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Nonlinear Tracking Control of Rigid Spacecraft under Disturbance using PID-type H_∞ Adaptive State Feedback*

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Future space programs will require an agile relative position and attitude control technology for spacecraft. Rendezvous and docking, capture of inoperative spacecraft and formation flight in orbit are the typical scenarios. One of the key technologies is designing the tracking controllers that can control the six degrees-of-freedom (d.o.f.) of spacecraft under the influence of physical parameter uncertainties and external disturbances. To achieve agility, the controller design must be formulated as nonlinear control problems where translational and rotational motions are dynamically coupled with each other. This paper proposes a tracking controller, proportional-integral-derivative (PID)-type H_∞ adaptive state feedback controller, that can attenuate disturbances. The proposed controller has positive definite gain matrices whose conditions to be satisfied are given by linear matrix inequalities. The properties of the proposed controller were evaluated through numerical studies and compared with those of existing controllers.

Key Words: Spacecraft, Tracking Control, PID-type H_∞ Adaptive State Feedback Control

Nomenclature

$\{i\}$: inertial frame
$\{c\}$: chaser body fixed frame
$\{t\}$: target body fixed frame
R^n: linear space of real vectors of dimension n
$\mathsf{R}^{n\times m}$: ring of matrices with n rows and m columns and elements in R
S^q: hypersphere of dimension q
$t \in \mathsf{R}$: time
$m,\ m_t \in \mathsf{R}$: mass of chaser and target
$J,\ J_t \in \mathsf{R}^{3\times 3}$: inertia matrix of chaser and target
$f,\ \tau \in \mathsf{R}^3$: control force and torque
$d_f,\ d_\tau \in \mathsf{R}^3$: disturbance force and torque
$r,\ r_t \in \mathsf{R}^3$: position vector of frame $\{c\}$ and $\{t\}$
$[\varepsilon^{\mathrm{T}}\ \eta]^{\mathrm{T}} \in \mathsf{S}^3$: attitude vector (quaternion) of frame $\{c\}$
$[\varepsilon_t^{\mathrm{T}}\ \eta_t]^{\mathrm{T}} \in \mathsf{S}^3$: attitude vector (quaternion) of frame $\{t\}$
$v,\ v_t \in \mathsf{R}^3$: linear velocity vector of frame $\{c\}$ and $\{t\}$
$\omega,\ \omega_t \in \mathsf{R}^3$: angular velocity vector of frame $\{c\}$ and $\{t\}$
$p_t \in \mathsf{R}^3$: constant vector fixed frame $\{t\}$
$x^\times \in \mathsf{R}^{3\times 3}$: skew symmetric matrix derived from vector $x \in \mathsf{R}^3$
$\|x\| = \sqrt{x^{\mathrm{T}}x}$: vector 2-norm
$X > 0$: positive definite matrix
$\lambda_X = \|X\|$: induced matrix 2-norm
I_n: a unit matrix of size $n \times n$
$O_{n\times m}$: a zero matrix of size $n \times m$
$a_1,\ a_2,\ b_1,\ b_2,\ \gamma,\ \sigma_\eta \in \mathsf{R}$: design parameter
$\sigma_r,\ \sigma_v,\ \sigma_\omega \in \mathsf{R}^{3\times 3}$: design parameter
$k_{p1},\ k_{p3},\ k_{i1},\ k_{i2} \in \mathsf{R}$: positive scalar feedback gain
$K_{p2},\ K_{d1},\ K_{d2} \in \mathsf{R}^{3\times 3}$: positive definite matrix feedback gain
$\Gamma_1 \in \mathsf{R}$: positive scalar adaptive gain
$\Gamma_2 \in \mathsf{R}^{6\times 6}$: positive definite matrix adaptive gain
$\mathrm{diag}\{a,\ b,\ c, \ldots\}$: diagonal matrix

1. Introduction

Future space programs will require an agile relative position and attitude control technology for spacecraft. Rendezvous and docking, capture of inoperative spacecraft and formation flight in orbit are the typical scenarios. One of the key technologies is designing the tracking controllers that can control the six degrees-of-freedom (d.o.f.) of spacecraft under the influence of physical parameter uncertainties and external disturbances. As for physical parameter uncertainties, we suppose that the mass and the inertia of the spacecraft vary as the results of fuel consumption when using thrusters. To achieve agility, the controller design must be formulated as nonlinear control problems where translational and rotational motions are dynamically coupled with each other.

Many studies have been carried out on nonlinear attitude control of rigid spacecraft. Among them, studies on passivity-based control[1–12)] appear to be the most promising because this control method is simple to implement and robustly stable against physical parameter uncertainties. In the same framework, attitude tracking using a proportional-derivative (PD)-type state feedback controller having positive scalar gains has been proposed[1,4–9)] and extended to backstepping control.[13,14)] However, these passivity-based control methods ensure only asymptotic stability of the relative attitude under a disturbance-free environment. To achieve tracking control under disturbances, most research-

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ers have focused on a nonlinear H_∞ controller[15–22] which is a PD-type state feedback controller whose scalar gains are tuned so as to make the L_2 gain of the closed-loop system from the disturbance to the controlled output less than a positive constant γ. However, although it generally requires high feedback gains to achieve high disturbance attenuation ability, it is not realizable because the maximum level of the control input is physically limited. Therefore, in order to suppress the disturbances, we consider it is not necessarily the best approach to employ the H_∞ PD-type controller.

In fact, an integral action must be added to the controller to eliminate the steady-state error caused by constant disturbance as the proportional-integral-derivative (PID) control for linear systems.[23] From this viewpoint, we propose a PID-type H_∞ state feedback controller for nonlinear tracking control problems assuming that the change rate in space environment disturbance is smaller compared to the control system dynamics.[24] We derived the conditions of the closed-loop asymptotic stability and L_2 gain less than or equal to γ described by linear matrix inequalities (LMIs).[24] Although it has achieved enough disturbance attenuation ability with moderate feedback gains, one problem still remains unsolved: the lack of closed-loop robust stability against physical parameter uncertainties.

Considering the above facts, this paper extended the results of Ikeda et al.[24] to a PID-type H_∞ adaptive state feedback tracking controller that has a parameter update law to cope with parameter uncertainties as well as to effectively attenuate constant signals included in the disturbance. The properties of the proposed controller were evaluated through numerical studies and compared with those of existing controllers.

2. Modeling and Problem Description

We consider a control problem in which a chaser spacecraft tracks a target point moving in an inertial frame under the influence of disturbances. Frames and vectors are defined in Fig. 1. Our objective is to control the chaser so that its mass center C tracks point P and frame $\{c\}$ tracks frame $\{t\}$.

