is a parameter to be deduced from data on resonance production [4]. The other parameter is $\beta_0 = 1/(k_B T_0)$, with T_0 explicitly governing the observed energy distribution and therefore identified with the *temperature of the hadronizing system*. In the followings we put $k_B = 1$. One of the aims in the study of multiparticle production processes is therefore the best possible estimation of this quantity. To this end we would like to investigate the measured transverse momentum (p_t) distributions integrated over longitudinal degrees of freedom. From Eq.(1), we have

$$S_0 \equiv \frac{d^2\sigma}{2\pi p_t dp_t} = C \int dm\rho(m) m_t K_1(m_t\beta_0) , \quad (3)$$

where $m_t = \sqrt{p_t^2 + m^2}$ is transverse mass and K_1 is the Bessel function.

However, as was recently demonstrated by us [5], this simple formula can explain the RHIC data only in the limited range of transverse momenta, namely for $p_t \leq 6$ GeV/c. For larger values of p_t data exhibit a power-like tail. There are many attempts to explain it using some kind of nonequilibrium approach like, for example, the flow or decay of resonances (see [6] for most recent review and further references); instead of trying to exclude them we would like to investigate the possibility that the observed nonexponential spectra could result from some form of equilibrium characteristic of nonextensive thermodynamics. In fact, as was shown in [5], using an approach based either on nonextensive statistics or on stochastic approach one can successfully account for the whole range of

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the observed transverse momenta. The reason for this success is the fact that in both approaches the resultant distributions are *intrinsically non-exponential*, ranging from a power-law like form (cf. Eq.(6) below) to a gaussian in transverse rapidity [7] (which can be regarded as another implementation of the effective power-law distribution)[1]. The fact that the proposed formulas can fit *the whole range* of p_t is by itself very interesting and important observation as it shows that the power-law is present not only in very hard scale physics but that it reflects also a possible nontrivial property of hadronic matter in equilibrium (like, for example, Quark Gluon Plasma) [9].

Such properties are best seen in an approach using a nonextensive statistical model in which two parameters are used: the action of the heat bath is described now by the mean temperature T_0 and by the nonextensivity parameter q, which can be identified with some specific intrinsic fluctuations of the temperature existing in the hadronizing system under consideration [10]. In the case when these fluctuations can be described by gamma distribution one can write exact formulas [10] telling us that (cf. [10])

$$[1-(1-q)\beta_0 H_0]^{\frac{1}{(1-q)}} = \int_0^\infty e^{-\beta H_0} f_\Gamma(\beta) d\beta\,, \quad (4)$$

where

$$f_\Gamma(\beta) = \frac{1}{\Gamma(\alpha)}\left(\frac{\alpha}{\beta_0}\right)^\alpha \beta^{\alpha-1} e^{-\frac{\alpha}{\beta_0}\beta}\,, \quad \alpha = \frac{1}{q-1}\,. \quad (5)$$

In general, one refers to the concept of so called *superstatistics* introduced in [11]. In our previous work [5], RHIC data were described by the following distribution with $H_0 = \sqrt{p_l^2 + m_t^2}$:

$$\frac{d^2\sigma}{2\pi p_t dp_t} = C\int_0^\infty dp_l \left[1 - \frac{1-q}{T_0}\sqrt{p_l^2+m_t^2}\right]^{\frac{1}{1-q}}\,. \quad (6)$$

As is seen in [5], this formula leads to very good agreement with all RHIC data [12,13,14].

It is important to notice that Eq. (6) has essentially the same form as the formula proposed long time ago and used with success in many QCD-inspired power-law fits to experimental data [15,16,17] (recently used also by RHIC collaborations [18]):

$$\left(1+\frac{p_t}{p_0}\right)^{-n} \longrightarrow \begin{cases} \exp\left(-\frac{n}{p_0}p_t\right) & \text{for } p_t \to 0, \\ \left(\frac{p_0}{p_t}\right)^n & \text{for } p_t \to \infty, \end{cases} \quad (7)$$

However, one has also to realize the important difference in physical pictures leading to Eq. (6) and Eq. (7). The underlying physical picture in Eq. (7) is that the small p_t region is governed by *soft physics* described by some unknown unperturbative theory or model, and the large p_t region is governed by *hard physics* believed to be described by perturbative QCD. Contrary to it, the nonextensive formula Eq.(6), which is valid in the whole range of p_t, does not claim to originate from any particular theory. It merely offers the kind of general unifying principle, namely the existence of some kind of complicated equilibrium involving all scales of p_t, which is described by two parameters, T_0 and q: the temperature T_0 describing its mean properties and the parameter q describing action of the possible nontrivial long range effects believed to be caused by fluctuations but essentially also by some correlations or long memory effects [19] [2].

