On Bound States in the Full Symmetry Theory. II
—*An Effect of the N-Λ Mass Difference*—

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1 Introduction

Recently IKEDA et al. and the present author have proposed a new approach (IKEDA et al., 1963*; MAEKAWA, 1965) for treating bound states in the framework of the quantum field theory, and applied (IKEDA et al., 1964*) it to the full symmetry theory of the Sakata model (SAKATA, 1956; OGAWA, 1959; IKEDA, OGAWA and OHNUKI, 1959; 1960; 1961; IKEDA, MIYACHI, OGAWA, SAWADA and YONEZAWA, 1961). Throughout the paper I, stress has been laid on the full-symmetric aspect of the model : It has been assumed that all the basic particles stand on an equal footing and that the total Hamiltonian is invariant under the full symmetry group $U(3)$. In reality, however, there is a mass difference of about ten percent between the nucleon and the Λ-particle. In order to make the model a physically interesting one, we have to take the N-Λ mass difference into account. As was pointed out in I, however, mere substitution of the observed values for the parameters m_p $(=m_n)$ and m_Λ in the formalism leads to some difficulty : e. g., except for π, some states are not eigenstates by themselves, and others cannot be split into the singlet and the triplet spin states. The purpose of the present paper is to overcome such a difficulty by introducing, in an adiabatic way, an interaction H' which violates the full symmetry. If H' is invariant under the space inversion, spatial rotations and isospin rotations, the states will have the same spin, parity and the isospin as in the equal mass limit, and the singlet and the triplet spin states will be separable. Along this line of reasoning, we assume in 2 a simple form for the symmetry-violating interaction H', and investigate its consequences after the method of normal modes developed earlier (IKEDA et al., 1963; 1964; MAEKAWA, 1965). For a two-body bound system of a particle and an antiparticle, the case of the singlet spin is discussed in 3 and 4 and the case of the triplet spin in 5. It is shown that the mass

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* These two papers (IKEDA et al., 1963 and 1964) are referred to as A and I respectively throughout the present paper.

relations among existing boson multiplets can be reproduced by assuming two kinds of interactions H_1 and H_2 (IKEDA et al., 1964) play an important role for the spin triplet and the spin singlet states respectively. The final section is devoted to the estimation of the accuracy of the approximations used here and to some remarks.

2 The Symmetry-Violating Interaction*

As a most simple form for the interaction H' which induces the N-Λ mass difference and violates the full symmetry, we assume

$$H' = \delta m \int \bar{\psi}_3(x)\psi_3(x) d^3x, \tag{1}$$

where δm is a parameter which we take to be of the order of the actual N-Λ mass difference ($\sim$175 Mev). In the momentum representation, (1) contains, among others, the following term:

$$\delta m \sum_r \int d^3p \frac{m}{\mathcal{E}_p} (a^{(r)}{}_{3,\,p}{}^* a^{(r)}{}_{3,\,p} + b^{(r)}{}_{3,\,p}{}^* b^{(r)}{}_{3,\,p}). \tag{2}$$

Now we add H' to the full-symmetric Hamiltonian H given in I and follow the method of normal modes developed in the previous paper A. The essence of the method is to find a normal coordinate A^* which satisfies the following condition in the vacuum state Ω_0 :

$$[H+H',\ A^*]_- = EA^*. \tag{3}$$

Since we are concerned only with a two-body bound system of a particle and an antiparticle, we take $A^* = fa^*b^*$. Then the amplitude f and the energy eigenvalue E must be subject to a certain integral equation. To find the integral equation, we have to calculate the commutator $[H+H',\ a^*b^*]_-$. The first term is given in I, while we obtain from (2)

$$[H', a^{(r)}{}_{\alpha,\,p}{}^* b^{(s)}{}_{\beta,\,-p}{}^*]_- = \frac{m}{\mathcal{E}_p} \delta m (\delta^3{}_\alpha + \delta^3{}_\beta) a^{(r)}{}_{\alpha,\,p}{}^* b^{(s)}{}_{\beta,\,-p}{}^*, \tag{4}$$

for the second term. Eq. (4) has the same form as for the free part of the Hamiltonian,

$$[H_0,\ a^{(r)}{}_{\alpha,\,p}{}^* b^{(s)}{}_{\beta,\,-p}{}^*]_- = 2\mathcal{E}_p\, a^{(r)}{}_{\alpha,\,p}{}^* b^{(s)}{}_{\beta,\,-p}{}^*.$$

