The Stagnant Turbid Layer Developed in the Thermocline of Lake Noziri II

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(Received Nov. 28, 1961)

I Introduction

Since 1954 at Lake Noziri the author has continued the observations of the vertical distributions of the light extinction coef., the hydrogen ion concentration, and the water temperature to make sure of the development of the turbid layer and the layer where the water is markedly alkaline as was observed in the thermocline of this lake and Lake Aoki. The results of observations are represented in the figures from (2) to (25).

The graphs in these figures show the vertical distribution of the light extinction coef. And the numerals appended to the right sides of the graphs show the values of the hydrogen ion concentration at respective depths, and the numerals appended to the left sides of the graphs show the water temperature at respective depths. Since 1958 the water temperature was measured more minutely than before.

From these continued observations it has been again affirmed that, in the thermocline in the period from late summer to early autumn there conspicuously develops in stagnent state both kinds of layer above mentioned, as was observed in this lake and Lake Aoki.(1)(2) And it has further been elucidated that the layers where the water is markedly alkaline (the author calls this kind of layer an alkaline layer in this paper) are not perfectly erased by the small circulation of the lake water which frequently occurs in late summer, though, of course, they are somewhat influenced by them.

The author wishes to report the results of observations in more detail and consider the reason why the turbid layer develops in the thermoclines of these lakes.

Part I

Results of Observations made at Lake Noziri

II Observations made in 1954

The observations of this lake in 1954 were performed on the 30th of May, the 17th of June, the 9th of August, the 5th of September, and the 16th of September. The alphabet A in the sketch map of this lake (Fig. 1) shows the place

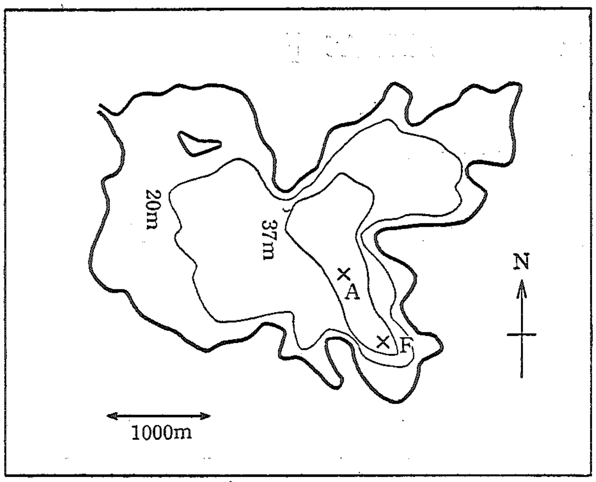

Fig. 1 Sketch-map of Lake Noziri

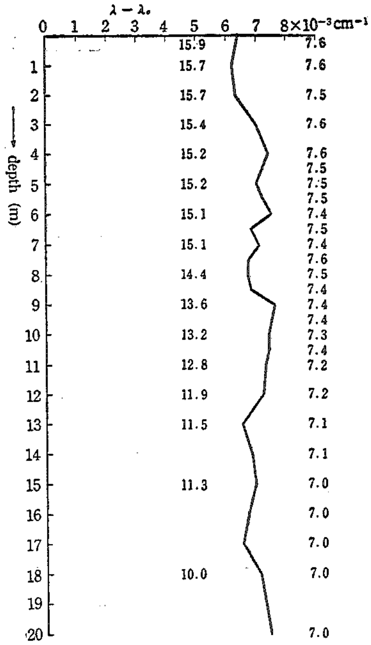

Fig. 2 May 30, 1954

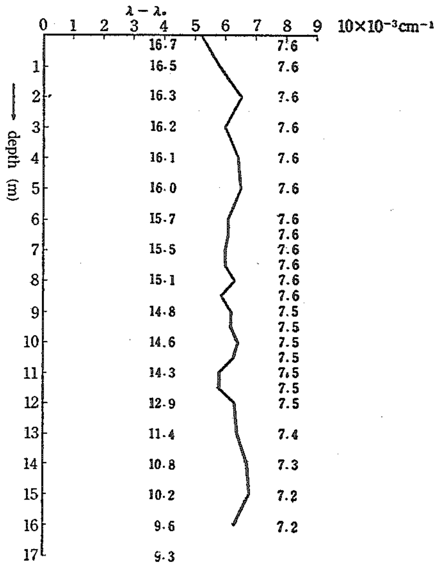

Fig. 3 June 17, 1954

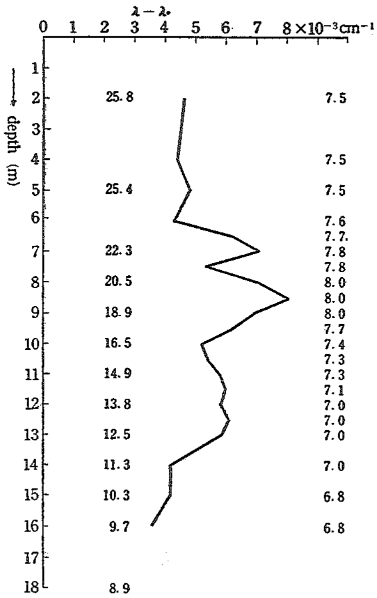

Fig. 4 Aug. 9, 1954

of observation. The results of the observations are represented in the figures (2), (3), (4), (5) and (6).

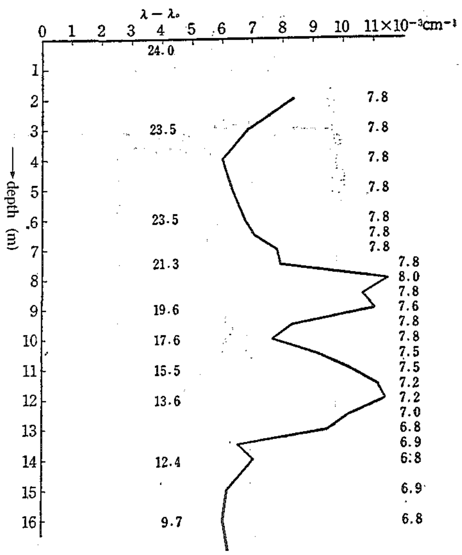

Fig. 5 Sept. 5, 1954

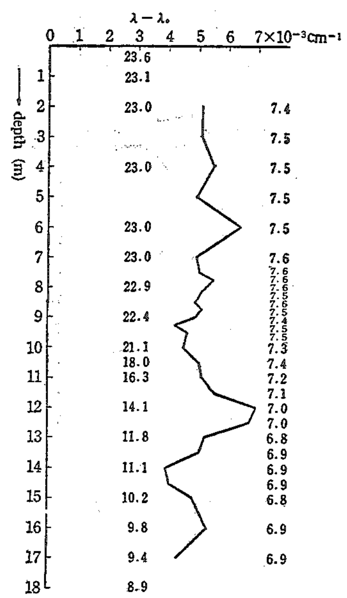

Fig. 6 Sept. 16, 1954

From them it is evident that no conspicuous layer was observed on the 30th of May and the 17th of June. On the 9th of August it was observed that a conspicuous alkaline layer had developed with the thickness of about 2 metres extending from the depth about 7 m. to that of 9 m. overlapping on a turbid layer. On the 5th of September the turbid layer still existed, but the alkaline layer was scarecely observed though faintly developed at the depth of about 8 metres. The water temperature in the thermocline on the 5th of September was a little higher than that on the 9th of August, while the water temperature in the epilimnion on the former was lower than that on the latter. From this fact, it is conjectured that a small circulation of the lake water had occurred on some days between the 9th of August and the 5th of September.

