A TWO-COMPONENT MODEL FOR HIGH ENERGY COLLISION : PURE BIRTH PROCESS AND POISSON PROCESS

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Abstract:

In order to improve several results on multiplicity distribution by means of formulae derived from a pure birth process, we consider a two-component model, i. e., the pure birth process with a Poisson process. We obtain better results for $C_q=\langle n^q\rangle/\langle n\rangle^q$ moments in high energy collisions, provided that the number of the "excited hadrons" at the initial stage is constant irrespective of the incident energy $\sqrt{s}$. A KNO scaling function in the two-component model is also derived and applied to data. Moreover, the foward-backward correlation is considered. Details of caluculations are presented.

§1. Introduction

A few years ago, the Furry process, i. e., the pure birth (PB) process was proposed as a suitable branching model for the description of multiplicity distribution in the soft hadronic process [9, 10, 11, 16, 20];

$$\partial P(m, t)/\partial t=\lambda(m-1)P(m-1, t)-\lambda mP(m, t), \tag{1}$$

where t is an evolution parameter, and λ is a production rate in the interval dt by the PB process.

Several authors have considered the initial condition (A) [16, 20]:

$$\text{(A)} \qquad P_k(m,\ t=0)=\delta_{m,k}. \tag{2}$$

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However, the authors of refs. [10] and [11] have considered the second one (B):

$$\text{(B)} \qquad P_{\langle m \rangle}(m, t=0) = \langle m \rangle^m \exp(-\langle m \rangle)/m!. \tag{3}$$

It has been shown that the formulae derived from eq. (1) with eq. (3) gives better explanations for the observed moment $C_q = \langle n^q \rangle / \langle n \rangle^q$ than that with eq. (2). This suggests that the fluctuation on the number of "excited hadrons" at $t=0$ is important.

Through analyses in refs. [10] and [11] it is found that the number of "excited hadrons" at $t=0$, i.e., $\langle m \rangle$, is weakly decreasing function of $\sqrt{s}$. However, $\langle m \rangle$ is expected to be constant in the scheme. (See Eq. (3)). Second, there are significant, but small differences between experimental data and theoretical calculations derived from eq. (1) with eq. (3). See Table I. Third, we should consider a simpler case than a two-component stochastic equation of refs. [4] and [23], in which a parameter k should be fixed a priori. To solve these problems, let us consider a possibly new approach.

Here we consider the following two-component model, i.e., eq. (1) with the stochastic Poisson process [9]:

$$\partial P(n, \ m;t)/\partial t = \mu P(n-1, \ m;t) - \mu P(n, \ m;t) + \lambda(m-1)P(n, \ m-1;t) - \lambda m P(n, m;t), \tag{4}$$

where μ is a production rate in the interval dt by the Poisson process, and λ

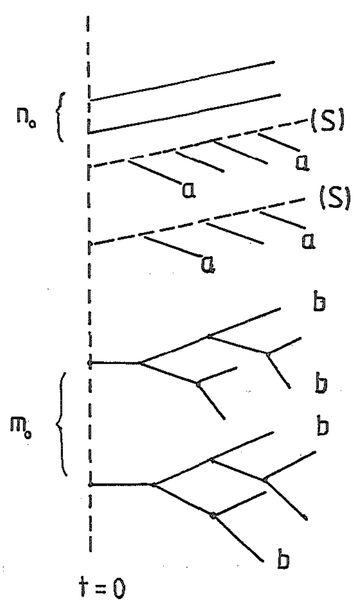

Fig. 1 Two kinds of the production proceses. The dashed line denotes the source (S) which cannot be observed. Two kinds (a and b) of observed particles are produced.

is that by the PB process. We have the following physical picture for eq. (4): a) The Poisson process mainly describes the multiplicity distribution of the fragmentation region. b) On the other hands, the PB process does that of the central region. A stochastic diagram of branching equation, eq. (4), is depicted in Fig. 1.

Contents of the present paper are as follows:

§2 Generating Function and Initial Conditions,
§3 Analysis of C_q moment,
§4 KNO Scaling Function and Fokker-Planck Equation of Eq. (4),
§5 Forward-Backward Correlation,
and §6 Concluding Remarks.

In Appendixes A～E, detailed derivations of several equations in §2～§4 are presented.

§2 Generating Function and Initial Conditions

2. A Generating Functions To solve eq. (4), we introduce the probability generating function,

$$\Pi(u,\ v;t)=\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}P(n,\ m;t)u^n v^m, \tag{5}$$

where u and v are variables corresponding to the Poisson and the PB process, respectively. Applying eq. (5) to eq. (4), we obtain the partial differential equation,

$$\partial\Pi/\partial t=\mu(u-1)\Pi+\lambda v(v-1)\ \partial\Pi/\partial v. \tag{6}$$

After some algebra, we have a solution for eq. (6) (see **Appendix** A);

$$\Pi(u,\ v;t)=f\left(\frac{v-1}{v}e^{\lambda t}\right)e^{\mu t(u-1)}, \tag{7}$$

where function $f(x)$ is determined by the initial condition.

2. B Initial Conditions We mention two initial conditions, i. e., eqs. (2) and (3), for eq. (1). For the present purpose, we consider at first the following case;

$$\text{(I)}\quad P(n,\ m;t=0)=n_0^{\ n}\exp(-n_0)/n!\cdot\delta_{m,k}. \tag{8a}$$

If n_0 approaches to zero, eq. (8a) reduces to

$$P(n,\ m;\ t=0)=\delta_{n,o}\cdot\delta_{m,k}\cdot \tag{8b}$$

Therefore, the initial condition (8b) is included in eq. (8a).

After some calculations, we find that results with eq. (8) cannot explain observed C_q moments. Thus, they are not presented in this paper.