The translation and rotation dynamics of the chaser fixed frame $\{c\}$ are given by the following equations[25]:

$$m\dot{v} + m\omega^\times v = f + d_f, \tag{1}$$

$$J\dot{\omega} + \omega^\times J\omega = \tau + d_\tau. \tag{2}$$

The position of mass center C and the attitude of $\{c\}$ with respect to $\{i\}$ are given by the following kinematics if a quaternion is used as the attitude parameterization:

$$\begin{aligned} \dot{r} &= v - \omega^\times r, \\ \dot{q} &= E(q)\omega = \frac{1}{2}\begin{bmatrix} \eta I_3 + \varepsilon^\times \\ -\varepsilon^{\mathrm{T}} \end{bmatrix}\omega, \end{aligned} \tag{3}$$

where $q = [\varepsilon^{\mathrm{T}}\ \eta]^{\mathrm{T}}$ satisfies the constraint $\|q\| = 1$, $\forall t \geq 0$.

On the other hand, the dynamics and kinematics of the target motion are described as follows:

$$m_t\dot{v}_t + m_t\omega_t^\times v_t = 0, \tag{4}$$

$$J_t\dot{\omega}_t + \omega_t^\times J_t\omega_t = 0, \tag{5}$$

$$\begin{aligned} \dot{r}_t &= v_t - \omega_t^\times r_t, \\ \dot{q}_t &= E(q_t)\omega_t = \frac{1}{2}\begin{bmatrix} \eta_t I_3 + \varepsilon_t^\times \\ -\varepsilon_t^{\mathrm{T}} \end{bmatrix}\omega_t. \end{aligned} \tag{6}$$

Then, the position and velocity of point P fixed in frame $\{t\}$ are given by

$$r_{p_t} = r_t + p_t, \quad v_{p_t} = v_t + \omega_t^\times p_t. \tag{7}$$

The objective of our tracking control problem is to find control laws such that

$$r = r_{p_t}, \quad q = q_t, \quad v = v_{p_t}, \quad \omega = \omega_t$$

as $t \to \infty$. To this end, an error system in $\{c\}$ is described as follows. Let the direction cosine matrix from $\{t\}$ to $\{c\}$ be

$$C = \left(\eta_e^2 - \varepsilon_e^{\mathrm{T}}\varepsilon_e\right)I_3 + 2\varepsilon_e\varepsilon_e^{\mathrm{T}} - 2\eta_e\varepsilon_e^\times \tag{8}$$

using the quaternion of relative attitude $q_e = [\varepsilon_e^{\mathrm{T}}\ \eta_e]^{\mathrm{T}}$, where ε_e and η_e are defined as

$$\varepsilon_e = \eta_t\varepsilon - \eta\varepsilon_t + \varepsilon^\times\varepsilon_t, \quad \eta_e = \eta\eta_t + \varepsilon^{\mathrm{T}}\varepsilon_t. \tag{9}$$

The relative position, linear velocity, and angular velocity are given in the same frame $\{c\}$ as

$$r_e = r - Cr_{p_t}, \quad v_e = v - Cv_{p_t}, \quad \omega_e = \omega - C\omega_t. \tag{10}$$

Substitution of Eq. (10) into Eqs. (1)–(3) using the identity $\dot{C} = -\omega_e^\times C$ yields the following relative motion equations:

$$\begin{aligned} m\dot{v}_e = &-m\left\{(\omega_e + C\omega_t)^\times v_e + C\dot{v}_{p_t} + (C\omega_t)^\times Cv_{p_t}\right\} \\ &+ f + d_f, \end{aligned} \tag{11}$$

$$\begin{aligned} J\dot{\omega}_e = &-(\omega_e + C\omega_t)^\times J(\omega_e + C\omega_t) - J\left(C\dot{\omega}_t - \omega_e^\times C\omega_t\right) \\ &+ \tau + d_\tau, \end{aligned} \tag{12}$$

$$\dot{r}_e = v_e - (\omega_e + C\omega_t)^\times r_e, \tag{13}$$

$$\dot{q}_e = E(q_e)\omega_e = \frac{1}{2}\begin{bmatrix} \eta_e I_3 + \varepsilon_e^\times \\ -\varepsilon_e^{\mathrm{T}} \end{bmatrix}\omega_e. \tag{14}$$

By transformation, the tracking control problem is reduced to a regulation problem to design control inputs f and τ such that

$$(r_e, \varepsilon_e, v_e, \omega_e) \to (0, 0, 0, 0)$$

as $t \to \infty$ under disturbance d_f and d_τ, according to

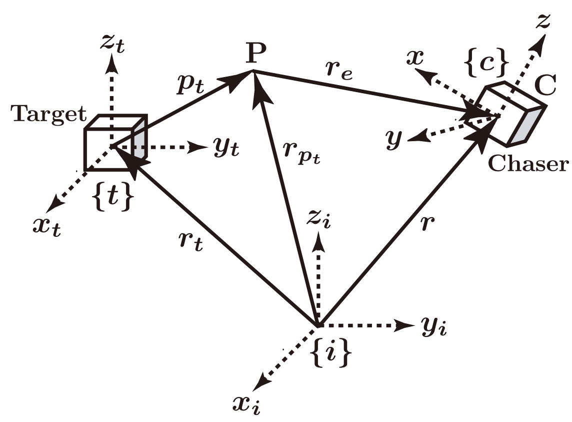


Fig. 1. Definitions of vectors and frames.

Eqs. (11)–(14).

Hereafter, we assume that the following variables

$$\text{chaser: } r,\ \varepsilon,\ \eta,\ v,\ \omega$$

$$\text{target: } r_t,\ \varepsilon_t,\ \eta_t,\ v_t,\ \omega_t,\ \dot{v}_t,\ \dot{\omega}_t$$

are directly measurable,† and m and J are unknown but nominal values m_0 and J_0 are known. In addition, regarding the target states, the following assumption is made.

Assumption 1

The target states r_t, ε_t, η_t, v_t, ω_t, $\dot{v}_t$, and $\dot{\omega}_t$ are uniformly continuous and bounded for all $t \in [0, \infty)$.

Remark 1

Quaternion η_e when $\varepsilon_e = 0$ exists as $\eta_e = \pm 1$ from the constraint of quaternion $\|q_e\| = 1$. In this paper, η_e, which should be stabilized, is set to $\eta_e = 1$.

3. PID-type H_∞ Adaptive State Feedback Controller

In this section, we derive a PID-type H_∞ adaptive state feedback controller. For this design, we further transform Eqs. (11)–(14) as

$$\bar{v}_e = v_e - (C\omega_t)^\times r_e, \tag{15}$$

$$\bar{f} = f - m_0\delta_r, \tag{16}$$

$$\bar{\tau} = \tau - \delta_q\alpha_0, \tag{17}$$

where $\bar{f}$, $\bar{\tau} \in \mathrm{R}^3$ are the new inputs, and

$$\begin{aligned}\delta_r &= 2(C\omega_t)^\times \bar{v}_e + (C\omega_t)^\times (C\omega_t)^\times r_e + (C\dot{\omega}_t)^\times r_e \\ &\quad + (C\omega_t)^\times C v_{p_t} + C\dot{v}_{p_t},\end{aligned} \tag{18}$$

$$\begin{aligned}\delta_q &= \delta_{q1}(\omega_e, C\omega_t) + \delta_{q1}(C\omega_t, \omega_e + C\omega_t) \\ &\quad + \delta_{q2}(C\dot{\omega}_t - \omega_e^\times C\omega_t),\end{aligned} \tag{19}$$