The author, therefore thinks that two kinds of layer observed on the 9th of August had once vanished by the circulation of the lake water, appearing again after the circulation. The distinct example of the reappearance of the alkaline layer after its disappearance brought about by the circulation, was observed on the 5th of September in 1959. (3)

On the 16th of September in 1954, either kind of layer was not observed; presumably erased by the autumnal circulation of the lake water.

III Observations made in 1955

The observations of this lake in 1955 were performed on the 23rd of June, the 18th of July, the 5th of August, the 23rd of August, the 1st of September, and the 17th of September. The place of observation was at A, and the results of the observations are represented in the figures (7), (8), (9), (10), (11) and (12).

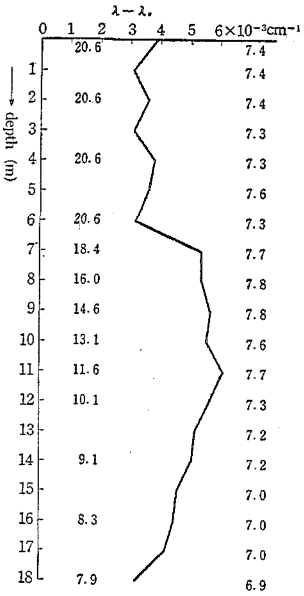

Fig. 7 June 23, 1955

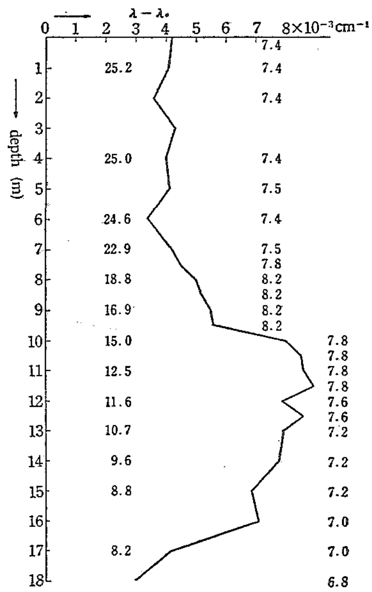

Fig. 8 July 18, 1955

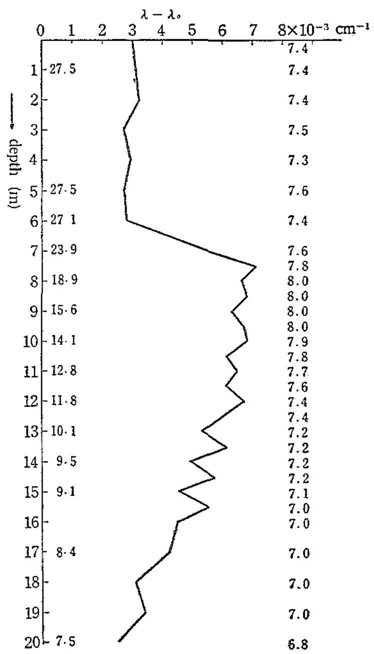

Fig. 9 Aug. 5, 1955

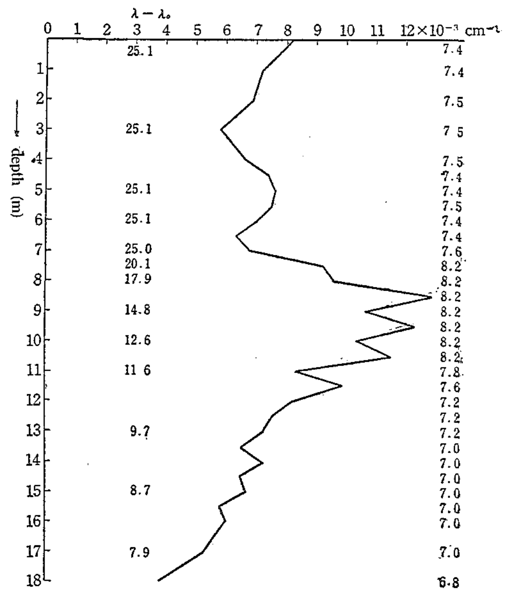

Fig. 10 Aug. 23, 1955

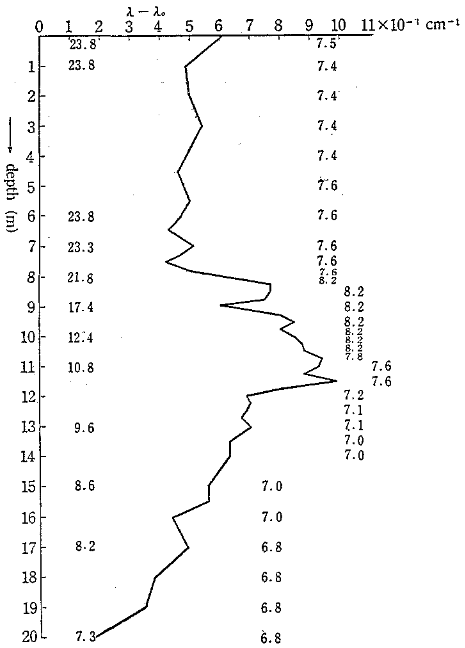

Fig. 11 Sept. 1, 1955

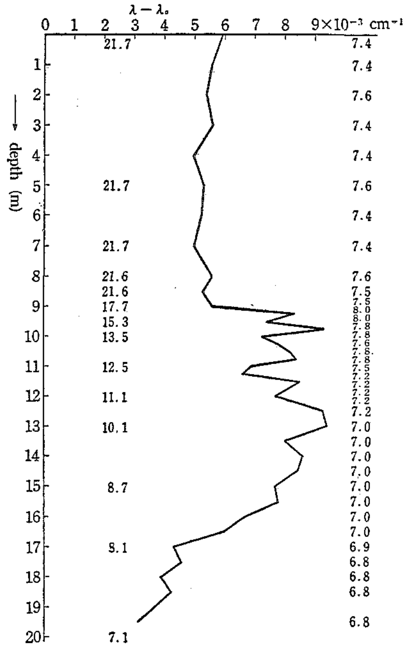

Fig. 12 Sept. 17, 1955

On the 23rd of June a small alkaline layer was observed between the depth of 8 m. and that of 9 m. By the 18th of July it had grown to the thickness of about 5 metres extending from 7.5 m depth to 12.5 m. depth overlapping on a conspicuously turbid layer. But on the 5th of August it was observed that its thickness had decreased itself, wholly overlapping on a turbid layer. On the 23rd of August both layers were observed with conspicuous growth, and in nearly the same states they were observed on the 1st and 17th of September. But the upper boundary of the layer, which was about 8 or 8.25 m. depth on the 1st of September was seen to have somewhat subsided on the 17th of the same month within the order of 1 metre; while the circulation of lake water was seen to have reached about 11 metres depth.

From this fact it may be said that in the period from late summer to early autumn, the growing power of the layer had been too intensive to be erased by the circulation of lake water.

IV Observations made in 1956

The observations of this lake in 1956 were performed on the 21st of May, the 5th of June, the 7th, the 11th and the 19th of August, and the 4th of September. The place of observations was at A except that of the one which was made on the 19th of August, on which day the observation was made at a place somewhat distant from and south-east of A. The results of the observations are represented in the figures from (13) to (18).