This fact implies that the introduction of H' is equivalent to the following substitution in the results obtained in I:

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* Unless stated otherwise, notations are the same as those used in A and I throughout the present paper.

$$2\varepsilon_p \rightarrow 2\varepsilon_p + \frac{m}{\varepsilon_p}\delta m\ (\delta^3{}_\alpha + \delta^3{}_\beta). \tag{5}$$

From (5) it is obvious that the separation of the singlet and the triplet spin states remains possible even in the presence of the symmetry-violating interaction H'.

3 Effect of H' on the Singlet Spin States

In order to see to what extent the interaction H'. modifies the results obtaind in I, we first consider the singlet spin states in this section. The case of the triplet spin will be discussed in 5.

According to the Sakata model (Sakata, 1956), π meson is composed of a nucleon and an antinucleon, and the normal coordinate does not include $a_3{}^*$ nor $b_3{}^*$. Therefore the integral equation is not changed through H';

$$(2\varepsilon_p - E_\pi) f_\pi(p) = \left(G_1 - \frac{G_2}{18}\right)\int G(\,|p-q|\,) M(p, q) f_\pi(q) d^3q. \tag{6}$$

The K meson state, however, includes either $a_3{}^*$ or $b_3{}^*$, and the integral equation is, according to (5),

$$\left(2\varepsilon_p + \frac{m}{\varepsilon_p}\delta m - E_K\right) f_K(p) = \left(G_1 - \frac{G_2}{18}\right)\int G(\,|p-q|\,) M(p, q) f_K(q) d^3q. \tag{7}$$

In any case, the π and the K states are normal states by themselves. This, however, is not the case with the $\pi_0{}'$ and the $\pi_0{}''$ states. For $\pi_0{}'$ we have

$$\begin{aligned}[H+H',\ e^*{}_{\pi'}(p)]_- &= [H,\ e^*{}_{\pi'}(p)]_- + [H',\ e^*{}_{\pi'}(p)]_- \\ &= 2\varepsilon_p e^*{}_{\pi'}(p) - \left(G_1 - \frac{G_2}{18}\right)\int d^3q G(\,|p-q|\,) M(p,\ q) e^*{}_{\pi'}(q) \\ &+ \frac{2}{\sqrt{6}} \cdot \frac{2m}{\varepsilon_p}\delta m\, e^*{}_{33}(p),\end{aligned} \tag{8}$$

where

$$e^*{}_{\pi'}(p) = \frac{1}{\sqrt{6}}(-a^{(r)}{}_{1,\,p}{}^* b^{(s)}{}_{1,\,-p}{}^* - a^{(r)}{}_{2,\,p}{}^* b^{(s)}{}_{2,\,-p}{}^* + 2a^{(r)}{}_{3,\,p}{}^* b^{(s)}{}_{3,\,-p}{}^*),$$

$$e^*{}_{33}(p) = a^{(r)}{}_{3,\,p}{}^* b^{(s)}{}_{3,\,-p}{}^*,$$

and use is made of Eq. (I. 3. 8)* for the first commutator. Of course, the spin indices r and s must be chosen so as to be compatible with the definition (I. 3 . 5a) of the singlet spin states. Similarly we have for $\pi_0{}''$

$$[H+H',\ e^*{}_{\pi''}(p)]_- = 2\varepsilon_p e^*{}_{\pi''}(p) - \left(G_1 + \frac{4}{9}G_2\right) \times$$

* (I. 3. 8) means "(3. 8) in I", and so on.

