# ON THE EQUIVARIANT MAPPING FROM THE SPHERE TO THE UNITARY GROUP WITH $Z_4$ AND $Zp^2$ ACTION FOR ODD PRIME $p$

Masato NAKAMURA

Let $k$ be the positive integer and $k \geqq 2$. And $\gamma$ be the rotation of $(2n+1)$-sphere $S^{2n+1} = [(z_0, z_1, \cdots\cdots, z_n)/\sum_{i=0}^{n}|z_i|^2 = 1]$ of the complex $(n+1)$-space $C^{n+1}$ given by $\gamma(z_0, z_1, \cdots\cdots, z_n) = (e^{2\pi i/k}z_0, e^{2\pi i/k}z_1, \cdots\cdots, e^{2\pi i/k}z_n)$. Then $\gamma$ generates the $Z_k$-action on $S^{2n+1}$. Let unitary group $U(m)$ also carry the $Z_k$-action

$$A| \longrightarrow e^{2\pi i/k}A$$

In this paper we consider the case $k=4$ and $p^2$ for odd primep. We shall prove the following theorem.

Theorem 1. If equivariant map $S^{2n+1} \longrightarrow U(m)$ exists with $Z_4$ action, then $2^{n+1}$ divides $m$.

Proof. Let $B^{2(n+1)}$ denote the unit ball in $C^n$ with $Z_4$-action $Z = (z_0, z_1, \cdots\cdots, z_n) | \longrightarrow e^{2\pi i/4}z = (e^{2\pi i/4}z_0, e^{2\pi i/4}z_1, \cdots e^{2\pi i/4}z_n)$. The product vector bundle $B^{2(n+1)} \times C^m \longrightarrow B^{2(n+1)}$ can be made a $Z_4$—vector bundle in two ways, corresponding to the $Z_4$-action $(z, v) | \longrightarrow (e^{2\pi i/4}z, v)$ and $(z, v) | \longrightarrow (e^{2\pi i/4}z, e^{2\pi i/4}v)$ ; the resulting $Z_4$-vector bundle over $B^{2(n+1)}$ will be denoted by $E^+$ and $E^-$. Suppose an equivariant map $f: S^{2n+1} \longrightarrow U(m)$ was given. Since $f(e^{2\pi i/4}z)v = e^{2\pi i/4}f(z)v$ for $z\ S^{2n+1}$ and $v \epsilon C^m$, the map $(z, v) | \longrightarrow (z, f(z)v)$ of $S^{2n+1} \times C^m$ onto itself is an isomorphism of the restrictions of the ~~the~~ $Z_4$-vector bundles $E^+$ and $E^-$ to $S^{2n+1}$. By means of the difference construction, we then obtain a difference bundle $d_f(E^+, E^-) \epsilon K_{Z_4}(B^{2(n+1)}, S^{2n+1})$. The composition of the induced homomorphisms

$K_{Z_4}(B^{2(n+1)}, S^{2(n+1)}) \longrightarrow K_{Z_4}(B^{2(n+1)}) \longrightarrow K_{Z_4}(S^{2(n+1)})$ must vanish. And $KZ_4(S^{2n+1}) = K(L^n(4))$, where $L((4))$is lens space. Let $\eta$ be the canonical complex line bundle over $L(4)$ and set

$$\sigma = 1 - \eta \epsilon \widetilde{K}(L^n(4)).$$

Then, following T. Kobayashi and M. Sugawara (see refference) order of $\sigma$ is $2^{n+1}$.

Now $d_f(E^+, E^-)$is carried by the above composition of homomorphisms to $m\sigma$ in $K(L^n(4))$, hence we conclude that $2^{n+1}$ divides $m$. This completes the proof of the theorem.

Next we consider the case $k = p^2$.

Theorem 2. Let $p$ be odd prime, $n$ be an integer and $n = a(p-1) + b$ $(o \leqq b < p$

$-1$). If the equivariant map $S^{2n+1} \longrightarrow U(m)$ exists with $Zp^2$—action, then $p^{2+a}$ divides $m$.

Proof. Proof is similar to the above theorem using the result of T. Kobayashi and M. Sugawara (see refference [2]) from which order of $\sigma$ is $p^{2+a}$. So the theorem follow.

(1977. 9. 29)

## Refference


[1] T. Kobayashi and M. Sugawara, $K_\Lambda$-Rings of Lens Spaces $L(4)$), Hiroshima Math. j. 1 (1971), 253-271.

[2] T. Kawaguchi and M. Sugawara, K-and KO-Rings of the Lens Space $L^n(p^2)$ for Odd Prime $p$, Hiroshima Math. j. 1 (1971) 273-286.