Next we consider the condition,

$$\text{(II)}\quad P(n,\ m;t=0)=n_o{}^n\exp(-n_o)/n!\cdot m_o{}^m\exp(-m_o)/m!. \tag{9}$$

Equation (9) means that there are two kinds of fluctuations at $t=0$. By making use of eq. (9), we obtain the following generating function from eq. (7) (see **Appendix** A);

$$\Pi(u,\ v;T)=\Pi_a(u;T)\Pi_b(v;T), \tag{10a}$$

$$\Pi_a(u;T)=\exp\ [(\mu T+n_o)\ (u-1)], \tag{10b}$$

$$\Pi_b(v;T)=\exp\ \left[\left[m_o\left(\frac{v}{1-p(v-1)}-1\right)\right]\right]. \tag{10c}$$

where $p=\exp(\lambda T)-1$ with $\lambda T=\int\lambda dt$ (T is the maximum value of t). From eq. (10), we have the multiplicity distribution

$$\begin{aligned} P(n) &=\frac{1}{n!}\frac{\partial^n}{\partial u^n}\Pi(u,\ v=u;T)\Big|_{u=0} \\ &=\sum_{n_a+n_b=n} P_a(n_a,\ \langle n_a\rangle)\ P_b(n_b,\ \langle n_b\rangle). \end{aligned} \tag{11}$$

In eq. (11), $P_a(n,\ \langle n_a\rangle)$ is the multiplicity distribution obtained from the Poisson process with the mean multiplicity $\langle n_a\rangle=\mu T+n_o$:

$$P_a(n,\ \langle n_a\rangle)=\frac{1}{n!}\frac{\partial^n}{\partial u^n}\Pi_a(u;T)\Big|_{u=0}=\langle n_a\rangle^n\ \exp\ [-\langle n_a\rangle]\ /n!. \tag{12}$$

$P_b\ (n,\ \langle n_b\rangle)$ is that from the PB process with $\langle n_b\rangle=m_o(1+p)$ (see **Appendix B**):

$$P_b(n,\ \langle n_b\rangle)=\frac{1}{n!}\frac{\partial^n}{\partial u^n}\Pi_b(u;T)\Big|_{u=0}=\exp[-m_o]\quad \text{for } n=0, \tag{13a}$$

$$=\frac{\langle n_b\rangle}{n}\frac{p^{n-1}}{(1+p)^{n+1}}\exp[-m_o]L_{n-1}^{(1)}(-m_o/p)\quad \text{for } n\geqq 1. \tag{13b}$$

A factorial moment is obtained from eq. (10) as (see **Appendix B**),

$$F^{(l)}=\langle n(n-1)\cdots(n-l+1)\rangle=\frac{\partial^l}{\partial u^l}\Pi(u,\ v=u;\ T)\Big|_{u=1}$$

$$=\langle n_a\rangle^l+\sum_{j=1}^{l}\binom{l}{j}\langle n_a\rangle^{l-j}\ \Gamma(j)\langle n_b\rangle p^{j-1}\ L_{j-1}^{(1)}\ (-\langle n_b\rangle/p), \tag{14a}$$

where

$$F^{(1)}=\langle n_a\rangle+\langle n_b\rangle=\langle n\rangle, \tag{14b}$$

$$F^{(2)}=\langle n\rangle^2+2p\langle n_b\rangle. \tag{14c}$$

§3 Analysis of C_q moment

In order to obtain calculated results, we must determine values of the parameters. We use the observed mean multiplicity $\langle n\rangle$, the observed second moment C_2, and m_0 as input. Then, the term $\exp(\lambda T)$ can be determined as,

$$e^{\lambda T}=p+1=\frac{1}{2}\left\{1+\sqrt{1+2\langle n\rangle^2(C_2-1-1/\langle n\rangle)/m_0}\right\}. \tag{15}$$

The mean multiplicity of the PB process and that of the Poisson process are given as $\langle n_b\rangle=m_0(1+p)$ and $\langle n_a\rangle=\langle n\rangle-\langle n_b\rangle$, respectively.

Our results with $m_0=3.8$ are compared with the experimental data [1~3, 6~8, 14, 15, 17, 22, 24] in Table I. In the present analysis, we find that $m_0=3.5\sim4$ gives almost the same results. We show ones with $m_0=3.8$. For the sake of reference, results by means of the PB process, eq. (1) with eq. (2), are also shown. As you see, calculated results by the two-component model describe the data better than those by the PB process with eq. (2).

The mean multiplicity $\langle n_a\rangle$ of the Poisson process, and that $\langle n_b\rangle$ of the pure birth process are plotted as a function of $\sqrt{s}$ in Figs. 2a) and 2b), respectively. Energy dependences of $\langle n_a\rangle$ and $\langle n_b\rangle$ are determined by the use of the method of the linear regression. Those are expressed as,

$$\langle n_a\rangle=\mu T+n_0=1.27\ \ln\sqrt{s/s_0}+1.84 \quad (\text{cc}=0.990), \tag{16a}$$

$$\langle n_b\rangle/m_0=\exp(\lambda T)=\exp\left[0.426\ \ln\sqrt{s/s_0}\right] \quad (\text{cc}=0.998), \tag{16b}$$

where $\sqrt{s_0}=8.09$, and cc denotes the correlation coefficient. T is expressed by $\ln\sqrt{s/s_0}\propto Y$ (rapidity of an incident particle in the center of mass system).

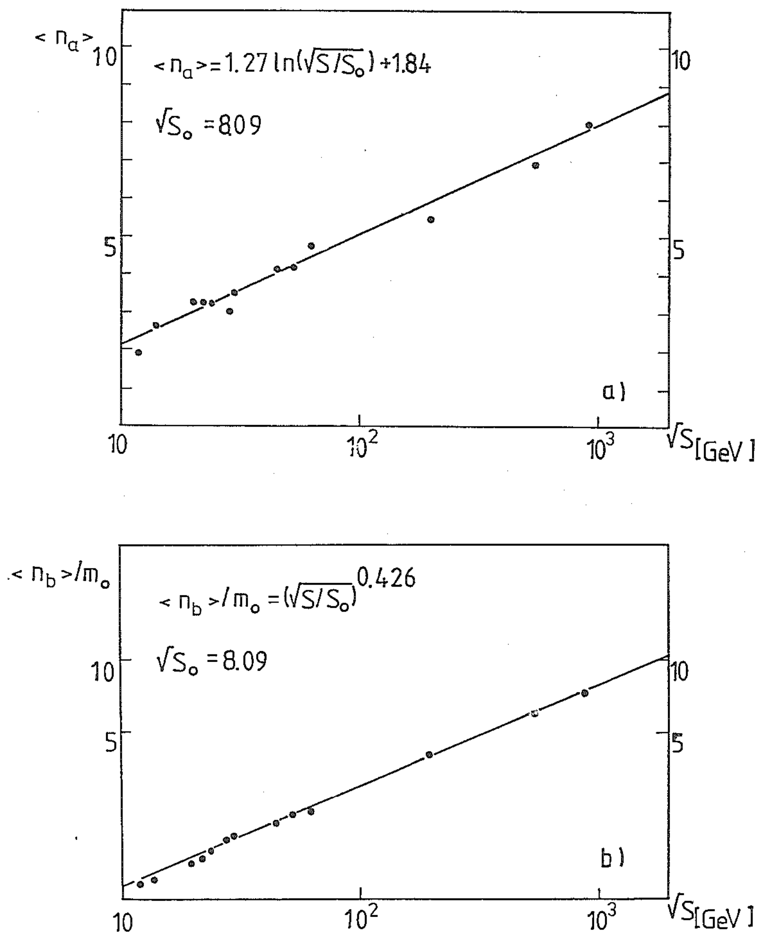

Fig. 2 Energy dependence of the mean multiplicity: a) that of the Poisson process, $\langle n_a \rangle$. b) that of the pure birth process, $\langle n_b \rangle$. Solid lines are determined by the method of the linear regression.

