

# Adaptive Control Based on Theoretical Analysis in RC-OFDM Systems

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## ABSTRACT

In the OFDM systems with repetition coding, we theoretically investigate an adaptive control to enable the best switching of both the number of diversity branches and the level of digital modulation to enhance the throughput performance.

## I. INTRODUCTION

OFDM is very attractive in the aspects of spectrum efficiency and robustness against multipath fading. In order to combat the deep fade over wireless channels, we have been focused on the OFDM with a repetition code on the frequency domain (RC-OFDM) to realize the diversity technique with one pair of transmit/receive antennas [1]. On the other hand, with a trade-off relationship, there is a demerit that the transmission rate is degraded in order to send the same signal. The purpose of this study is to improve the communication quality and the transmission rate by determining the relative merits for each subcarrier using the adaptive control. In this paper, we employ a theoretical adaptive control to enable the best switching of both the number of diversity branches and the level of digital modulation.

## II. RESEARCH OUTLINE

### A. System summary

Figure 1 shows a system model, where  $C$  is the number of subcarriers. At a transmitter, digital modulated signals are subject to serial-to-parallel conversion, then the same signal is repeatedly coded (duplicated) with equal frequency spacing in the subcarrier mapping. Figure 1 shows an example of two-branch diversity with half the subcarriers spacing. The OFDM modulated signal with a guard interval (GI) is transmitted and subject to Rayleigh fading with frequency selectivity and additive white Gaussian noise (AWGN). At a receiver, removal of the GI, serial-to-parallel conversion, OFDM demodulation and coherent detection are successively performed. Digital data are demodulated by applying the maximum ratio combining (MRC). Instantaneous carrier to noise power ratio (CNR) of each subcarrier after MRC is individually calculated. The adaptive control, that is, the adequate selection of digital modulation and repetition coding for MRC can be performed at the transmitter by using the CNR. In the practical case, the transmitter needs to know the CNR which is determined at the receiver in order to perform the adaptive control. The CNR is assumed to be perfectly known at the transmitter.

### B. Determination of instantaneous CNR

Assuming the average CNR over fading channels to be  $\Gamma$  and the number of diversity branches to be  $L$ , the instantaneous CNR  $\gamma_i$  of the  $i$ -th subcarrier after MRC ( $i = 0, 1, \dots, C/L - 1$ ) can be determined using the following equation:

$$\gamma_i = \sum_{l=0}^{L-1} |h_{i+\frac{C}{L}l}|^2 \Gamma, \quad (i = 0, 1, \dots, C/L - 1), \quad (1)$$

where  $h_{i'}$  ( $i' = 0, 1, \dots, C - 1$ ) is the fading coefficient of the  $i'$ -th subcarrier. The summation in Equ. (1) means the diversity combining of the signals with  $C/L$  subcarriers spacing.

### C. Theoretical adaptive control

In order to theoretically control the number of diversity branches, the packet success probability is used as an index. According to  $\gamma_i$  in Equ. (1), instantaneous bit error rate (BER)  $P_i$  over fading channels can be theoretically calculated by

$$P_i = \begin{cases} \frac{1}{2} \text{erfc} \sqrt{\gamma_i} & (\text{BPSK}) \\ \frac{1}{2} \text{erfc} \sqrt{\frac{\gamma_i}{2}} & (\text{QPSK}) \\ \frac{1}{3} \text{erfc} \left( \sqrt{\frac{\gamma_i}{2}} \times \sin \frac{\pi}{8} \right) & (8\text{PSK}) \\ \frac{3}{8} \text{erfc} \sqrt{\frac{\gamma_i}{10}} & (16\text{QAM}) \end{cases} \quad (2)$$

Equ. (2) is the formula for the BER over AWGN channels [2]. The modulation can be theoretically decided with a threshold of the instantaneous BER, which will be explained in Section III. On the other hand, the BER  $P_i$  also depends on the instantaneous CNR  $\gamma_i$ , that is, the number of branches  $L$ , then the packet success probability also depends on  $L$ . Now we define the BER as  $P_i^j(l)$  ( $l = 1, 2, \dots, L$ ), where  $j$  is index number of bit within the packet. The packet success probability  $Q(l)$  for each number of branch  $l$  can be calculated by

$$Q(l) = \prod_{j=0}^{F-1} \{1 - P_i^j(l)\}, \quad (l = 1, 2, \dots, L), \quad (3)$$

where  $F$  is the packet size (bits). We choose the number of branches  $l$  whose  $Q(l)$  becomes the highest value. If  $Q(l)$  has taken the same value, we choose the lowest  $l$  in order to suppress the degradation of the transmission rate.