$$\alpha_0 = [J_{0,11}\ J_{0,12}\ J_{0,13}\ J_{0,22}\ J_{0,23}\ J_{0,33}]^{\mathrm{T}},$$

$$\delta_{q1}(x, y)\alpha_0 = x^\times J_0 y, \quad \delta_{q2}(x)\alpha_0 = J_0 x, \quad \forall x, y \in \mathrm{R}^3,$$

$$\delta_{q1}(x, y) = \begin{bmatrix} 0 & -x_3y_1 & x_2y_1 & -x_3y_2 & x_2y_2 - x_3y_3 & x_2y_3 \\ x_3y_1 & x_3y_2 & x_3y_3 - x_1y_1 & 0 & -x_1y_2 & -x_1y_3 \\ -x_2y_1 & x_1y_1 - x_2y_2 & -x_2y_3 & x_1y_2 & x_1y_3 & 0 \end{bmatrix},$$

$$\delta_{q2}(x) = \begin{bmatrix} x_1 & x_2 & x_3 & 0 & 0 & 0 \\ 0 & x_1 & 0 & x_2 & x_3 & 0 \\ 0 & 0 & x_1 & 0 & x_2 & x_3 \end{bmatrix},$$

$J_{0,ij}$ ($i \le j$, $i = 1, 2$, $j = 1, 2, 3$) is the (i, j) element of matrix J_0. Then, Eqs. (11)–(14) are transformed into

$$m\dot{\bar{v}}_e = -m\omega_e^\times \bar{v}_e - \Delta m\delta_r + \bar{f} + d_f, \tag{20}$$

$$J\dot{\omega}_e = -\omega_e^\times J\omega_e - \delta_q\Delta\alpha + \bar{\tau} + d_\tau, \tag{21}$$

$$\dot{r}_e = \bar{v}_e - \omega_e^\times r_e, \tag{22}$$

$$\dot{q}_e = E(q_e)\omega_e, \tag{23}$$

where $\Delta m = m - m_0$, and $\Delta\alpha = \alpha - \alpha_0$. For the above system Eqs. (20)–(23), the following Lemma holds.

Lemma 1

The system Eqs. (20)–(23) are passive w.r.t input $u = [\bar{f}^{\mathrm{T}}\ \bar{\tau}^{\mathrm{T}}]^{\mathrm{T}}$ and output $y = [\bar{v}_e^{\mathrm{T}}\ \omega_e^{\mathrm{T}}]^{\mathrm{T}}$ when $\Delta m = 0$, $\Delta\alpha = 0$, $d_f = 0$ and $d_\tau = 0$. □

Proof: Let us define the storage function as

$$S = \frac{m}{2}\|\bar{v}_e\|^2 + \frac{1}{2}\omega_e^{\mathrm{T}} J\omega_e. \tag{24}$$

The time derivative of Eq. (24) along the trajectories of system Eqs. (20)–(23) with $\Delta m = 0$, $\Delta\alpha = 0$, $d_f = 0$ and $d_\tau = 0$ become

$$\begin{aligned}\dot{S} &= \bar{v}_e^{\mathrm{T}}\left(-m\omega_e^\times \bar{v}_e + \bar{f}\right) + \omega_e^{\mathrm{T}}\left(-\omega_e^\times J\omega_e + \bar{\tau}\right) \\ &= \bar{v}_e^{\mathrm{T}}\bar{f} + \omega_e^{\mathrm{T}}\bar{\tau} \\ &= y^T u.\end{aligned}$$

Therefore, system Eqs. (20)–(23) are passive w.r.t input u and output y when $\Delta m = 0$, $\Delta\alpha = 0$, $d_f = 0$ and $d_\tau = 0$. ■

By transforming Eqs. (15)–(17), the control problem is now ready to regulate system Eqs. (20)–(23) in order to design control inputs $\bar{f}$ and $\bar{\tau}$ such that

$$(r_e, \varepsilon_e, \eta_e, \bar{v}_e, \omega_e) \to (0, 0, 1, 0, 0)$$

as $t \to \infty$.

Now, let us consider the PID-type adaptive state feedback controller that has positive definite gain matrices as follows:

$$\begin{cases} \bar{f} = -\dfrac{1}{a_2}\left(k_{p1} r_e + K_{d1}\bar{v}_e\right) - k_{i1}\xi_1 + \Delta\hat{m}\delta_r \\ \xi_1 = \displaystyle\int_0^t \left(r_e + \frac{a_2}{a_1}\omega_e^\times r_e\right) \mathrm{d}t \\ \Delta\dot{\hat{m}} = -\Gamma_1\delta_r^{\mathrm{T}}(a_1 r_e + a_2\bar{v}_e), \end{cases} \tag{25}$$

$$\begin{cases} \bar{\tau} = -\dfrac{1}{b_2}\left\{K(q_e)\varepsilon_e + K_{d2}\omega_e\right\} - k_{i2}\xi_2 + \delta_q\Delta\hat{\alpha} \\ \xi_2 = \displaystyle\int_0^t \left[\varepsilon_e + \frac{b_2}{2b_1}\left\{\left(2 - \eta_e\right)I_3 - \varepsilon_e^\times\right\}\omega_e\right] \mathrm{d}t \\ \Delta\dot{\hat{\alpha}} = -\Gamma_2\delta_q^{\mathrm{T}}(b_1\varepsilon_e + b_2\omega_e), \end{cases} \tag{26}$$

$$K(q_e) = \left(\eta_e I_3 - \varepsilon_e^\times\right)K_{p2} + k_{p3}\left(1 - \eta_e\right)I_3,$$

where $\Delta\hat{m}$ and $\Delta\hat{\alpha}$ are estimated values of Δm and $\Delta\alpha$, respectively, and the output to be controlled is defined as $z = \Sigma\zeta$, where Σ is the weighting matrix,

$$\Sigma = \mathrm{diag}\{\sigma_r, \sigma_v, \sigma_\eta, \sigma_\omega\},$$

$$\zeta = [r_e^{\mathrm{T}}\ \bar{v}_e^{\mathrm{T}}\ 2\cos^{-1}(|\eta_e|)\ \omega_e^{\mathrm{T}}]^{\mathrm{T}}.$$

From the definition of quaternion, $2\cos^{-1}(|\eta_e|)$ of the element of ζ represents the eigen-angle around the unit vector (eigen-axis) with respect to relative attitude.[17] Then, the following theorem can be obtained.