Table I Analysis of Cq moments with eq. (14) derived from the two-component model. m_0 is fixed at 3.8.

$\sqrt{s}$	$\langle n \rangle / \langle m \rangle$/or m_0	C_2	C_3	C_4	C_5
11.5	$\langle n \rangle = 6.35 \pm 0.08$	1.192 ± 0.009	1.63 ± 0.03	2.49 ± 0.08	4.2 ± 0.2
PB(B)	$\langle m \rangle = 5.72$	(input)	1.62	2.45	4.05
TWO	$m_0 = 3.8$(fixed)	(input)	1.62	2.45	4.06
13.8	$\langle n \rangle = 7.21 \pm 0.06$	1.175 ± 0.006	1.57 ± 0.02	2.33 ± 0.04	3.8 ± 0.1
PB(B)	$\langle m \rangle = 6.38$		1.56	2.30	3.71
TWO			1.56	2.30	3.71

19.7	⟨n⟩=8.56±0.11	1.174±0.010	1.57±0.03	2.34±0.08	3.8 ±0.2
PB(B)	⟨m⟩=6.88		1.56	2.30	3.71
TWO			1.56	2.32	3.76
22.0	⟨n⟩=8.93±0.16	1.18±0.01	1.59±0.04	2.4±0.1	
PB(B)	⟨m⟩=6.85		1.58	2.36	3.86
TWO			1.59	2.38	3.93
23.9	⟨n⟩=9.25±0.19	1.19±0.02	1.62±0.06	2.47±0.14	4.17±0.34
PB(B)	⟨m⟩=6.71		1.64	2.46	4.10
TWO			1.62	2.49	4.20
27.6	⟨n⟩=9.77±0.16	1.21±0.01	1.72±0.05	2.76±0.13	5.0 ±0.4
PB(B)	⟨m⟩=6.40		1.69	2.66	4.63
TWO			1.70	2.70	4.78
30.4	⟨n⟩=10.54±0.14	1.20±0.01	1.68±0.03	2.64±0.10	4.58±0.28
PB(B)	⟨m⟩=6.78		1.66	2.56	4.38
TWO			1.66	2.61	4.53
44.5	⟨n⟩=12.08±0.13	1.20±0.01	1.67±0.03	2.63±0.01	4.6 ±0.3
PB(B)	⟨m⟩=7.07		1.66	2.57	4.40
TWO			1.67	2.62	4.60
52.6	⟨n⟩=12.76±0.14	1.21±0.01	1.70±0.03	2.70±0.09	4.8 ±0.3
PB(B)	⟨m⟩=6.94		1.69	2.67	4.66
TWO			1.71	2.74	4.90
62.6	⟨n⟩=13.63±0.16	1.20±0.01	1.67±0.03	2.60±0.08	4.4 ±0.2
PB(B)	⟨m⟩=7.32		1.66	2.57	4.40
TWO			1.67	2.64	4.65
200	⟨n⟩=21.4±0.4	1.26±0.03	1.88±0.08	3.2 ±0.3	6.2 ±0.5
PB(B)	⟨m⟩=6.52		1.88	3.22	6.17
TWO			1.91	3.35	6.68
546	⟨n⟩=29.4±0.9	1.29±0.02	2.01±0.09	3.66±0.29	7.6 ±0.8
PB(B)	⟨m⟩=6.17		2.00	3.57	7.20
TWO			2.03	3.73	7.85
900	⟨n⟩=35.6±0.9	1.30±0.03	2.08±0.09	3.9 ±0.3	8.2 ±0.8
PB(B)	⟨m⟩=6.10		2.03	3.69	7.56
TWO			2.07	3.86	8.27

§4 KNO Scaling Function and Fokker-Planck Equation of Eq. (4)

By making use of the double inverse Poisson transform in eq. (4), we obtain the Fokker-Planck equation (see **Appendix** *C*),

$$\frac{\partial \phi}{\partial t}=-\frac{\mu}{\langle n_a\rangle}\frac{\partial}{\partial z_a}\phi+\frac{\partial}{\partial z_b}\left\{-\lambda z_b+\frac{\lambda}{\langle n_b\rangle}\frac{\partial}{\partial z_b}z_b\right\}\phi. \tag{17a}$$

From eq. (10), we obtain a two-component KNO scaling function (see **Appendix C**):

$$\begin{aligned}\phi(z_a,\ z_b;T)=&\left(\frac{1}{2\pi i}\right)^2\int_{c_1-i\infty}^{c_1+i\infty}\int_{c_2-i\infty}^{c_2+i\infty}\Pi(1-s_1/\langle n_a\rangle, 1-s_2/\langle n_b\rangle;T)\\ &\times\exp(s_1z_a+s_2z_b)\ ds_1ds_2\\ &=\delta(z_a-1)\delta(z_b-(+0))e^{-\xi}\\ &+\delta(z_a-1)\xi z_b^{-1/2}\exp[-\xi(z_b+1)]\cdot I_1(2\xi\sqrt{z_b}),\end{aligned} \tag{17b}$$

where $z_a=n_a/\langle n_a\rangle$, $z_b=n_b/\langle n_b\rangle$, $\xi=\langle n_b\rangle/p$, and $I_1(x)$ is the modified Bessel function.

Next, we consider the KNO scaling function corresponding to the multiplicity distribution $P(n)$ defined by eq. (11). In this case, both n_a particles produced in the Poisson process and n_b particles produced in the PB process contribute to $P(n)$ (note that $n=n_a+n_b$). Then, the scaling variable z is written as $z=(n_a+n_b)/(\langle n_a\rangle+\langle n_b\rangle)=rz_a+(1-r)z_b$, where $r=\langle n_a\rangle/\langle n\rangle=1-\langle n_b\rangle/\langle n\rangle$.