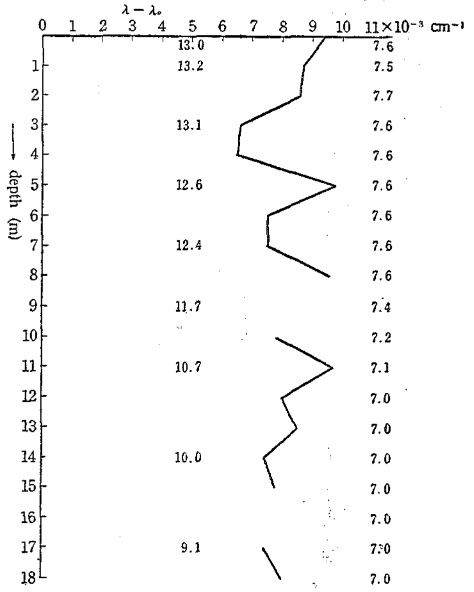

Fig. 13 May 21, 1956

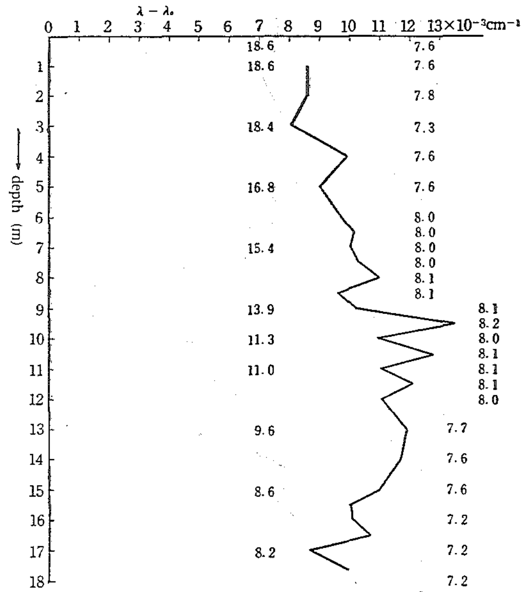

Fig. 14 June. 5, 1956

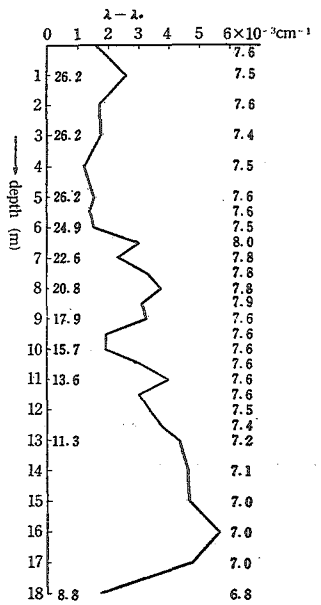

Fig. 15 Aug. 7, 1956

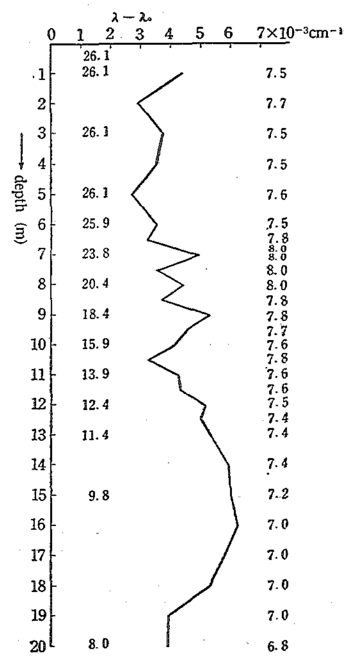

Fig. 16 Aug. 11, 1956

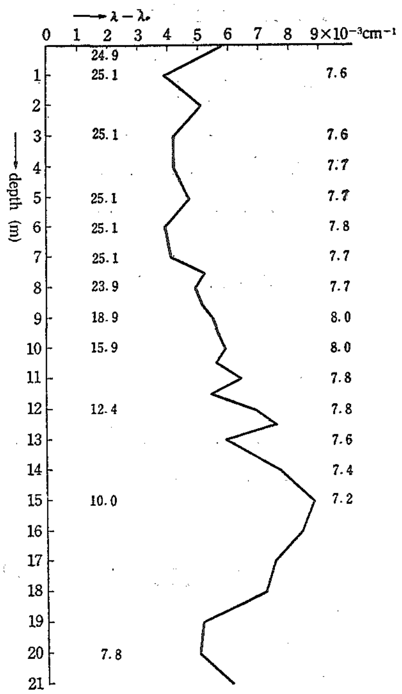

Fig. 17 Aug. 19, 1956

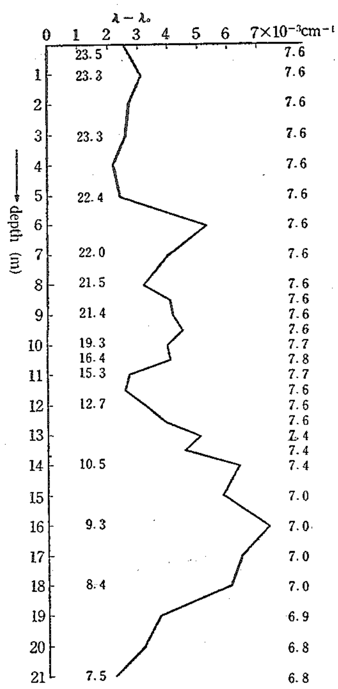

Fig. 18 Sept. 4, 1956

On the 21st of May no layer was observed. But on the 5th of June a thick alkaline layer was observed in the range covering from the depth of 6 m. to that more than 12 m. On the 7th of August, however, the value of pH of the water in the deeper side of the layer decreased from 8.1 to about 7.6. The same phenomenon was seen in the observations made on the 18th of July and the 5th of August in 1955. As the observations made in 1954 had lacked the observation in July, it is not clear whether the same phenomenon had occurred two years before, as was observed in 1956. But from the fact that the value of pH measured on the 17th of June was comparatively large, viz. amounted to 7.5 or 7.6 extending from the surface of the lake to the depth of about 12 m., the author thinks it probable that the same phenomenon as was observed in 1956, had also occurred in 1954. The results of the observation made on the 11th of August in 1956 are much like those observed on the 7th of this month.

The place where the observations were made on the 19th of August is somewhat distant from A, however the author thinks that the states of the phenomena observed here must have been quite like those at A, because every kind of layer generally extends widely over the lake. By this day, it was observed, the value of pH of the water in the upper part of the alkaline layer which had been observed on the 11th of August had diminished, and on the other hand, the value of pH of the water in its lower part had increased. Comparing the vertical distributions of the water temperature on these two days, it is conjectured that the circulation of the lake water had occurred between these two dates, and the author attributes the diminishing of the value of pH to this circulation of the lake water. The circulation was, to the author's consideration, caused by the typhoon which passed near the lake on the 17th of that month. On the 4th of September, the circulation was observed to have further advanced with the tendency of the dissolution of the layer

In this year the turbidity at the alkaline layer was generally not so conspicuous as had been observed before.

V Observations made in 1958

The observations of this lake in 1958 were made on the 18th of July and the 6th of August. The place of observations was at A and the results are shown in the figures (19) and (20).