†When $\dot{v}_t$ and $\dot{\omega}_t$ are not directly measurable, they can be obtained from Eqs. (4) and (5) as

$$\dot{v}_t = -\omega_t^\times v_t, \quad \dot{\omega}_t = -J_t^{-1}\omega_t^\times J\omega_t$$

if r_t, q_t, v_t, ω_t and the moment of inertia ratio of the disturbance-free target can be estimated using image information of the target obtained by a camera on-board the chaser.[26] Even when the target is under disturbance, it is possible to estimate $\dot{v}_t$ and $\dot{\omega}_t$ by constructing a Kalman filter under disturbances.[27,28]

Theorem 1

Given a_1, a_2, b_1, b_2, and $\gamma > 0$, the closed-loop system of Eqs. (20)–(23) with Eqs. (25), (26) satisfies the L_2 gain less than or equal to γ from disturbance input $d = [d_f^{\mathrm{T}}\, d_\tau^{\mathrm{T}}]^{\mathrm{T}} \in L_2[0, T]$ to controlled outputs z if feedback gains satisfy the following conditions:

$$F > 0, \quad 2k_{p3}I_3 > K_{p2} > k_{p3}I_3, \quad Q > 0, \tag{27}$$

$$Q - \bar{\Sigma}^{\mathrm{T}}\bar{\Sigma} - \frac{1}{4\gamma^2}W^{\mathrm{T}}W > 0, \tag{28}$$

$$F = \mathrm{diag}\{F_1,\ F_2\}, \quad Q = \mathrm{diag}\{Q_1,\ Q_2\},$$

$$F_1 = \begin{bmatrix} k_{p1}I_3 & a_1 m I_3 & a_2 k_{i1} I_3 \\ * & a_2 m I_3 & 0 \\ * & * & a_1 k_{i1} I_3 \end{bmatrix},$$

$$F_2 = \begin{bmatrix} 2K_{p2} & b_1 J & b_2 k_{i2} I_3 \\ * & b_2 J & 0 \\ * & * & b_1 k_{i2} I_3 \end{bmatrix},$$

$$Q_1 = \begin{bmatrix} \dfrac{a_1}{a_2}k_{p1}I_3 - a_2 k_{i1} I_3 & \dfrac{a_1}{2a_2}K_{d1} \\ * & K_{d1} - a_1 m I_3 \end{bmatrix},$$

$$Q_2 = \begin{bmatrix} \dfrac{b_1}{b_2}\big(2k_{p3}I_3 - K_{p2}\big) - \left(b_2 + \dfrac{b_2^2}{2b_1}\right)k_{i2}I_3 & \dfrac{b_1}{2b_2}K_{d2} - \dfrac{b_2^2}{2b_1}k_{i2}I_3 \\ * & K_{d2} - \dfrac{3}{2}b_1\lambda_J I_3 - \dfrac{b_2^2}{8b_1}k_{i2}I_3 \end{bmatrix},$$

$$W = \begin{bmatrix} a_1 I_3 & a_2 I_3 & O_{3\times3} & O_{3\times3} \\ O_{3\times3} & O_{3\times3} & b_1 I_3 & b_2 I_3 \end{bmatrix},$$

$$\bar{\Sigma} = \mathrm{diag}\{\sigma_r, \sigma_v, \pi\sigma_\eta I_3, \sigma_\omega\},$$

where symbol $*$ denotes a symmetric element. Moreover, the state variable of the closed-loop system becomes

$$(r_e,\ \varepsilon_e,\ \eta_e,\ \bar{v}_e,\ \omega_e,\ \xi_1,\ \xi_2,\ \Delta\hat{m},\ \Delta\hat{\alpha}) \\ \to (0,\ 0,\ 1,\ 0,\ 0,\ 0,\ 0,\ c_1,\ c_2)$$

as $t \to \infty$ for arbitrary initial state when $d = 0$, where $c_1 \in \mathrm{R}$ and $c_2 \in \mathrm{R}^6$ are a constant scalar and a constant vector, respectively, determined by an initial condition of parameter update law. □

Proof: Let us define the candidate of the Lyapunov function as

$$\begin{aligned} V &= \frac{1}{2}\chi^{\mathrm{T}}F\chi + k_{p3}(\eta_e - 1)^2 + \frac{1}{2\Gamma_1}\Delta\tilde{m}^2 \\ &\quad + \frac{1}{2}\Delta\tilde{\alpha}^{\mathrm{T}}\Gamma_2^{-1}\Delta\tilde{\alpha}, \\ \chi &= [r_e^{\mathrm{T}}\ \bar{v}_e^{\mathrm{T}}\ \xi_1^{\mathrm{T}}\ \varepsilon_e^{\mathrm{T}}\ \omega_e^{\mathrm{T}}\ \xi_2^{\mathrm{T}}]^{\mathrm{T}}, \end{aligned} \tag{29}$$

where $\Delta\tilde{m} = \Delta m - \Delta\hat{m}$ and $\Delta\tilde{\alpha} = \Delta\alpha - \Delta\hat{\alpha}$ are estimated errors. Therefore, $V > 0$ if $F > 0$. The time derivative of Eq. (29) along the trajectories of the closed-loop system becomes

$$\begin{aligned} \dot{V} &= -\left(\frac{a_1}{a_2}k_{p1} - a_2 k_{i1}\right)\|r_e\|^2 - \frac{a_1}{a_2}r_e^{\mathrm{T}}K_{d1}\bar{v}_e \\ &\quad - \bar{v}_e^{\mathrm{T}}(K_{d1} - a_1 m I_3)\bar{v}_e - \frac{b_1}{b_2}\varepsilon_e^{\mathrm{T}}K(q_e)\varepsilon_e \\ &\quad + b_2 k_{i2}\|\varepsilon_e\|^2 - \varepsilon_e^{\mathrm{T}}\left(\frac{b_1}{b_2}K_{d2} - \frac{b_2^2}{b_1}k_{i2}I_3\right)\omega_e \\ &\quad - \omega_e^{\mathrm{T}}K_{d2}\omega_e + \frac{b_1}{2}\omega_e^{\mathrm{T}}JT(q_e)\omega_e - b_1\varepsilon_e^{\mathrm{T}}\omega_e^{\times}J\omega_e \\ &\quad - \frac{b_2^2}{2b_1}k_{i2}\eta_e\varepsilon_e^{\mathrm{T}}\omega_e + \big(a_1 r_e^{\mathrm{T}} + a_2\bar{v}_e^{\mathrm{T}}\big)d_f \\ &\quad + \big(b_1\varepsilon_e^{\mathrm{T}} + b_2\omega_e^{\mathrm{T}}\big)d_\tau, \end{aligned} \tag{30}$$

where $T(q_e) = \eta_e I_3 + \varepsilon_e^{\times}$. In Eq. (30),

$$\|T(q_e)\| = 1, \quad \|\varepsilon_e\| \le 1, \quad \|\omega_e^{\times}\| = \|\omega_e\|,$$

$$\left|\frac{b_1}{2}\omega_e^{\mathrm{T}}JT(q_e)\omega_e - b_1\varepsilon_e^{\mathrm{T}}\omega_e^{\times}J\omega_e\right| \le \frac{3}{2}b_1\lambda_J\|\omega_e\|^2,$$

$$-\frac{b_2^2}{2b_1}k_{i2}\eta_e\varepsilon_e^{\mathrm{T}}\omega_e \le \frac{b_2^2}{2b_1}k_{i2}\left(\|\varepsilon_e\|^2 + \frac{1}{4}\|\omega_e\|^2\right),$$

and the identity $\varepsilon_e^{\mathrm{T}}T(q_e) = \varepsilon_e^{\mathrm{T}}\eta_e$ yields

$$\begin{aligned} -\varepsilon_e^{\mathrm{T}}K(q_e)\varepsilon_e &= -\varepsilon_e^{\mathrm{T}}\big\{\eta_e\big(K_{p2} - k_{p3}I_3\big) + k_{p3}I_3\big\}\varepsilon_e \\ &= -\varepsilon_e^{\mathrm{T}}G(\eta_e)\varepsilon_e, \end{aligned} \tag{31}$$

where $G(\eta_e)$ is described as follows according to η_e.