As seen in eq. (17b), the variable z_a is contained only in the delta function $\delta(z_a-1)$. Therefore, it becomes constant ($z_a=1$), and its role is only to shift the minimum value of the scaling variable z from zero. Using the following relations,

$$\phi_b(z_b;T)=\int_0^\infty \phi(z_a,\ z_b;T)dz_a,$$

and

$$\phi(z;T)dz=\phi_b(z_b;T)\ dz_b,$$

we obtain the KNO scaling function (see **Appendix D**):

$$\begin{aligned}\phi(z;T)=\delta(z-r)e^{-\xi}+\xi\{(1-r)(z-r)\}^{-1/2}\exp\left[-\xi\left(\frac{z-r}{1-r}+1\right)\right]\\ \cdot I_1(2\xi\sqrt{(z-r)/(1-r)}\,),\end{aligned} \tag{18a}$$

where $r=\langle n_a\rangle/\langle n\rangle=1-\langle n_b\rangle/\langle n\rangle$ and $z=r+(1-r)z_b$. In the actual analysis the first delta function should be approximated by

$$\delta(z-r)\exp(-\xi)=\lim_{\alpha\to\text{large}}\sqrt{\frac{\alpha}{2\pi}}\exp[(-\alpha(z-r)^2/2-\xi)]. \tag{18b}$$

Equation (18a) is also directly obtained by the inverse Laplace transform of eq. (10a) (see **Appendix D**);

$$\psi(z;T)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\Pi(u=1-s/\langle n\rangle,\ v=1-s/\langle n\rangle;T)\ e^{sz}\ ds\ =\ \text{eq. (18)}. \tag{19}$$

Note that $u=v$ in eq. (19). Moments formula is obatined as (see Appendix E),

$$C_q=\langle z^q\rangle=\int_0^\infty\psi(z;T)z^q\ dz,$$

$$=r^q+\sum_{j=1}^{q}\binom{q}{j}r^{q-j}\ \Gamma(j)\ (1-r)^j\ \xi^{-j+1}\ L_{j-1}^{(1)}(-\xi), \tag{20a}$$

$$C_0=C_1=1, \tag{20b}$$

$$C_2=1+2(1-r)^2/\xi. \tag{20c}$$

In analysis of the C_q moments and the multiplicity distributions by means of the KNO scaling function, $\exp(\lambda T)$ is also determined by $\langle n\rangle$, C_2 moment and m_0. However, we can take somewhat different value for m_0 (for KNO) in eq. (15), due to neglection of $O(1/\langle n\rangle)$ in the second moment:

$$e^{\lambda T}=p+1=\frac{1}{2}\left\{1+\sqrt{1+2\langle n\rangle^2\ (C_2-1)/m_0}\right\}. \tag{21}$$

In analysing the data with eqs. (19) and (20), we fix m_0 (for KNO) at 4.0. KNO scaling functions at $\sqrt{s}=30.4$ and 546 GeV are compared with the experimental data [2, 7, 14] in Fig. 3. A typical example of eq. (18b) is displayed in Fig. 3. (c). Calculated moments obtained from eq. (20) are compared with the data [1~3, 6~8, 14, 15, 17, 22, 24] in Table II. It can be said that our KNO scaling function describe experimental data from $\sqrt{s}=11.5$ to 900 GeV as well as those with eq. (14).

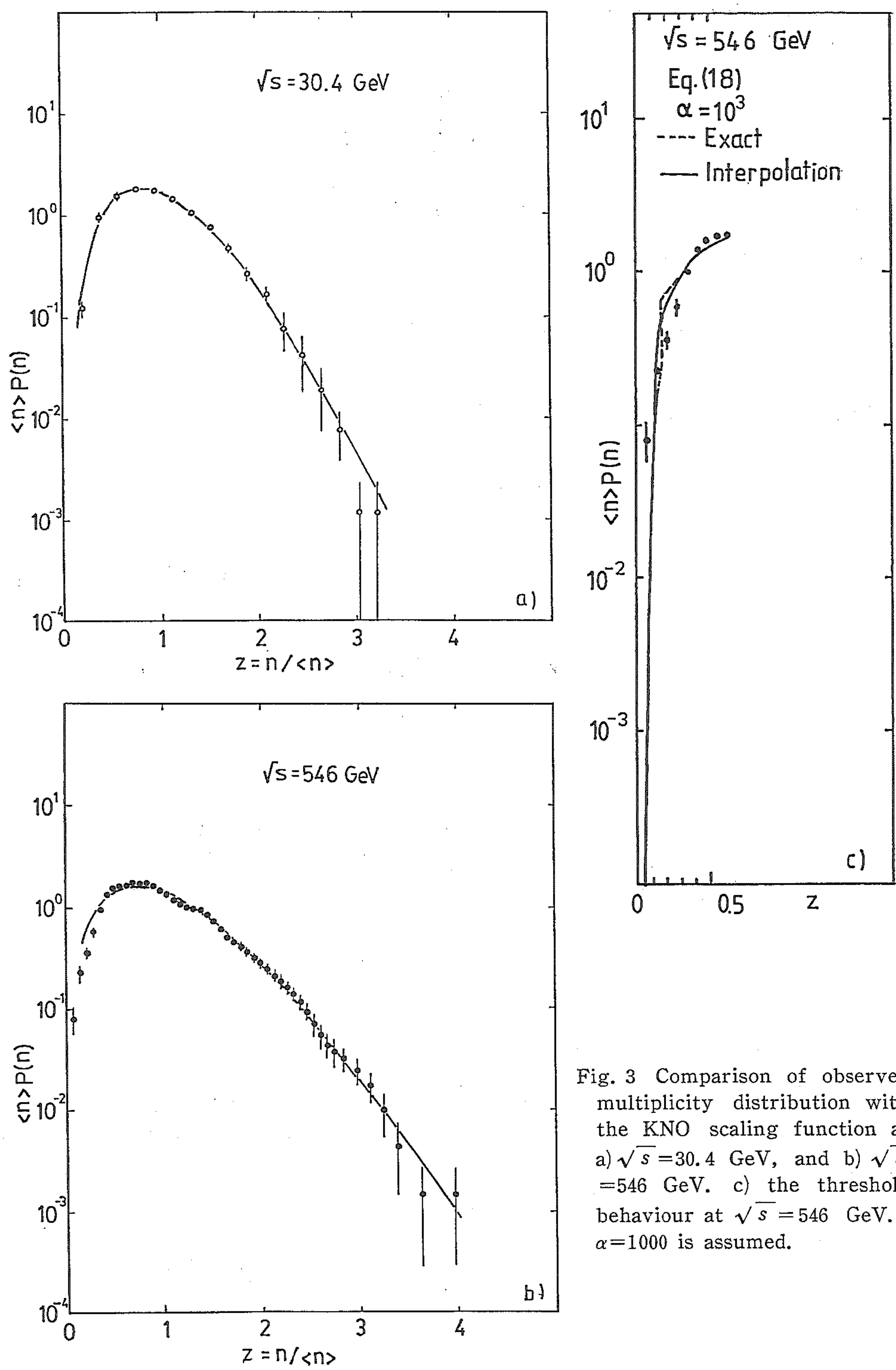

Fig. 3 Comparison of observed multiplicity distribution with the KNO scaling function at a) $\sqrt{s}=30.4$ GeV, and b) $\sqrt{s}=546$ GeV. c) the threshold behaviour at $\sqrt{s}=546$ GeV. $\alpha=1000$ is assumed.