2 Calculations and results

In this paper, we would like to compare results of an analysis of p_t spectra measured at RHIC experiments [12,13,14] performed by using three approaches: the original Hagedorn model, Eq. (3), the QCD-inspired power-like formula, Eq. (7), and the modified Hagedorn formula including temperature fluctuation given by :

$$S_{tot} \equiv \frac{d^2\sigma}{2\pi p_t dp_t} = C\int dy \cosh y \int dm\rho(m) m_t \cdot$$
$$\cdot [1-\beta_0(1-q)m_t\cosh y]^{\frac{1}{1-q}}\,. \quad (8)$$

It can be written also in the form of series ($\alpha = 1/(q-1)$):

$$S_{tot} = \frac{4C}{\alpha-1}\int_{m_\pi}^\infty dm\rho(m)\frac{\beta_0 m_t^2/\alpha}{(1+\beta_0 m_t/\alpha)^\alpha}\cdot \quad (9)$$
$$\cdot\sum_{k=0}^\infty \frac{\Gamma(k+3/2)\Gamma(\alpha+1+k)}{\Gamma(\alpha+k+1/2)\Gamma(k+1)}\left(\frac{1-\beta_0 m_t/\alpha}{1+\beta_0 m_t/\alpha}\right)^k\,,$$

or, accounting for the smallness of $q-1$ encountered in our fits and of the fact that we are interested only in midrapidity region (i.e., for small y) one can write it also as[3]

$$S_{tot} \simeq C\int dy\cosh y\int dm\rho(m)m_t\cdot$$
$$\cdot\left[1+\frac{1}{2}(q-1)\beta_0^2 m_t^2 y^2\right]\cdot$$
$$\cdot\exp\left[-\beta_0 m_t\cosh y + \frac{1}{2}(q-1)\beta_0^2 m_t^2\right]\,. \quad (10)$$

Equations (9) and (10) are used to check the numerical integration of Eq.(8).

At first the STAR data [12] were analyzed using of Eq. (3) (which corresponds to $q = 1$ in Eq. (8)) and modified Hagedorn formula, Eq. (8). Results are shown in Fig. 1

[1] See also [8], where flow effect is included and relation between Gaussian-like distribution in transverse rapidity and power law behavior in p_t is discussed.

[2] The origin of such fluctuations and/or correlations must be most probably traced back to the nonperturbative QCD, cf., for example, [20].

[3] In our case, because we are integrating over the whole mass spectrum $\rho(m)$ in Hagedorn formula, we cannot simply expand in $(q-1)$ and keep only linear term as it is done on such occasions in the literature, cf., for example [21], because for large masses m the series becomes divergent.

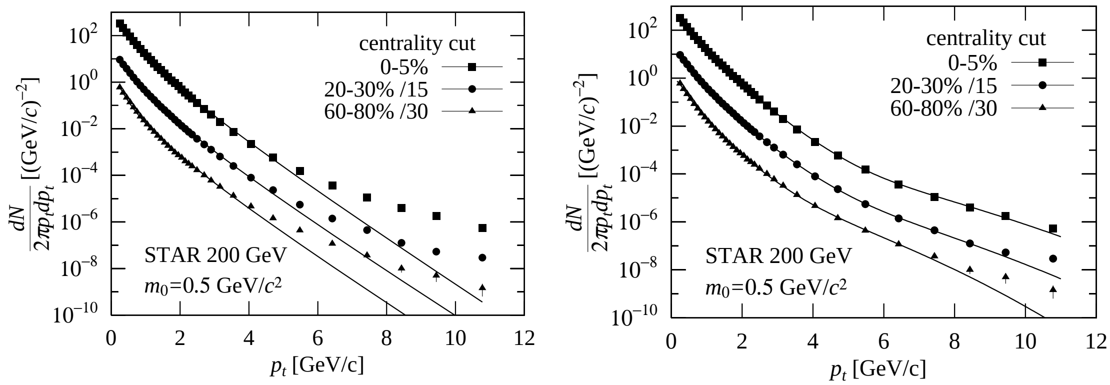

Fig. 1. Analysis of STAR data [12] by using usual Hagedorn formula (Eq.(3), left panel) and its nonextensive generalization (Eq. (8), right panel).