$$G(\eta_e) = \begin{cases} K_{p2}, & \eta_e = 1 \\ \eta_e\big(K_{p2} - k_{p3}I_3\big) + k_{p3}I_3, & 1 > \eta_e > 0 \\ k_{p3}I_3, & \eta_e = 0 \\ -|\eta_e|\big(K_{p2} - k_{p3}I_3\big) + k_{p3}I_3, & 0 > \eta_e > -1 \\ 2k_{p3}I_3 - K_{p2}, & \eta_e = -1. \end{cases}$$

From the above equation, $G(\eta_e) > 0$ for all η_e if $2k_{p3}I_3 > K_{p2} > k_{p3}I_3$. Furthermore, the minimum value of $G(\eta_e)$ is

$$\min_{\eta_e} G(\eta_e) = G(-1) = 2k_{p3}I_3 - K_{p2}.$$

Therefore, if $2k_{p3}I_3 > K_{p2} > k_{p3}I_3$, then Eq. (31) becomes

$$-\varepsilon_e^{\mathrm{T}}K(q_e)\varepsilon_e \le -\varepsilon_e^{\mathrm{T}}\big(2k_{p3}I_3 - K_{p2}\big)\varepsilon_e,$$

and Eq. (30) satisfies

$$\begin{aligned} \dot{V} &\le -\left(\frac{a_1}{a_2}k_{p1} - a_2 k_{i1}\right)\|r_e\|^2 - \frac{a_1}{a_2}r_e^{\mathrm{T}}K_{d1}\bar{v}_e \\ &\quad - \bar{v}_e^{\mathrm{T}}(K_{d1} - a_1 m I_3)\bar{v}_e \\ &\quad - \varepsilon_e^{\mathrm{T}}\left\{\frac{b_1}{b_2}\big(2k_{p3}I_3 - K_{p2}\big) - \left(b_2 + \frac{b_2^2}{2b_1}\right)k_{i2}I_3\right\}\varepsilon_e \\ &\quad - \varepsilon_e^{\mathrm{T}}\left(\frac{b_1}{b_2}K_{d2} - \frac{b_2^2}{b_1}k_{i2}I_3\right)\omega_e \\ &\quad - \omega_e^{\mathrm{T}}\left(K_{d2} - \frac{3}{2}b_1\lambda_J I_3 - \frac{b_2^2}{8b_1}k_{i2}I_3\right)\omega_e \\ &\quad + \big(a_1 r_e^{\mathrm{T}} + a_2\bar{v}_e^{\mathrm{T}}\big)d_f + \big(b_1\varepsilon_e^{\mathrm{T}} + b_2\omega_e^{\mathrm{T}}\big)d_\tau \\ &= -\bar{\chi}^{\mathrm{T}}Q\bar{\chi} + \bar{\chi}^{\mathrm{T}}W^{\mathrm{T}}d, \end{aligned}$$

$$\bar{\chi} = [r_e^{\mathrm{T}}\ \bar{v}_e^{\mathrm{T}}\ \varepsilon_e^{\mathrm{T}}\ \omega_e^{\mathrm{T}}]^{\mathrm{T}}. \tag{32}$$

By completing the square, we obtain

$$\dot{V} + \|z\|^2 - \gamma^2\|d\|^2 \le -\bar{\chi}^{\mathrm{T}} Q\bar{\chi} + \frac{1}{4\gamma^2}\bar{\chi}^{\mathrm{T}} W^{\mathrm{T}} W\bar{\chi} - \gamma^2\left\| d - \frac{1}{2\gamma^2}W\bar{\chi}\right\|^2 + \zeta^{\mathrm{T}}\Sigma^{\mathrm{T}}\Sigma\zeta.$$

If we note that

$$d^* = \frac{1}{2\gamma^2}W\bar{\chi}$$

is the worst-case disturbance and that

$$\zeta^{\mathrm{T}}\Sigma^{\mathrm{T}}\Sigma\zeta \le \bar{\chi}^{\mathrm{T}}\bar{\Sigma}^{\mathrm{T}}\bar{\Sigma}\bar{\chi}$$

from $2\cos^{-1}(|\eta_e|) \le \pi\|\varepsilon_e\|$,[17] the inequality

$$\dot{V} + \|z\|^2 - \gamma^2\|d\|^2 \le -\bar{\chi}^{\mathrm{T}}\left(Q - \bar{\Sigma}^{\mathrm{T}}\bar{\Sigma} - \frac{1}{4\gamma^2}W^{\mathrm{T}}W\right)\bar{\chi}$$

holds for all $d \in L_2[0, T]$. Therefore, the condition in Eq. (28) implies

$$\dot{V} \le \gamma^2\|d\|^2 - \|z\|^2$$

indicating that the L_2 gain of the closed-loop is less than or equal to γ.[29]

With regard to the asymptotic stability, when $d = 0$, Eq. (32) becomes

$$\dot{V} \le -\bar{\chi}^{\mathrm{T}} Q\bar{\chi}$$

and $\dot{V} \le 0$ if $Q > 0$. Therefore,

$$V(x(t)) \le V(x(0)), \quad \forall t \ge 0 \quad \left(x = [\chi^{\mathrm{T}}\ \eta_e\ \Delta\hat{m}\ \Delta\hat{\alpha}^{\mathrm{T}}]^{\mathrm{T}}\right)$$

and x is bounded because V is radially unbounded. Furthermore, control inputs Eqs. (25) and (26) are bounded by the conditions in Assumption 1. Since $\dot{x}$ is also bounded,

$$\ddot{V} \le -2\bar{\chi}^{\mathrm{T}} Q\dot{\bar{\chi}}$$

is bounded and $\dot{V}$ is uniformly continuous with respect to t. Additionally, since V is lower bounded from $V \ge 0$,

$$\dot{V} \to 0 \Rightarrow \bar{\chi} \to 0$$

as $t \to \infty$ from the Lyapunov-like lemma.[30] Hence, $(\xi_1, \xi_2) \to (0, 0)$ and $\eta_e \to 1$ when $V = 0$. In addition, since the closed-loop system becomes

$$\Delta\tilde{m} = 0, \quad \Delta\tilde{\alpha} = 0,$$

$\Delta\hat{m} = 0$ and $\Delta\hat{\alpha}$ converge to a constant scalar c_1 and a constant vector c_2, respectively. Therefore, the state variable of the closed-loop system becomes