Table II Analysis of C_q moments with eq. (20) derived from the KNO scaling function. m_0 (for KNO) is fixed at 4.0.

$\sqrt{s}$	$\langle n \rangle / m_0$(KNO)	C_2	C_3	C_4	C_5
11.5	$\langle n \rangle$=6.35±0.08	1.192±0.009	1.63±0.03	2.49±0.08	4.2 ±0.2
TWO	m_0=4.0 (fixed)	(input)	1.63	2.50	4.24
	ξ=10.63				
13.8	$\langle n \rangle$=7.21±0.06	1.175±0.006	1.57±0.02	2.33±0.04	3.8 ±0.1
TWO	ξ=10.01		1.57	2.36	3.89
19.7	$\langle n \rangle$=8.56±0.11	1.174±0.010	1.57±0.03	2.34±0.08	3.8 ±0.2
TWO	ξ=8.66		1.57	2.37	3.92
22.0	$\langle n \rangle$=8.93±0.16	1.18±0.01	1.59±0.04	2.4 ±0.1	
TWO	ξ=8.30		1.60	2.43	4.08
23.9	$\langle n \rangle$=9.25±0.19	1.19±0.02	1.62±0.06	2.47±0.14	4.17±0.34
TWO	ξ=7.96		1.63	2.52	4.33
27.6	$\langle n \rangle$=9.77±0.16	1.21±0.01	1.72±0.05	2.76±0.13	5.0 ±0.4
TWO	ξ=7.45		1.70	2.73	4.87
30.4	$\langle n \rangle$=10.5±0.14	1.20±0.01	1.68±0.03	2.64±0.10	4.58±0.28
TWO	ξ=7.23		1.67	2.64	4.63
44.5	$\langle n \rangle$=12.08±0.13	1.20±0.01	1.67±0.03	2.63±0.01	4.6 ±0.3
TWO	ξ=6.71		1.67	2.65	4.68
52.6	$\langle m \rangle$=12.76±0.14	1.21±0.01	1.70±0.03	2.70±0.09	4.8 ±0.3
TWO	ξ=6.45		1.71	2.75	4.96
62.6	$\langle n \rangle$=13.63±0.16	1.20±0.01	1.67±0.03	2.60±0.08	4.2 ±0.2
TWO	ξ=6.34		1.68	2.66	4.72
200	$\langle n \rangle$=21.4 ±0.4	1.26±0.03	1.88±0.08	3.2 ±0.3	6.2 ±0.5
TWO	ξ=5.18		1.90	3.33	6.62
546	$\langle n \rangle$=29.4 ±0.9	1.29±0.02	2.01±0.09	3.66±0.29	7.6 ±0.8
TWO	ξ=4.78		2.02	3.70	7.73
900	$\langle n \rangle$=35.6 ±0.9	1.30±0.03	2.08±0.09	3.9 ±0.3	8.2 ±0.8
TWO	ξ=4.62		2.06	3.84	8.15

§5 Forward-Backward Correlation

From the generating function, eq. (10), some formulae for the forward-backward (FB) multiplicity correlation are derived. Following the discussions in refs. [11] and [12], we can write the generating function of the FB multiplicity distribution as

$$\Pi_{FB}(x, y)=\Pi_a(\alpha x+\beta y)\cdot\Pi_b(\alpha x+\beta y) \tag{22}$$

where α (β) is a probability that a particle emitted in a final states enter into a F (B) region. The multiplicity distribution that n particles are in the B region and m particles are in the F region is derived as

$$P(n,\ m)=\frac{1}{n!}\frac{1}{m!}\frac{\partial^n}{\partial x^n}\frac{\partial^m}{\partial y^m}\Pi_{FB}(x, y)\Big|_{x=y=0}. \tag{23}$$

The conditional moment of the backward multiplicity that m particles are in the F region is written as

$$\langle n(n-1)\cdots\cdots(n-j+1)\rangle_m=\frac{(m+j)!}{m!}(\alpha/\beta)^j P^{(F)}(m+j)/P^{(F)}(m), \tag{24}$$

$$P^{(F)}(m)=\frac{1}{m!}\frac{\partial^m}{\partial y^m}\Pi_{FB}(x, y)\Big|_{y=0}$$
$$=\sum_{n_a+n_b=m}P_a{}^{(F)}(n_a,\ \beta\langle n_a\rangle)\ P_b{}^{(F)}(n_b,\ \beta\langle n_b\rangle), \tag{25}$$

where

$$P_a{}^{(F)}(n,\ \beta\langle n_a\rangle)=(\beta\langle n_a\rangle)^n\ \exp\left[-\beta\langle n_a\rangle\right]/n!, \tag{26}$$

$$P_b{}^{(F)}(0,\ \beta\langle n_b\rangle)=\exp\left\{-\frac{\beta\langle n_b\rangle}{1+\beta\langle n_b\rangle/\xi}\right\}, \tag{27a}$$

$$P_b{}^{(F)}(n,\ \beta\langle n_b\rangle)=\frac{\xi}{n}\frac{(\beta\langle n_b\rangle/\xi)^n}{(1+\beta\langle n_b\rangle/\xi)^{n+1}}\exp\left\{-\frac{\beta\langle n_b\rangle}{1+\beta\langle n_b\rangle/\xi}\right\}$$
$$\cdot L_{n-1}^{(1)}\left(-\frac{\xi}{1+\beta\langle n_b\rangle/\xi}\right)\qquad \text{for } n\geq 1. \tag{27b}$$

The observed conditional backward multiplicity $\langle n\rangle_m$ at $\sqrt{s}=900$ GeV [5] is compared with our calculation in Fig. 4. In our calculation the observed mean multiplicity $\langle n\rangle$, the observed second moment C_2, and m_0 are used as input. m_0 is fixed at 3.8 as before. A dashed line is obtained from eqs. (24)~(27)