$$\times\int G(|p-q|)M(p,q)e_{\pi''}^{*}(q)d^3q+\frac{1}{\sqrt{3}}\cdot\frac{2m}{\varepsilon_p}\cdot\delta m\cdot e^{*}_{33}(p), \tag{9}$$

where

$$e^{*}_{\pi''}(p)=\frac{1}{\sqrt{3}}(a^{(r)}{}_{1,\,p}{}^{*}b^{(s)}{}_{1,\,-p}{}^{*}+a^{(r)}{}_{2,\,p}{}^{*}b^{(s)}{}_{2,\,-p}{}^{*}+a^{(r)}{}_{3,\,p}{}^{*}b^{(s)}{}_{3,\,-p}{}^{*}).$$

Thus the π_0' and the π_0'' states are not normal states by themselves. This result is naturally to be expected if we remember that the interaction H' violates the full symmetry.

In order to proceed further, we assume that the interaction H' is a small perturbation and require that an appropriate linear combination of π_0' and π_0'' be a normal coordinate:

$$[H+H',\ a\Psi^{*}_{\pi'}+b\Psi^{*}_{\pi''}]_{-}={}^{1}E(a\Psi^{*}_{\pi'}+b\Psi^{*}_{\pi''}). \tag{10}$$

Here a and b are constants and

$$\Psi^{*}_{\pi'}=\int d^3p f'(p)e^{*}_{\pi'}(p),$$

$$\Psi^{*}_{\pi''}=\int d^3p f''(p)e^{*}_{\pi''}(p),$$

denote the normal coordinates (in the equal mass limit) for π_0' and π_0'' respectively, f' and f'' satisfying the following integral equations:

$$\left.\begin{aligned}&(2\varepsilon_p-\overline{E})f(p)=\overline{G}\int d^3q G(|p-q|)M(p,\ q)f(q),\\&\overline{G}=G_1-G_2/18,\ \ \overline{E}={}^{1}E_8\quad\text{for}\quad\pi_0',\\&\overline{G}=G_1+4G_2/9,\ \ \overline{E}={}^{1}E_1\quad\text{for}\quad\pi_0''.\end{aligned}\right\} \tag{11}$$

From (8) and (9) we obtain for the left-hand side of (10)

$$\int d^3p\Big[a{}^{1}E_8 f'(p)e^{*}_{\pi'}(p)+b{}^{1}E_1 f''(p)e^{*}_{\pi''}(p)$$
$$+\frac{2}{3}B\Big\{(2af'(p)+\sqrt{2}\,bf''(p))e^{*}_{\pi'}(p)+(\sqrt{2}\,af'(p)+bf''(p))e^{*}_{\pi''}(p)\Big\}\Big],$$

where $B=\delta m\cdot\dfrac{m}{\varepsilon_p}$ and use is made of (11). Putting this expression equal to the right-hand side of (10), we obtain

$$({}^{1}E_8+\frac{4}{3}B-{}^{1}E)af'(p)+\frac{2\sqrt{2}}{3}Bbf''(p)=0, \tag{12a}$$

$$\frac{2\sqrt{2}}{3}Baf'(p)+({}^{1}E_1+\frac{2}{3}B-{}^{1}E)bf''(p)=0. \tag{12b}$$

Multiplying (12a) and (12b) by $f'^*(p)$ and $f''^*(p)$ respectively and taking the normalization of the f's into account, we have

$$({}^1E_8+\frac{4}{3}B'-{}^1E)a+\frac{2\sqrt{2}}{3}bB'''^*=0,$$

$$\frac{2\sqrt{2}}{3}aB'''+({}^1E_1+\frac{2}{3}B''-{}^1E)b=0,$$

where

$$B'=\delta m\int\frac{m}{\varepsilon_p}|f'(p)|^2d^3p,$$

$$B''=\delta m\int\frac{m}{\varepsilon_p}|f''(p)|^2d^3p,$$

$$B'''=\delta m\int\frac{m}{\varepsilon_p}f''^*(p)f'(p)d^3p.$$

The results obtained in I show that B', B'' and B''' are of the same order of magnitude in the case of a contact interaction. If we assume this is true also in the present case and put them equal to δm_s for brevity's sake, we have the energy eigenvalue

$${}^1E=\frac{{}^1E_8+{}^1E_1+2\delta m_s\pm\sqrt{({}^1E_8-{}^1E_1)^2+\frac{4}{3}\delta m_s({}^1E_8-{}^1E_1)+4(\delta m_s)^2}}{2}. \tag{13}$$