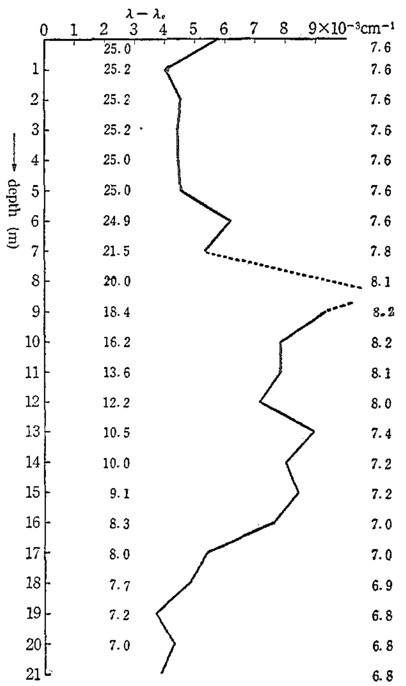

Fig. 19 July 18, 1958

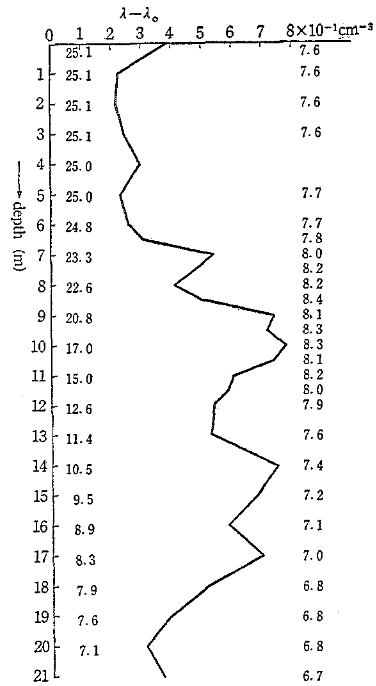

Fig. 20 Aug 6, 1958

On the 18th of July it was observed that an alkaline layer had developed covering the range from 7 m. depth to 12 m. depth wholly falling on a turbid layer. On the 6th of August this alkaline layer still existed having grown to a somewhat larger scale. And a turbid layer was seen in this alkaline layer, while the small circulation of the lake water was conjectured to have occurred before the day. The growth of the alkaline layer observed on this day, notwithstanding the circulation of the lake water shows, the author thinks, the alkaline layer to have the conspicuous power to grow.

VI Observations made in 1960

The observations of the lake in 1960 were made on the 8th, the 14th, the 19th, and the 27th of August and the 9th of September. The place of observations was at A, and the results are shown in the figures (21), (22), (23), (24) and (25).

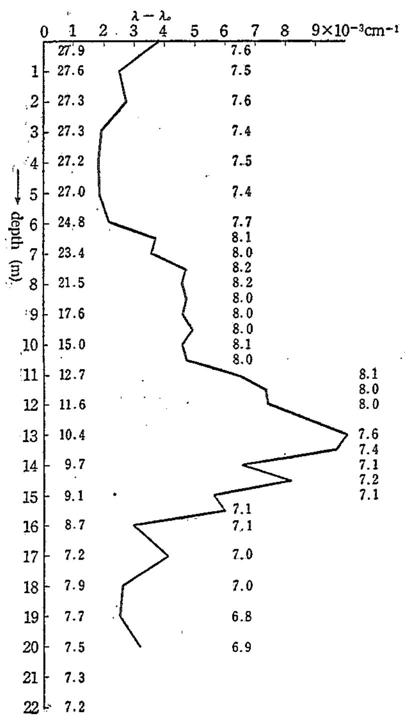

Fig. 21 Aug. 8, 1960

On the 8th of August a thick alkaline layer was observed to have developed covering the range from about 6.5 m. depth to about 13 m. depth as had been observed in 1958. The conspicuous state lasted at least to the 9th of September, though by this date the upper boundary of the layer had gradually sunk to the depth of 9m. Comparing the vertical distributions of the water temperature on the 8th of August and the 9th of September, we see that in the epilimnion, the water temperature on the latter day is lower than that on the former day, while in the thermocline the water temperature on the latter is higher than that on the former, the warmer part reaching the depth of about 12 m. The fact that the markedly alkaline layer still existed in late summer in the space where the circulation of the lake water had extended, shows that the growing power of the layer was more intensive than the destructive power of the circulation.

A thick turbid layer which was seen on the 19th of August in the alkaline layer was not observed on the 27th of this month; and the author attributes its disappearance to the circulation of the lake water.

Again, in every time of observation of this year another turbid layer was observed with its centre at the depth af about 12 m. or 13 m. somewhat shifted from the centre of the alkaline layer.

VII Alkaline layer

The values of pH in the epilimnion and the hypolimnion of Lake Noziri amount to about 7.6 and 6.8 respectively. And year after year, in the thermocline of this lake in summer stagnant period, there develops a layer, where the water is markedly alkaline with the pH value of about 8.2 covering nearly the whole lake and in many cases wholly or partly falling on the turbid layer.

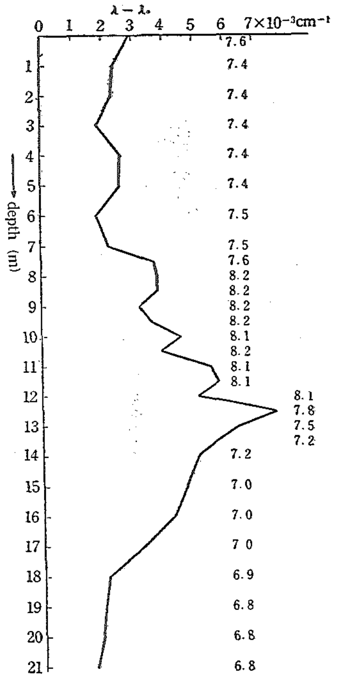

Fig. 22 Aug. 14, 1960

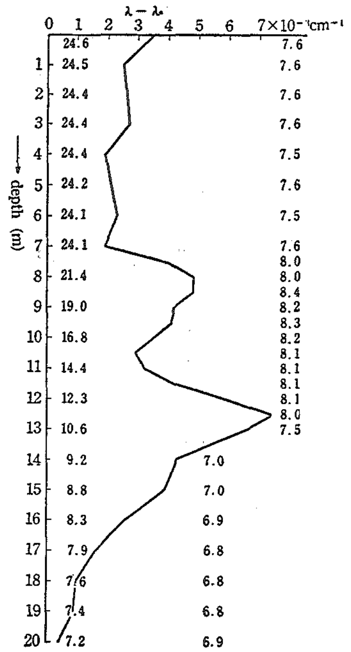

Fig. 23 Aug. 19, 1960

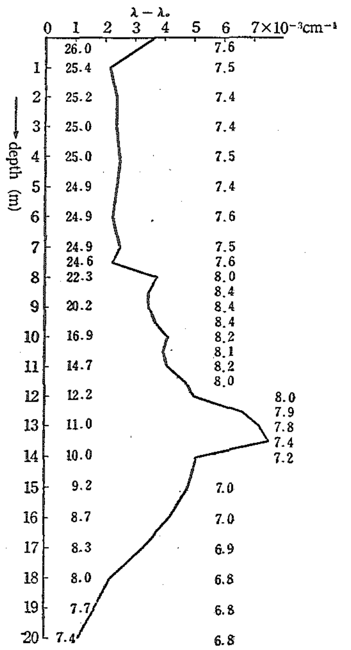

Fig. 24 Aug. 27, 1960

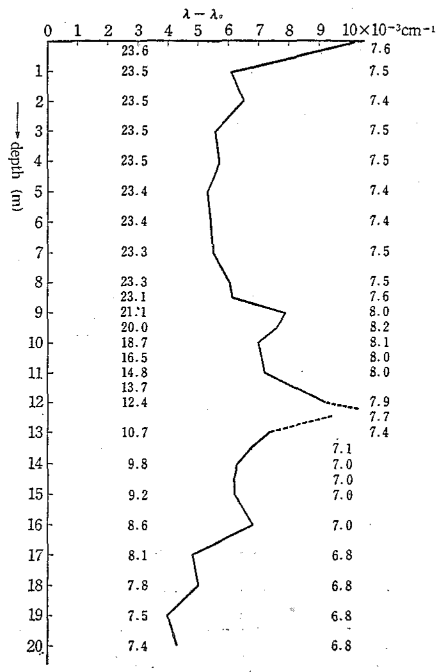

Fig. 25 Sept. 9, 1960

Table 1

depth (m)	water temperature(°C)	pH	$O_2\left(\frac{cc}{l}\right)$	degree of saturation of oxygen	total conc. of dissolved CO_2	consumption of potassium permanganate mg/l	nitrogen NO_3-N $\mu g/l$
0	24.8	7.6	5.75	105	10.64	7.59	2
5	26.4	7.6	5.65	106	11.31	8.23	20
7	26.5	7.6	5.78	109	12.12	6.33	15
8	25.1	8.0	6.99	128	11.02	5.70	0
9	21.4	7.8	7.55	129	11.14	9.18	0
10	17.5	7.9	8.08	128	—	7.91	0
11	15.9	7.8	7.46	114	11.54	8.23	0
13	12.0	7.2	6.93	98	13.05	6.28	0
15	10.3	7.0+	6.17	84	—	6.96	0
20	8.0	6.8	5.39	60.5	15.77	6.33	12
25	7.4	6.8	3.41	38	15.36	5.70	7
30	6.45	6.8	2.78	34	15.13	7.59	10

observed on Aug. 19, 1956

Table 2

depth(m)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
number	588	580	568	576	608	1020	900	948	524	528	204	124	124	44	60	48	48	160	40	32

number of phytoplanktons (per c.c.) at Lake Noziri. observed on July 18, 1958.