Table 1. Parameters of our analysis presented in Fig. 1 (left panel) by the use of Eq. (3) which corresponds to $q-1=0$ in Eq.(8). Those for right panel with $q-1 \neq 1$ in Eq.(8) can be found in Table 4. Other parameters are $m_0 = 0.5$ GeV (fixed), $\delta T_H = 0.0001$-0.002 and $\delta T_0 = 0.0001$-0.002. Notice that very large values of $\chi^2$are obtained for fits with $q-1=0$.

C. C. (%)	C	T_H (GeV)	T_0 (GeV)	χ^2/n.d.f.
0-5	816± 15	0.086	0.085	532/32
20-30	382±7	0.077	0.076	249/32
60-80	106± 2	0.037	0.037	308/32

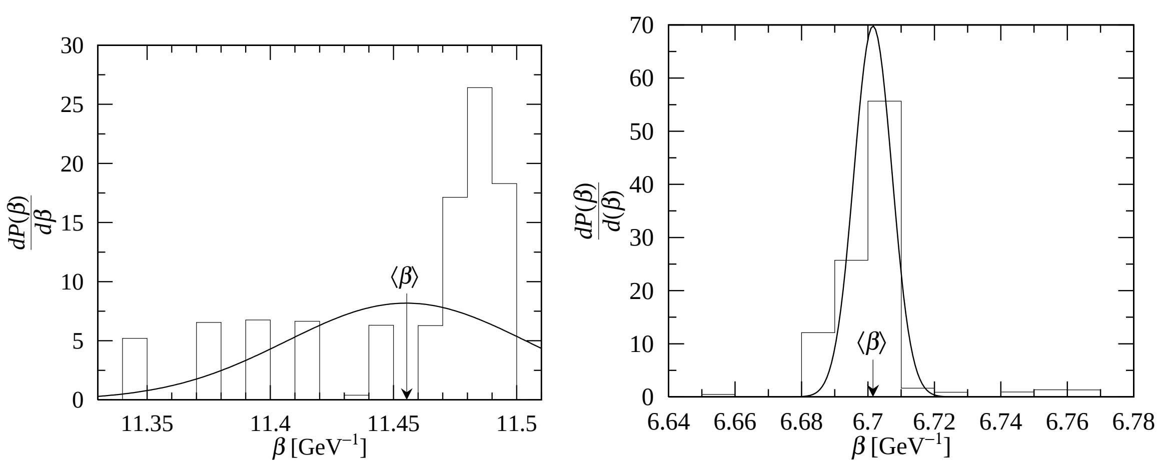

Fig. 2. Temperature fluctuation in STAR data (for C.C. $= 0-5$ %) [12] are analyzed by the use of Eq. (3) with $q-1=0$ (left panel) and Eq. (8) with $q-1 \neq 0$ (right panel).

and Table 1. The corresponding results for BRAHMS and PHENIX data [13,14] are very similar. As can be seen in Fig. 1, whereas distributions in the small p_t region can be explained by simple formula (3), data including the larger p_t region can only be explained by using the modified Hagedorn formula, Eq. (8) (or Eqs. (9) and (10)). The nonzero values of $|q-1|$ are then interpreted as an indication of the sizeable temperature fluctuations existing in the hadronizing system [10,11].