Thus, we can summarize the effect of H' on the singlet spin states as follows: 1) The π meson state is not affected at all. 2) The energy eigenvalue for the K meson is determined by the integral equation (7). (The energy shift will be estimated in the next section.) 3) The π_0' and π_0'' states are mixed up in general, the energy of the new states being given approximately by (13).

4 Estimation of the Mass Splitting

On the basis of the results obtained in the preceding section, we now investigate the mass splitting of individual states in an approximate way.

1) *π meson state*

From (6) the energy remains unaltered even in the presence of H' and is given by

$$E_\pi={}^1E_8, \tag{14}$$

where ${}^1E_8$ is the (degenerate) energy of the spin singlet and $U(3)$ octet states in the limit of vanishing δm.

2) *K meson state*

In this case the energy is determined by solving the integral equation (7). If δm is not too large, we may regard the second term on the left-hand side of (7) as a small perturbation. We thus put

$$\left.\begin{aligned} f_K(p) &= f_0(p)+u(p), \\ E_K &= {}^1E_8+\Delta E, \end{aligned}\right\} \quad (15)$$

and look for the first order change of the energy level of a K meson due to the symmetry-violating interaction H'. Here $f_0(p)$ is a solution of the "unperturbed" equation

$$\left.\begin{aligned} &2\varepsilon_p f_0(p)-\Big(G_1-\frac{G_2}{18}\Big)\int d^3q G(\,|p-q|\,)M(p,q)f_0(q)={}^1E_8\, f_0(p), \\ &\int |f_0(p)|^2 d^3p=1, \end{aligned}\right\} \quad (16)$$

and $u(p)$ and ΔE are small corrections. Substituting (15) into (7) and taking (16) into account, we get, to the lowest order approximation,

$$2\varepsilon_p u(p)-\Big(G_1-\frac{G_2}{18}\Big)\int d^3q G(\,|p-q|\,)M(p,q)u(q)$$
$$+\delta m\frac{m}{\varepsilon_p}f_0(p)={}^1E_8 u(p)+\Delta E f_0(p).$$

We multiply the above by $f_0^*(p)$ and integrate over p, making use of the fact* that the kernel $G(\,|p-q|\,)M(p,q)$ in (16) is symmetric with respect to p and q (IKEDA et al., 1963). Then we have, by virtue of (16),

$$\Delta E = \delta m\int\frac{m}{\varepsilon_p}|f_0(p)|^2 d^3p, \quad (17)$$

which is of the same order of magnitude as δm_s defined in the preceding section. Thus the K meson mass is given approximately by

$$E_K = {}^1E_8+\delta m_s. \quad (18)$$

3) *π_0' and π_0'' states*

The two states are mixed up in general by the interaction H'. If, however, we tentatively assume

$${}^1E_1-{}^1E_8\gg\delta m_s \quad (19)$$

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* This is true also for the axial-vector coupling. We have verified it by direct calculations and the results are to be published in the forthcoming issue of the "*Scientific Reports f the Research Institute for Theoretical Physics, Hiroshima University*".

for the singlet spin states under consideration, and confine ourselves only to the lowest order term in δm_s, π_0' and π_0'' remain to be eigenstates even in the presence of H'. Then, from (13), their energy eigenvalues are given approximately by

$$\left.\begin{aligned} E_{\pi'} &= {}^1E_8+\frac{4}{3}\delta m_s, \\ E_{\pi''} &= {}^1E_1+\frac{2}{3}\delta m_s, \end{aligned}\right\} \tag{20}$$

respectively.