## III. PERFORMANCE EVALUATION

### A. Error correcting coding versus repetition coding

Fast of all, we briefly compare the repetition coding and the error correcting coding, such as low density parity check (LDPC) coding and Turbo coding, which also combats the

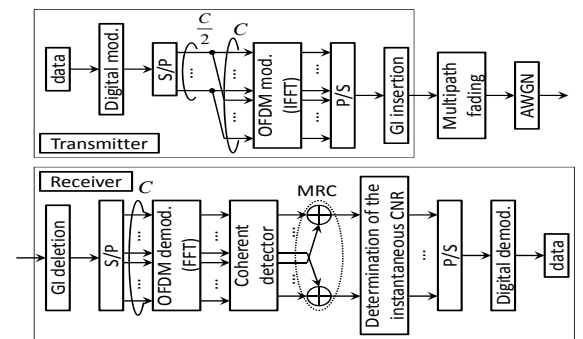


Fig. 1. System model as example of two-branch diversity.

deep fade over wireless channels at the price of the transmission rate. Figure 2 shows the average BER performance as a parameter of normalized delay spread in the case where the average CNR is 20 dB and Doppler frequency is 0 Hz (without time selectivity). We use QPSK modulation in 48 subcarriers with 64 points FFT and 16 samples of GI. The transmission rate with the same shape of plot in the figure is the same, for example, repetition coding with  $L = 2$  and LDPC coding with  $R = 1/2$  become the same rate. It is found from the figure that the BER with the repetition coding can be more improved even if the fading selectivity is low, that is, imperfect interleaving effect against the deep fade.

### B. Instantaneous BER performance

Table I shows the simulation specifications and Figure 3 shows the instantaneous BER performance. The curve in the figure shows the theoretical values of the BER over AWGN channels (without fading). The plots show the simulation results over Rayleigh fading channels. It is found from the figure that the simulation values are consistent with the theoretical ones. The instantaneous BER can be accurately counted by using the instantaneous CNR of the receiver. The filled plot in the figure shows the BER when implementing the adaptive modulation by setting the threshold to  $10^{-4}$ , which is illustrated in the figure. At high instantaneous CNR, the highest-level modulation can be chosen while satisfying the threshold condition.

### C. Average throughput performance

Figure 4 shows the average throughput performance, where the filled circle plot shows in the case of the adaptive branch control and the outline plots show in the case of the fixed number of branches. Adaptive modulation threshold is set to  $10^{-4}$  in the instantaneous BER performance and the packet size is assumed to be 1000 bits regardless of the number of branches. It is found from the figure that the best number of branches cannot be selected. The high CNR with the 4 branches leads to high packet success probability, hence the 4 branches force to be selected.

To determine the minimum number of branches that satisfy the condition, the threshold of the packet success probability is provided in order to select the best branch, which is indicated by the other filled plots in the figure. If we set the threshold

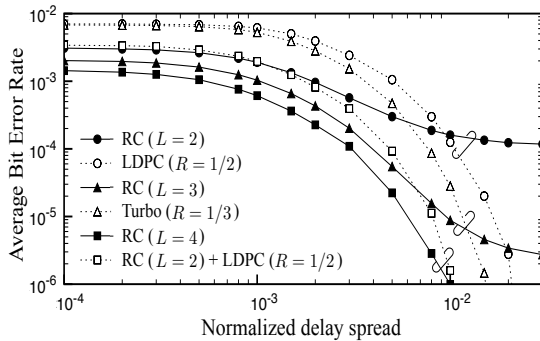


Fig. 2. Comparison between error correcting coding and repetition coding.

of 99% success, the throughput is drastically improved against the one in the case of ‘w/o threshold’. By loosening the threshold to 80%, the throughput is further improved, that is, we can select the more optimal number of branches. It should be noted that the loss packet needs to be re-transmitted, which includes the average throughput. In this scheme, it is necessary to set an optimum threshold for the best trade-off between the communication quality and the transmission rate.

## IV. CONCLUSION

In this paper, we have investigated a theoretical adaptive control to enable the best switching of both the number of diversity branches and the level of digital modulation. As future works, we intend to investigate the adaptive control over time selective fading channels.

## ACKNOWLEDGMENT

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## REFERENCES

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- [2] J.G. Proakis, Digital Communications, McGraw-Hill, New York, 1983.

TABLE I  
SIMULATION SPECIFICATIONS.

Modulation	BPSK, QPSK, 8PSK, 16QAM
Number of FFT points, Subcarriers, GI	256,192,64
Normalized delay spread	0.01 (Exponential decay)

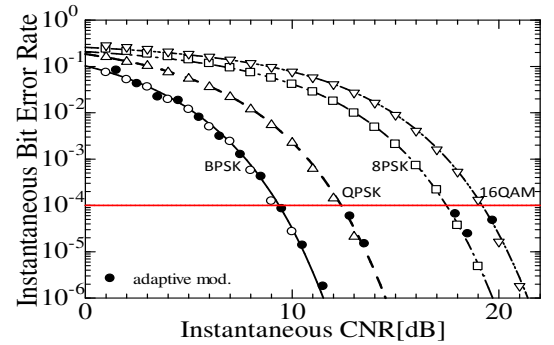


Fig. 3. Instantaneous CNR vs. instantaneous BER over fading channels.

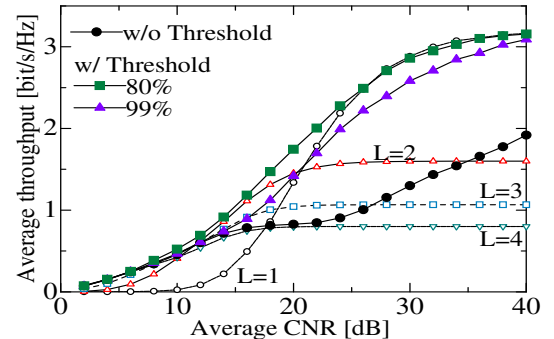


Fig. 4. Throughput performance with adaptive control over fading channels.