博 士 論 文 の 内 容 の 要 旨

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論文題目	On some topological invariants for one-dimensional discrete-time quantum walks with chiral symmetry (カイラル対称性を持つ 1 次元離散時間量子ウォークの位相不変量)

（博士論文の内容の要旨）

Quantum walk theory is a quantum-mechanical counterpart of the classical random walk theory. Despite its apparent simplicity, this ubiquitous concept has found many useful applications. In general, a concrete quantum walk model is characterised by an associated unitary time-evolution operator. This differs from the usual setting of Schrödinger operators in which we are required to construct the time-evolution operator from a given unbounded hamiltonian via the spectral theorem.

The major themes of this dissertation belong to the broad subject of index theory for (discrete-time) chirally symmetric quantum walks. To a chirally symmetric quantum walk model, we wish to assign a certain well-defined index satisfying the following two properties: (i) The index needs to be robust in the sense that it is stable against a wide range of perturbations; (ii) The index gives a lower bound for the number of so-called edge-states. Given (i), if the index turns out to be non-zero, then the associated time-evolution operator has at least one edge-state. This implication, known as the topological protection of edge-states, is an important feature of the bulk-edge correspondence. The present thesis consists of the following two main theorems:

Theorem A states that we can assign two indices satisfying (i), (ii) to a certain variant of itagawa’s split-step quantum walk on the one-dimensional integer lattice. We impose the so-called asymptotically periodic assumption, the scope of which is beyond that of the existing literature on the bulk-edge correspondence for 2-phase quantum walks. As such, we take a completely new approach by making use of Toeplitz operators.

Theorem B states that we can assign a yet another index to a non-unitary version of the split-step model we consider in Theorem A. This index satisfies (i), but it is not known whether or not (ii) also holds true. The main difficulty of this construction lies in the non-unitary feature of the given model. Theorem B forms a basis for future mathematical research into the bulk-edge correspondence for non-unitary quantum walks.