The results (14), (18) and (20) satisfy the well known Okubo formula (Okubo, 1962)

$$3E_{\pi'}+E_{\pi}=4E_K, \tag{21}$$

which is valid experimentally with reasonable accuracy (Rosenfeld et al., 1964). It should be remarked here that the assumption (19) implies $G_2 \neq 0$, since otherwise π_0' and π_0'' would become completely degenerate as has been pointed out in I. Thus, we may say that the interaction H_2 plays an important role at least for the spin singlet bound states. It is also to be noted that the results obtained above are independent of the concrete expression of $G(|p-q|)$ in so far as it is spherically symmetric.

5 Triplet Spin States

The foregoing discussions concerning the effect of H' on the singlet spin states are also applicable to the triplet spin states without a substantial modification, although state vectors or integral equations for the triplet states are much more complicated than those for the singlet states. Since calculations are similar in both cases, we only give the results here, with special mention of the points characteristic of the triplet spin case. To avoid confusion, we denote hereafter the triplet spin counterparts of π, K, π_0' and π_0'' by ρ, K^*, φ_0 and ω_0 respectively.

1) *ρ meson state*

Just as in the π meson case, the energy eigenvalue is not changed through H' and is given by

$$E_{\rho}={}^3E_8, \tag{22}$$

where ${}^3E_8$ is the (degenerate) energy of the spin triplet and $U(3)$ octet states in the limit of vanishing δm.

2) *K^* state*

The integral equation is given by (A. 3. 16) and (I. 3. 14) with $2\varepsilon_p+B$ substituted for $2\varepsilon_p$. In contrast to the singlet spin case, the kernels $P(p,q)$, $U(p,q)$ and $X(p,q)$

of the integral equation are not symmetric with respect to p and q (IKEDA et al., 1963). Therefore the approximation used for the K meson is not applicable here. We, however, assume that the qualitative features for K^* will not be much different from those for K and take the energy shift ΔE to be given by (17) with $g(p)$ substituted for $f_0(p)$. Denoting this ΔE by δm_t, we have

$$E_{K^*} = {}^3E_8 + \delta m_t. \tag{23}$$

3) *φ_0 and ω_0 states*

On the analogy of (10), we require

$${}^3\Psi^* = a\Psi^*_{\varphi_0} + b\Psi^*_{\omega_0}$$

be a normal coordinate. Then the energy eigenvalue is given by

$${}^3E = \frac{{}^3E_8 + {}^3E_1 + 2\delta m_t \pm \sqrt{({}^3E_8 - {}^3E_1)^2 + \frac{4}{3}\delta m_t({}^3E_8 - {}^3E_1) + 4(\delta m_t)^2}}{2}, \tag{24}$$

where we have assumed $\delta m\int \frac{m}{\varepsilon_p}|g_{\varphi_0}(p)|^2 d^3p$, etc. are of the same order of magnitude and put them equal to δm_t as before.

Contrary to the spin singlet case, we here assume

$${}^3E_8 = {}^3E_1, \tag{25}$$

which is an opposite extreme to the spin singlet case (19). Then (24) gives

$$\left.\begin{aligned} E_\varphi &= {}^3E_8 + 2\delta m_t, \\ E_\omega &= {}^3E_1 (= {}^3E_8). \end{aligned}\right\} \tag{26}$$

It is easily verified that the normal coordinates for φ and ω are given by

$$\Psi^*_\varphi = \frac{1}{\sqrt{3}}(\sqrt{2}\Psi^*_{\varphi_0} + \Psi^*_{\omega_0}),$$

$$\Psi^*_\omega = \frac{1}{\sqrt{3}}(\Psi^*_{\varphi_0} - \sqrt{2}\Psi^*_{\omega_0}),$$

respectively. In the conventional notation they are written as

$$\varphi = \Lambda\bar{\Lambda},$$

$$\omega = \frac{1}{\sqrt{2}}(p\bar{p} + n\bar{n}),$$

as is expected from the assumption (25).