In fact RHIC data allow us to investigate the fluctuation of temperature in more detail, cf. Fig. 2. The centrality cut C.C. $= 0-5$ % region of STAR data [12] was fitted by using, respectively, Eq. (3) (with $q-1=0$, left panel) and Eq. (8) (with q as in Table 4, right panel). Fit was performed by fixing all parameters in Eqs. (3) and (8) except β_0; $\beta(=\beta_0)$ is then calculated for each of 35 data points and it is assumed that the reciprocal of the each error bar calculated by the fitting program MINUIT is proportional to the corresponding probability of this value of β, $P(\beta)$. In this way a probability distribution for β is obtained and presented in the form of a histogram in Fig.2. The histogram in each panel is then fitted to the Gamma distribution with $\alpha = 55000$ (shown as solid curves). The mean value $\langle\beta\rangle$ is also shown in each panel in Fig.2. As can

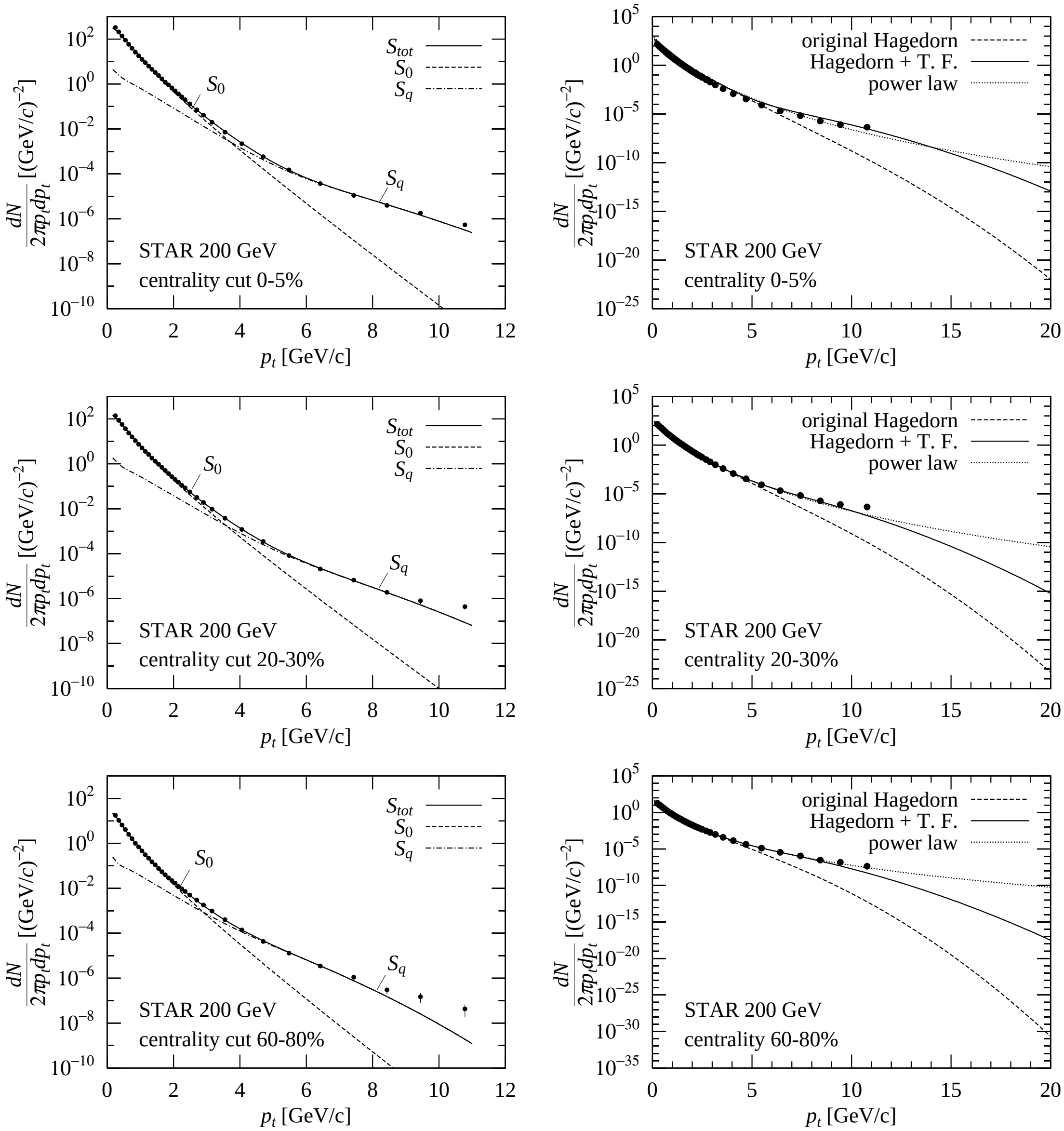

Fig. 3. Example of the visualization of results presented in Tables 2 and 4 using STAR [12] results. Left panels show in detail contribution of different mechanisms represented by S_{tot} Eq. (8), by S_0 Eq. (3) and by their difference denoted by $S_q = S_{tot} - S_0$. Right panels show results on original Hagedorn model, Eq.(1) by dashed line, the modified the Hagedorn formula including temperature fluctuation, Eq.(8), by solid line, and Eq.(7) by dotted line.