Table 3

depth(m)	0	1	2	3	4	5	6	6.5	7	7.5	8	8.5	9
number	596	740	656	760	616	500	548	800	1588	1656	1836	1964	1900
	9.5	10	10.5	11	11.5	12	13	14	15	16	17	18	19
	2596	2728	1460	1536	976	984	764	368	332	148	364	276	152

number of phytoplanktons (per c.c) at Lake Noziri observed on Aug. 6, 1958.

The same kind of phenomenon had been observed in the thermocline of Lake Sai, and also at Lake Aoki before the water of River Kashima was led into this lake. (1)

The amount of oxygen dissolved in the water in the alkaline layer was observed in the state of supersaturation. The amount of nitrogen dissolved in this layer was also larger than that in the other layer, however the degree of excess was much more conspicuous for oxygen than for nitrogen. (1)

The table (1) shows the results of the observations performed by Mr. Tadashiro Koyama and his partners on the 19th of August in 1956 near F (Fig. 1) The table shows not only that, at this markedly alkaline layer, the dissolved oxygen was in the state of supersaturation, but also that the total concentration of dissolved carbon dioxide was less than that in the other layer.

The tables (2) and (3) show the vertical distribution of the number of phytoplanktons at A on the 18th of July and the 6th of August in 1958 respectively. The counting was performed by Mr. Sadawo Kojima and Miss Yoshiye Kaneko. The vertical distributions of the extinction coef, of water, the water temperature and the hydrogen ion concentration on both days are shown in the figures (19) and (20). On the 6th of August, both layers, the one where the water was markedly alkaline and the other where the number of phytoplanktons was larger, well coincides with each other. The same result was also observed by Mr. K. Hogetsu and Mr. S. Ichimura on the 11th of September 1952. [2]

From the results of these observations, the author thinks it probable that the alkaline layer is the layer where the assimilation by phytoplanktons is actively carried; and he further premises that the season when the alkaline layer most conspicuously develops is the period from late summer to early autumn, But the author thinks that there is no direct relation between the turbid layer that has hitherto been observed and the alkaline layer.

Of course, the latter itself forms a turbid layer by the generation of phytoplanktons, and this kind of layer was actually observed. The layer observed on the 5th of September 1959 at A, to the author's consideration distinctly belongs to this kind of layer. [3]

Part II

Reason Why the Turbid Layer Develops in the Thermocline.

VIII Coagulation of suspended particles.

Generally the lake water contains a great number of suspended particles, which are various in size, shape and sort. And the author thinks that the most of these particles have a tendency to coagulate growing into larger particles. To disperse these particles into smaller ones, therefore, one must give them the necessary energy.

The table (4) shows the increase of the extinction coef. of the water which

Table 4

Water temperature (°C)	10.2	11.8	13.4	26.6	32.2
A	0.0104			0.0135	
B		0.0080	0.0091		0.0132

increase of extinction coef. (measured with the unit cm^{-1}) of the water drawn out of Lake Kizaki in accordance with the increase of its temperature.

Table 5

water temperature(°C)	26.6	20.4	18.0
A	0.0135	0.0128	0.0126

decrease of extinction coef. (measured with the unit cm^{-1}) of once warmed water of Lake Kizaki in accordance with the decrease of its temperature.

was drawn from the two places A and B on the surface of Lake Kizaki, in accordance with the increase of its temperature; and table (5) shows the decrease of the extinction coef. of the once warmed water, in accordance with the decrease of its temperature. When the temperature of the water increases, the kinetic energy of the water molecules increases in average. And by the increase of the molecular energy, some of the suspended particles are thought to be dispersed into smaller ones, and hence, the author thinks the increase of the extinction coef. is brought. Similarly, the decrease of the extinction coef. is brought by the decrease of temperature of the lake water.

The dispersion of the suspended particles is also brought by the agitation of the lake water, but the water in the thermocline and hypolimnion is nearly in the state of rest, and moreover its temperature is fairly low; consequently the particles in these spaces are not dispersed by the agitation of the water, rather coagulating each other due to the lowness of the water temperature.

Generally the lake water becomes clear when it is put in restful state. An example of this kind of phenomenon is shown in the figure (26), in which the graph I shows the vertical distribution of the extinction coef. of the water in the thermocline of Lake Kizaki, and the graph II shows the extinction coef. of the water after it had been left in a restful state for 24 hours, and the graph III shows the extinction coef. of the water which was filtrated by filter paper after it had been left in a restful state for 24 hours. From these graphs, it is shown that the water in the most turbid part became clearer than the water in any other layer; this fact, to the author's consideration, may be considered one of the proofs of the assumption that the suspended particles have a tendency to coagulate with each other.

The water in the hypolimnions of Lake Aoki, Lake Noziri etc. are clear in a wide range. This kind of phenomenon, the author thinks, is also a fact to affirm the assumption above mentioned, considering the fact that the hypolimnion is the place where any kind of disturbance of the lake water is absent for a long period and the temperature of the water is kept very low throughout the year.

Table 6

depth (m)	0	5	10	15	20	25	30	36	42	48
extinction coef.	2.44	0.90	1.19	0.94	0.86	1.17	0.86	1.19	0.94	1.22

extinction coef. of the water under ice-sheet. measured with the unit cm^{-1}, represented multiplied by 10^3. observed on March 23, 1944, at Lake Aoki.

The table (6) shows the extinction coef. of the water of Lake Aoki when it is frozen over; its values from depth of 5 m. to that of 48 m. are of the order

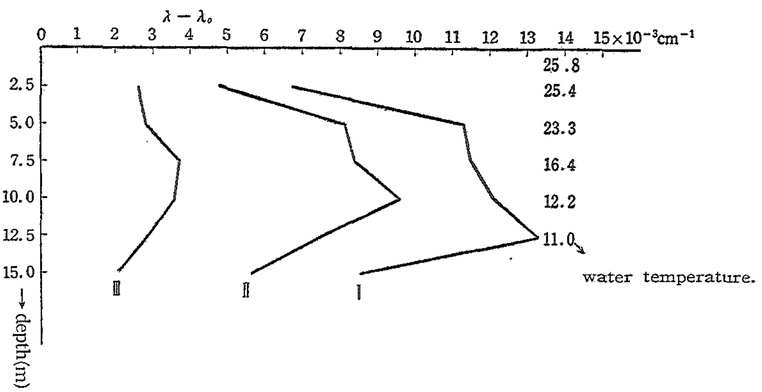

Fig. 26 I vertical distribution of extinction coef. on the 19th of July 1944, at Lake Kizaki.
II extinction coef. of water left restful state for 24 hours
III extinction coef. of water filtrated after left restful state for 24 hours.

from. 0.00086 cm^{-1}. to 0.0012 cm^{-1}., while the values before the freezing of the lake are of the order 0.003cm^{-1}. to 0.004cm^{-1}. The extinction coef. of water just below the ice-sheet is 0.0024cm^{-1}. which is somewhat large compared with that in any other layer; its largeness is, to the author's consideration, due to the mingling of the water contained in the snow on the ice-sheet with the surface water when the author drew it. If the assumption above mentioned be correct, the exceeding clearness of the calm water under the ice-sheet, from the surface of the lake to nearly its bottom, is a matter of course.