$$(r_e, \varepsilon_e, \eta_e, \bar{v}_e, \omega_e, \xi_1, \xi_2, \Delta\hat{m}, \Delta\hat{\alpha}) \to (0, 0, 1, 0, 0, 0, 0, c_1, c_2)$$

as $t \to \infty$ for arbitrary initial state when $d = 0$. ■

Remark 2

1) Since physical parameters m and J are in non-diagonal elements of matrix F such as $a_1 m I_3$ and $b_1 J$ and in diagonal elements of matrix Q such as $-a_1 m I_3$ and $-(3/2)b_1\lambda_J I_3$, the influence of parameter uncertainties can be reduced by setting a_1 and b_1 to small values. In addition, by setting γ and $\bar{\Sigma}$ to appropriate values, the term of $-\bar{\Sigma}^{\mathrm{T}}\bar{\Sigma} - (1/4\gamma^2)W^{\mathrm{T}}W$ in condition Eq. (28) does not become large. Therefore, by setting a_1, b_1, k_{i1} and k_{i2} to small values and a_2, b_2, k_{p1}, K_{p2}, k_{p3}, K_{d1} and K_{d2} to large values under the condition $2k_{p3}I_3 > K_{p2} > k_{p3}I_3$ and γ, $\bar{\Sigma}$ to appropriate values, the feedback gains satisfy conditions Eqs. (27) and (28) for small parameter uncertainties.

2) The obtained conditions in Eqs. (27) and (28) are LMIs with respect to feedback gains that are effectively solved using convex optimization tools.[31]

3) If we set $K_{p2} = k_{p3}I_3$, matrix $K(q_e)$ becomes a positive scalar constant k_{p3}.

4) It can be seen that the position and attitude can track their targets without offset errors when the disturbance is constant. At the steady state, $\bar{v}_e = 0$ and $\omega_e = 0$ hold, assuming that r_e and ε_e are very small. Therefore, as

$$K(q_e) \approx K_{p2}, \quad \xi_1 \approx \int_0^t r_e\,\mathrm{d}t, \quad \xi_2 \approx \int_0^t \varepsilon_e\,\mathrm{d}t,$$

$$\delta_r \approx 0, \quad \delta_q \approx 0,$$

the closed-loop system is

$$\frac{1}{a_2}k_{p1}r_e + k_{i1}\int_0^t r_e\,\mathrm{d}t - d_f = 0, \tag{33}$$

$$\frac{1}{b_2}K_{p2}\varepsilon_e + k_{i2}\int_0^t \varepsilon_e\,\mathrm{d}t - d_\tau = 0. \tag{34}$$

If we define

$$e_1 = \int_0^t r_e\,\mathrm{d}t - \frac{1}{k_{i1}}d_f, \quad e_2 = \int_0^t \varepsilon_e\,\mathrm{d}t - \frac{1}{k_{i2}}d_\tau,$$

then Eqs. (33) and (34) become

$$\dot{e}_1 = -\frac{a_2 k_{i1}}{k_{p1}}e_1, \quad \dot{e}_2 = -b_2 k_{i2} K_{p2}^{-1} e_2. \tag{35}$$

Since a_2, b_2, k_{p1}, k_{i1} and $k_{i2} > 0$ and $K_{p2} = K_{p2}^{\mathrm{T}} > 0$, $(e_1, e_2) \to (0, 0)$ as $t \to \infty$. Therefore, from Eqs. (33) and (34), it can be considered that $r_e \to 0$ and $\varepsilon_e \to 0$ as $t \to \infty$.

4. Numerical Study

The properties of the proposed controller were compared and discussed in this numerical study. For this purpose, we set the physical parameters of the target and chaser spacecraft as

$$m_t = 300\,\mathrm{kg}, \quad J_t = \mathrm{diag}\{50,\ 275,\ 275\}\,\mathrm{kgm^2},$$

$$m_0 = 200\,\mathrm{kg}, \quad J_0 = \begin{bmatrix} 75.0 & -28.1 & -28.1 \\ * & 75.0 & -28.1 \\ * & * & 75.0 \end{bmatrix}\mathrm{kgm^2},$$

$$\Delta m = 60\,\mathrm{kg}, \quad \Delta J = \begin{bmatrix} 22.5 & -8.44 & -8.44 \\ * & 22.5 & -8.44 \\ * & * & 22.5 \end{bmatrix}\mathrm{kgm^2}.$$

The target position in frame $\{t\}$ is given as $p_t = [0\ 5\ 0]^{\mathrm{T}}$. The initial conditions for the chaser spacecraft are

$$r(0) = [10\ 10\ 10]^{\mathrm{T}}\ \mathrm{m}, \quad v(0) = [0\ 0\ 0]^{\mathrm{T}}\ \mathrm{m/s},$$

$$q(0) = [0.06\ 0.69\ 0.06\ 0.72]^{\mathrm{T}}, \quad \omega(0) = [0\ 0\ 0]^{\mathrm{T}}\ \mathrm{rad/s},$$

and those for the target are

$$r_t(0) = [3\ 3\ 3]^{\mathrm{T}}\ \mathrm{m}, \quad v_t(0) = [0\ 0\ 0]^{\mathrm{T}}\ \mathrm{m/s},$$

$$q_t(0) = [0\ 0\ 0\ 1]^{\mathrm{T}}, \quad \omega(0) = [0.2\ 0.2\ 0.2]^{\mathrm{T}}\ \mathrm{rad/s}.$$

4.1. Tracking performance under a disturbance-free environment

First, we show the six d.o.f. tracking ability of the conventional controller[24)]

$$\begin{cases} \bar{f} = -\dfrac{1}{a_2}\left(k_{p1} r_e + K_{d1}\bar{v}_e\right) - k_{i1}\xi_1 + m_0\delta_r \\ \xi_1 = \displaystyle\int_0^t \left(r_e + \frac{a_2}{a_1}\omega_e^{\times} r_e\right) \mathrm{d}t, \end{cases}$$

$$\begin{cases} \bar{\tau} = -\dfrac{1}{b_2}\left\{K(q_e)\varepsilon_e + K_{d2}\omega_e\right\} - k_{i2}\xi_2 + \delta_q\alpha_0 \\ \xi_2 = \displaystyle\int_0^t \left[\varepsilon_e + \frac{b_2}{2b_1}\left\{\left(2-\eta_e\right)I_3 - \varepsilon_e^{\times}\right\}\omega_e\right] \mathrm{d}t, \end{cases}$$

and proposed controller in Eqs. (25) and (26) when $d = 0$. The conventional and proposed controller gains are selected to satisfy only the conditions in Eqs. (27) and (28), respectively. The controller gains are set as

$$a_1 = 0.2, \quad b_1 = 0.1, \quad a_2 = b_2 = 1,$$

$$k_{p1} = 18, \quad K_{p2} = 31 I_3, \quad k_{p3} = 31,$$

$$K_{d1} = 180 I_3, \quad K_{d2} = 300 I_3,$$

$$k_{i1} = 1.0, \quad k_{i2} = 0.4,$$

$$\Gamma_1 = 40, \quad \Gamma_2 = 600 I_6.$$

Figure 2 shows the responses of relative positions and relative quaternions of the conventional and proposed controllers. In the conventional controller, the chaser cannot track the target under the influence of physical parameter uncertainties. On the other hand, in the proposed controller, the chaser can track the target because the influence of physical parameter uncertainties is compensated by the parameter update law.