be seen, a good fit can be obtained *only* when Eq. (8) is used and in this case the resultant distribution of temperatures is very narrow. This result suggests that accounting for intrinsic fluctuations considerably narrows the distribution of temperatures (actually its reverse, $\beta = 1/T$) and minimizes what can be regarded as a kind of systematic error in deduction of β_0 from experimental data. Therefore it strongly suggests that the modified Hagedorn formula, Eq. (8), should be used whenever possible.

The results of our fits to RHIC data [12,13,14] performed by using Eq. (6) (as given by nonextensive statistical approach), Eq. (7) (representing the QCD-inspired power-law formula) and Eq. (8) (given by the modified Hagedorn formula proposed by us here) are presented in, respectively, Tables 2, 3 and 4. The results for STAR data are then also shown in Fig.3. In particular, the left hand panels of Fig. 3 demonstrate contribution of different mechanism represented, respectively by S_{tot} and S_0. It is clear that data for the larger p_t region can be explained only by S_{tot}, which can be attributed to the intrinsic primordial temperature fluctuations in the hadronizing system. However, at present it is difficult to treat this as a possible signal of a Quark-Gluon Plasma. Notice that the temperature parameter $T_0 = 1/\beta_0$ in Table 2 and 4 was estimated by the use of Eq. (8) from the whole region of transverse momenta, whereas $\tilde{T}_0 = p_0/n$, which corresponds to temperature in Eq. (7), shown in Table 3 governs only the small p_t region. RHIC data show that we always have $\tilde{T}_0 > T_0$, i.e., that inclusion of fluctuations and long-range correlations present in the hadronizing system lowers the estimated value of its mean temperature. From Table 4, we can see that both temperatures, T_H and T_0, estimated by the use of Eq. (8) decrease as the centrality cut, C.C., increases (i.e., it can be argued that they increase with the volume of interaction; similar effect concerning T_H has been also found in [22]). It should be emphasized that when one uses the modified Hagedorn formula, Eq. (8), then $T_H \sim T_0 \sim m_\pi$, i.e., estimated values of T_H and T_0 are almost equal to m_π, which we regard as very reasonably result[4].

3 Summary

We have presented a systematic analysis of RHIC data [12, 13, 14] on transverse momenta distributions, which allow, in principle, the deduction of the parameter believed to represent the temperature T_0 of the hadronizing system. We have shown that in order to fit the whole range of p_t one has to use a nonextensive approach, which accounts for temperature fluctuations present in the hadronizing system. This has been compared with approach using the old QCD-inspired power-like formulas introduced long time ago. We have demonstrated that gradual accounting for the intrinsic dynamical fluctuations in the hadronizing system by switching from Eq. (6) (as given by nonextensive statistical approach) to the modified Hagedorn formula including temperature fluctuation, Eq. (8), substantially lowers the values of parameter $q-1$. This is because part of the fluctuations ascribed in Eq. (6) to q are accounted for by the resonance spectrum $\rho(m)$ present in the Hagedorn formula. It also changes the temperature we are looking for. Therefore one has to be very careful when interpreting the temperature parameter obtained in such fits, especially when attempting to address any questions concerning Quark Gluon Plasma production issues[5]. If data with larger p_t are available, we can further investigate whether the modified Hagedorn formula including temperature fluctuation is really applicable or not.

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[4] Actually, analysis performed assuming both thermal and chemical equilibrium and including also baryons performed by GSI group [23] gives $T = 170$ MeV. In our case we are considering only pions and get $T \simeq m_\pi$. This difference is important for the description of phase diagram and we plan to address it elsewhere. One more remark is in order here. The T_0 parameters obtained by us are in a range of $T_c = 170$ MeV, the QCD crossover temperature. On the other hand, traditional exponential fits for the low p_T part of pion spectra used to give $T = 340$ MeV, pointing to a transverse flow with a Doppler blue-shift factor of two. However, we do not claim that there is no transverse flow in RHIC experiments, we only show that nonextensive approach can mimic this effect as well.