Moreover, if fine weather and calmness of water in the epilimnion of the lake last, in the period when the water temperature is fairly stagnant, the author thinks that the water should have the tendency to become clear. In the late summer of 1939 fine weather lasted with the calmness of the water at Lake Kizaki, and the transparency of the water exceedingly increased. This result, to the author's consideration falls within the purview of the phenomenon mentioned above. (4)

IX Fundamental equation

The temperature of the water in the epilimnion and hypolimnion are considered, in the first approximation, uniform respectively. And it is thought that the particles suspended in the water are slowly descending with the tendency to coagulate with each other. Now the author wishes to obtain an equation which represents the vertical distribution of the concentration of the suspended particles which are concerned to the scattering of light.

Take the origin of the coordinates axes at a point on the boundary between the epilimnion and thermocline, and the z-axis vertically downward. There is a tendency that the epilimnion increases its thickness with the lapse of time therefore the boundary slightly subsides with the lapse of time. But the author thinks that this kind of phenomenon, at present, has no serious influence on promoting the theoretical consideration.

Now consider a volume element at a distance z from the boundary with two horizontal faces of a unit area and thickness dz. If we take suspended particles, whose power to scatter light is most conspicuous, the author thinks they are also to be considered to have nearly the equal sinking velocity. Denote the number of such particles contained in a unit volume by n. Then the increase of the number of the particles in the volume element above mentioned in a unit time is given by

$$\frac{\partial n}{\partial t} dz$$

The number of the particles which falls into the volume element from its upper face and that of those which falls out of it from its lower face are given by

$$nv$$

$$nv + \frac{\partial (nv)}{\partial z} dz$$

respectively, where v is the sinking velocity of the particles.

It is probable to consider the number of the particles which coagulate in a unit time is proportional to n, therefore the number of particles which coagulate in the volume element is given by

$$rn \cdot dz$$

where r is the coagulation coef. which is perhaps the function of the temperature of the water.

A particle grown in some degree by the coagulation sinks away with a somewhat larger velocity and also loses nearly all of its power to scatter light. Consequently, we obtain a differential equation

$$\frac{\partial n}{\partial t} = -\frac{\partial (nv)}{\partial z} - rn$$

Transform it, then we have

$$\frac{Dn}{Dt} = -\frac{\partial v}{\partial z} n - rn \cdots\cdots\cdots\cdots\cdots\cdots\cdots (1)$$

By the equation (1) the distribution of the number of the particles, which most conspicuously scatter the light, is determined.

X Meaning of the observation of vertical distribution of turbidity

$$z = \int v\, d\, t + \text{constant}$$

is a characteristic equation of each suspended particle. If we trace it in zt-plane, it gives a group of curves as shown in the figure (27).

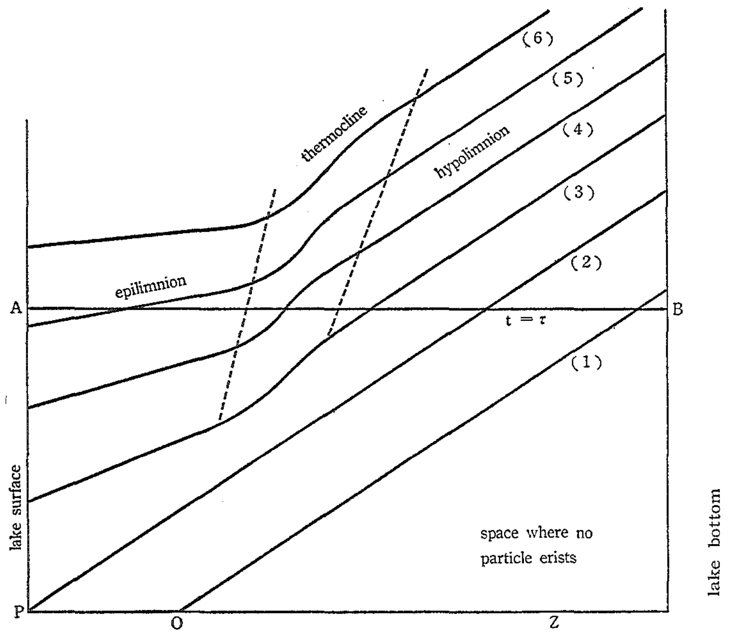

Fig. 27

Now take the origin of time at a certain moment in spring when all water of the lake is in uniform temperature. Then PO in the figure (27) corresponds to the thickness of epilimnion at

$$t = o$$

though there is no thermal stratification yet.

Approximately, the temperatures of the water in the epilimnion and hypolimnion are uniform respectively, therefore the sinking velocity of a particle is thought to be constant in each space, and therefore the characteristic curves corresponding to these spaces are nearly straight.

The curve (1) in the figure shows the characteristic concerning the motion of a particle which starts from the point

$$z = o$$

at the moment

$$t = o.$$

And the curve (2) shows that of a particle which starts from the surface of the lake at the moment

$$t = o.$$

Curves (3) (4) ………… show those of particles which start from the surface at moments t_1 t_2 t_3 ……………

where

$$0 < t_1 < t_2 < t_3 < \cdots\cdots\cdots\cdots\cdots$$

Now take as the initial condition

$$t \leqq 0 \qquad n = 0$$

then the space in the figure (27) bounded by the curve (1) and z-axis is a space in which no particle exists.

Now

$$\frac{dt}{dz} = \frac{1}{v}$$

And the temperature of water in the epillmnion ascends keeping the uniformity in the period from spring to summer, therefore the magnitude of the sinking velocity of a particle which started later is larger than that of a particle which started before. And therefore the value of $\frac{1}{v}$ gradually decreases with the lapse of time in this period, therefore the inclinaton of the characteristic curve corresponding to the epilimnion gradually decreases as is shown in the figure (27).

However in the hypolimnion, the temperature of water in it is approximately constant throughout the year, therefore the value of $\frac{1}{v}$ is constant, accordingly the value of $\frac{dt}{dz}$ is constant throughout the year, therefore the characteristic curves corresponding to this space are parallel to each other.

In the thermocline, at first the absolute value of the temperature gradient increases as z increases, then it reaches its maximum, and then it gradually decreases, and finally it nearly vanishes.

Now the sinking velocity of a particle is inversely proportional to the viscosity of water. Accordingly, at first the rate of increase of $\frac{1}{v}$ gradually increases, then it gradually decreases, and finally it reaches the constant value, therefore the characteristic curve is at first convex to the z-axis and finally concave to it showing a point of inflexion at about the middle point of the thermocline.

Moreover, there is a tendency that the thickness of the epilimnion gradually increases as aforementioned, with the lapse of time, therefore the characteristic curves corresponding to the thermocline gradually shift to the positive direction of z-axis, according as the time t increases as is shown in the figure (27).

Now the observation of the vertical distribution of the extinction coef. of lake water means the observation of the vertical distribution of the number of suspended particles, which can scatter light most conspicuously than the other particles. And the observation performed at some moment τ on some day means the observation made along a straight line parallel to the z-axis, for example along a straight line AB in the figure (27), where

$$AP = \tau$$

And the number of the descending particles corresponding to any point on AB is determined by the equation (1).

XI Solution of the fundamental equation

The water in the epilimnion is more or less in motion, therefore the suspended particles in it are supposed to be in uniform distribution. But its concentration is variable to the variation of the meteorological conditions. And its variation should be determined by the observations. Denote it by $n_o(t)$.