4.2. Tracking under constant disturbance

Then, we examine the six d.o.f.. tracking ability of the conventional and proposed controllers under constant disturbances

$$d_f = [3\ 3\ 3]^{\mathrm{T}}\ \mathrm{N}, \quad d_\tau = [3\ 3\ 3]^{\mathrm{T}}\ \mathrm{Nm}.$$

To compare only the six d.o.f.. tracking ability under constant disturbances, the physical parameter uncertainties Δm and ΔJ are set to zero. The simulation results are shown in Fig. 3. Both controllers can eliminate the steady-state error caused by a constant disturbance, and the chaser tracks the target. However, time responses of relative position and attitude of the proposed controller vibrate. This phenomenon can be explained as follows. Since δ_r and δ_q in the parameter update law are functions of ω_t and the target quickly moves in a short period, the responses of the estimated values $\Delta\hat{m}$ and $\Delta\hat{\alpha}$ are vibrating before convergence. Additionally since the proposed control law uses $\Delta\hat{m}$ and $\Delta\hat{\alpha}$, the control inputs vibrate. As a result, the time responses of relative position and attitude vibrate.

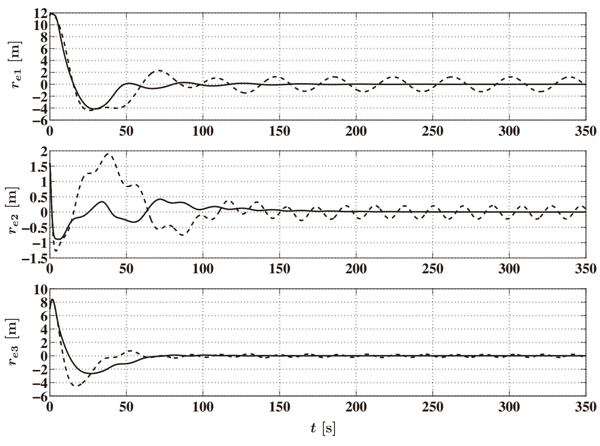
The simulation results with the physical parameter uncertainties under disturbances are shown in Fig. 4. The tracking performance of the conventional controller deteriorates under the influence of physical parameter uncertainties. On the other hand, the tracking performance of the proposed controller is almost the same as in the case of Fig. 3 except that ε_{e3}.

4.3. Disturbance attenuation ability

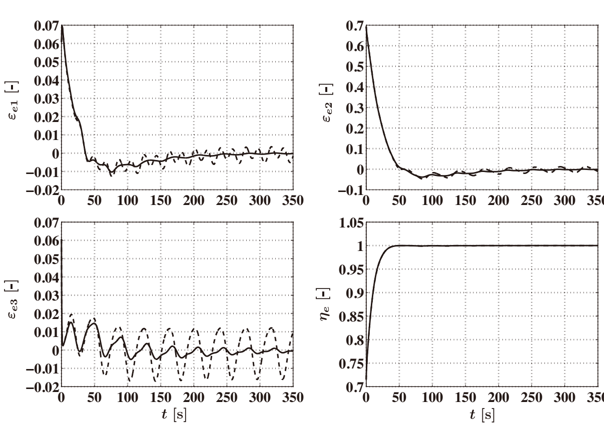
Finally, we examine the disturbance attenuation ability. We compare the conventional and proposed controllers, both of which exhibit the H_∞ property when $\gamma = 0.8$ and $\gamma = 0.2$. The design parameters are set as

$$a_1 = 8, \quad b_1 = 4, \quad a_2 = b_2 = 40,$$

$$\sigma_r = 6 I_3, \quad \sigma_v = 1 I_3, \quad \sigma_\eta = 3, \quad \sigma_\omega = 1 I_3,$$

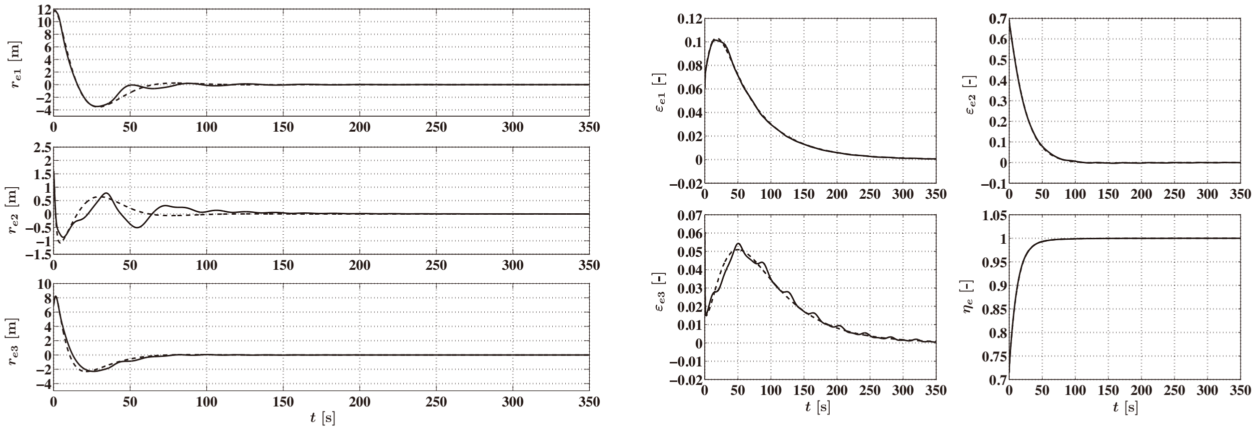


(a) r_e



(b) q_e

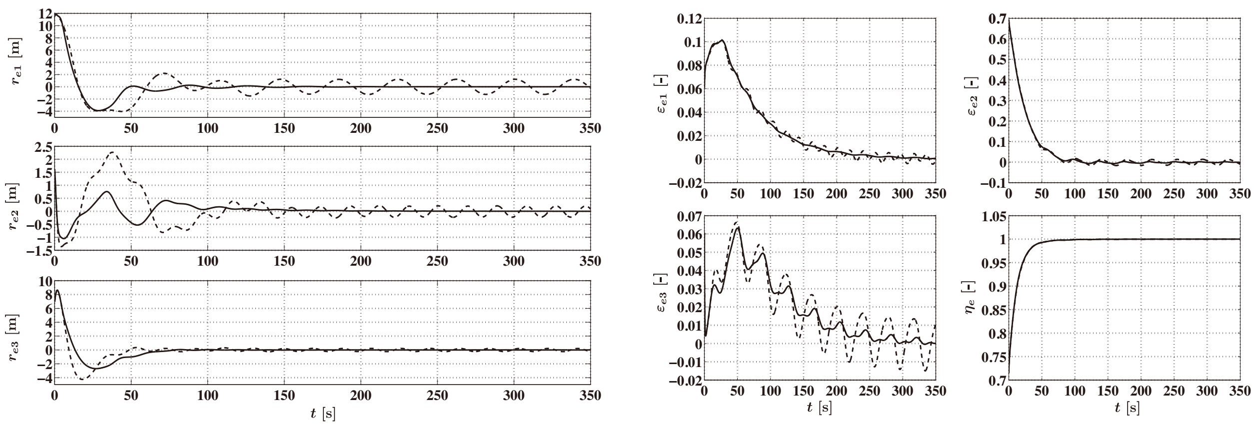
Fig. 2. Simulation results under a disturbance-free environment (solid line: proposed controller, dashed line: conventional controller).