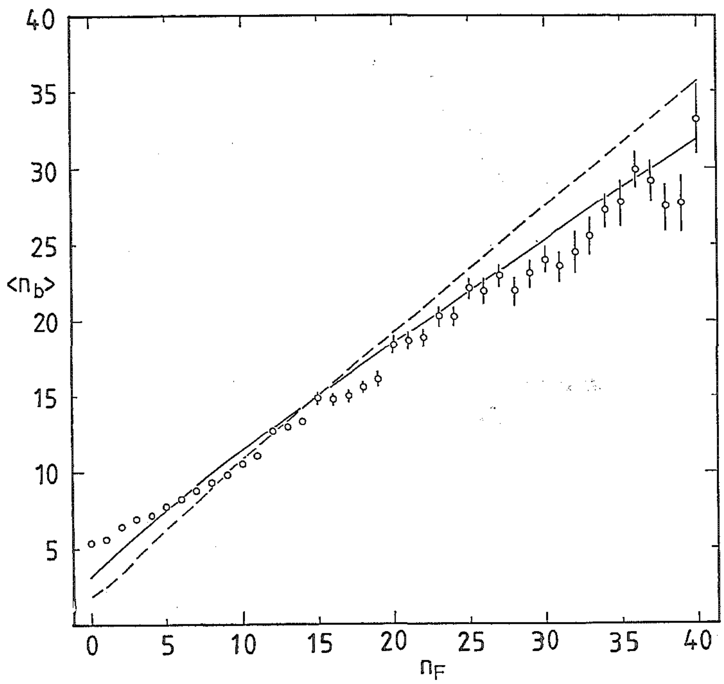

Fig. 4 Comparison of observed conditional backward maltiplicity $\langle n\rangle_m$ at $\sqrt{s}=900$ GeV with our calculation. m_0 is fixed at 3.8. Dashed line is obtained with $\langle n\rangle=28.3$ and $C_2=1.42$. Solid line is obtained with $\langle n\rangle=14.15$ and $C_2=1.42$, under an additional assumption that particles are distributed in pairs into the F or B region.

by the use of $\langle n\rangle=28.3$ and $C_2=1.42$ [6]. A solid line is obtained from eqs. (24)~(27) with the additional assumption that particles are distributed in pairs into the F or B region. In this case, we use $\langle n\rangle=14.15$ and $C_2=1.42$.

§6 Concluding Remarks

We consider the two-component model, the PB process with the Poisson process, to improve results derived from the PB process (eq. (1)) with eq. (2) as the initial condition [10, 11]; In the analysis by means of the PB process, the mean number of "excited hadrons" at the initial stage decreases with energy in the S$\bar{\text{P}}$PS collider energy region. Observed higher moments at some energies are not well expressed by the formula obtained from the PB process.

We can analyse C_q moments by the two-component model with fixed m_0 (the mean number of "excited hadrons" at t=0). Indeed we have better results for C_q moments by the two-component model than those by the PB process. Moreover, we find that the KNO scaling function obtained from the two-component model have a threshold effect at high energies due to the Poisson component (See eq. (18)). This behaviour is essentially different from other two-component models. Various two-component models have been proposed by Gupta and Sarma [19], Lim and Phua [21], Cai Xu et al [25]., Fowler et al [18]., and Blazek [13]. Some formulae for the FB multiplicity correlation are also obtained, and the observed conditional mean multiplicity at $\sqrt{s}=900$ GeV is analysed.

To distinguish those two-component models from our present approach, we can use KNO scaling functions and the conditional multiplicity moment in the FB correlation in analyses of data. In a near future this will be examined by experiments at Tevatron.

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Appendix A: General solution (7) of eq. (4) and its particular solution (10)

According to the general method for a linear differential equation of the first order, we obtain the following characteristic equation from eq. (6):

$$\frac{dt}{1}=\frac{-dv}{\lambda v(v-1)}=\frac{d\Pi}{\mu(u-1)}. \tag{A1}$$

From the first and second terms of eq. (A1), we have

$$t=\frac{1}{\lambda}\ln\frac{v}{v-1}+b_1,$$

$$B_1=\frac{v-1}{v}e^{\lambda t}, \tag{A2}$$

where B_1 $(=\exp(-b_1))$ is an arbitrary constant. Similarly from the first and third terms of eq. (A1), we have

$$t=\frac{1}{\mu(u-1)}\ln\Pi+b_2,$$

$$\Pi=B_2\exp\{\mu t(u-1)\}. \tag{A3}$$

where B_2 $(=\exp(-b_2))$ is an arbitrary constant. By making use of an arbitrary function $B_2=\mathrm{f}(B_1)$, we obtain eq. (7).

Substituting the intial condition, eq. (9), we obtain

$$\Pi(u,\ v;\ 0)=\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{{n_0}^n}{n!}e^{-n_0}\frac{{m_0}^m}{m!}e^{-m_0}\ u^n v^m$$
$$=\exp\{n_0(u-1)+m_0(v-1)\}. \tag{A4}$$

Comparing eq. (A4) with eq. (7), we get

$$f\left(\frac{v-1}{v}\right)=\exp\{n_0\ (u-1)+m_0(v-1)\}, \tag{A5a}$$

$$f(w)=\exp\left\{n_0(u-1)+m_0\left(\frac{1}{1-w}-1\right)\right\}. \tag{A5b}$$

Finally from eqs. (7) and (A5b), eq. (10) is obtained:

$$\Pi(u,\ v;\ t)=\exp\{(\mu t+n_0)\ (u-1)\}\cdot\ \exp\left\{m_0\left(\frac{v}{1-p(v-1)}-1\right)\right\}. \tag{A6}$$

Appendix B: Derivation of eqs. (13) and (14)

Let us start with the following function

$$f(u)=m_0\left\{\frac{u}{1-p(u-1)}-1\right\}. \tag{B1}$$

Performing the differentiation of eq. (B1) to the third order, we have

$$P_b(3,\langle n_b\rangle)=\frac{1}{3!}\ \frac{\partial^3}{\partial u^3}\exp\{f(u)\}\Big|_{u=0}$$
$$=\frac{1}{3!}\{f^{(3)}(0)+3f''(0)f'(0)+[f'(0)]^3\}\exp\{f(0)\}. \tag{B2}$$

In eq. (B2), $f^{(n)}(0)$ is given in the following

$$f(0)=-m_0, \tag{B3a}$$

$$f^{(n)}(0)=m_0\frac{n!p^{n-1}}{(1+p)^n}. \tag{B3b}$$

By making use of similar calculations, we have the following expressions for $P_b(n,\ \langle n_b\rangle)$

$$P_b(0,\ \langle n_b\rangle)=e^{-m_0}. \tag{B4a}$$

$$\cdots\cdots\cdots\cdots,$$

$$P_b(3,\ \langle n_b\rangle)=\frac{\langle n_b\rangle}{3}\,\frac{p^2}{(1+p)^4}e^{-m_0}\cdot(6+6m_0/p+m_0^2/p^2),$$

$$=\frac{\langle n_b\rangle}{3}\,\frac{p^2}{(1+p)^4}e^{-m_0}\cdot\Gamma(3)\cdot L_2^{(1)}(-m_0/p), \tag{B4b}$$

$$\cdots\cdots\cdots\cdots.$$

Thus we infer eq. (13), as the Laguerre polynomial is used.