As the boundary condition, take

$$z \gtreqless 0 \qquad n = n_o(t)$$

As the initial condition, take as aforementioned

$$t \gtreqless 0 \qquad n = 0$$

Under these conditions, solve the differential equation (1).

Put $$n = TZ \cdots\cdots\cdots\cdots (2)$$

where T is the function of t only, and Z is the function of z only. Substitute (2) in (1), and arrange, then

$$\frac{1}{T}\frac{\partial T}{\partial t} = -\frac{v}{Z}\frac{\partial Z}{\partial z} - \frac{\partial v}{\partial z} - r \cdots\cdots\cdots\cdots (3)$$

where v and r are the functions of the water temperature, and therefore the function of z.

Therefore, in the equation (3) its left side is the function of t only, and right side of it is the function of z only; therefore both sides must be a constant, say k, then

$$\frac{1}{T}\frac{dT}{dt} = k \cdots\cdots\cdots\cdots (4)$$

$$-\frac{v}{Z}\frac{dZ}{dz} - \frac{dv}{dz} - r = k \cdots\cdots\cdots\cdots (5)$$

In these equations $\frac{\partial}{\partial t}$ and $\frac{\partial}{\partial z}$ are replaced by $\frac{d}{dt}, \frac{d}{dz}$ respectively.

From (4)

$$\log T = kt + \text{constant} \cdots\cdots\cdots\cdots (6)$$

From (5)

$$\log Z = -\int\left(\frac{1}{v}\frac{dv}{dz} + \frac{r}{v} + \frac{k}{v}\right)dz + \text{constant} \cdots\cdots\cdots (7)$$

Add (6) and (7), and arrange, then

$$TZ = n = \text{constant} \cdot exp\left\{kt - \int\left(\frac{1}{v}\frac{dv}{dz} + \frac{r}{v} + \frac{k}{v}\right)dz\right\} \cdots\cdots\cdots (8)$$

In (8) $\int\frac{dz}{v}$ is the time required by a particle to descend the distance z.

Now denote by t' the moment at which the particle started from the point,

$$z = 0$$

then

$$t - t' = \int\frac{dz}{v}$$

$$t-\int\frac{dz}{v}=t'$$

Substitute this in (8), then

$$n=\text{constant}\cdot exp\,(kt')\cdot exp\left\{-\int\left(\frac{1}{v}\frac{dv}{dz}+\frac{r}{v}\right)dz\right\}\cdots\cdots\cdots\cdots(9)$$

A particle which is situated at the point whose coordinate is z, must have started from the boundary at the moment t', therefore the *constant* $\cdot$ *exp* (kt') in the equation (9) must be equal to $n_0(t')$ or to

$$n_0\left(t-\int\frac{dz}{v}\right)$$

therefore the equation (9) must be as following

$$n=n_0\left(t-\int\frac{dz}{v}\right)exp\left\{-\int\left(\frac{1}{v}\frac{dv}{dz}+\frac{r}{v}\right)dz\right\}\cdots\cdots\cdots(10)$$

or

$$n=n_0\left(t-\int\frac{dz}{v}\right)\cdot\frac{v_0}{v}\,exp\left(-\int\frac{r}{v}\,dz\right)\cdots\cdots\cdots\cdots\cdots(11)$$

where v_0 is the sinking velocity of a particle in the epilimnion.

XII Reason why the turbid layer develops.

The equation (10) or (11) consists of the product of three factors, viz. the product of $n_0\left(t-\int\frac{dz}{v}\right)$, $exp\left(-\int\frac{1}{v}\frac{dv}{dz}dz\right)$ or $\frac{v_0}{v}$ and $exp\left(-\int\frac{r}{v}dz\right)$. Among them $exp\left(-\int\frac{1}{v}\frac{dv}{dz}\right)dz$ or $\frac{v_0}{v}$ increases with the increase of z,

because

$$\frac{dv}{dz}<0$$

for all values of z except near the bottom of the lake. And $exp\left(-\int\frac{r}{v}dz\right)$ decreases with the increase of z.

Near the epilimnion

$$v\doteqdot v_0$$

$$\frac{dv}{dz}\doteqdot 0$$

therefore the equations (10) and (11) are reduced to the form

$$n=n_0\left(t-\int\frac{dz}{v}\right)\;exp\left(-\int\frac{r}{v}\,dz\right)$$

If $n_0\left(t-\int\frac{dz}{v}\right)$ be constant or approximately constant, which is frequently the case, n decreases with the increase of z.

In the hypolimnion

$$v_0>v$$

and

$$\frac{v_0}{v} \doteqdot \text{constant}$$

Therefore, if $n_0\left(t - \int \frac{dz}{v}\right)$ be constant or nearly constant, n decreases with the increase of z.

In the thermocline

$$\frac{dv}{dz} < 0$$

However, if

$$\left|\frac{dv}{dz}\right| < r$$

n also decreases with the increase of z, as is evident from the equation (10).
If

$$\frac{dv}{dz} > r$$

n increases with the increase of z, as is evident from the equation (10). And if

$$\left|\frac{dv}{dz}\right| = \text{r} \cdots\cdots\cdots\cdots\cdots\cdots\cdots (12)$$

then

$$\frac{dn}{dz} = 0$$

therefore, n is maximum or minimum.

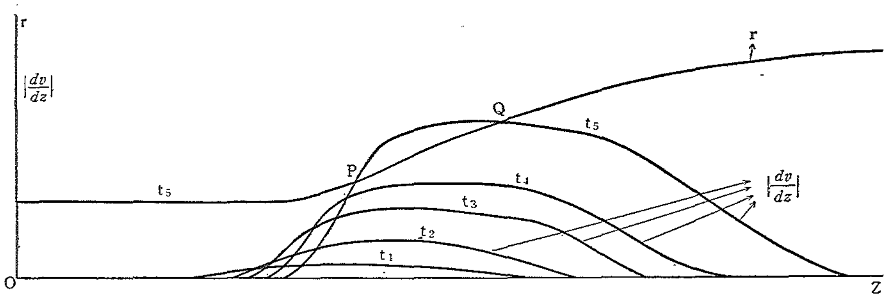

Fig. 28

The series of curves in the figure (28) graphically show the change of $\left|\frac{dv}{dz}\right|$ and r with the increase of z and the lapse of time.
At the momet

$$t = 0$$

the temperature of the water is uniform from the surface of the lake to its bottom, and therefore throughtout the lake the coagulation coef. is also constant, But with the lapse of time as t_1, t_2, t_3, ………… in the figure, the temperature of the water in the epilimnion gradually increases, and at the same time the temperature gradient in the thermocline increases. And the period comes when both kinds of graph intersect with each other.

Denote by P and Q the points of intersection as shown in the figure (28). Between P and Q

$$\left|\frac{dv}{dz}\right| > r$$

therefore in this space, n increases with the increase of z. Out of the space PQ

$$\left|\frac{dv}{dz}\right| < r$$

therefore in these spaces, n decreases with the increase of z. As z increases from zero, at first the decrease of n occurs and at P, n turns to increase, therefore at P, n is minimum. At Q, n turns to decrease, therefore at Q, n is maximum. With the further increase of z, n suffers approximately the exponential decrease. Assuming $n_0(t)$ is constant, the change of n is shown with typical graph in the figure (29).