Eqs. (22), (23) and (26) give the relation

$$E_\varphi + E_\rho = 2E_{K^*},$$
$$E_\rho = E_\omega, \tag{27}$$

which is also valid experimentally with reasonable accuracy (ROSENFELD et al., 1964). We can see that the relation (27) is intimately connected with (25) which presumes the complete degeneracy of the $U(3)$ octet and the $U(3)$ singlet states for the triplet spin case. It is interesting to notice that we are automatically led to the assumption (25) by putting $G_2=0$, as has been remarked in the preceding section. Thus we may say, in contrast to the case of the spin singlet bound states, the interaction H_1 plays an important role at least for the spin triplet bound states.

6 Concluding Remarks

The present paper deals with the effect of the N–Λ mass difference in the full symmetry theory of the Sakata model. For this purpose we have introduced an interaction

$$H' = \delta m \int \bar{\psi}_3(x)\psi_3(x)d^3x, \tag{1}$$

which induces the N–Λ mass difference and violates the full symmetry, and investigated its consequences after the method of normal modes developed earlier (IKEDA et al., 1963; 1964). Then it has been shown that the splitting of mass levels of existing boson multiplets can be understood in terms of the mass difference among constituent basic particles and that the mass relations among various boson states are reproduced by assuming the interaction H_2 or H_1 plays a predominant role for the spin singlet and the spin triplet state respectively.

In deriving above results, we have made approximate calculations, regarding H' as a small perturbation. To test the validity of the normal mode method and the approximation used in this paper, we have solved numerically the integral equation (7) for the K meson states in the case of a contact interaction of the vector type. The results are given in **Fig. 1** for $\delta m/m = 175/940 = 0.186$. According to (17), we expect ΔE to be less than δm, since the amplitude $f_0(p)$ is normalized and $m/\varepsilon_p \leqq 1$. Indeed, **Fig. 1** shows that ΔE is nearly constant and is of the order of 150 Mev, which is consistent with the above prediction. To get a quantitative agreement with the observed K–π mass difference, we have only to choose δm larger than 175 Mev. In any case, we should take the results only qualitatively. Thus the normal mode method seems a promising approach in exploring dynamical aspects of the composite model.

Finally we give a brief mention of the interaction H_1 and H_2. As is well known, Fierz transformation (FIERZ, 1936) is possible in the special case of a

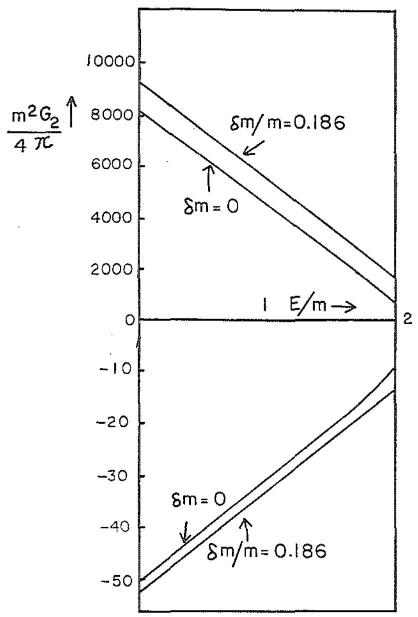

Fig. 1. G_2 versus E for the K meson. The cutoff momentum is taken to be m.

contact interaction. We have taken, however, both H_1 and H_2 into account, since the interaction in the present formalism may contain a form factor with a coupling type fixed. It should be stressed in this connection that the assumptions (19) and (25) can be readily understood if we accept that the interactions H_1 and H_2 play predominant roles for the triplet and the singlet spin states respectively. Since the mass relations (21) and (27), which are based on (19) and (25) respectively, are valid experimentally with reasonable accuracy, we are inclined to think that the singlet spin states are composed through the interaction H_2, while the triplet spin states through H_1. At present, there is no justification for this reasoning from a theoretical point of view and we don't know why such a circumstance is realized in nature. It will be of much interest to investigate this problem from the viewpoint of the structure of interactions among basic particles.

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Note added in proof :

After submitting the manuscript for publication, we noticed a paper by S. Ishida [Ishida, S. (1964) *Prog. Theor. Phys.*, 32, p. 922]. The same problem is treated in a different formalism and the mass relations (21) and (27) are obtained under the same assumptions (19) and (25). No argument, however, is made there concerning the implications of these assumptions.