The factorial moment is obtain as

$$F^{(l)}=\frac{\partial^l}{\partial u^l}\Pi(u,\ v=u;T)\Big|_{u=1}$$

$$=\sum_{j=0}^{l}\binom{l}{j}\frac{\partial^{l-j}}{\partial u^{l-j}}\Pi_a(u;T)\Big|_{u=1}\cdot\frac{\partial^j}{\partial u^j}\Pi_b(u;T)\Big|_{u=1}$$

$$=\sum_{j=0}^{l}\binom{l}{j}\cdot\langle n_a\rangle^{l-j}\frac{\partial^j}{\partial u^j}\exp\{f(u)\}\Big|_{u=1}. \tag{B5}$$

An explicit expression for $f^{(n)}(1)$ is given as

$$f(1)=0, \tag{B6a}$$

$$f^{(n)}(1)=n!\,p^{n-1}\langle n_b\rangle. \tag{B6b}$$

By making use of eq. (B6), we know concrete expressions in eq. (B5) as

$$\frac{\partial^3}{\partial u^3}\exp\{f(u)\}\Big|_{u=1}=f^{(3)}(1)+3f''(1)f'(1)+[f'(1)]^3$$

$$=6p^2\langle n_b\rangle+6p\langle n_b\rangle^2+\langle n_b\rangle^3. \tag{B7a}$$

By making use of the Laguerre polynomial, the differential in eq. (B5) is expcessed as

$$\frac{\partial^j}{\partial u^j}\exp\{f(u)\}\Big|_{u=1}=\Gamma(j)\langle n_b\rangle p^{j-1}L_{j-1}^{(1)}\ (-\langle n_b\rangle/p) \qquad (j\geq 1). \tag{B7b}$$

Thus we arrive at eq. (14).

Appendix C: Derivation of eqs. (17a) and (17b)

We know inverse poisson transform and the poisson transform between the discrete distribution and the KNO scaling function

$$P(n,\ m,\ t) \xrightleftharpoons[\text{Poisson trans.}]{\text{inverse Poisson trans.}} \phi\Big(\frac{w_1}{\langle n_a\rangle/\alpha_1},\ \frac{w_2}{\langle n_b\rangle/\alpha_2};T\Big)$$
$$\times(\langle n_a\rangle\langle n_b\rangle/\alpha_1\alpha_2.)^{-1} \qquad \text{(C1)}$$

We apply the double inverse poisson transform to eq. (4)

$$\phi\Big(\frac{w_1}{\langle n_a\rangle/\alpha_1},\ \frac{w_2}{\langle n_b\rangle/\alpha_2};T\Big)/(\langle n_a\rangle\langle n_b\rangle/\alpha_1\alpha_2)$$
$$=\frac{1}{(2\pi)^2}\exp(\alpha_1 w_1+\alpha_2 w_2)\cdot\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}dx_1dx_2\exp(-ix_1w_1-ix_2w_2)$$
$$\cdot\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\Big(\frac{ix_1}{\alpha_1}\Big)^n\Big(\frac{ix_2}{\alpha_2}\Big)^m P(n,\ m;t). \qquad \text{(C2)}$$

All the terms on the right hand side (RHS) in eq. (4) are calculated as

$$\text{The 1st term}=\mu\frac{1}{(2\pi)^2}\exp(\alpha_1x_1+\alpha_2x_2)$$
$$\cdot\Big(-\frac{1}{\alpha_1}\Big)\frac{\partial}{\partial w_1}\iint dx_1dx_2\exp(-ix_1w_1-ix_2w_2)$$
$$\cdot\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\Big(\frac{ix_1}{\alpha_1}\Big)^{n-1}\Big(\frac{ix_2}{\alpha_2}\Big)^m P(n-1,\ m;T)$$
$$=\mu\Big(-\frac{1}{\alpha_1}\frac{\partial}{\partial w_1}+1\Big)\phi/(\langle n_a\rangle\langle n_b\rangle/\alpha_1\alpha_2). \qquad \text{(C3a)}$$

$$\text{The 2nd term}=-\mu\phi/(\langle n_a\rangle\langle n_b\rangle/\alpha_1\alpha_2). \qquad \text{(C3b)}$$

$$\text{The 3rd term}=\lambda\frac{1}{(2\pi)^2}\exp(\alpha_1w_1+\alpha_2w_2).$$
$$\cdot\alpha_2w_2\iint dx_1dx_2\exp(-ix_1w_1-ix_2w_2)$$
$$\cdot\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\Big(\frac{ix_1}{\alpha_1}\Big)^n\Big(\frac{ix^2}{\alpha_2}\Big)^{m+1} P(n,m-1;t)$$
$$+\lambda(-2)\Big(-\frac{1}{\alpha_2}\frac{\partial}{\partial w_2}+1\Big)\phi/(\langle n_a\rangle\langle n_b\rangle/\alpha_1\alpha_2)$$
$$=\lambda\Big\{\frac{w_2}{\alpha_2}\frac{\partial^2}{\partial w_2{}^2}-2\Big(w_2-\frac{1}{\alpha_2}\Big)\frac{\partial}{\partial w_2}+\alpha_2w_2-2\Big\}\phi/(\langle n_a\rangle\langle n_b\rangle/\alpha_1\alpha_2).$$
$$\text{(C3c)}$$

$$\text{The 4-th term}=-\lambda\frac{1}{(2\pi)^2}\exp(\alpha_1w_1+\alpha_2w_2)$$
$$\cdot\alpha_2w_2\iint dx_1dx_2\exp(-iw_1x_1-iw_2x_2)$$

$$\cdot\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\left(\frac{ix_1}{\alpha_1}\right)^n\left(\frac{ix_2}{\alpha_2}\right)^{m+1}\cdot P(n, m;t)$$

$$-(-1)\lambda\phi/(\langle n_a\rangle\langle n_b\rangle/\alpha_1\alpha_2)$$

$$=\lambda\left\{w_2\frac{\partial}{\partial w_2}-w_2\alpha_2+1\right\}\phi/(\langle n_a\rangle\langle n_c\rangle/\alpha_1\alpha_2). \tag{C3d}$$

Summing the all terms, we have the following Fokker–Planck equation,

$$\partial\phi/\partial t=-\mu\frac{1}{\alpha_1}\frac{\partial}{\partial w_1}\phi+\lambda\left\{\frac{w_2}{\alpha_2}\frac{\partial^2}{\partial w_2{}^2}+\left(-w_2+\frac{2}{\alpha_2}\right)\frac{\partial}{\partial w_2}-1\right\}\phi. \tag{C4}$$

Intoducing two scaling variables $z_a=w_1/(\langle n_a\rangle/\alpha_1)$ and $z_b=w_1/(\langle n_b\rangle/\alpha_2)$, we obtain eq. (17a).