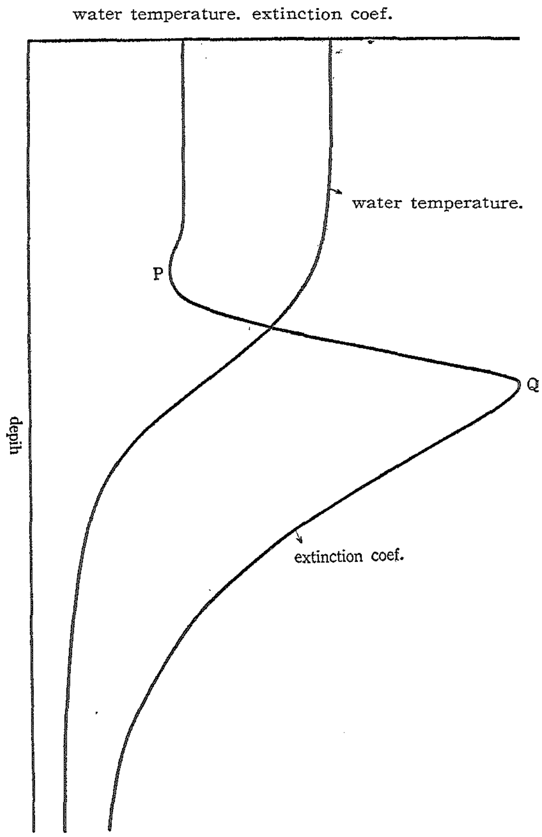

Fig. 29

The results of observations performed at Lake Aoki before the water of River Kashima has been led into the lake, much resemble the result deduced above.

The reason why both results coincides, is attributed, in the author's consideration, to the fact that the configuration of Lake Aoki is simple, and the weather surrounding it is comparatively moderate.

In the thermocline of Lake Noziri, as in that of Lake Aoki, year after year there develops a conspicuous turbid layer in the period from late summer to early autumn, and some results much resemble the result which is theoretically deduced. But frequently several maxima of turbidity appear.

In Lake Noziri, there are several inlets. And every inlet has its own individuality. Further the weather surrounding the lake is somewhat severe. The author thinks the fact above mentioned is the reason, why the complicated variation of $n_0(t)$ with the lapse of time is brought, and why several maxima appear in the vertical distribution of the descending particles.

XIII Stagnancy of the turbid layer

The temperature of the water in the epilimnion ascends keeping uniformity in the period from spring to summer, and at the same time the temperature gradient in the thermocline increases. However the period comes when the temperature of water in the epilimnion and the temperature gradient in the thermocline last in stagnancy. In the stagnant period, the temperature of water in the epilimnion of Lake Noziri amounts to about 25°C.

In the stagnant period both graphs representing the coagulation coef. r and $\frac{dv}{dz}$ are almost settled. Therefore P and Q in the figure (28) are supposed to be almost fixed at respective points. And therefore the turbid layer once appeared lasts in stagnancy.

Likewise, the minimum of turbidity is also supposed to exist in stagnancy. But it is too near the epilimnion, therefore the author thinks, that it is frequently erased by the small disturbances of the lake water.

Now the descending velocity v of the particle is given by

$$v=\frac{(D-\rho)\,Vg}{k\eta}$$

where D, V denote the density of the descending particle, and its volume respectively, k is a constant proper to the individual particle, ρ and η are the density of the lake water and its viscosity, g is the acceleration of gravity.

In the period from late summer to early autumn, the temperature of the water in the thermoclines of lakes in central Japan extends from about 10°C to 25°C covering the breadth of about 15°C. The change of density corresponding to the change of temperature mentioned above is

$$0.9997\sim 0.9971\ gr/cm^3$$

The difference between them is 0.0026 gr/cm^3

The lake water is not pure, however the change of its density is supposed to be almost equal to that of pure water. Denote the difference by $\Delta\rho$, then

$$\frac{\Delta\rho}{\rho_m}=0.0026$$

where ρ_m is the mean value of ρ.

Of the viscosity of the water, its change corresponding to the change of temperature is

$$13.1\times10^{-3}\sim8.9\times10^{-3}\ gr/sec^2\,deg.$$

The difference of them is $4.2\times10^{-3}\ gr/sec^2\,deg.$

Denote the difference by $\Delta\eta$, then

$$\frac{\Delta\eta}{\eta_m}\doteqdot0.38$$

where η_m is the mean value of η.

The viscosity of the lake water is supposed to be nearly equal to that of pure water. Therefore, if $D-\rho$ is not extremely small, as aforementioned, the descending velocity of the particle is said to be inversely proportional to the viscosity of the water surrounding just outside of the particle.

Denote the water temperature by θ, then η is given by

$$\eta=\frac{0.01779}{1+0.03368\theta+0.000220\theta^2}$$

However, in the range of the water temperature from 10°C to 25°C, the viscosity is approximately given by

$$\eta=\frac{0.01779}{1+0.03368\theta}$$

neglecting the term which contains θ^2

Substitute this in the formula of v, then we have

$$v=K(1+0.03368\theta)\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\ (13)$$

where

$$K=\frac{(D-\rho)\,Vg}{0.01779k}$$

From (13)

$$\left|\frac{dv}{dz}\right|\doteqdot0.03368\,K\left|\frac{d\theta}{dz}\right|$$

Substitute this in (12), then

$$0.03368\left|\frac{d\theta}{dz}\right|=\frac{r}{K}$$

where $\frac{r}{K}$ is a quantity proper to the particle. The data obtained in the observations made in 1960 are rough to determine the value of $\left|\frac{d\theta}{dz}\right|$, but the order of the value of $\frac{r}{K}$ can be calculated. And the result of the calculation gives

$$\frac{r}{K}=3.4 \times 10^{-4} \sim 5.7 \times 10^{-4}\, cm^{-1}$$

XIV Summary and conclusion

The results of the observations performed since 1954 at Lake Nozìri and the theoretical consideration on the reason why the turbid layer develops are summarised as following.

1) The development of the turbid layer and the alkaline layer in the thermocline of Lake Noziri in the period from late summer to early autumn, has been again affirmed.

2) In the period above mentioned, the growing power of the alkaline layer is too intensive to be erased by the small circulation of the lake water, which frequently occurs in late summer.

3) The alkaline layer is supposed to be a layer where the assimilation by phytoplanktons is actively carried.

4) There is no direct relation between the turbid layer and the alkaline layer.

5) The particles in the lake water are supposed to be in a state of sinking with a very small velocity, and have a tendency to coagulate with each other.

Therefore, the concentration of the particles which most conspicuously scatter light is, on one side, in a tendency to increase itself by the decrease of the sinking velocity, and is on the other side, in a tendency to decrease itself by their coagulation. And therefore, if the relation

$$\left|\frac{dv}{dz}\right| = r$$

appear in the thermocline by its growth, the maximum and minimum of the turbidity appear.

Acknowledgments

The author wishes to thank the Ministry of Education for the Grant in Aid for Fundamental Scientific Research with which he was enabled to make such observations as he liked; also he is grateful to Prof. S. Hayami, to Prof. K. Seno, and to Prof. K. Sugawara for their helpful discussions and kind advices for the performance of the observations and for the promotion of the theoretical consideration. And the aid offered by Mr. T. Koyama and his partners, Mr. S. Kojima, and Miss Y. Kaneko is also acknowledged with thanks.

References

(1) MUKAI, M. (1952) The Stagnant Turbid Layers Developed in the Thermocline of Lake Aoki and Lake Noziri. *Jour. Fac. Lib. A. Sci. Shinshu Univ., No. 2, 1–32.*

(2) MUKAI, M. (1956) The Stagnant Turbid Layer Developed in the Thermocline of Lake Noziri. *ditto, No. 6, 1–10.*

(3) MUKAI, M. (1959) The Turbid Layer Developed in the Thermocline of Lake Noziri in the Period of Lingering Summer Heat. *Jour. Japanese Society of Limnology, Vol. 21, No. 2, 99–103.* (in Japanese)

(4) MUKAI, M. (1940) The Vertical Distribution of Turbidity of Lake Kizaki and its Seasonal Variation. *Jour. Meteorological Society of Japan, Vol. 19, No. 4.* (in Japanese)