(a) r_e

(b) q_e

Fig. 3. Simulation results under constant disturbance without parameter uncertainties (solid line: proposed controller, dashed line: conventional controller).



(a) r_e

(b) q_e

Fig. 4. Simulation results under constant disturbance with parameter uncertainties (solid line: proposed controller, dashed line: conventional controller).

and feedback gains of both controllers are derived by solving LMIs Eqs. (27) and (28). In addition, to prevent relative error from vibrating and integral gains k_{i1} and k_{i2} from becoming very small, the following conditions are applied:

$$K_{d1} > 10k_{p1}I_3, \quad K_{d2} > 10K_{p2}, \quad K_{d2} > 10k_{p3}I_3,$$
$$k_{i1} > 1.0, \quad k_{i2} > 0.4.$$

The disturbance input $d \in L_2[0, T]$ is

$$d_f = 3\sin\left(\frac{\pi}{40}\right)[1\ 1\ 1]^{\mathrm{T}}\ \mathrm{N},$$

$$d_\tau = 3\sin\left(\frac{\pi}{40}\right)[1\ 1\ 1]^{\mathrm{T}}\ \mathrm{Nm},$$

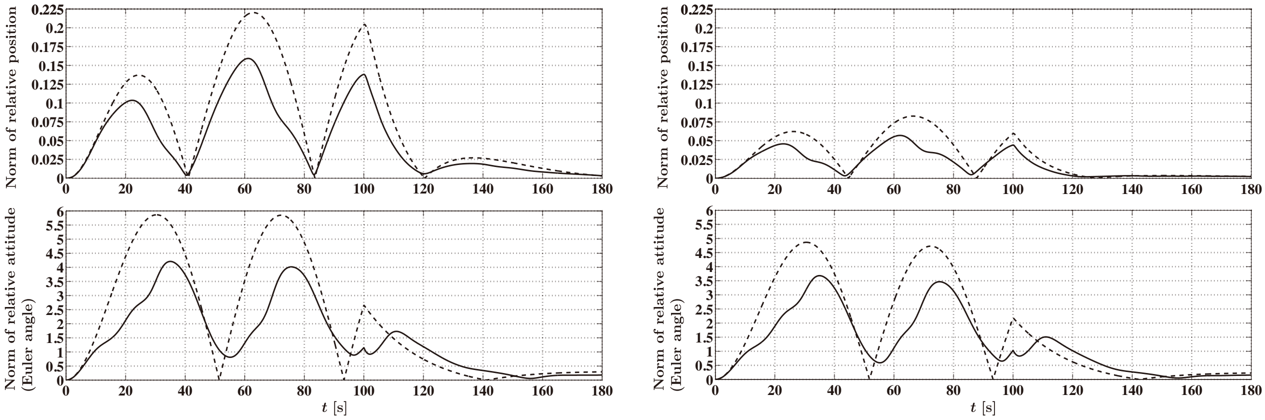
where $T = 100$. For the same initial conditions, the responses of relative position and attitude norms described by Euler angles of 3-2-1 system are obtained.

First, as in previous subsection, in order to compare only the disturbance attenuation ability, physical parameter uncertainties Δm and ΔJ are set to zero. The simulation results are shown in Fig. 5. Figure 5 show that the disturbance attenuation ability is higher in the proposed controller than in the conventional controller. This is considered that the parameters $\Delta\hat{m}$ and $\Delta\hat{\alpha}$ are estimated to reduce the relative errors because the parameter update law uses the relative error information. Similar results are obtained for other periodic disturbances.

The simulation results with the physical parameter uncertainties are shown in Fig. 6. Although the disturbance attenuation performance of the conventional controller deteriorates due to the influence of physical parameter uncertainties, that of the proposed controller is almost the same as in the case of Fig. 5.

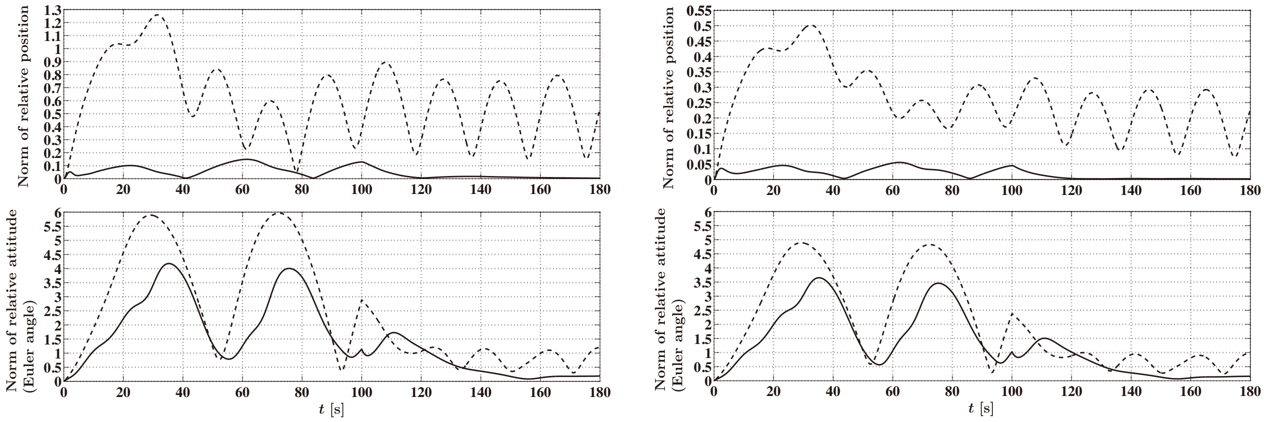
5. Conclusion

We investigated six d.o.f. nonlinear tracking control technologies of spacecraft under external disturbance in order to prepare for future space missions, and a PID-type H_∞ adaptive state feedback controller was proposed. Conditions of the asymptotic stability of error systems and the L_2 gain properties of a closed-loop system were obtained. The performances of the proposed controller were compared and discussed through numerical studies.



(a) $\gamma = 0.8$ (b) $\gamma = 0.2$

Fig. 5. Simulation results under disturbance without parameter uncertainties (top: norm of relative position, bottom: norm of relative attitude, solid line: proposed controller, dashed line: conventional controller).



(a) $\gamma = 0.8$ (b) $\gamma = 0.2$

Fig. 6. Simulation results under disturbance with parameter uncertainties (top: norm of relative position, bottom: norm of relative attitude, solid line: proposed controller, dashed line: conventional controller).

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