Here we have to obtain a solution of eq. (17a). By making use of eq. (10), we can calculate the KNO scaling function as

$$\phi(z_a,\ z_b;T)=\frac{1}{2\pi i}\int_{c_1-i\infty}^{c_1+i\infty}\Pi_a\left(1-\frac{s_1}{\langle n_a\rangle};T\right)e^{s_1z_a}\ ds_1$$

$$\cdot\frac{1}{2\pi i}\int_{c_2-i\infty}^{c_2+i\infty}\Pi_b\left(1-\frac{s_2}{\langle n_a\rangle};T\right)e^{s_2z_b}\ ds_2, \tag{C5a}$$

$$\Pi_a(1-s_1/\langle n_a\rangle)=\exp(-s_1), \tag{C5b}$$

$$\Pi_b(1-s_2/\langle n_b\rangle)=\exp\left\{\frac{\xi^2}{\xi+s_2}-\xi\right\}, \tag{C5c}$$

where $\langle n_b\rangle=m_0(1+p)$ and $\xi=\langle n_b\rangle/p$. By use of a new variable $s'=s_2+\xi$, we can perform the integration

$$\Psi(z_a,\ z_b;T)=\frac{1}{2\pi i}\int_{c_1-i\infty}^{c_1+i\infty}e^{s_1}e^{s_1z_1}ds_1$$

$$\cdot\frac{1}{2\pi i}\exp\left\{-(z_b+1)\xi\right\}\int_{c'-i\infty}^{c'+i\infty}\exp(\xi^2/s')e^{s'z_b}ds'$$

$$=\delta(z_a-1)\exp\{-(z_b+1)\xi\}\cdot\left\{\frac{\partial}{\partial z_b}I_0\ (2\xi\sqrt{z_b})+\delta(z_b-(+0))\right\}. \tag{C6}$$

Finally we get eq. (17b).

Appendix D: Derivation of eqs. (18a) and (19)

i) Derivation of eq. (18a).

$$\Psi(z_b;T)=\int_0^\infty \Psi(z_a,z_b;T)dz_a$$

$$=\exp\{-(z_b+1)\xi\}\xi z_b^{-1/2}\cdot I_1(2\xi\sqrt{z_b})+\delta(z_b-(+0))e^{-\xi}$$

$$=\exp\left\{-\left(\frac{z-r}{1-r}\right)\xi\right\}\cdot\xi\left\{(z-r)/(1-r)\right\}^{-\frac{1}{2}}\cdot I_1\{2\xi\sqrt{(z-r)/(1-r)}\}$$

$$+\delta\{(z-r)/(1-r)\}\cdot e^{-\xi}\quad (z\geq r), \tag{D1}$$

where we use $z_b=(z-r)/(1-r)$. Noticing $dz_b/dz=1/(1-r)$, eq. (18a) is obtained.

ii) **Derivation of eq.** (19)

The generating function Π, i.e., eq. (10a) is expressed as

$$\Pi(1-s/\langle n\rangle,\ 1-s/\langle n\rangle;T)$$

$$=\exp\left\{-\langle n_a\rangle\frac{s}{\langle n\rangle}\right\}\exp\left\{m_0\left(\frac{1-s/\langle n\rangle}{1+ps/\langle n\rangle}-1\right)\right\}$$

$$=\exp(-rs)\cdot\exp\left\{\frac{\xi}{1-r}\Big/\left(\frac{\xi}{1-r}+s\right)-\xi\right\}, \tag{D2}$$

where $r=\langle n_a\rangle/\langle n\rangle$, $\xi=m_0(1+p)/p$ and $p/\langle n\rangle=(1-r)/\xi$. KNO scaling function Ψ is calculated by the Laplace transformation

$$\Psi=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\exp\left\{\frac{\xi^2}{1-r}\Big/\left(\frac{\xi}{1-r}+s\right)-\xi\right\}e^{s(z-r)}\ ds. \tag{D3a}$$

Introducing new variable $z'=z-r$ and $s'=s+\xi/(1-r)$, we can performed integration

$$\Psi=\frac{1}{2\pi i}\exp\left\{-\xi\left(\frac{z'}{1-r}+1\right)\right\}\int_{c'-i\infty}^{c'+i\infty}\exp\left\{\frac{\xi^2/(1-r)}{s'}\right\}e^{s'z'}\ ds'$$

$$=\exp\left\{-\xi\left(\frac{z'}{1-r}+1\right)\right\}\left\{\frac{\partial}{\partial z'}I_0\ (2\sqrt{z'/(1-r)})-\delta(z')\right\}. \tag{D3b}$$

Finally we get eq. (19).

Appendix E: Derivation of eq. (20)

$$C_q=\int_{r-0}^\infty z^q\ \Psi(z)dz$$

$$=\int_{-0}^\infty (z+r)^q\Psi(z+r)dz$$

$$=\int_0^\infty dz(z+r)^q\exp\left\{-\xi\left(\frac{z}{1-r}+1\right)\right\}\xi\left\{z(1-r)\right\}^{-\frac{1}{2}}$$

$$\cdot I_1(2\xi\sqrt{z/(1-r)}) + r^q e^{-\xi}$$

$$=\sum_{j=0}^{q}\binom{q}{j} r^{q-j}\ e^{-\xi}/\{k!(k+1)!\}$$

$$\cdot\sum_{k=0}^{\infty}\xi^{2k+2}\ (1-r)^{-k-1}\int_0^{\infty} dz\ \exp\left(-\xi\frac{z}{1-r}\right) z^{k+j} + r^q e^{-\xi}$$

$$=\sum_{j=0}^{q}\binom{q}{j} r^{q-j} e^{-\xi}(1-r)^j\ \xi^{1-j}\sum_{k=0}^{\infty}\frac{(k+j)!}{k!(k+1)!}\xi^k + r^q e^{-\xi}. \tag{E1}$$

Here we calculate the second summantion,

$$A_j=\sum_{k=0}^{\infty}\frac{(k+j)!}{k!(k+1)!}\xi^k, \tag{E2a}$$

then

$$A_0=\xi^{-1}(e^{\xi}-1), \tag{E2b}$$

$$A_1=e^{\xi},$$

$$A_2=e^{\xi}(\xi+2)=e^{\xi}\ L_1{}^{(1)}(-\xi),$$

$$A_3=e^{\xi}(\xi^2+6\xi+6)=2e^{\xi}L_2{}^{(1)}(-\xi), \tag{E2c}$$

$$\cdots\cdots\cdots\cdots,$$

$$A_j=e^{\xi}\,\Gamma(j)L_{j-1}^{(1)}(-\xi)\qquad (j\geq 1). \tag{E2d}$$

Thus we obtain eq. (20).

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