[5] One should be aware of the fact that there is still an ongoing discussion on the meaning of the temperature in nonextensive systems. However, the small values of the parameter q deduced from data allow us to argue that, to first approximation, T_0 can be regarded as the hadronizing temperature in such system. One must only remember that in general what we study here is not so much the state of equilibrium but rather some kind of stationary state. For a thorough discussion of the temperature of nonextensive systems, see [24]. It is also worth to be aware that in addition to the possibility of long-range correlations and memory effects to be at work in relativistic heavy-ion reactions (which were so far not yet proven) one can also view $q > 1$ as a general, leading order finite-size effect, $q = 1 + O(1/N)$ as proposed in [25].

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Table 2. Analysis of RHIC data [12,13,14] by means of nonextensive approach as given by Eq. (6). For comparison the results for pp collisions are also shown.

Coll. & C.C.	c	T_0 [GeV]	q	$1/(q-1)$	χ^2/n.d.f
BRAHMS 0-10%	936±68	0.227±0.005	1.0394±0.0026	25.4	10.2/23
10-20%	716±56	0.217±0.005	1.0455±0.0029	22.0	12.9/23
20-40%	468±41	0.208±0.006	1.0507±0.0033	19.7	12.8/23
40-60%	265±32	0.185±0.007	1.0607±0.0044	16.5	10.6/23
60-80%	36.2±4.2	0.165±0.005	1.0764±0.0024	13.1	2.76/23
PHENIX 0-5%	1530±359	0.195±0.012	1.0461±0.0060	21.7	5.00/29
5-15%	1200±276	0.193±0.012	1.0472±0.0057	21.2	3.56/29
15-30%	760±180	0.189±0.012	1.0503±0.0058	19.9	5.50/29
30-60%	384±96	0.170±0.011	1.0613±0.0055	16.3	2.60/29
60-80%	120±39	0.144±0.012	1.0728±0.0067	13.7	10.5/29
80-92%	59.2±32.0	0.114±0.017	1.0879±0.0106	11.4	8.99/29
STAR 0-5%	3980±186	0.164±0.002	1.0651±0.0009	15.4	172/32
5-10%	2900±148	0.169±0.002	1.0622±0.0011	16.1	64.5/32
10-20%	2340±114	0.164±0.002	1.0662±0.0011	15.1	66.4/32
20-30%	1630±81	0.162±0.002	1.0684±0.0011	14.6	40.7/32
30-40%	1170±61	0.158±0.002	1.0709±0.0011	14.1	38.9/32
40-60%	739±39	0.146±0.002	1.0772±0.0010	13.0	14.7/32
60-80%	328±19	0.130±0.002	1.0850±0.0011	11.8	9.39/32
pp (nsd)	49.9±5.5	0.111±0.003	1.0894±0.0014	11.2	10.1/29

Table 3. Analysis of RHIC data [12,13,14] by means of the QC-inspired power-like formula (7). For comparison the results for pp collisions are also shown.

Coll. & C.C.	c	n	p_0 [GeV]	$\tilde{T} = p_0/n$	χ^2/n.d.f
BRAHMS 0-10%	353±19	32.2±3.3	8.89±1.09	0.276	7.96/23
10-20%	260±15	26.4±2.5	7.05±0.83	0.267	14.7/23
20-40%	163±11	22.8±2.1	5.87±0.70	0.257	13.9/23
40-60%	83.7±7.5	17.9±1.8	4.13±0.56	0.231	11.7/23
60-80%	11.1±1.0	12.8±0.5	2.58±0.17	0.202	2.86/23
PHENIX 0-5%	536±398	23.8±34.0	5.54±31.26	0.233	4.69/29
5-15%	417±276	23.0±23.7	5.30±21.15	0.231	3.58/29
15-30%	260±149	21.3±13.2	4.84±11.01	0.227	5.54/29
30-60%	120±924	16.6±91.8	3.40±90.58	0.205	2.66/29
60-80%	32.1±38.2	13.5±31.5	2.38±30.46	0.177	10.3/29
80-92%	12.8±35.4	10.8±31.6	1.53±30.44	0.142	8.83/29
STAR 0-5%	1140±41	15.4±0.3	3.10±0.09	0.201	194/32
5-10%	843±33	16.5±0.4	3.44±0.12	0.208	68.6/32
10-20%	660±25	15.3±0.3	3.12±0.10	0.203	72.3/32
20-30%	457±18	14.7±0.3	2.94±0.09	0.200	42.8/32
30-40%	319±13	14.1±0.3	2.77±0.09	0.196	38.1/32
40-60%	190±8	12.6±0.2	2.30±0.07	0.182	13.9/32
60-80%	75.4±3.3	11.3±0.2	1.84±0.06	0.163	7.18/32
pp (nsd)	10.8±0.9	10.4±0.2	1.42±0.06	0.136	11.6/29

Table 4. Analysis of RHIC data [12,13,14] by means of nonextensive modification of the Hagedorn formula as given by Eq. (8). Maximum m is fixed at 70 GeV (therefore in (9) one always has $(1-\beta_0 m_t/\alpha)>0$). Numbers of divisions for y and m in computations are given in the last column. For comparison the results for pp collisions are also shown.

Coll. & C.C.	c	$q-1$	T_H [GeV]	T_0 [GeV]	χ^2/n.d.f.	no. of div.
BRAHMS 0-10%	156±3	0.00	0.192±0.000	0.178±0.000	15.4/22	6×6
10-20%	106±5	$(4.76\pm0.62)\times10^{-4}$	0.206±0.007	0.187±0.005	13.1/22	6×7
20-40%	67.7±4.9	$(8.49\pm21.05)\times10^{-5}$	0.177±0.013	0.166±0.010	11.6/22	5×3
40-60%	32.5±2.9	$(2.57\pm0.63)\times10^{-4}$	0.168±0.010	0.157±0.008	9.54/22	5×4
60-80%	5.00±0.14	$(8.12\pm0.45)\times10^{-5}$	0.124±0.000	0.120±0.000	3.19/22	6×6
PHENIX 0-5%	226±56	$(1.21\pm2.31)\times10^{-4}$	0.16±0.02	0.152±0.019	4.98/29	5×6
5-15%	157±34	$(4.01\pm0.02)\times10^{-4}$	0.183±0.023	0.167±0.017	3.32/29	6×5
15-30%	87.5±10.4	$(4.26\pm0.80)\times10^{-4}$	0.187±0.010	0.170±0.008	4.31/29	5×3
30-60%	50.3±8.7	$(1.64\pm0.47)\times10^{-4}$	0.140±0.012	0.133±0.010	2.54/29	6×7
60-80%	27.8±2.5	$(1.99\pm0.43)\times10^{-5}$	0.0731±0.0002	0.0719±0.0002	9.91/29	5×6
80-92%	10.0±1.2	$(1.24\pm0.30\times10^{-5}$	0.0565±0.0002	0.0558±0.0001	8.71/29	12×12
STAR 0-5%	477±13	$(1.48\pm0.05)\times10^{-4}$	0.140±0.001	0.132±0.001	56.6/31	6×6
5-10%	443±15	$(1.08\pm0.06)\times10^{-4}$	0.127±0.002	0.122±0.002	38.0/31	7×6
10-20%	326±17	$(1.02\pm0.10)\times10^{-4}$	0.126±0.004	0.121±0.003	33.8/31	6×5
20-30%	236±14	$(8.15\pm1.00)\times10^{-5}$	0.119±0.004	0.115±0.004	30.0/31	6×6
30-40%	169±10	$(7.13\pm0.09)\times10^{-4}$	0.113±0.004	0.109±0.004	25.5/31	6×6
40-60%	109±4	$(4.40\pm0.22)\times10^{-5}$	0.0961±0.0014	0.0937±0.0013	24.5/31	6×7
60-80%	46.0±1.0	$(2.80\pm0.08)\times10^{-5}$	0.0797±0.0001	0.0782±0.0000	23.4/31	6×7
pp (nsd)	4.98±0.15	$(2.87\pm0.08)\times10^{-5}$	0.0725±0.0000	0.0711±0.0000	38.2/28	20×22